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# Analysis of the $\gamma N\Delta$ and $\Delta N\pi$ interactions for pion photoproduction

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**Abstract**

In this paper we analyze for the first time the $\gamma N\Delta$ excitation vertex from the point of view of the dynamics of the $\Delta$ field, $\Psi_\mu$. That is, we look for the value of the $Z$ parameter, present in all contact invariant $\Delta$-field interaction Lagrangians and usually regarded as redundant, and shift it to contact background non-$\Delta$ resonant amplitudes (in this sense it is called "redundant") by imposing that the $\Psi_0$ has no dynamics, instead of readjusting the background coupling constants. We do this within an unitarized model that comprises the $\Delta$-direct amplitude plus background contributions including the $\Delta$-cross term, nucleon Born and meson exchange ones, already implemented in previous works of $\pi N$ scattering, photo-production and weak-$\pi$ production. Also we analyze the use of a $\pi N\Delta$ decay vertex interaction containing both first ($I_1$) and second ($I_2$) order derivative contributions, as required by renormalization and power counting considerations, in building the $\gamma N \to \Delta \to \pi N$ amplitude, in contrast with other works where only $I_1$ or $I_2$ is adopted. It is shown that the description of the $\gamma N \to \pi N$ process, following these prescriptions, is improved. This is a first step: we plan to introduce final state interactions (FSI) in the future, following our previous work in which FCI were introduced using the $I_1$ interaction alone.


*Keywords:* Pion-nucleon-Delta interaction; Effective model; Pion photoproduction

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## 1. Introduction

The effective Lagrangian describing the photoexcitation of the $\Delta$ (1232 MeV) within an effective approach respects relevant symmetries, like Lorentz and electromagnetic gauge invariance, by construction. The Rarita-Schwinger (RS) spinor, $\Psi_\mu$, is a non-unitary representation of the Poincare group arising from projection of $[(1/2,0) \oplus (0,1/2)] \otimes (1/2,1/2)$, being $\Psi_\mu \equiv \psi \otimes W_\mu$, where $\psi$ a Dirac spinor field and $W_\mu$ a Dirac 4-vector [1]. In this way the field $\Psi_\mu$ will contain by construction a physical spin 3/2 sector and a spurious spin 1/2 one. The Lagrangian of the free $\Psi_\mu$ is set to lead to the spin 3/2 equations of motion, including the constraints $\partial_\mu \Psi^\mu = \gamma_\mu \Psi^\mu = 0$, which, on shell, projects out the spurious spin 1/2 sector. As a consequence, an arbitrary parameter $A$ is introduced, resulting in a type II constraint which fixes this sector as a function of the spin 3/2 sector and $A$. Changing $A$ amounts to change $\Psi_\mu$ through the contact transformation $\Psi^\nu \to R_{\mu\nu}(a)\Psi^\nu$, $R_{\mu\nu}(a) = g_{\mu\nu} + a\gamma_\mu\gamma_\nu$, with $a = -\frac{1}{2}(1+A)$, $A \neq -1/2$ which generates, when applied to the canonical RS Lagrangian, the well known family of equivalent Lagrangians $\mathcal{L}_{free}(A)$ [2]. The interactions should be set so $A$-dependence is avoided in the amplitudes, which is guaranteed by introducing contact invariant interaction terms. This is achieved if any interaction term involves $\Psi_\mu$ through the combination $R_{\mu\nu}(b)\Psi_\nu$ with $b = (2Z + (1+4Z)A)/2$. The family of Lagrangians $\mathcal{L}_{free}(A)$ are connected by the contact transformation $\Psi^\mu \to R^{\mu\nu}\Psi_\nu$, $A \to \frac{A-2a}{1+4a}$ ($A \neq -\frac{1}{2}$ to avoid the singularity) which change the proportion of the 1/2 states while leaving the equations of motion invariant. The same is true also for the interaction term, regardless the value of $Z$. As will be shown below, with this choice for $R(b)$ the amplitudes are independent of $A$ for any $Z$.[1] This is a general property that is even valid when we have $Z$-independent interactions from the beginning.

Criteria for fixing the value for $Z$ (we use $Z$ for strong vertexes while $Z'$ for electromagnetic ones) have been discussed extensively in the past [3], but unconclusively. On the other hand, in a previous work within the framework of Chiral Perturbation Theory (CHPT) [4], it was shown that $Z$ parameters present in the $\pi N \Delta$ Lagrangians are redundant in the sense that contributions depending on it can be absorbed into the $\pi N$ Lagrangian. To show this, a $\Delta$ functional integration was done on the total $\Delta$-Lagrangian, which is equivalent to making the transformation (for $A = -1$) $\Psi_\mu \to G_{\mu\nu} R^{\nu\alpha}(-\frac{1}{2} - Z)(\partial_\alpha \phi)\psi$ with $\phi$, $\psi$ and $G$ being the $\pi$ and $N$ fields, and $\Delta$ propagator, respectively, to get an "equivalent" Lagrangian where the $\Delta$ field is eliminated while the $Z$ parameter is shifted to the $\pi N$ sector. The price to be paid is that one gets a nonlocal Lagrangian where it is hard to maintain the various symmetries and to organize the Lagrangian according to the naive dimensional analysis. In that reference was concluded that it is needed to treat the $\Delta$ as a dynamical degree of freedom, in agreement with the usual approach where $Z$ is present. Nevertheless, as $\gamma_\mu G^{\mu\nu}$ and $G^{\mu\nu}\gamma_\nu$ do not contain pole-$\Delta$ contributions, the $Z$ dependent terms are again shifted to contact $\pi N$ ones, with coupling constants to be fitted. These authors have chosen *by convenience* (since, in fact, for the choice $A = -1$ there is not shifted contributions to the $\pi N$ sector) $Z = -1/2$ with $b(A = -1, Z = -1/2) = 0$ without affecting the background by presence of the $\Delta$. This choice is supported by the fact that, as stated, it is not relevant to the physics, as shown by the introduction of the previous mentioned non-local Lagrangian. With this choice the authors have calculated $\pi N$ phase shifts [5] within a naive isobar model, where $\rho$ and $\sigma$ exchange are not included as in more evolved calculations [6]. The

[1] As can be seen from the factorization in Eq. (1), it is not true the claim that the $Z$ parameters have been introduced in order to preserve the contact $A$-invariance as done in Ref. [4].

$\pi N$ sector fitted parameters $\beta_\pi(-1/2)$, $\kappa_{\pi,1,2}(-1/2)$ and $\lambda_{1,\dots,5}(-1/2)$ (which already contain contact terms) have been fitted to reproduce the phase shifts that really have not any contribution from de $\Delta$-sector for $Z=-1/2$. Of course, if based in the 'irrelevance' of $Z$ they chose another $Z \neq -1/2$ value, the contact amplitude in the $\pi N$ sector would be affected ($b \neq 0$) and they should refit its parameters to reproduce the data. The $Z$-redundance for the $\Delta$ interaction was also analyzed in a more general fashion in Ref. [7], where it is shown that all off-shell parameters which appear in the chiral effective Lagrangian $Z$-dependent contributions with explicit $\Delta$ isobar degrees of freedom can be absorbed into redefinitions of certain low-energy constants (LEC's), to be fixed by the experiments [7,8].

The first-order interaction derivative in the pion field $I_1$ [9] respects chiral invariance and dominates at small energies. Among all possible second order derivative interactions $I_2$ we will adopt the one called "consistent" in the literature [10], obtained from $\mathcal{L}_\Delta^{free}(A)$ by making the transformation $\Psi_\mu \to \Psi_\mu - g_2(\partial_\mu\phi)\psi$, that preserves the counting of degrees of freedom when $m_\Delta \to 0$, as can be seen from the constraint analysis [11]. $\mathcal{L}_{I_2}$ is denominated "spin 3/2" gauge invariant interaction since it remains unchanged under the transformation $\Psi_\mu \to \Psi_\mu + \partial_\mu\chi$, where $\chi$ is an arbitrary spinor, and leaves $\mathcal{L}_{free}(m_\Delta=0)$ also invariant. As a consequence, in $\pi N$ elastic scattering at tree level, it decouples the off-shell spin $\frac{1}{2}$ contribution present in the $\Delta$-propagator for any value of $A$. Besides, since $I_2$ is obtained from $\mathcal{L}_\Delta^{free}$ (independent of $Z$) it is clear that it doesn't appear a dependence with $Z$ in the combination $R(a)\times I_2$ in $\mathcal{L}_{I_2}$. In contrast to the consistent interaction $I_2$, the $I_1$ couples to the $1/2$ components in the propagator and in general it depends on $Z$ through the combination $R(b)\times I_1$ in $\mathcal{L}_{I_1}$.

For comparative purposes, we remark that when one adds one-loop radiative corrections to the $\pi N$ scattering amplitude corresponding to $I_2$, we are forced to introduce also the interaction $I_1$ together with $I_2$, acting as counterterm to avoid divergences [12] which reintroduces the unwanted coupling to the $1/2$ sector. Nevertheless, the presence of the virtual spin $1/2$ contribution in the $\Delta$ propagator is totally analogous to that of the spin-0 sector in the W propagator, where the projectors $P_0$ (spin 0) and $P_1$ (spin 1) appear, and the $W \to \pi$ decay vertex goes as $p_\pi^\mu$, $p_W^\mu$ [3] being $p_\pi^\mu, p_W^\mu P^0_{\mu\nu} \neq 0$: it would be impossible for the pion to decay without the off-shell spin-0 piece of the W propagator.

