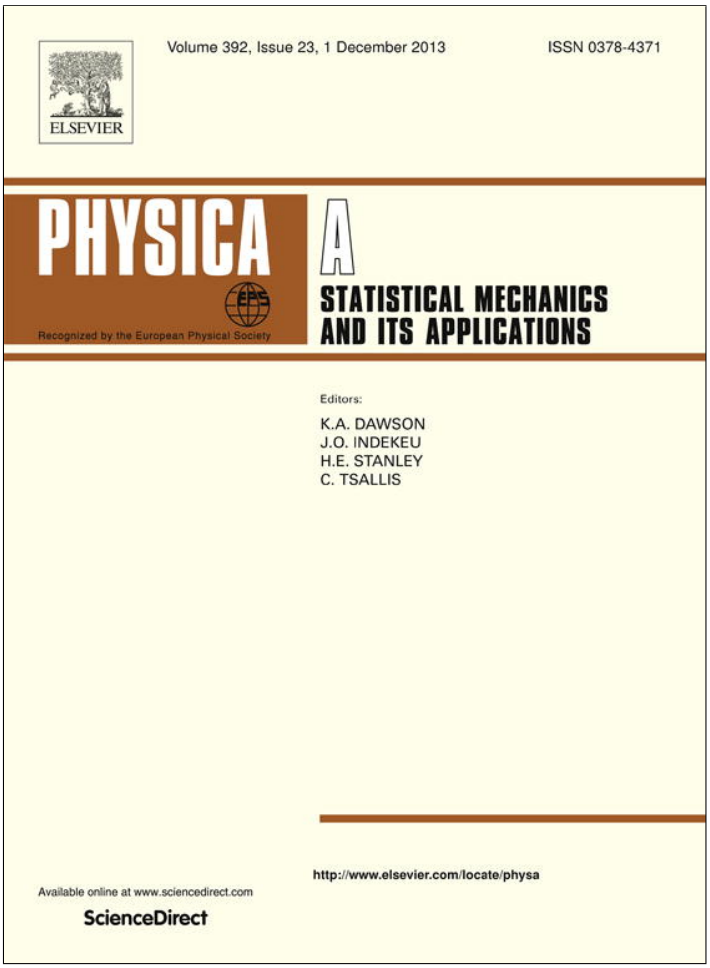

Physica A 392 (2013) 6155–6160

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Remanent quantum correlations in dissipative qubits

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HIGHLIGHTS

- We solved analytically a Markov Dissipative Quantum Walk.
- We define and solve a non-Markov dissipative qubit.
- We calculate the concurrence and the quantum discord.
- We compare quantum correlations versus classical correlations.

ARTICLE INFO

Article history:
Received 30 April 2013
Received in revised form 6 June 2013
Available online 2 August 2013

Keywords:
Quantum Walk
Master equation
Non-Markovian decoherence
Quantum correlations
Qubits

ABSTRACT

Starting from the exact evolution of a Markovian dissipative quantum walk, a non-Markovian decoherence of two qubits interacting with a phonon thermal bath has been investigated analytically using quantum information tools. Concurrence and quantum discord are affected in a complex way, showing that entanglement decreases with dissipation. At the limit where dissipation dominates, quantum correlations survive in time as $\propto t^{-1/2}$. Thus, even under the influence of dissipation, two qubits retain their quantumness for a long time. Quantum correlations could be therefore observed for a long time in related photonic experiments.

1. Introduction

Within the theory of Quantum Information (QI), we are mainly concerned with some important questions about the preservation of quantum coherence in computation protocols. In the context of quantum circuits, we often speak about *applying* unitary gates to a single qubit, but unitary operations describe only the evolution of a *closed* quantum system. To begin tackling this problem, even when the systems are not closed, unitary operators in quantum computation are applied. In a second approximation, the time-evolution of the system $\mathcal{S}$ (i.e., a set of *qubits*) is studied under different decoherence of mechanisms [1]. Several mechanisms are used to emulate the coupling between $\mathcal{S}$ and the environment $\mathcal{E}$, among which: quantum internal states [2], coupling with a set of numerable external qubits [3], thermal bath [4–6] and random interaction [7] can be found in the literature. These decoherence of mechanisms diminish the superposition of states and in many cases destroy the quantum entanglement in $\mathcal{S}$. The main task, therefore, is to predict the temporal decay of coherence (power law, exponential, etc.) in a set of qubits in order to prevent disentanglement in the computation protocol, since entanglement is a fundamental resource in QI and Quantum Computation (QC).

