A Method to Improve Flow-Velocity Measurements From an Array of Partially Cosine Response Sensors

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***Abstract*—A groundwater flowmeter whose working principle is based on an array of thermal sensor axes was studied to analyze the errors introduced when the angular response of the sensor axes is partially cosine and varies with flow velocity. It is shown that, although errors due to the partially cosine response can be reduced by increasing the number of sensor axes, the increment in the number of axes introduces some additional errors. A method is proposed to keep the advantages of increasing the number of axes and to minimize the observed additional errors. This method could be extended to other type of arrays whose sensors have a partial cosine response varying with the physical quantity of interest.**

***Index Terms*—Angular response, flow-velocity measurement, sensor arrays, thermal flowmeter.**

I. Introduction

THERE ARE several traditional methods of measuring groundwater velocity. The working principles of these methods are described in texts on groundwater hydrology (see, for example, [1] and references therein), but flowmeters needing just only one borehole to measure horizontal groundwater flow constitute a particular sort [2]. Some of these flowmeters are based on the application of heat by means of a central heater surrounded by a number of sensor elements [3], [4]. The heat sensing flowmeter [4], [5] is placed in a saturated porous medium in which the heater produces a heat pulse that diffuses radially. Heat diffusion is affected by the groundwater flow, which causes cooling upstream of the heater and heating downstream. A few minutes after the beginning of the pulse, the temperature difference between two opposite sensors (sensor axis) in the flow direction reaches a maximum that is proportional to the flow velocity within the porous medium. This temperature difference is converted into a velocity component through a calibration curve. The total flow-velocity vector is found by adding its components along every sensor axis [5]. Some flowmeters use two orthogonal sensor axes [6], [7], others require more axes [5], [8], [9]. In all of these instruments, the response of a sensor axis as a function of the angle with the flow-velocity vector is known as the angular response. This response is called a cosine response if it is proportional to the magnitude of the flow velocity times the cosine of the angle between the axis and the flow-velocity vector [7].

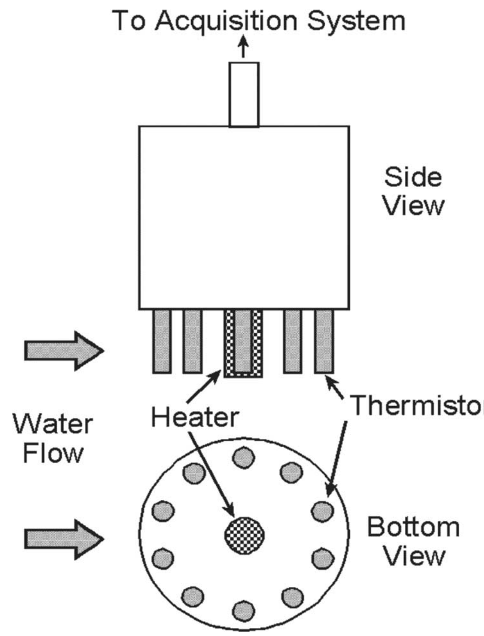

Fig. 1. Schematic of the KVGF Model 30 thermal ground water flowmeter probe.

The K-V Geo Flowmeter (KVGF) Model 30 is a thermal groundwater flowmeter with five sensor axes [5]. A new model from the same manufacturer (Geoflow 40) has four axes [4], [9]. Both models are used to measure groundwater velocities less than 150 m/d [2]. An analysis supporting the need of such a number of axes was not found in the flowmeter literature. Using the Model 30 as a case study, this paper aims to analyze the relation between the amount of sensor axes and the error in the calculation of groundwater flow velocity. Based on experimental data, different axis configurations were studied, and a method was developed to keep the errors as low as possible.