From CHPT both interactions could be considered to be of the same order. In fact, each pion momentum gives a contribution to the power counting in the expansion parameter $\delta \sim (m - m_N \sim 2m_\pi)/\Lambda_{\chi PT}$ ($\Lambda_{\chi PT} = 1$ GeV) or $\delta^2 \sim m_\pi/\Lambda_{\chi PT}$, and depending of its value both interactions are of the same order since momentum coming from $\partial_\mu\Psi_\nu$ behaves as order $\sim 1$ at threshold [10,12]. Both interactions $I_1$ and $I_2$ share formal problems related with the appearance of negative indefinite norm states when quantization is achieved. In fact, it was shown in Ref. [11] that, when background fields are present, some physical negative norm states arise for $I_2$, which shows that $I_2$ has problems just like $I_1$. Nevertheless, perturbative series make perfect sense in amplitude calculations, and one should consider both $I_1$ and $I_2$ together (the Lagrangians $\mathcal{L}_{I_k}$, depending on a coupling constant $g_k$, together with the kinematical one $\mathcal{L}_{free}$, will be defined in the next section). We consider $I_2$ of higher order in consideration to the dimension of the coupling constant and the number of derivatives in the interaction term, in line with [13].

Since the $I_2$ interaction is "spin 3/2" gauge invariant and $Z$-independent, it is the most physically sound. On the other hand, $I_1$ has been repeatedly adopted in many past and present works, and it is always possible to make a transformation $I_1(\Psi_\mu \to \Psi_\mu + ig_2(\partial_\mu\phi)\psi) \to I_2 + contact\,terms$ and consider that the $I_1$ contribution is shifted to the background. Then the background could be fitted to get coincidence with the data. Nevertheless, if the background is fixed using low energy phenomenology, it would be possible to keep $I_1$ and fix $g_1, Z$ by other criteria: this is our point of view. This does not mean that one *needs* $I_1$ to reproduce data, but it is a different way of fixing the background. Recently in Ref. [11] we introduced a new criterion to look for $Z$ in $b(A, Z)$ based on the dynamics of the time component $\Psi_0$, absent in $\mathcal{L}_{\Delta}^{free}(A)$, that was applied to the $\pi N \Delta$ vertex.

We calculated in Ref. [14] the total $\pi^- p$ and elastic (for which background contributions are far more relevant than in the $\pi^+ p$ due to isospin coefficients) scattering cross sections. It was shown that each interaction term fits poorly the data, while a judicious mixture of both interactions leads to a better description of this channel while keeping a good description of the $\pi^+ p$ one, and without ad-hoc manipulations of the background [15].

In this paper we analyze the particular case of pion photoproduction where, in addition to some non-resonant contributions to the amplitude generated from a Lagrangian of the form $\mathcal{L}_{non.res.} = \mathcal{L}_{N,M}^{free} + \mathcal{L}_{NM} + \mathcal{L}_{\gamma NM} + \mathcal{L}_{\gamma NN'\pi}$ $(M = \pi, \rho, \omega)$, we have a resonant one $\gamma N \rightarrow \Delta \rightarrow \pi N'$ obtained from $\mathcal{L}_{res.} = \mathcal{L}_{\Delta}^{free} + \mathcal{L}_{\gamma N\Delta}(Z') + \mathcal{L}_{\pi N\Delta}(Z)$. Thus, we need to combine electromagnetic photoexcitation vertexes, propagators and strong decays.

The first approach analyzing different contributions in the $\gamma N \Delta$ interaction Lagrangian was done in Refs. [16,17]. There is concluded, on the basis of field theoretic arguments originally formulated by Fierz and Pauli [18], that only appears a first order derivative contribution with a $Z_1' = 0$ and the second order one is dropped. One consequence of this result is that the dynamical freedom of two independent electromagnetic multipoles at the $\gamma N \Delta$ vertex is lost. Thus, the electric quadrupole (E2) to the magnetic dipole (Ml) amplitude ratio (EMR) for the $\Delta$ radiative decay is fixed kinematically, and it gives

$$EMR = -(m_{\Delta} - m_N)/(3m_{\Delta} + m_N) = -6\%,$$

which is very different from the experimental and quark model values [3]. More recently, the Sachs parametrization for the $\gamma N \Delta$ vertex introduced by Jones and Scadron [19] was used in several works where the values of the parameters $Z$ and $Z'$ mentioned above are assumed without any analysis. This parametrization implies contributions from second and four order derivatives in the $\gamma N \Delta$ Lagrangian, with a common $Z'$ value. For example, Refs. [20,21] seem to adopt the values $A = -1, Z = Z' = -1/2$, while in Ref. [22] the values $A = -1/3, Z = Z' = 1/2$ were used. In all mentioned references the usual first order derivative pion field $\pi N \Delta$ interaction $I_1$ was adopted.

Following the above discussion, the $Z, Z'$-dependent terms included in the $\gamma N \rightarrow \Delta \rightarrow \pi N'$ amplitude could in fact be absorbed as contact vertices in $\mathcal{L}_{\gamma NN'\pi}$ where the parameters should be fitted to reproduce the data. Then $\gamma N \rightarrow \Delta \rightarrow \pi N'$ is free of any $Z, Z'$ dependence, now transferred to the non resonant amplitude. Nevertheless, in our model the parameters present in $\mathcal{L}_{\gamma NM}$ are taken from low energy phenomenology and are fixed [22]. When we absorb the above mentioned terms, we change $\mathcal{L}_{\gamma NM} \rightarrow \mathcal{L}_{\gamma NM} + \mathcal{L}'_{\gamma N\pi}(Z, Z')$ and the new parameters should be fitted to reproduce the data. Without avoiding the fact of redundance of $Z, Z'$ for the $\Delta$-amplitude, we follow here a different approach as above for $\pi N \Delta$ vertex. We want to apply it now for the $\gamma N \Delta$ case in order to give a value for $Z'$ in the non-resonant amplitude. Then, we obtain the parameters $Z, Z'$ of the contact non resonant amplitude by a procedure which is alternative, but equally valid, to the one in LEC's of CHPT.

What is the purpose of doing this? In a very detailed previous works on photoproduction [20–22] the parameter $Z' = 1/2$ was used not considering the previous observation done in Refs. [4,5] or the $\psi_0$ criteria, which as we will see below, leads to the same $Z' = -1/2$ as in Refs. [4,5].

Then, in the first place, we want to write the tree level amplitudes by shifting the $Z'$-dependence to the contact terms and check for any improvement in the full amplitude by changing to the new $Z' = -1/2$. We will do this by introducing corrections such as unitarization, already present in Refs. [20–22]. Since the introduction of regularizing form factors to treat FSI would make the analysis of the basic tree amplitude obscure, that procedure will be omitted here. In the second place, we analyse the effect of considering $I_1 + I_2$ in the decay vertex of the amplitude $\gamma N \to \Delta \to \pi N'$.

On the other hand, the $\gamma N \to \Delta$ Sachs parametrization vertex is self (electromagnetic) gauge invariant by construction since it cannot be obtained from $\mathcal{L}_{free}$ through minimal coupling, and also is spin 3/2-gauge invariant since its second order contribution could be obtained from $\mathcal{L}_{free}$ making the transformation $\gamma_\nu \Psi_\mu \to \gamma_\nu \Psi_\mu + i\gamma_5 \partial_\nu A_\mu$, being $A_\mu$ the photon field. Nevertheless, because is not so clear that the Sachs fourth order contribution can be derived from $\mathcal{L}_{free}$ and since this parametrization was used with different values of $Z'$, we will enable the factor $R(b')$ and we will recover the spin $3/2$ gauge invariance for the appropriate value of $Z'$.

The paper will be organized as follows. In Section 2 we show the Lagrangians involved in the construction of the resonant contribution of the pion photoproduction amplitude. In the Section 3 the amplitudes are built and the differences obtained for the contact terms with different values of $Z'$ are discussed. In Section 4 the numerical results are shown. Finally, our conclusions are summarized in Section 5.

## 2. $\gamma N \Delta$ and $\Delta \pi N$ Lagrangians

We will work with the free $A$-dependent Lagrangian and the $A$ and $Z$-dependent strong interactions above mentioned [14]. It is clear that the A-dependence in any physical amplitude derived from these Lagrangians should cancel. This is achieved since the factors $R^{-1}\left(-\frac{1}{2}(1+A)\right)$ present in the $\Delta$ propagator (coming from the invertion of the free Lagrangian) cancel up with those in $R(b)$ from the interactions, since it can be shown that (idem for $Z'$)

$$R\left(b = \frac{1}{2}(1+4Z)A + Z\right)^{\mu\nu} = R\left(-\frac{1}{2}(1+A)\right)^{\mu\alpha} R\left(-\frac{1}{2}(1+2Z)\right)_{\alpha}^{\nu}. \tag{1}$$

Then we can use reducted $A$-independent Lagrangians

$$\mathcal{L}_{free} = \overline{\psi}_\mu(x)\mathcal{K}(\partial)^{\mu\nu}\Psi_\nu(x), \tag{2}$$

$$\mathcal{L}_{I_1}(Z_1) = g_1 \bar{\psi}\partial_\mu \boldsymbol{\phi}^\dagger \cdot \mathbf{T} R\left(-\frac{1}{2}(1+2Z_1)\right)^{\mu\nu} \Psi_\nu + h.c., \tag{3}$$

$$\mathcal{L}_{I_2} = -g_2\, \bar{\Psi}\partial_\mu \boldsymbol{\phi}^\dagger \cdot \mathbf{T}\epsilon^{\mu\nu\rho\beta}\gamma_\beta\gamma_5\partial_\rho\Psi_\nu + h.c. \tag{4}$$

with

$$\mathcal{K}(\partial)_{\mu'\nu'} = \epsilon_{\mu'\nu'\alpha\beta}\partial^\alpha\gamma^\beta\gamma_5 + im\sigma_{\mu'\nu'} \tag{5}$$

being $\sigma_{\mu,\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$, $\epsilon_{0123} = 1$, $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and $\mathbf{T}^\dagger$ are the $N \to \Delta$ isospin excitation operators defined in the Appendix A.[2] Note that, as mentioned above, $\mathcal{L}_{I_2}$ is obtained from $\mathcal{L}_{free}$ through the transformation $\Psi_\mu \to \Psi_\mu - g_2(\partial_\mu\phi)\psi$.