The coherent nature of Quantum Walk (QW) has been recently explored, providing interesting results for a wide range of applications in QI, QC and solid state. Among these, diverse implementations have been shown to suit the realizations of QW in a lattice (e.g., trapped ions, neutral atoms, photon, waveguide, nuclear-magnetic-resonance), and therefore experimental

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0378-4371/$ – see front matter
http://dx.doi.org/10.1016/j.physa.2013.07.055

and theoretical QW architecture have been shown to be important tools in the study of controlled time-dependent quantum operations [8]. Dissipative Quantum Walks (DQWs) [9–11] have also been studied and they provide a universal platform for QI analysis in the presence of quantum error [1,12]. All these studies have been enlightening as regards the interplay of superposition and entanglement in quantum mechanics, and have led to new work in the realm of real applications and the development of new quantum devices.

In a first approximation, the evolution of an open system is given by a quantum master equation (QME) [5], which can be derived from first principle tracing-out of the variables of the thermal bath, considering that initially $\mathcal{S}$ and $\mathcal{E}$ are in a separable state. In particular, tight-binding-like models in interaction with a phonon bath have been used for the derivation of the QME [10]. Here, *one particle* in the lattice is considered, allowing us to define two qubits in interaction with the bath (the qubit is associated with the empty or occupied state in a lattice site). Starting with the QME associated with a DQW we have defined a bipartite system, which we solve analytically, thus the exact non-Markovian evolution in continuous-time of two qubits interacting with a bath has been found. We stress how quantum correlations (i.e., concurrence [13], and QD [15,14]) depend on the Initial Condition (IC) of the DQW. We predict a power-law decay for the long-time quantumness of correlation, a fact explained in terms of the non-local IC of our open system. The present results enable us to understand the dynamics of realistic quantum circuits more precisely. In addition, our approach allows us to tackle the important problem of two particles interacting with an environment, which may induce or reduce correlation between the particles; this issue will be presented elsewhere.

2. The dissipative QW model

A free particle in a one-dimensional infinity lattice interacting with a thermal bath $\mathcal{B}$ is characterized by the total Hamiltonian [6]:

$$H_{\mathcal{T}} = \left(E_0\mathbf{1} - \Omega\frac{a+a^\dagger}{2}\right) + \sum_{\nu=1}^{2} V_\nu \otimes B_\nu + H_{\mathcal{B}}. \tag{1}$$

The first term corresponds to the (one-particle) *tight-binding* Hamiltonian H_S where $a = \sum_{s=-\infty}^{s=+\infty} |s-1\rangle\langle s|$ and $a^\dagger = \sum_{s=-\infty}^{s=+\infty} |s+1\rangle\langle s|$ are translational operators in the Wannier $|s\rangle$ basis and $\mathbf{1}$ is the identity operator (see Appendix A in Ref. [10]). The second term considers a linear interaction and describes the coupling between phonon operators $B_1 = B_2^\dagger = \sum_k v_k \mathcal{B}_k$ and system operators $V_1 = V_2^\dagger = \hbar\Gamma a$, where $\Gamma > 0$ is the coupling parameter. The third term is the phonon Hamiltonian $\sum_k \hbar\omega_k \mathcal{B}_k^\dagger \mathcal{B}_k$. E_0 is the *tight-binding* energy of site and Ω the next neighbor hopping energy. The QME for the DQW can be obtained by eliminating the quantum variables of $\mathcal{B}$ [10,11]:

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{-i}{\hbar}\left[H_{\mathit{eff}}, \rho\right] + D\left(a\rho a^\dagger + a^\dagger\rho a - 2a^\dagger a\rho\right), \tag{2}$$

where $H_{\mathit{eff}} = H_S - \hbar\omega_c a^\dagger a$ is a trivial effective Hamiltonian because $a^\dagger a = \mathbf{1}$ [10]. The additive energy $\hbar\omega_c$ is a cut-off in the Ohmic approximation [11]. The diffusion constant D is given in terms of the bath temperature T and the coupling constant Γ in the form: $D \propto \Gamma^2 k_B T/\hbar$. For simplicity we can add $-E_0 + \omega_c\hbar + \Omega$ to H_S. This assumption does not change the general results, and finally we get: $H_{\mathit{eff}} = \Omega\left(\mathbf{1} - \frac{a+a^\dagger}{2}\right)$. From Eq. (2) when $D \to 0$ ($T \to 0$) the von Neumann equation is recovered. The opposite limit is the classical one ($\Omega \to 0$), in this case and from any diagonal $\rho_{ss}(0)$ the usual random walk profile is reobtained. The exact solution of the Markovian DQW can be written from Eq. (2) using Wannier indices $\{s\}$ as follows:

$$\rho_{s_1,s_2}(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\mathrm{d}k_1\int_{-\pi}^{\pi}\mathrm{d}k_2\rho(0)_{k_1,k_2}\mathrm{e}^{\mathcal{F}(k_1,k_2)\,t + i(k_1s_1 - k_2s_2)}, \tag{3}$$

where k_1, k_2 are Fourier indices and $\mathcal{F}(k_1,k_2) = \frac{-i}{\hbar}\left(E_{k_1} - E_{k_2}\right) + 2D[\cos(k_1-k_2) - 1]$, where $E_k = \Omega(1-\cos k)$ is the continuous eigenenergy [10]. Note that for the IC $\rho(0) = |k_0\rangle\langle k_0|$ the system does not show decoherence in time. In general, evaluating Eq. (3) and using the basis $|s\rangle$ we can rewrite the *operator* $\rho(t)$ using outer product notation.

Using outer product notation we can do the algebra to work out different definitions for bipartite systems. In the inset of Fig. 1 we show the party AB and C, in particular we will be interested in the two-level system defined by the set of elements $\{\pm s_0\} \in AB$. In the present work, in order to consider a non-local problem we introduce the following *mirror* IC for the vector state $|\psi_0^\pm\rangle \equiv \frac{1}{2}(|s_0\rangle \pm |-s_0\rangle)$, then we will use $\rho^\pm(0) = |\psi_0^\pm\rangle\langle\psi_0^\pm|$ for the preparation of the density matrix. This IC has the maximum entanglement for the particular bipartition that we will consider, i.e.,

$$\rho^\pm(0) = \frac{1}{2}\{|s_0\rangle\langle s_0| + |-s_0\rangle\langle -s_0| \pm |-s_0\rangle\langle s_0| \pm |s_0\rangle\langle -s_0|\}.$$