II. Theory

Flow measurement in the KVGF Model 30 is based on thermal transmission within a porous medium under the influence of interstitial liquid flow [8]. The probe consists of a central heater surrounded by a circular array of ten equally spaced thermistors [10] Fig. 1. Any couple of opposite thermistors constitutes a sensor axis. The heater produces a heat pulse that radially propagates outward. If the groundwater velocity is null, a symmetrical temperature field is generated, and the

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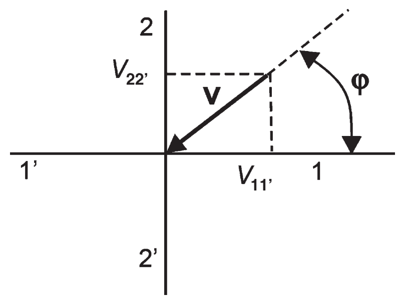

Fig. 2. Flow velocity components along two orthogonal sensor axes.

thermistors, which are located at the same distance from the heater, measure the same temperature increment. When the heat pulse is influenced by an interstitial water movement, an asymmetry in the temperature field arises, and a temperature difference ΔT between any pair of opposite thermistors is detected. This difference is maximum in the flow direction and proportional to the flow velocity. When a sensor axis is normal to the direction of water flow, $\Delta T = 0$. The relation between the temperature difference and the flow velocity is stable over a wide range of velocities, i.e., 0.06 to 30 m/d [11]. Melville *et al.* [10] suggest that this relation is linear in a smaller range, i.e., 0.03–3 m/d. Kerfoot and Skinner [11] say that, for uniform flow, ΔT is proportional to the cosine of the angle between the flow and the axis formed by the two thermistors. This is tantamount to saying that the axes have a cosine response, in which case the instrument would require only two axes to properly determine the flow velocity [7].

If two orthogonal axes $11'$ and $22'$ with cosine responses are used to measure the horizontal flow-velocity vector $\mathbf{V}$, the velocity components $V_{11'}$ and $V_{22'}$ are given by Fig. 2

$$V_{11'} = CV\cos\varphi \quad V_{22'} = CV\sin\varphi \tag{1}$$

where V is the length of $\mathbf{V}$, φ is the angle between the reference axis $11'$ and $\mathbf{V}$, and C is a constant that relates V with the magnitude measured by the sensor axis (ΔT in this case) when $\varphi = 0$. V and φ are determined from

$$V = \sqrt{V_{11'}^2 + V_{22'}^2} \quad \varphi = \arctan\left|\frac{V_{22'}}{V_{11'}}\right|. \tag{2}$$

If the angle between the two sensor axes $11'$ and $22'$ is $\theta < \pi/2$, the orthogonal projections of $\mathbf{V}$ along both directions are (Fig. 3)

$$V_{11'} = CV\cos\varphi \quad V_{22'} = CV\cos(\varphi + \theta). \tag{3}$$

For a probe with N sensor axes equally spaced by an angle $\theta < \pi/2$, the component $V_{kk'}$ along the kk' sensor axis ($kk' = 11', 22', \ldots NN'$) is

$$\begin{aligned} V_{kk'} &= CV\cos\alpha_k \\ \alpha_k &= \left[\varphi + (k-1)\frac{\pi}{N}\right], \qquad k = 1, 2, \ldots, N \end{aligned} \tag{4}$$

where α_k is the angle that the kk' sensor axis makes with $\mathbf{V}$, and $V_{kk'}$ is taken positive when the projection of $\mathbf{V}$ onto the kk' axis is from k to k'.

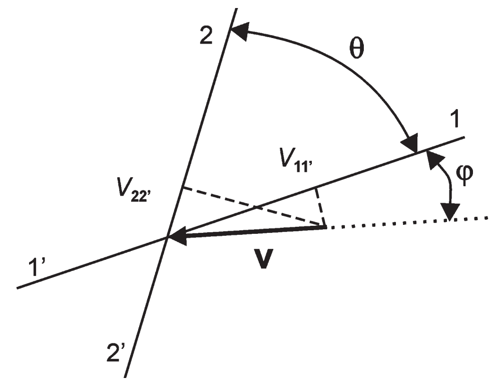

Fig. 3. Flow velocity components along two sensor axes forming an angle $\theta < \pi/2$.