[2] We adopt the Bjorken and Drell conventions.

For the electromagnetic lagrangian terms we adopt the Sachs parametrization introduced by Jones and Scadron [19]. Those terms read:

$$\begin{aligned}\mathcal{L}_{\gamma N\Delta}(Z') &= i\bar{\Psi}^{\mu} R\left(-\frac{1}{2}(1+2Z'))\right)_{\mu}^{\lambda}\left[e(G_M-G_E)\epsilon_{\lambda\nu\alpha\beta}\left(\partial^{\alpha}A^{\nu}\right)\left(\partial^{\beta}\psi\right)\right.\\ &\quad\left.+\ G_E i\gamma_5\epsilon_{\mu\delta\alpha\beta}\epsilon^{\delta}_{\nu\alpha'\beta'}\left(\partial^{\alpha}\partial^{\alpha'}A^{\nu}\right)\left(\partial^{\beta}\partial^{\beta'}\psi\right)\right]+h.c.\\ &= -ie\bar{\Psi}^{\mu}R\left(-\frac{1}{2}(1+2Z'))\right)_{\mu}^{\lambda}\\ &\quad\times\left[(G_M-G_E)\tilde{F}_{\lambda\beta}+G_E i\gamma_5\left(\partial^{\alpha}\widetilde{\partial_{\beta'}\tilde{F}^{\delta\beta'}}\right)_{\lambda\beta}\right]\partial^{\beta}\psi, \end{aligned}\tag{6}$$

where $\tilde{F}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}(\partial^{\alpha}A^{\beta}-\partial^{\beta}A^{\alpha})$ is the dual electromagnetic tensor and $G_E\equiv G_E(k^2=0)$ and $G_M\equiv G_M(k^2=0)$ are the electric (E) and magnetic (M) form factors at $k^2=0$, as corresponds for photoproduction. $\gamma_5=\frac{i}{4!}\epsilon_{\mu\nu\alpha\beta}\gamma^{\mu}\gamma^{\nu}\gamma^{\alpha}\gamma^{\beta}$ in our convention for $\epsilon_{\mu\nu\alpha\beta}$.

By inverting the kinetic operator $\mathcal{K}(\partial)^{\mu\nu}$ we get the reducted propagator, which can be put in terms of the well known projectors $P^{3/2}$, $P^{1/2}_{11}$, $P^{1/2}_{22}$, $P^{1/2}_{21}$ and $P^{1/2}_{12}$ on the spin 3/2 and 1/2 sectors (see Appendix A) or alternatively developing them as

$$\begin{aligned}G\left(p\right)_{\mu\nu} &= -\frac{\not{p}+m_{\Delta}}{p^2-m_{\Delta}^2}\left[P^{3/2}_{\mu\nu}-\frac{2}{3m_{\Delta}^2}(\not{p}+m_{\Delta})(P^{1/2}_{22})_{\mu\nu}+\frac{1}{\sqrt{3}m_{\Delta}}(P^{1/2}_{12}+P^{1/2}_{21})_{\mu\nu}\right],\\ &= \frac{\not{p}+m_{\Delta}}{p^2-m_{\Delta}}\left[-g_{\mu\nu}+\frac{1}{3}\gamma_{\mu}\gamma_{\nu}+\frac{1}{3m_{\Delta}}(\gamma_{\mu}p_{\nu}-p_{\mu}\gamma_{\nu})+\frac{2}{3m_{\Delta}}p_{\mu}p_{\nu}\right].\end{aligned}\tag{7}$$

We observe that the dependence on $Z, Z'$ persists as a free parameter of the interactions $\mathcal{L}_{I_1}, \mathcal{L}_{\gamma N\Delta}$ [2].

Nevertheless, note that

$$R\left(b\equiv-\frac{1}{2}[1+2(Z,Z')]\right)_{\alpha}^{\nu}=g_{\alpha}^{\nu}+b\gamma^{\nu}\gamma_{\alpha},\tag{8}$$

and

$$G\left(p\right)_{\mu\nu}\gamma^{\nu}=\frac{1}{3m_{\Delta}}\left[\gamma_{\mu}-\frac{2}{m_{\Delta}}p_{\mu}\right],\gamma^{\mu}G\left(p\right)_{\mu\nu}=\frac{1}{3m_{\Delta}}\left[\gamma_{\nu}-\frac{2}{m_{\Delta}}p_{\nu}\right],$$

which shifts the $Z$, $Z'$ dependence present in $R\left(b,b'\right)$ at (1) to contact background terms without poles in the $\pi N$ sector, as we will see in the next section, where $Z$, $Z'$ need to be fixed fitting the data. We will argue below that those parameters can be fixed through other criteria.

Finally, we mention that the electromagnetic Lagrangian $\mathcal{L}_{\gamma N\Delta}$ from Eq. (6), in contrast with $\mathcal{L}_{I_1}+\mathcal{L}_{I_2}$ that represents a first order derivative term plus a second order one, has a second order plus four order structure. We stress that $\mathcal{L}_{\gamma N\Delta}$ for $Z'=-1/2$ (the original form proposed in Ref. [19]) is spin 3/2 gauge invariant, as mentioned above, since it remains unchanged under $\Psi_{\mu}\rightarrow\Psi_{\mu}+\partial_{\mu}\chi$, $\chi$ being an arbitrary spinor. The reason for setting $Z'\neq-1/2$ is to compare with other works adopting the same setting. The Sachs parametrization can be connected with the so called "normal parity" (NP) one [25] which exhibit first order plus second order derivative terms with $g_1', g_2'=f(G_{M,}G_E)$ but now with $g_2'\neq 0$, in contrast with the original work by Nath [17]. Nevertheless, in order to establish this connection we must assume the $\Delta$ to be on-shell and make some approximations using the RS constraints.

## 3. Pion photoproduction cross section

The total cross section for the $\pi$-photoproduction process $\gamma(k) + N(p_i) \to \pi(q) + N'(p_f)$, with $k$, $p_i$, $q$ and $p_f$ being the photon, initial nucleon, pion and final nucleon 4-momentum, respectively, receives contribution from both, background (B) first seven terms in Fig. 1 and the pole resonant (R) eighth term. This total cross section will be calculated as:

$$\sigma(E_\gamma) = \frac{|\vec{q}|}{|\vec{k}|} \frac{m_N^2}{32\pi s} \int_0^\pi \sum_{m_s m_s', pol.} d\theta^* \sin\theta^* \left|\bar{u}(p_f, m_s') \left(\mathcal{M}_B + \mathcal{M}_R\right) u(p_i, m_s)\right|^2, \tag{9}$$

with $u(p, m_s)$ being the Dirac spinor of the nucleon with mass $m_N$ and the integration is performed over the c.m. angle $\theta^*$ between $\pi$ and $N'$. In this c.m. frame, we can write all the momenta in terms of the photon energy in laboratory frame, $E_\gamma$, as follows:

$$k = (E, \quad 0, \quad 0, \quad E_k), \; p_i = (E_i, \quad 0, \quad 0 \quad -E_k),$$
$$q = (E_q, \quad \mathrm{q}\sin\theta^*, \quad 0, \quad \mathrm{q}\cos\theta^*), \; p_f = (E_f, \quad -\mathrm{q}\sin\theta^*, \quad 0, \quad -\mathrm{q}\cos\theta^*)$$

where $E = \frac{s - m_N^2}{2\sqrt{s}}$, $E_i = \frac{s + m_N^2}{2\sqrt{s}}$, $E_q = \frac{s - m_N^2 + m_\pi^2}{2\sqrt{s}}$, $E_f = \frac{s + m_N^2 - m_\pi^2}{2\sqrt{s}}$. Here $m_\pi$ is the pion mass and $\sqrt{s} = \sqrt{m_N(m_N + 2E)}$ is the total energy. It is interesting to mention here that the "threshold" of the reaction occurs for the following minimum energy:

$$(E)_{min} = \frac{(m_\pi + m_N)^2 - m_N^2}{2m_N} \simeq 0.149 \text{ GeV}.$$

The idea of the manuscript is not to describe again in detail the photoproduction observables (total and differential cross sections, multipole amplitudes, asymmetries, etc.) since this was done in Refs. [8,10,20,21], and by us in Ref. [23]. These works built the amplitude with chiral effective Lagrangians introducing the $N, \Delta, \pi, \rho, \omega$ degrees of freedom (in Refs. [8,10] the $\rho, \omega$ contributions are absent), keeping unitarity through the rescattering contributions as in [23,20, 21], or within CHPT with the $N$, $\Delta$ and $\pi$ fields [8,10]. In spite of this we use the unitaryzed model from Ref. [23] (referred as UIBA) which introduces the $\pi N$ phase shifts to evaluate the effect of different $Z, Z'$ contact contributions. Also we wish to know the effect of using $I_1 + I_2$ in place of $I_1$ or $I_2$ alone in the strong vertex. According to this, we will adopt a simplified model (compared with that in [23]) for the amplitude without FSI, since their inclusion needs form factors. Additionally, it is important to remark that we choose to use the total cross section and some multipole amplitudes as observables.