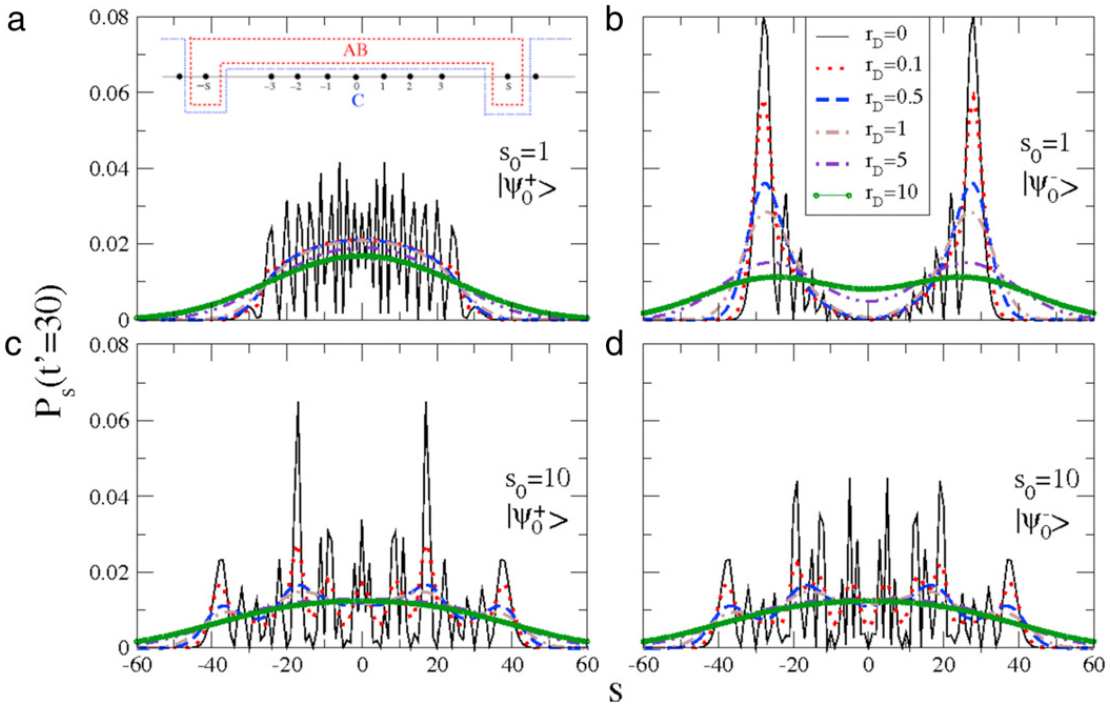

Fig. 1. Probability as function of position s for $t' = 30$, and for values of $r_D = 0, 0.1, 0.5, 1, 5, 10$. We display plots from a non-local initial condition $|\psi_0^+\rangle$ with $s_0 = 1$ (Fig. a) and $s_0 = 10$ (Fig. c); and also for $|\psi_0^-\rangle$ with $s_0 = 1$ (Fig. b) and $s_0 = 10$ (Fig. d). The inset shows the *AB* party and its complement, the party *C*.

In the next section a left–right bipartition will be introduced for the study of entanglement. Thus, we now introduce the previous IC $\rho^{\pm}(0)$ in Eq. (3) to obtain the exact time-evolution for the density matrix in the lattice.

$$\rho^{\pm}_{s_1,s_2}(t) = \frac{i^{s_2-s_1}}{2e^{2Dt}} \sum_n \left\{ J_{s_1+s_0+n}\left(\frac{\Omega t}{\hbar}\right) J_{s_2+s_0+n}\left(\frac{\Omega t}{\hbar}\right) \pm (-1)^{s_0} J_{s_1+s_0+n}\left(\frac{\Omega t}{\hbar}\right) J_{s_2-s_0+n}\left(\frac{\Omega t}{\hbar}\right) \right.$$
$$\left. \pm (-1)^{s_0} J_{s_1-s_0+n}\left(\frac{\Omega t}{\hbar}\right) J_{s_2+s_0+n}\left(\frac{\Omega t}{\hbar}\right) + J_{s_1-s_0+n}\left(\frac{\Omega t}{\hbar}\right) J_{s_2-s_0+n}\left(\frac{\Omega t}{\hbar}\right) \right\} I_n(2Dt)\,. \tag{4}$$

Hermiticity, positivity and normalization of $\rho(t)$ can be checked from Eq. (4) using the properties of the Bessel functions. The first important consideration to be analyzed is the probability profile as a function of D.

In the non-dissipative case ($D = 0$) the expression for the QW probability profile $P_s^{\pm}(\Omega, D, s_0, t) \equiv \rho^{\pm}(t)_{s,s}$ is quite simple and shows the expected interference phenomena coming from the non-local IC $\rho^{\pm}(0)$, i.e.: $P_s^{\pm}(\Omega, 0, s_0, t) = \frac{1}{2}\left[J^2_{s+s_0}\left(\frac{\Omega t}{\hbar}\right) + J^2_{s-s_0}\left(\frac{\Omega t}{\hbar}\right)\right] \pm (-1)^{s_0} J_{s+s_0}\left(\frac{\Omega t}{\hbar}\right) J_{s-s_0}\left(\frac{\Omega t}{\hbar}\right)$. From this expression, and due to the symmetry of the problem, it is possible to write a simple formula for the time-dependent interference pattern at the origin. The pattern is constructive for the symmetric IC $|\psi_0^+\rangle$ and destructive for the antisymmetric one $|\psi_0^-\rangle$, i.e., the probability at the origin is $\left\{P_0^+ = 2J^2_{s_0}\left(\frac{\Omega t}{\hbar}\right); P_0^- = 0\right\}$.