To obtain $\mathbf{V}$, some authors consider the orthogonal components $V_{kk'}$ as vectors $\mathbf{V}_{kk'}$ and add them graphically [5], [12]. This method is valid for the sensor axes with cosine response, but if it is used with nonstrictly cosine responses, some degree of error is introduced. In this paper, a different method is adopted to calculate $\mathbf{V}$ with a computer. The vectors $\mathbf{V}_{kk'}$ are projected along two orthogonal axes, namely, north–south (NS) and east-west (EW), and $\mathbf{V}$ is obtained from the corresponding sums of the projections [7]. If the directions $11'$ and NS are made to coincide, the sums of the projections of the vectors $\mathbf{V}_{kk'}$ along both directions V_{NS} and V_{EW} are

$$\begin{aligned} V_{\mathrm{NS}} &= \sum_{k=1}^{N} V_{kk'}\cos\left[(k-1)\frac{\pi}{N}\right] \\ V_{\mathrm{EW}} &= \sum_{k=1}^{N} V_{kk'}\sin\left[(k-1)\frac{\pi}{N}\right] \end{aligned} \tag{5}$$

and V and φ are obtained from

$$V = \sqrt{V_{\mathrm{NS}}^2 + V_{\mathrm{EW}}^2} \quad \varphi = \arctan\left|\frac{V_{\mathrm{EW}}}{V_{\mathrm{NS}}}\right|. \tag{6}$$

The signs of V_{NS} and V_{EW} define the quadrant of $\mathbf{V}$. This method might also be used with angular responses that slightly deviate from a cosine if some degree of error is accepted.

An angular response will be called partially cosine if, for small angles between the sensor axis and the flow vector, it is close to the cosine function but below the cosine function as the angle increases. An example of a partially cosine response for different velocities is shown in Fig. 4. It is a normalized orthogonal projection of the water flow velocity V onto one of the sensor axes of the Model 30 from a laboratory test. To obtain this transfer function, the temperature difference between the opposite thermistors was measured with an accuracy better than 1%. The thermistors were previously calibrated in a water bath. Projections in Fig. 4 are normalized to that for which the flow velocity is parallel to the sensor axis. Hereafter, it will be assumed that all of the sensor axes have the same response. Although this is not completely true for a real probe because of constructive problems, it was verified that the other axes have partially cosine responses too. In addition, it was constructed a probe of size and form comparable to that of the Model 30,

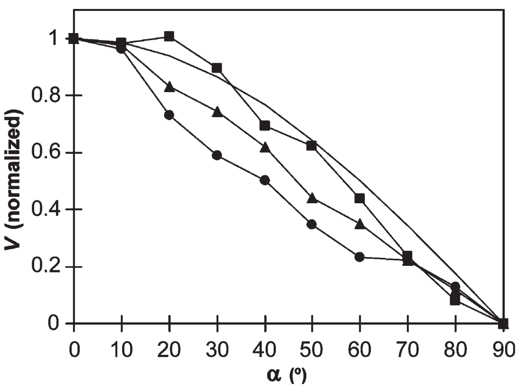

Fig. 4. Normalized orthogonal projection of water velocity V as a function of the angle α between $\mathbf{V}$ and one of the sensor axes for three velocities. A true cosine curve (simple solid line) is shown for comparison purposes. References: ■ 2.3 m/d; ▲ 8.6 m/d; ● 18.4 m/d.

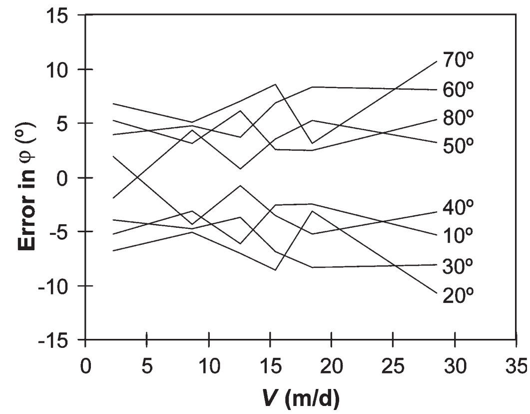

Fig. 5. Errors in the determination of φ as a function of flow velocity V for φ varying in steps of 10°. Probe with two sensor axes ($\theta = 90°$, four thermistors).