The background ($\mathcal{M}_B$) and resonant ($\mathcal{M}_R$) contributions to the amplitude are written, as usual, in the form $\mathcal{M}_B = \mathcal{M}_B^{N,s} + \mathcal{M}_B^{N,u} + \mathcal{M}_B^t + \mathcal{M}_B^c + \mathcal{M}_B^\rho + \mathcal{M}_B^\omega + \mathcal{M}_R^{\Delta,u}$ and $\mathcal{M}_R = \mathcal{M}_R^{\Delta,s}$, with each contribution corresponding to a given Feynman diagram shown in Fig. 1. They are the nucleon s- and u-channels ($\mathcal{M}_B^{N,s}$ and $\mathcal{M}_B^{N,u}$), pion in flight or t-channel ($\mathcal{M}_B^t$), Kroll-Rudermann or contact term ($\mathcal{M}_B^c$), the $\rho$ and $\omega$ vector mesons exchange ($\mathcal{M}_B^\rho$ and $\mathcal{M}_B^\omega$) and the $\Delta(1232)$ resonance s- and u-channels ($\mathcal{M}_R^{\Delta,u}$ and $\mathcal{M}_R^{\Delta,s}$). For the non-resonant background terms we show the propagators, Lagrangians and amplitudes in Appendix B.

The s-channel contribution of the $\Delta$ resonance will be evaluated as:

$$\mathcal{M}_R^{\Delta,s} \equiv i\left[g_1 V_1^\sigma R_{\sigma\alpha}(b(Z_1)) + g_2 V_2^\sigma g_{\sigma\alpha}\right]$$
$$\times\, i G^{\alpha\beta}(p_\Delta = p_i + k, -1) R_\beta{}^\delta\left(b(Z')\right)\Gamma_{\delta\nu}(k)\epsilon^\nu$$

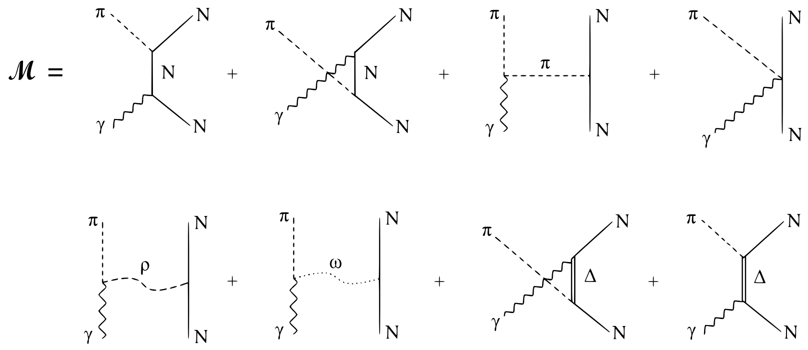


Fig. 1. Different contributions to the amplitude. The first seven contributions are the background while the last one is the resonant pole contribution.

$$
\begin{aligned}
&= i\,[g_1 V_{1\alpha} + g_2 V_{2\alpha}]\, i G^{\alpha\beta}(p_\Delta, -1)\Gamma_{\beta\nu}(k)\epsilon^\nu \\
&+ i g_1 b(Z_1) V_{1\alpha} \frac{i}{3m_\Delta}\gamma^\alpha \left(\gamma^\beta - \frac{2}{m_\Delta} p_\Delta^\beta\right)\Gamma_{\beta\nu}(k)\epsilon^\nu \\
&+ i\,[g_1 V_{1\alpha} + g_2 V_{2\alpha}]\frac{i}{3m_\Delta}\left(\gamma^\alpha - \frac{2}{m_\Delta} p_\Delta^\alpha\right) b(Z')\gamma^\beta \Gamma_{\beta\nu}(k)\epsilon^\nu \\
&+ i g_1 b(Z_1) V_{1\alpha}\frac{i}{3m_\Delta}\gamma^\alpha\left(4 - \frac{2}{m_\Delta}\not{p}_\Delta\right) b(Z')\gamma^\beta\Gamma_{\beta\nu}(k)\epsilon^\nu
\end{aligned} \tag{10}
$$

where the polarization vectors are $\epsilon_\nu(+1) = (0, 1, 0, 0)$ and $\epsilon_\nu(-1) = (0, 0, 1, 0)$, and the strong $\Delta\pi N$ vertexes obtained from the Lagrangians given in Eqs. (3) and (4) are:

$$
V_1^\sigma = -q^\sigma,\; V_2^\sigma = -i q_\delta \epsilon^{\delta\sigma\eta\rho}\gamma_\rho\gamma_5 (p_\Delta)_\eta. \tag{11}
$$

The $\gamma N \to \Delta$ vertex obtained from Eq. (6) is

$$
\Gamma_{\mu\nu} = \left[-e(G_M - G_E)\epsilon_{\mu\nu\alpha\beta}P^\alpha k^\beta - eG_E i\gamma_5 \epsilon_{\mu\delta\alpha\beta}P^\alpha k^\beta \epsilon^\delta_{\;\nu\alpha'\beta'} P^{\alpha'} k^{\beta'}\right]\epsilon^\nu \tag{12}
$$

$$
G_M = \frac{3}{2m_N(m_N + m_\Delta)}
$$

$$
G_E = \frac{6}{(m_N + m_\Delta)^2 (m_N - m_\Delta)^2},
$$

being $P = \frac{p_i + p_\Delta}{2} = p_\Delta - \frac{k}{2} = p_i + \frac{k}{2}$ for the s-channel. From Eq. (10) it is clear that the resonant first term contribution is independent on $Z_1$, $Z'$,($b(Z_1, Z' = -\frac{1}{2}) = 0$) and only the background contact terms depend on them as in Ref. [4]. The contributions of $I_2$ are independent of $Z$ since it is obtained form $\mathcal{L}_{free}$ that is independent of $Z$. In Ref. [5] the authors have additional contact contributions (not coming from $Z's$) together with a nucleon exchange term similar to $\mathcal{M}_B^{N,s}$ and their corresponding parameters are fitted for $Z = -\frac{1}{2}$ to reproduce $\pi N$ phase shifts. Of course, for another $Z$ the shifted terms should affect now the contact terms and the fitting. In our model we fix the background parameters through low-energy phenomenology and have also a Kroll-Rudermann fixed contact term that is a $Z$-independent contributions in $\mathcal{M}_B$, being necessary a criterion to fix $b(Z_1, Z')$ different from fitting for the additional contact contributions. Here, the free parameters $Z_1$, $Z'$ are set such that Lagrange multiplier field ($\Psi_0$) of the free theory do not acquire dynamics due to the interaction, as explained in Ref. [11]. This means that we need to avoid $\partial_0\Phi$ contributions (with $\Phi = A^\mu,\ \psi$) in the interaction vertexes, for the factor $S_0$ coupled to $\bar{\Psi}_0$ in $\mathcal{L}_{int} = \bar{\Psi}^0 S_0 + \bar{S}_0 \Psi^0 \cdots$ since it could generate dynamics for $\bar{\Psi}_0$ through the $\partial_0(\bar{\Psi}_0 \frac{\partial S_0}{\partial(\partial_0\Phi)})$ term in the equation of motion. Note that in the second term of the Lagrangian (6) we have only second order ($\partial_\alpha\partial_\beta\Phi$) contributions and thus the first order term alone ($\partial_0\Phi$) is not present. After choosing $Z_1$, $Z'$ they are used in our additional contact terms as done in Ref. [4]. Note that we do not need to fix any $Z_2$ as mentioned above, but if we had assumed a form $I_2 \times R(b)$ (which is, in general, not consistent) the consistent $I_2$ obtained with the $\Psi_0$ criteria, would be that corresponding to the value $Z_2 = -1/2$. We fixed $Z_1 = 1/2$ for $I_1$ as in Ref. [9], and assume the $\mathcal{L}_{I_2}$ originally proposed by [10]. Following the same procedure applied to the first term in Eq. (6) we also get $Z' = -1/2$, and this value lead to $b' = 0$ and to a consistent electromagnetic vertex.

Note that in Ref. [23], when still the "$\Psi_0$ criterion" was not used, we adopted the values $Z_1 = Z' = 1/2$ to get the simplest form for $\mathcal{L}_{I_1}$ and $\mathcal{L}_{\gamma N\Delta}$ Lagrangians. In this present work, since we have the same $Z_1$ value but a different $Z' = -1/2$ value, we will analyze the effect of using $Z' = \pm\frac{1}{2}$ in order to evaluate the importance of adopting a different criterion to fix this parameter. Of course, this only affects the shifted background contributions, letting the resonant pole one unchanged.