As expected, at the limit $\Omega \to 0$ we recover from Eq. (4) the classical profile on the lattice: $P_s^{\pm}(0, D, s_0, t) = \frac{1}{2}e^{-2Dt}\left[I_{s+s_0}(2Dt) + I_{s-s_0}(2Dt)\right]$ [16,17]. Nevertheless, if we calculate the off-diagonal elements of $\rho^{\pm}(t)$, at the limit $\Omega \to 0$ from Eq. (4) we get a non null lattice structure for the off-diagonal elements:

$$\rho^{\pm}(t)_{s_1,s_2} = \frac{1}{2}e^{-2Dt} I_{s_1 \pm s_0}(2Dt) \quad \text{iif} \quad 2s_0 = \pm(s_2 - s_1). \tag{5}$$

So at the highly dissipative limit there will be a remanent quantumness correlation from the entanglement of $|\psi_0^{\pm}\rangle$, which decays asymptotically in time as $\rho^{\pm}(t)_{s_1,s_2} \sim t^{-1/2}$.

Now we define a new parameter $r_D = \frac{2D}{\Omega/\hbar}$ (rate of characteristic energy scales in the system) and $t' = \frac{\Omega}{\hbar}t$ a dimensionless time in order to plot the figures, thus the classical limit will be $r_D \gg 1$ and the non-dissipative limit will be $r_D \ll 1$. In Fig. 1 we plot the probability profile $P_s(t' = 30) \equiv P_s(\Omega, D, s_0, t)$ as a function of s for different values of the dissipation parameter r_D and $t' = 30$, for the two cases of symmetry $\rho^{\pm}(0)$ and two values of separation s_0 at the IC. The comparison of the probability profile $\rho^{\pm}(t)_{s,s}$ for the two symmetries involved in the IC, $\rho^{\pm}(0) = |\psi_0^{\pm}\rangle\langle\psi_0^{\pm}|$, is shown in this figure. It can be seen that for small values of separation s_0, the quantum probability profile is different for the symmetric case as in the

case antisymmetric (Fig. 1(a)–(b)), but the probability profile has a similar behavior for both symmetries for large values of s_0 (Fig. 1(c)–(d)).

For small values of r_D the quantum correlations are notorious in the system (Fig. 1(a)–(d)), but for large values of r_D the probability tends towards the Gaussian behavior. To quantify the quantumness of correlations present in the system, we will analyze the concurrence [13] and Quantum Discord (QD) [15].

3. Bipartition from the DQW

To study how quantum correlations on the lattice are affected by the dissipation we now introduce the following bipartition procedure from our Markovian DQW. Consider two fixed sites $\pm s$, tracing-out over the remaindering sites on the infinite 1D lattice (see the inset in Fig. 1). We can define a two-level system as our bipartite system "*AB*". To quantify the concurrence [13] and QD [15], we need to know the reduced density matrix for two qubits (in our case as a function of $\pm s$). Using the outer product notation for ρ and the Wannier basis (state of one particle) in the form $|-s\rangle = |1_A\rangle \otimes |0_B\rangle \otimes |0_C\rangle$ and $|s\rangle = |0_A\rangle \otimes |1_B\rangle \otimes |0_C\rangle$ (where $|0_R\rangle$ and $|1_R\rangle$ are the empty and occupied states in the subsystem R, with $R = A, B, C$), and tracing over the sites $s_j \neq \pm s$ we end up with a 4×4 reduced (non-Markovian) density matrix $\rho^{\pm}_{AB}$ (simplifying the notation for the basis in the system *AB*, we can write $|1_A 0_B\rangle \equiv |1_A\rangle \otimes |0_B\rangle$, and we can use a similar notation for other states in *AB*):

$$\begin{array}{c} \\ |1_A1_B\rangle \\ |1_A0_B\rangle \\ |0_A1_B\rangle \\ |0_A0_B\rangle \end{array} \begin{array}{c} \begin{array}{cccc} |1_A1_B\rangle & |1_A0_B\rangle & |0_A1_B\rangle & |0_A0_B\rangle \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \rho^{\pm}_{-s,-s}(t) & \rho^{\pm}_{-s,s}(t) & 0 \\ 0 & \rho^{\pm}_{s,-s}(t) & \rho^{\pm}_{s,s}(t) & 0 \\ 0 & 0 & 0 & 1-\sum_{j=\pm s}\rho^{\pm}_{j,j}(t) \end{pmatrix} \end{array}. \tag{6}$$