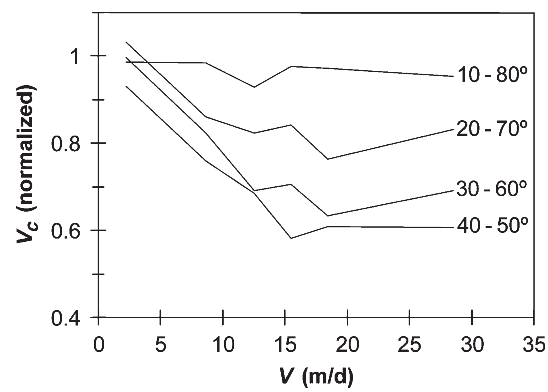

Fig. 6. Normalized calculated water velocity V_c as a function of flow velocity V for φ varying in steps of 10°. Probe with two sensor axes ($\theta = 90°$, four thermistors).

and it was found that it had the same type of partially cosine response. This shows that the lack of cosine response is a problem inherent to this type of thermal flowmeters.

III. Results and Discussion

A field evaluation showed that the heat-pulse flowmeter fails to consistently provide repeatable measurements of velocity and direction [2]. This behavior could be due to the use of vector addition when the response of the sensors is not of a cosine type. It was therefore desired to estimate the error introduced by the vector addition (5) when applied to a probe with N partially cosine response sensor axes. For a flow velocity $\mathbf{V}$, the angle α_k between the kk' axis and $\mathbf{V}$ is defined in (4). With α_k, the components $V_{kk'}$ are obtained from the angular response of the axis (Fig. 4) and introduced into (5). Finally, the velocity is calculated with (6) and compared with $\mathbf{V}$. This procedure was first used to study a probe with two orthogonal axes (four thermistors). Figs. 5 and 6 show the error in the determination of φ and the magnitude of the calculated velocity V_c, both as a function of V, for φ varying in steps of 10°. The depicted magnitude of V_c is normalized to that for which $\mathbf{V}$ is parallel to the sensor axis. It can be seen that errors in φ are in the range of $\pm 10°$ and that the relative magnitude of V_c depends on φ and V. These results are due to the fact that the partially cosine response varies with the flow velocity.

As shown in Fig. 4, the response of an axis begins to deviate from a cosine response for angles larger than a certain α, for example, α_0; the greater the flow velocity, the smaller the angle α_0. For example, for the flow velocity of 2.3 m/d, the response is relatively of cosine type up to $\alpha_0 \cdot 60°$. Because the cosine is an even function, the curve in Fig. 4 would be symmetrical with respect to the ordinate axis up to $\alpha_0 \cdot -60°$. Thus, the portion of the curve that can be considered as having an approximate cosine response for the flow velocity of 2.3 m/d spans an angle of about 120°. In general, for a given α_0, the portion of the response curve that could be taken as nearly of cosine type would be 2 α_0. Therefore, the number of axes N that would be needed in spanning the whole circumference would be $N = \pi/\alpha_0$.

The aforementioned procedure was used to evaluate the errors arising in the determination of φ as a function of V for a probe with six sensor axes ($\theta = 30°$) and φ varying in steps of 10°. It is shown in Fig. 7 that the maximum error is about $\pm 1.4°$. For a probe with only three axes ($\theta = 60°$), the error in φ reached $\pm 5.2°$ (not shown). Increasing the number of axes not only considerably improves the accuracy in the determination of φ but also reduces the dispersion in the normalized calculated velocity magnitude V_c (Fig. 8). However, this causes a loss in the magnitude sensitivity, which is a function of N and V (Fig. 8). Therefore, when the sensor axes have a partially cosine response variable with flow velocity, the vector addition produces an attenuation in the array response.