Now we express $V_1^\sigma$ and $V_2^\sigma$ in terms of the projectors $P_{\mu\nu}$ following the same procedure as in Ref. [14] and evaluating the $\Delta$ propagator within the complex mass scheme (CMS) approximation [26] (see next section) where the replacement $m_\Delta \to \tilde{m}_\Delta = m_\Delta - i\frac{\Gamma_\Delta}{2}$ is done, being $\Gamma_\Delta$ the resonance width. It is important to remark that (10) will contain a pole term proportional to $g_1 + \tilde{m}_\Delta g_2$, and contact terms contributing to the background. As in Ref. [14], it is better to redefine the couplings in terms of $g$ and $\kappa$ as follows: $g = g_1 + m_\Delta g_2$, $g_2 = \frac{\kappa g}{m_\Delta}$ where the width was dropped, or, equivalently $g_1 = (1-\kappa)g$, $g_2 = \frac{\kappa g}{m_\Delta}$ and $g = \frac{f_{\pi N\Delta}}{m_\pi}$. From Eq. (10) we get (with 'cons.' we indicate 'consistent' couplings)

$$\mathcal{M}_R^{\Delta,s} = \left\{ \begin{array}{ll} -g_1 q^\sigma \left[ P^{3/2}_{\sigma\beta} \frac{(\not{p}_\Delta + \tilde{m}_\Delta)\tilde{m}_\Delta}{p_\Delta^2 - \tilde{m}_\Delta^2} - \frac{1}{\tilde{m}_\Delta} P^{1/2}_{11,\sigma\beta} \right] & I_1(\kappa = 0) \\ -g q^\sigma \left[ P^{3/2}_{\sigma\beta} \frac{(\not{p}_\Delta + \tilde{m}_\Delta)(\tilde{m}_\Delta(1-\kappa) + \kappa\not{p}_\Delta)}{p_\Delta^2 - \tilde{m}_\Delta^2} - \frac{1-\kappa}{\tilde{m}_\Delta} P^{1/2}_{11,\sigma\beta} \right] & I_1 + I_2 (0 < \kappa < 1) \\ -g_2 q^\sigma P^{3/2}_{\sigma\beta} \frac{(\not{p}_\Delta + \tilde{m}_\Delta)\not{p}_\Delta}{p_\Delta^2 - \tilde{m}_\Delta^2} & I_2(\kappa = 1)(\text{cons.}) \end{array} \right\}$$
$$\times \Gamma^{\beta\nu}(k, P_\Delta)\epsilon_\nu, \ \text{for } Z' = -\frac{1}{2}(\text{cons.}), \tag{13}$$

$$\mathcal{M}_R^{\Delta,s} = \left\{ \begin{array}{ll} -g_1 q^\sigma \Big[ P^{3/2}_{\sigma\beta} \frac{(\not{p}_\Delta + \tilde{m}_\Delta)\tilde{m}_\Delta}{p_\Delta^2 - \tilde{m}_\Delta^2} & \\ \quad + \frac{2}{\tilde{m}_\Delta^2} \left( \not{p}_\Delta + \tilde{m}_\Delta \right) P^{1/2}_{11,\sigma\beta} + \frac{\sqrt{3}}{\tilde{m}_\Delta} P^{1/2}_{21,\sigma\beta} \Big] & I_1(\kappa = 0) \\ -g q^\sigma \Big[ P^{3/2}_{\sigma\beta} \frac{(\not{p}_\Delta + \tilde{m}_\Delta)(\tilde{m}_\Delta(1-\kappa) + \kappa\not{p}_\Delta)}{p_\Delta^2 - \tilde{m}_\Delta^2} & \\ \quad + \frac{2}{\tilde{m}_\Delta^2} \left( \not{p}_\Delta + \tilde{m}_\Delta(1-\kappa) \right) P^{1/2}_{11,\sigma\beta} & \\ \quad + \frac{\sqrt{3}(1-\kappa)}{\tilde{m}_\Delta} P^{1/2}_{21,\sigma\beta} \Big] & I_1 + I_2(0 < \kappa < 1) \\ -g_2 q^\sigma \left[ P^{3/2}_{\sigma\beta} \frac{(\not{p}_\Delta + \tilde{m}_\Delta)\not{p}_\Delta}{p_\Delta^2 - \tilde{m}_\Delta^2} + \frac{2}{\tilde{m}_\Delta^2} \not{p}_\Delta P^{1/2}_{11,\sigma\beta} \right] & I_2(\kappa = 1)(\text{cons.}) \end{array} \right\}$$

$$\times\, \Gamma^{\beta\nu}(k, P_\Delta)\epsilon_\nu, \text{ for } Z' = +\frac{1}{2}(\text{not cons.}), \tag{14}$$

with $P_\Delta = k + p_i$ and where it must be multiplied times $\begin{cases} \frac{2}{3} \text{ for } \gamma p \to p\pi^0 \\ \frac{\sqrt{2}}{3} \text{ for } \gamma p \to n\pi^+ \end{cases}$.

The third line in Eqs. (13) and (14) corresponds to the use of $I_2$ strong vertex combined with the lectromagnetic one for $Z' = -\frac{1}{2}$ ($b'(Z' = -\frac{1}{2}) = 0$) and $Z' = \frac{1}{2}$ ($b'(Z' = \frac{1}{2}) \neq 0$), respectively. In Eq. (13) there are no spin 1/2 background contributions since all interactions are consistent ($V_2^\mu p_{\Delta\mu} = p_{\Delta\mu}\Gamma^{\mu\nu} = 0$), while in Eq. (14) this is not the case since $p_{\Delta\mu} R(b')^{\mu\nu}\Gamma_{\nu\beta} \neq 0$. In the first line of the mentioned equations we show the contribution of $I_1$ ($Z_1 = 1/2$) alone for $Z' = -1/2$ and $Z' = 1/2$. We remark that we have background spin-$1/2$ contributions in both cases, which are different (in spite that for $Z' = -1/2$ we have an electromagnetic consistent vertex) due to the coupling of $I_1$ to the $1/2$ sector and also because $b'(-1/2) \neq 0$. Finally, in the middle lines of that equations we combine $I_1 + I_2$ and we have also $1/2$ background terms which are different for $Z' = \mp 1/2$ due to the same reasons.

Following the same procedure, we can evaluate the u-channel contribution of the $\Delta$-resonance (which contributes to the background) using the relation:

$$\mathcal{M}_R^{\Delta,u}(k,q) = \gamma_0 \mathcal{M}_R^{\Delta,s}(-k,-q)^\dagger \gamma_0 \begin{cases} \frac{2}{3} \text{ for } \gamma p \to p\pi^0 \\ -\frac{\sqrt{2}}{3} \text{ for } \gamma p/n \to n/p\pi^+ \end{cases},$$

with $\mathcal{M}_R^{\Delta,s}(k,q)$ given in Eqs. (13) and (14). Since in this case we found contributions with $p_\Delta^2 = 0$, it is not convenient to write this amplitude in terms of the projectors, which contain factors of the form $\frac{1}{p_\Delta^2}$ that would lead to divergences that cancel out when the projectors are developed. We get (omitting now isospin factors)

$$\begin{aligned}
\mathcal{M}_R^{\Delta,u} &= g(\epsilon^\nu)^\dagger \Gamma_{\nu\beta}(k) \Bigg[ \frac{\not{p}_\Delta + \tilde{m}_\Delta}{p_\Delta^2 - \tilde{m}_\Delta^2} \left( g^{\beta\sigma} - \frac{1}{3}\gamma^\beta\gamma^\sigma - \frac{1}{3\tilde{m}_\Delta}\gamma^\beta p^\sigma \right) \\
&\quad + \frac{\kappa}{\tilde{m}_\Delta}\left( g^{\beta\sigma} - \frac{1}{3}\gamma^\beta\gamma^\sigma \right) - \frac{(1-\kappa)}{3\tilde{m}_\Delta}\gamma^\beta\gamma^\sigma \Bigg] q_\sigma, \text{ for } Z' = -\frac{1}{2}, \\
\mathcal{M}_R^{\Delta,u} &= g(\epsilon^\nu)^\dagger \Gamma_{\nu\beta}(k) \Bigg[ \frac{\not{p}_\Delta + \tilde{m}_\Delta}{p_\Delta^2 - \tilde{m}_\Delta^2} \left( g^{\beta\sigma} - \frac{1}{3}\gamma^\beta\gamma^\sigma - \frac{1}{3\tilde{m}_\Delta}\gamma^\beta p^\sigma \right) \\
&\quad + \frac{2}{3\tilde{m}_\Delta^2}\left( \not{p}\gamma^\beta\gamma^\sigma + \gamma^\beta p^\sigma \right) \\
&\quad + \frac{\kappa}{\tilde{m}_\Delta}\left( g^{\beta\sigma} - \frac{1}{3}\gamma^\beta\gamma^\sigma \right) + \frac{2(1-\kappa)}{3\tilde{m}_\Delta}\gamma^\beta\gamma^\sigma \Bigg] q_\sigma, \text{ for } Z' = +\frac{1}{2}.
\end{aligned}$$

As we expected, this result shows no divergence for $p_\Delta^2 = 0$.