Noting that the reduced density matrix represents a genuine non-Markov process. Here $\rho^{\pm}_{s_1,s_2}(t)$ is given by the exact solution presented in Eq. (4). The non-Markov dynamics given in Eq. (6) will have an important implication on the behavior of the concurrence and quantum discord, as will be seen later.

3.1. Concurrence for two dissipative qubits

The quantum entanglement between A and B is $C^{\pm}(-s, s, t) = 2|\rho^{\pm}_{-s,s}(t)|$, see Ref. [18]. To measure the total entanglement in our system, we now consider the sum of concurrences between all sites $-s$ and s, thus we call this measure the mirror concurrence C_M as in Refs. [19,20]. For the contribution of the entanglement of all sites $-s$ and its mirror s we get the analytical expression:

$$C^{\pm}_M(\Omega, D, s_0, t) = 2\sum_{s=1}^{\infty}\left|\rho^{\pm}_{-s,s}(t)\right|. \tag{7}$$

In the non-dissipative case $D = 0$, using (4) we get from the symmetric IC: $C^{+}_M(\Omega, 0, s_0, t) = 1$, and for the antisymmetric one $C^{-}_M(\Omega, 0, s_0, t) = 1 - 2J^2_{s_0}(\frac{\Omega t}{\hbar})$, i.e., with initial maximum entanglement.

As mentioned previously, in the classical asymptotic limit ($r_D \gg 1$) using Eq. (5) in (7) for the mirror concurrence we get: $C^{\pm}_M(0, D, s_0, t) = e^{-2Dt} I_0(2Dt)$, indicating that there is a remanent correlation that asymptotically decays in time as $C^{\pm}_M(0, D, s_0, t \to \infty) \sim t^{-1/2}$.

In Fig. 2 we plot $C^{\pm}_M$ as a function of t' for different values of the dissipation parameter r_D, two cases of symmetry $\rho^{\pm}(0)$ and several values of separation s_0 at the IC $|\psi^{\pm}_0\rangle$. The case $r_D = 0$ is plotted in Fig. 2, and the validity of our asymptotic expression can be verified. It can be seen that for $s_0 = 1$ and $|\psi^{+}_0\rangle$ (Fig. 2(a)) and $|\psi^{-}_0\rangle$ (Fig. 2(b)) the concurrence has a different behavior, but for $s_0 = 10$ the behavior is similar at long time. In all cases concurrence decreases with r_D, and for large values of r_D an asymptotic long-time behavior of C_M can be appreciated. The analytical case $r_D \to \infty$ can be obtained by inserting solution (5) into Eq. (7). This phenomenon is intriguing and is due to the power-law decay of $\left|\rho^{\pm}_{-s,s}(t)\right|$, and so the entanglement does not show a sudden death [21]. In contrast, from a localized IC the entanglement is zero [22] (the off-diagonal elements of $\rho(t)$ are null).

3.2. Mirror quantum discord

We have also calculated QD, which is an alternative measurement of the quantum correlations in a bipartite system. We stress the different nature of the QD in comparison with the entanglement measured from the concurrence (QD is a measure of the difference between two natural, yet different in quantum analogs, of classical mutual information, see Refs. [15,14]). In our system, the Classical Correlations (CCs) can be defined explicitly (from the mutual information), and the mirror QD can be calculated between sites s and $-s$ using the reduced density matrix given in Eq. (6). Following Ref. [23] we have obtained