The simplest way to prevent signal attenuation would be to use a unique sensor axis and rotate it in small steps until it coincides with the direction of flow velocity. Unfortunately, this is not possible for a heat-pulse flowmeter because each measurement takes about 30 min to be performed [2], and it would take 18 h to sweep 360° in steps of, for example, 10°. The sensor array, instead, has the advantage of measuring in several directions simultaneously. Following the aforementioned discussion, a measuring method for the sensor array is proposed that takes advantage of the improvement in the calculation of φ, prevents magnitude attenuation, and is more efficient with

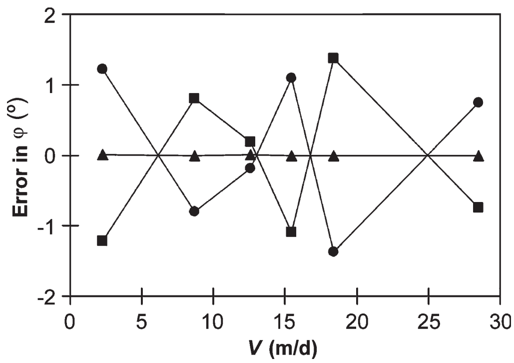

Fig. 7. Errors in the determination of φ as a function of flow velocity V for a probe with six sensor axes ($\theta = 30°$, 12 thermistors) and φ varying in steps of 10°. References: ■ 20° − 50° − 80°; ▲ 30° − 60°; • 10° − 40° − 70°.

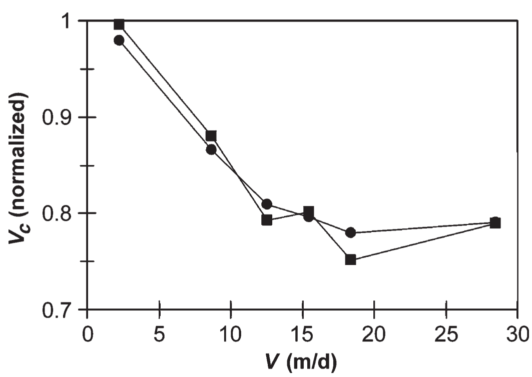

Fig. 8. Normalized calculated water velocity V_c as a function of water velocity V for φ varying in steps of 10°. Probe with six sensor axes ($\theta = 30°$, 12 thermistors). References: ■ 30° − 60°; • 10° − 20° − 40° − 50° − 70° − 80°.

respect to the time required to perform a measurement. The method has two steps.

Step 1) Estimate φ with (5) and (6).

Step 2) Turn the instrument to the angle φ estimated in Step 1). In this way, one of the sensor axes will be very close to the flow direction. Then, measure the magnitude of the flow velocity with this sensor axis only.

If it is not possible to turn the instrument, the second step could be split into two steps.

Step 2.1) Measure the projection of $\mathbf{V}$ with the axis nearest to the flow direction.

Step 2.2) Correct the value measured in Step 2.1) using the value of φ from Step 1) and the angular response of the axis.

The last procedure is valid only if the instrument has several axes such that φ is small, so that the angular response is quite similar for all velocities. It should be noted that, in this paper, the responses of all the axes were supposed to be identical. In a real case, due to manufacturing processes, their responses are slightly different. Although the method to prevent magnitude attenuation has been shown for a thermal flowmeter, the aforementioned results can be extended to other types of sensor systems with partially cosine responses (e.g., optical and acoustic sensors), in which the angular response varies with the physical quantity of interest (light or sound intensity).

IV. Conclusion

When the response of the sensor axes of an array to a given physical quantity is partially cosine and varies with it, the quality of the results can be improved by using more than two sensor axes. Increasing the number of axes improves the calculation of the direction and reduces the dispersion in the determination of the magnitude, but the magnitude itself is affected. It is possible to reduce this negative outcome by trying to measure the magnitude of the physical quantity of interest with only one axis using the two-step method previously proposed. These general results have been proved for the particular case of a thermal groundwater flowmeter.

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