## 4. Results

Following the ideas of the previous sections we calculate here the $\gamma p \to p\pi^0$ and $\gamma p \to n\pi^+$ total cross sections and some multipole amplitude contributions in order to visualize the quality of our results. We used a minimal realistic model for the non resonant background including the $N$, $\pi$, $\rho$ and $\omega$ fields, while the unstable character of the $\Delta$ has been taken into account through the CMS scheme, where we make the replacement $m_\Delta \to m_\Delta - i\Gamma_\Delta/2$ in the *full* propagator, being $\Gamma_\Delta$ the $\Delta$ width, which is assumed to be constant. It can be easily shown that if this change is done only in the denominator of the propagator or if the width $\Gamma$ is not constant, the Ward identity for the $\Delta\gamma\Delta$ vertex is violated by terms of order $\Gamma/m_\Delta$.[3] The loss of unitarity produced by this prescription is compensated by an additional phase (see [23,26,28] for details). As explained before: i) our expressions are independent of $A$; ii) the values of $Z_1$ and $Z'$ are fixed in order to $R_{0\sigma}(-1, Z_1)V_1^\sigma$ and $R_{0\sigma}(-1, Z')\Gamma^{\sigma\nu}$ not containing the zero component of any momenta at first order (we get $Z_1 = \frac{1}{2}$ and $Z' = -1/2$ [11]); iii) the $I_2$ interaction does not depend on $Z$ since it is obtained from the $\mathcal{L}_{free}$ Lagrangian; iv) $\mathcal{L}_{I_2}$ and $\mathcal{L}_{\gamma N\Delta}(Z' = -1/2)$ are both spin 3/2 gauge invariant. Nevertheless, in a previous work where the primary interactions considered here were improved by including Final State Interactions (FSI) [23], we used the $I_1$ interaction alone and the same $\gamma N\Delta$ interaction. We adopted there the values $Z_1 = Z' = \frac{1}{2}$ which lead to the simplest form, not leading to a spin 3/2 invariant $\gamma N\Delta$ vertex. In order to compare those results with the present ones, we also show here our results with $Z' = \frac{1}{2}$. This should only change, as mentioned before, the contact background terms and not the resonant contribution. Finally, we mention that by making the on-shell approximation [25] on the $\Delta$ we get the NP parametrization ($k \cdot \epsilon = 0$)

$$\Gamma_{\mu\nu} = g_1' \left(g^{\mu\nu}\not{k} - k^\mu\gamma^\nu\right) + g_2' \left(g^{\mu\nu}k \cdot p_\Delta - k^\mu p_\Delta^\nu\right),$$

where $g'_{1,2} \equiv g'_{1,2}(G_M, G_E)$. This is derived from a first order ($I_1'$) plus a second order derivative contribution ($I_2'$). In this sense, we return to the Nath proposal in Ref. [17] but now our prescription for fixing $Z'$ in the Sachs parametrization enables a nonzero second order contribution and the contribution of $G_E$. Nevertheless, it is important to note that this vertex is an approach obtained from the on-shell $\Delta$ assumption where the constraints $p_\Delta^\mu\Psi_\mu = \gamma^\mu\Psi_\mu = 0$ are used to connect both parameterizations [25].

For consistency, the strategy we follow is to use the sets of $\Delta$ mass, width, the strong coupling $g$ and $\kappa$ values previously fixed for the $\pi^+p$ and $\pi^-p$ scattering data [14]. Those values are $m_\Delta = 1211.41$; 1210.67; 1210.14 MeV, $\Gamma_\Delta = 88.0$; 84.60; 80.77 MeV, and $g = 0.32$; 0.31; 0.30 for $\kappa = 0$, 1 and 0.45, respectively. Given those values, we now explore all possibilities: $I_1$ alone, $I_2$ alone and the same combination of both interactions used in $\pi N$ scattering. For the non-resonant background contributions we use the same strong parameters as before: $g_\rho = 6.04$, $g_\omega = 9.05$, $\kappa_\rho = 3.7$ and $\kappa_\omega = -0.12$. Otherwise, for the electromagnetic ones we use $g_{\gamma\pi\rho} = 0.101$ and $g_{\gamma\pi\omega} = 0.32$ [20,21]. For the electromagnetic parameters $G_{M,E}$ we assume the same values as the previously obtained in the improved model for pion photoproduction from Ref. [23], which includes also rescattering: $G_M = 2.97$ and $G_E = 0.055$. These are fully consistent with those $\pi N$ scattering and $\gamma N \to N\pi$ photoproduction obtained recently in $\chi_{EFT}$ [29] calculations: $G_M = 2.95$ and $G_E = 0.070$. This indicates that our values effectively include the pionic loop corrections to the $\gamma N \to \Delta$ vertex as in the $\chi_{EFT}$ approach. The sensibility of the $\pi N$ and $\gamma N \to \pi N'$ cross sections with the adopted parameters has been previously analyzed in Refs. [23,27], paying special attention to the $\sigma$ and $\Delta$ masses, $\Delta$ width and coupling constants (that are strongly correlated) in $\pi N$ scattering, and the $G_{E,M}$ values in pion photoproduction process. From the results shown in Ref. [14] we can see that the difference in the adopted values for $g$, $m_\Delta$ and $\Gamma_\Delta$ to reproduce the $\pi^+p$ cross section (the leading contribution) generates a much smaller

[3] A more accurate procedure would be the use of the energy dependent $\Delta$ self-energy taking into account the $\Delta$ mixing with $\pi N$ states at one or higher loop bubbles to all orders [26]. Nevertheless, since we have used the UIBA unitaryzed model from Ref. [23], which includes phase shifts, it is enough to use the simpler CMS prescription.

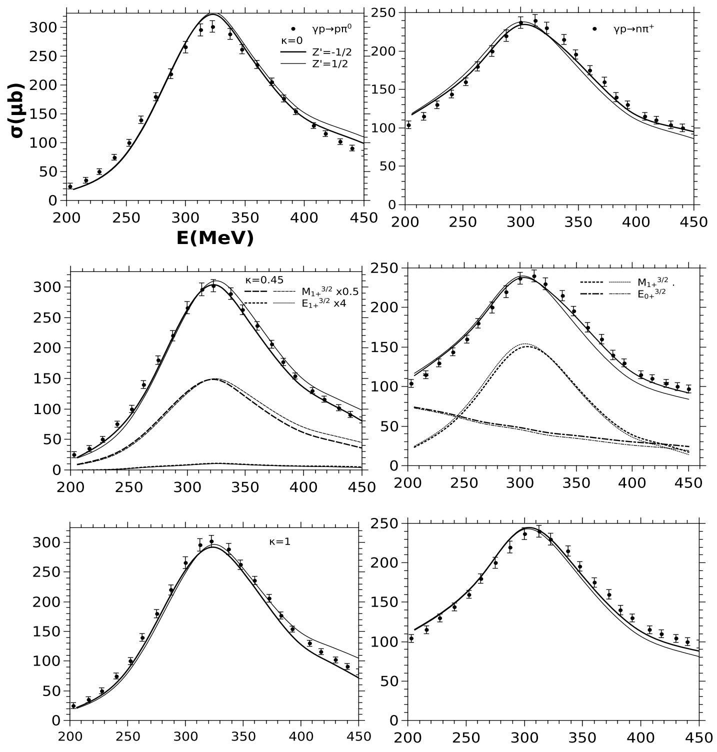


Fig. 2. In the upper plot we show the total cross section for $\kappa = 0$ $Z' = -\frac{1}{2}$ ($\frac{1}{2}$) with thick (thin) lines, both for $\gamma p \to p\pi^0$ (left) and $\gamma p \to n\pi^+$ (right) processes. In the middle plot we show the corresponding results for $\kappa = 0.45$ (some of the main multipole contributions are shown). In the lower plot we show our results for $\kappa = 1$. Experimental data were taken from Ref. [24].

dispersion of the results than that coming from different $Z'$ values in our present calculation. This clearly indicates that the theoretical uncertainties are under control. As previously explained, we adopt here the unitarized tree model from Ref. [23] for pion photoproduction without the inclusion of FSI (in spite that part of the rescattering is taken into account through the $\Delta$ width), which is enough in order to analyze the effect of using different values for the $Z'$ parameter in the shifted contact terms from the $\Delta$ sector. Also we analyze the effect of introducing the consistent $I_2$ interaction independent of $Z$. We use the "$\psi_0$" criterion to fix $Z_1$, $Z'$, instead of fitting it to reproduce the data. We calculated the cross section for the $\gamma p \to p\pi^0$ and $\gamma p \to n\pi^+$ processes adopting the value $Z_1 = \frac{1}{2}$ for the strong vertex and: i) $Z' = \frac{1}{2}$ used in the previous work [23], where we considered the simplest $\gamma N\Delta$ vertex, or ii) $Z' = -\frac{1}{2}$ which avoids any dynamics for the $\Psi_0$ component and leads to a consistent vertex.