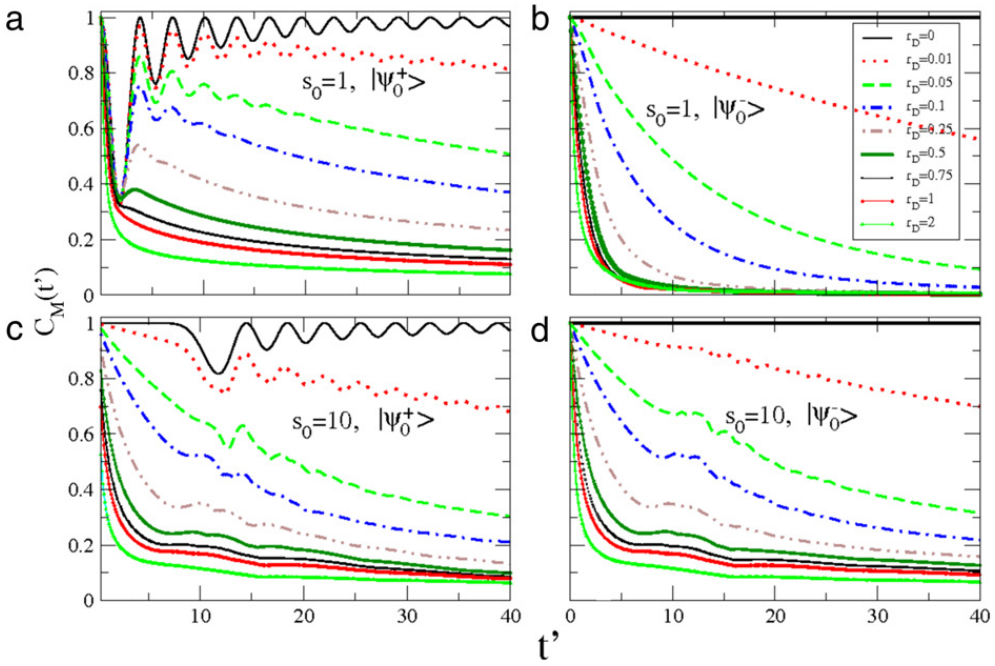

Fig. 2. The quantum concurrence for the bipartition shown in the inset of Fig. 1 as a function of t', for values of $r_D = 0, 0.01, 0.005, 0.1, 0.25, 0.5, 0.75, 1, 2$, and non-local initial conditions $|\psi_0^{\pm}\rangle$ with values of $s_0 = 1, 10$.