In Fig. 2 (upper pannel) we show our results for the total cross section with $Z' = \pm\frac{1}{2}$ for the $\gamma p \to p\pi^0$ (left) and $\gamma p \to n\pi^+$ (right) channels, for $\kappa = 0$ ($I_1$), while for the middle and bottom ones we report the same but for $\kappa = 0.45$ ($I_1 + I_2$) and $1.0(I_2)$ respectively. As it can be seen, the experimental data are better described in both channels for the background contact terms with $Z' = -\frac{1}{2}$ and combined interactions. In spite we are not making a fitting, we can report the $\chi^2/dof$ where we consider $dof = N_{data}$ to qualify the difference with the data. We

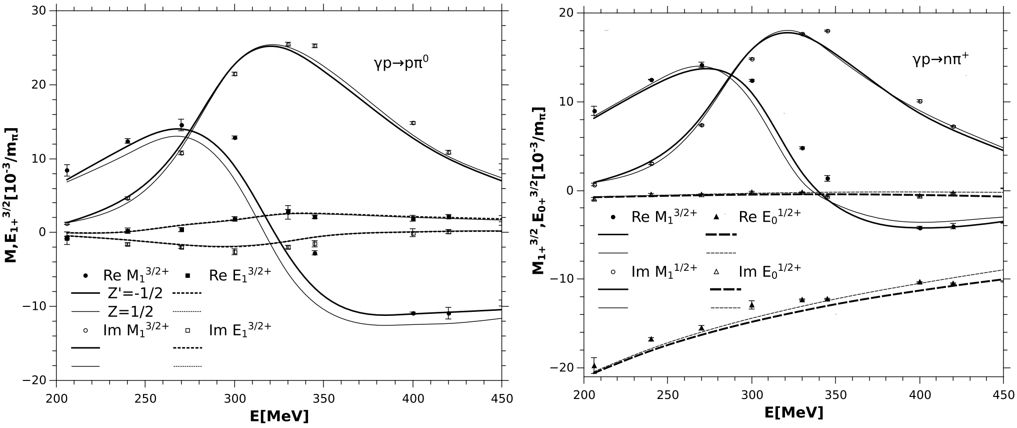


Fig. 3. We show here the real and imaginary parts of the multipole amplitudes considered in Fig. 2. Again, thick lines are used for results with $Z' = -\frac{1}{2}$, while thin ones for $Z' = \frac{1}{2}$, and data are taken from [24].

get the values $\chi^2/dof = 5.9$, 1.38 and 4.97 (1.72, 1.62, 1.9) for $\gamma p \to p\pi^0$ ($\gamma p \to n\pi^+$) and $\kappa = 0$, 0.45 and 1, respectively, for $Z' = -\frac{1}{2}$.

For the best result with $\kappa = 0.45$, $Z' = -\frac{1}{2}$ we show the multipole contributions to the total cross section in order of importance, and in Fig. 3 the multipole amplitudes compared with the corresponding data. Our results show that the adopted unitarized model (UIBA from Ref. [23]) works quit well. Note that when we use the values $\kappa = 0$, $Z' = \frac{1}{2}$ adopted in our previous work from Ref. [23] (upper pannel in Fig. 2) we get $\chi^2/dof = 6.45$ and 3.54 for the $\gamma p \to p\pi^0$ and $\gamma p \to n\pi^+$ channels, respectively, showing a worse coincidence. This would be compensated by the introduction of FSI as done in that work. Otherwise, for $\kappa = 1$, $Z' = -\frac{1}{2}$ (lower pannel in Fig. 2) which should be the analog to the corresponding calculation from Ref. [5] and both consistent interactions, we get a worse result than for $\kappa = 0.45$, $Z' = -\frac{1}{2}$. This indicates that the inclusion of both interactions and our consistent method of looking for $Z_1$, $Z'$ works as well as the crude fitting and avoids the inclusion of FSI that requires the use of form factors, which generate model dependence. Additionally, the present calculation provides a controlled method for adding the necessary 1/2 backgrounds to the consistent $I_2 + \gamma N\Delta(Z')$ amplitude.

We observe that the dispersion of results is smaller for the second channel. This is because the interference between background and resonance contributions is very different for each channel since we have some contributions (pion in flight and contact term) absent in the first one but present in the other, while the $\omega$ exchange is present in the first but absent in the second one. Also the $\mathcal{M}_R^{\Delta,u}$ has different signs for each channel. The above results do not change appreciably if we try to fit $G_M$ ($G_E = 0.025 G_M$ fixed for simplicity from the experimental $R_{EM}$ ratio) since the adopted value $G_M = 2.90$ is very close to that reported experimentally and, due to the mentioned ratio, we have a $G_M$ overall factor in Eq. (12) that makes the resulting cross section oscillate around the previous results without changing appreciably the $\chi^2/dof$ values.

## 5. Conclusions

We have implemented two new ideas for the $\gamma N \to N'\pi$ pion photoproduction process: i) A method for choosing the value for the $Z'$ parameter present in the shifted contact terms coming form the $\Delta$-sector in the contact invariant $\gamma N\Delta$ vertex. ii) The use of first order plus second order derivative strong interactions in the $\Delta\pi N$ vertex already used to describe $\pi N$ scattering.

In i) we have adopted the same method used to fix the parameters $Z_1$ of the contact terms generated from the contact invariant $I_1$ interaction as in a previous work: choosing them so the interactions do not generate dynamics for the $\Psi_0$ component of the $\Delta$ field, since it is a Lagrange multiplier of the constrained problem. We have chosen here a consistent $Z$-independent interaction $I_2$. We have compared the use of $I_1 + I_2$ with different coefficients for $I_1$ and $I_2$, adopting the value $Z' = \frac{1}{2}$, previously used in a calculation where, as usually, $I_1$ was considered alone; Additionally, we adopted $Z' = -\frac{1}{2}$ in accordance with the new fixing procedure described above. As it can be seen in the results shown in Figs. 2 and 3 the calculations with $Z' = -\frac{1}{2}$ improve notably the description of the data over the previous choice $Z' = \frac{1}{2}$, at least at tree level including unitarity by means of the method described in Ref. [23] but without including rescattering effects, already considered before in that reference. On the other hand, we have understood why the values chosen in Refs. [4,5], $Z_2 = Z' = -1/2$, lead to correct values for $Z$ (or a valley) around which additional $Z$ changes do not get appreciable modification on the contact term parameters or LEC's. The value $Z' = -1/2$ also corresponds to the original spin 3/2 gauge invariance version of the electromagnetic Lagrangian. We have enabled also the $Z' = 1/2$ value for which this invariance is violated *only* to compare with previous calculations. We also remark that it is not as clear how to obtain the fourth-order contribution to the Sachs vertex through a transformation on $\mathcal{L}_{free}$, analogously to what is done with the second-order one.

Once again, it is important to note that we are not fitting any parameter in this work, we took the parameters previously adopted (coupling constants and $Z_1$) in the description of $\pi N$ scattering [14] and $G_{E,M}$ from [23]. We have certain dispersion between the results for $\kappa = 0$, 0.45 and 1 that switch between the calculations with $I_1$ alone, $I_1 + I_2$ and $I_2$ alone, respectively. Once the change to $Z' = -\frac{1}{2}$ is achieved, we clearly see that the results for $\kappa = 0$ and 0.45 are close and lead to better description for $\gamma p \to p\pi^0$ and $\gamma p \to n\pi^+$ channels, while for $\kappa = 1$ the departure from the data is larger. This indicates that the presence of the 1/2 contribution coming from $P_{11}^{1/2}$ is necessary and is suppressed for the $\kappa = 1$ case where we have two consistent interactions. Thus, the electromagnetic vertex $\Gamma_{\mu\nu}$ in the Sachs parametrization has (at least in the most important $G_M - G_E$ term) the same gauge invariant spin 3/2-structure that $V_2^\sigma$. This is not the same situation than the $\pi N$ scattering case where both initial and final vertexes have both types of interactions. We note that this method of including both $I_1$ and $I_2$ interactions is a robust way of adding a background to the $I_2$ interaction alone together with the Sachs parametrization, that has the same spin 3/2 gauge symmetry. This is very different than adding "ad-hoc" background contributions to reproduce the cross section data as done in Ref. [15].

Summarizing, the use of the Sachs parametrization is consistent with the choice $Z' = -\frac{1}{2}$ and the best coincidence with data is achieved for $\kappa = 0.45$, while results obtained using the interaction $I_1$ or $I_2$ alone show a larger departure from experimental data. This also could be seen as a confirmation of the use of spin 3/2 gauge invariant interactions ($I_2$, $\mathcal{L}_{\gamma N\Delta}(-1/2)$) and, as we can transform $I_1 \to I_2+$contact term, a controlled way of fixing the contact parameters when the other parameters of the background are already fixed by low energy phenomenology. This is also useful since in the past many calculations where done using $I_1$ alone. This is in full consistence with the previous $\pi N$ scattering case, where $\kappa = 0.45$ should be chosen. In the future, other parameterizations could be probed for the electromagnetic vertex using independent parameters for the first and second derivative contributions in the NP parametrization. Nevertheless, this could be difficult since, as it was shown, both parameterizations should be connected if one wish to maintain the $\Psi_0$ criterion implemented for the Sachs one. In addition we have shown that the procedure of integrating out the $\Delta$-field (which leads to a nonlocal Lagrangian plus additional $Z$-dependent terms in the $\pi N$ sector) is not an argument against our procedure of looking for $Z$ through preserving the Lagrange multiplier character of $\Psi_0$ in the Lagrangian.