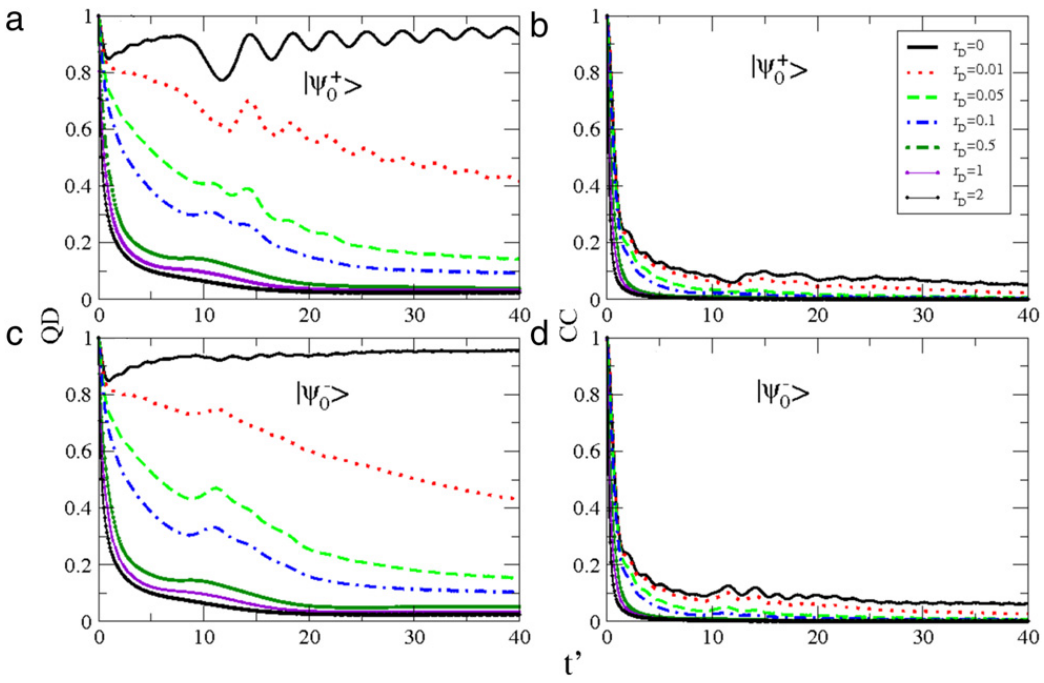

Fig. 3. The sum of quantum discord for the sites s and $-s$ (QD) as a function of t', for values of $r_D = 0, 0.01, 0.005, 0.1, 0.5, 1, 2$ ((a) and (c)). In (b) and (d) the total classical correlations (CC) can be observed. For non-local initial conditions $|\psi_0^{\pm}\rangle$ and with $s_0 = 10$.

an expression for QD and CC as functions of r_D using Eq. (4) in (6). We stress that in this instance Eq. (6) is not a Werner state [24], where the classical correlations CC(s) between the sites s and $-s$ read as follows, $\mathrm{CC}(s,t) = H_{bin}(p_s) - H_{bin}(p'_s)$, where $H_{bin}(x) = -x\log_2(x) - (1-x)\log_2(1-x)$, $p_s = \rho_{s,s}(t)$ and $p'_s = [1 + \sqrt{4\rho^2_{s,s}(t) + (2\rho_{s,-s}(t) - 1)^2}]/2$. Finally, QD is calculated analytically using Eq. (25) from Ref. [23].

Fig. 3 shows the total QD calculated as the sum of the mirror QD between sites s and $-s$ [22] (using our bipartition, see inset of Fig. 1) for different IC $|\psi_0^{\pm}\rangle$ and values of r_D. The total QD is in good agreement with the total mirror concurrence analysis (see Fig. 2), as both decay asymptotically in time following a similar law.

We note that these correlations (quantum and classical) decrease with dissipation (r_D). As expected, the behavior of CC is converse to the increase in entropy of the DQW interacting with the quantum bath [10].

4. Discussion

Starting with a QW in interaction with a phonon bath, we have presented an analytical bipartition to measure the quantum correlations of two qubits interacting with a thermal bath. The mirror concurrence has been shown to depend in a non-trivial way on the rate of energies of the system: $r_D = \frac{2D}{\Omega/\hbar}$. This result is in agreement with the calculation of other measures of entanglement such as negativity [22]. The non-dissipative limit ($r_D \ll 1$), and the classical limit ($r_D \gg 1$) have been found to be in agreement with the physical expectations of the system.

The importance of the present model is the opportunity it offers us to understand a non-Markovian dynamics of realistic quantum circuits more precisely, taking into account dissipative effects, and we have done this by analyzing classical and quantum correlations separately. Our approach also enables us to tackle the important subject of the analysis of the QD phase diagram in terms of the observables of the system. Interestingly, we have determined that the negativity also shows that the mirror concurrence and mirror QD describe well the quantum to classical transition [22].

An important aspect of our results is that for non localized initial states (with maximum entanglement as in the bipartition we studied) quantum correlations are present in the system even in the long-time regimen with characteristic power-law behavior, even though the dissipation is longer than the bandwidth of the QW ($r_D \gg 1$). This means that the time evolution of quantum correlations (concurrence and QD) strongly depends on the IC superimposed at $t = 0$ (for a localized IC the concurrence and QD is zero for all times [22]). This fact might be observed in the lab by using, for instance, photons in a prepared spatial IC with a suitable entanglement initial state, (as in Perets et al. and/or in Peruzzo et al. [8]). Therefore, quantum correlations should be observed over a long time in the experiment, and are possible scenarios where our present results can be applied.

Acknowledgments

We thank Dr. D. Frustaglia for fruitful discussions. M.O.C. is thankful for grants from SECTyP, Univ. Nac. de Cuyo, Argentina, and CONICET, PIP 90100290, Argentina. M.N. is thankful for a Post-Doc fellowship from CONICET, Argentina.

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