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## 6. Appendix

### 6.1. Spin projectors and isospin excitation operators

We have introduced $P^k_{ij}$ which projects on the $k = 3/2, 1/2$ sector of the representation space, with $i, j = 1, 2$ indicating the subsectors of the $1/2$ subspace, which are defined as:

$$\begin{aligned}
(P^{3/2})_{\mu\nu} &= g_{\mu\nu} - \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3p^2}\left[\not{p}\gamma_\mu p_\nu + p_\mu\gamma_\nu\not{p}\right],\\
(P^{1/2}_{22})_{\mu\nu} &= \frac{p_\mu p_\nu}{p^2},\\
(P^{1/2}_{11})_{\mu\nu} &= g_{\mu\nu} - P^{3/2}_{\mu\nu} - (P^{1/2}_{22})_{\mu\nu}\\
&= (g_{\mu\alpha} - \frac{p_\mu p_\alpha}{p^2})(1/3\gamma^\alpha\gamma^\beta)(g_{\beta\nu} - \frac{p_\beta p_\nu}{p^2}),\\
(P^{1/2}_{12})_{\mu\nu} &= \frac{1}{\sqrt{3}p^2}(p_\mu p_\nu - \not{p}\gamma_\mu p_\nu),\\
(P^{1/2}_{21})_{\mu\nu} &= \frac{1}{\sqrt{3}p^2}(-p_\mu p_\nu + \not{p}p_\mu\gamma_\nu).
\end{aligned} \tag{15}$$

On the other hand we define the isospin $\Delta$ excitation operators as

$$\mathbf{T}^\dagger \cdot \boldsymbol{\phi}_{+,-,0} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\sqrt{3}} \\ 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ -\frac{1}{\sqrt{3}} & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ \sqrt{\frac{2}{3}} & 0 \\ 0 & \sqrt{\frac{2}{3}} \\ 0 & 0 \end{pmatrix},$$

that acts on $|p, n\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $|\Delta_{++,+,0,-}\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ states.

### 6.2. Lagrangians and amplitudes involved in non resonant background

The propagators and interaction used to built amplitudes will be summarized here. First, the propagators which come from the inversion of the kinetic operators present in the free Lagrangians are:

$$S(p) = \frac{\not{p} + m_N}{p^2 - m^2} \text{ (nucleon)},$$
$$\Delta(p) = \frac{1}{p^2 - m_\pi^2} \text{ (pion)},$$
$$D_{\mu\nu}(p) = \frac{-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_V^2}}{p^2 - m_V^2} \text{ (vector-meson)},$$

while the strong interacting Lagrangians are:

$$\mathcal{L}_{\pi NN}(x) = -\frac{f_{\pi NN}}{m_\pi}\bar{\psi}(x)\gamma_5\gamma_\mu \boldsymbol{\tau} \cdot \left(\partial^\mu \boldsymbol{\phi}(x)\right)\psi(x),$$
$$\mathcal{L}_{VNN}(x) = -\frac{g_V}{2}\bar{\psi}(x)\left[\gamma_\mu \left\{ \begin{array}{c} \boldsymbol{\rho}^\mu(x)\cdot\boldsymbol{\tau} \\ \omega^\mu(x) \end{array} \right\} - \frac{\kappa_V}{2m_N}\sigma_{\mu\nu}\left(\partial^\nu \left\{ \begin{array}{c} \boldsymbol{\rho}^\mu(x)\cdot\boldsymbol{\tau} \\ \omega^\mu(x) \end{array} \right\}\right)\right]\psi(x).$$

The electromagnetic Lagrangians are:

$$\begin{aligned}
\mathcal{L}_{\gamma NN}(x) &= -\psi(x)\left[\hat{e}\gamma_\mu A^\mu(x) - \frac{\hat{\kappa}}{2m_N}\sigma_{\mu\nu}\left(\partial^\nu A^\mu(x)\right)\right]\psi(x), \\
\mathcal{L}_{\gamma\pi\pi}(x) &= -e\left[\boldsymbol{\phi}(x)\times\partial_\mu\boldsymbol{\phi}(x)\right]_3 A^\mu(x), \\
\mathcal{L}_{\gamma\pi NN}(x) &= -e\frac{f_{\pi NN}}{m_\pi}\bar{\psi}(x)\gamma_5\gamma_\mu(\boldsymbol{\tau}\times\boldsymbol{\phi}(x))_3\psi(x)A^\mu(x), \\
\mathcal{L}_{\gamma\pi\rho}(x) &= -\frac{g_{\gamma\pi\rho}}{m_\pi}\epsilon_{\mu\alpha\lambda\nu}\left(\partial^\mu A^\alpha(x)\right)\left(\partial^\lambda\boldsymbol{\phi}(x)\right)\cdot\boldsymbol{\rho}^\nu(x) \\
&= +\frac{g_{\gamma\pi\rho}}{m_\pi}\epsilon_{\mu\alpha\lambda\nu}\left(\partial^\mu A^\alpha(x)\right)\boldsymbol{\phi}(x)\cdot\left(\partial^\lambda\boldsymbol{\rho}^\nu(x)\right) \\
\mathcal{L}_{\gamma\pi\omega}(x) &= -\frac{g_{\gamma\pi\omega}}{m_\pi}\epsilon_{\mu\alpha\lambda\nu}\left(\partial^\mu A^\alpha(x)\right)\left(\partial^\lambda\boldsymbol{\phi}(x)\right)_3\omega^\nu(x) \\
&= +\frac{g_{\gamma\pi\omega}}{m_\pi}\epsilon_{\mu\alpha\lambda\nu}\left(\partial^\mu A^\alpha(x)\right)\boldsymbol{\phi}(x)_3\left(\partial^\lambda\omega^\nu(x)\right),
\end{aligned}$$

where the last two equivalent forms were obtained from:

$$\begin{aligned}
\partial^\lambda\left[\left(\partial^\mu A^\alpha(x)\right)\boldsymbol{\phi}(x)\cdot\boldsymbol{\rho}^\nu(x)\right] &= \left(\partial^\lambda\partial^\mu A^\alpha(x)\right)\boldsymbol{\phi}(x)\cdot\boldsymbol{\rho}^\nu(x) + \left(\partial^\mu A^\alpha(x)\right)\left(\partial^\lambda\boldsymbol{\phi}(x)\right)\cdot\boldsymbol{\rho}^\nu(x) \\
&\quad + (\partial^\mu A(x))\boldsymbol{\phi}(x)\cdot\partial^\lambda\boldsymbol{\rho}^\nu(x) \\
\left(\partial^\mu A^\alpha(x)\right)\left(\partial^\lambda\boldsymbol{\phi}(x)\right)\cdot\boldsymbol{\rho}^\nu(x) &\doteq -\left(\partial^\mu A^\alpha(x)\right)\boldsymbol{\phi}(x)\cdot\partial^\lambda\boldsymbol{\rho}^\nu(x),
\end{aligned}$$

since the total derivative does not contribute because it does not affect the action, nor the second derivative in the photon due the antisymmetric tensor. Next, we get the background amplitude obtained from the previous Lagrangians, which correspond to nonresonant background amplitude that is the sum of Born, contact, pion-in-flight and meson exchange terms ($p$, $q$ and $k$ denote here the nucleon, pion and photon momentum, respectively)

$$\begin{aligned}
\mathcal{M}_B &= ie\frac{f_{\pi NN}}{m_\pi}\gamma_5\not{q}S(p+k)\left(\hat{e}\not{\epsilon}^* + i\frac{\hat{\kappa}}{2m_N}\sigma_{\mu\nu}k^\nu\epsilon^\mu\right)\left\{ \begin{array}{c} 1, \gamma p\to p\pi^0 \\ \mp\sqrt{2}, \gamma p/n\to n/p\pi^\pm \end{array} \right. \\
&+ ie\frac{f_{\pi NN}}{m_\pi}\left(\hat{e}\not{\epsilon}^* + i\frac{\hat{\kappa}}{2m_N}\sigma_{\mu\nu}k^\nu\epsilon^\mu\right)S(p-q)\gamma_5\not{q}\left\{ \begin{array}{c} 1 \\ \mp\sqrt{2} \end{array} \right. \\
&+ ie\frac{f_{\pi NN}}{m_\pi}\gamma_5\not{\epsilon}^*\left\{ \begin{array}{c} 0 \\ \sqrt{2} \end{array} \right.
\end{aligned}$$

$$+ie\frac{f_{\pi NN}}{m_\pi}(k+p-p')\cdot\epsilon^*\Delta(p-p')\gamma_5(\not p-\not p')\left\{\begin{array}{c}0\\-\sqrt{2}\end{array}\right.$$
$$-\frac{g_{\gamma\pi\rho}g_\rho}{2m_\pi}\epsilon^{\mu*}\epsilon_{\mu\alpha\lambda\nu}q^\alpha(k=q+p-p')^\lambda D_\rho^{\nu\beta}(p-p')$$
$$\times\left[\gamma_\beta-i\frac{\kappa_\rho}{2m_N}\sigma_{\beta\nu}(p-p')^\nu\right]\left\{\begin{array}{c}1\\\mp\sqrt{2}\end{array}\right.$$
$$-\frac{g_{\gamma\pi\omega}g_\omega}{2m_\pi}\epsilon^{\mu*}\epsilon_{\mu\alpha\lambda\nu}q^\alpha(k=q+p-p')^\lambda D_\omega^{\nu\beta}(p-p')$$
$$\times\left[\gamma_\beta-i\frac{\kappa_\omega}{2m_N}\sigma_{\beta\nu}(p-p')^\nu\right]\left\{\begin{array}{c}1\\0\end{array}\right. .$$

## CRediT authorship contribution statement

*Category 1*

Conception and design of study: A. Mariano, C. Barbero;
acquisition of data: D. Badagnani;
analysis and/or interpretation of data: C. Barbero, D.F. Tamayo Agudelo.

*Category 2*

Drafting the manuscript: C. Barbero, A. Mariano;
revising the manuscript critically for important intellectual content: A. Mariano, C. Barbero, D. Badagnani.

*Category 3*

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## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

No data was used for the research described in the article.

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