A New Property of Transient Currents of Induction Motors

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***Abstract*— This work presents, demonstrates and analyzes a property that relates the transient response of stator and rotor currents to the speed of induction motors and their parameters. The straightforward application of this property allows determining the rotation speed of the motor and detecting a failure or variation of its parameters. This property is demonstrated analyzing the induction motor modeled by two parts: one linear and another non-linear. Based on the operating principles of induction machines and the theory of differential equations, a hypothesis is stated on the behavior of currents in the transient state. Then, by symbolically computing the eigenvalues, the hypothesis is verified so that it validates the stated property.**

***Keywords*—Electric motors model, Induction motors, Speed estimation.**

1. INTRODUCTION

Electrical drives containing a squirrel cage induction motor (IM) are widely used in many industrial applications. In general, the accuracy level of the IM model strongly depends on the application. Consequently, several IM models can be found in the literature. For instance, in [1], IM radial and axial nonuniformity was taken into account for modeling inductances. In [2], a model specifically designed to study the machine-inverter interaction was introduced. A model built with finite elements techniques was used in [3] to study iron losses.

However, the most widely used model in the electrical drives control field assumes that the stator windings are sinusoidally distributed, and the rotor as a three-phase lumped wound [4][5]. This model is accurate enough for describing the motor dynamics in sensorless speed control and fault detection applications when the IM is running at both medium and high speed. Although the model has been widely used, some of its properties are still unexplored. The goal of the paper is twofold. First, a new property of the conventional IM model is presented. Second, this property is used for estimating the rotor speed. It is worth noting that speed estimation is a widely studied topic in sensorless speed control of electrical drives ([6], [7] and references therein). In the cited bibliography several methods to estimate the speed of the IM are presented. Among these methods, maybe the most spread one is the one that uses observers to determine the speed. These observers are well known and simple to use tools but they need an updated model of the IM (see [8][9] and references therein). The parameters of the IM are very sensitive to the variation of temperature, especially the rotoric resistance [4]. It is clear that the obtained precision depends on the correlation of the parameters of the model with those of the observed motor. The presented property and the method proposed allows calculating speed of the IM from calculations on the results of an identification made by any method. It is presented and it is demonstrated by an identification of the model of the IM in states space.

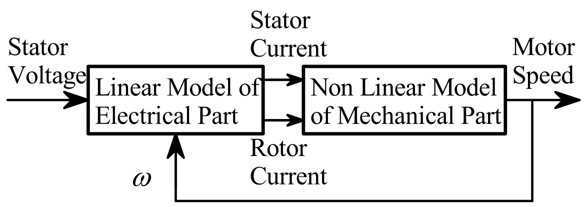

Fig. 1: Induction motor model outline

2. THE INDUCTION MOTOR MODEL

The model of the three-phase induction motor is multi-variable, with strong couplings, and non-linear. The first two characteristics stem from its physical structure and from its operating principle. The non-linearity is due to the fact that the motor torque is a non-linear function of the stator and rotor currents and, consequently, the speed is a nonlinear function of the currents.

In this paper, an IM non-linear model breaks into two parts is considered. One part is linear, and the other one is nonlinear. The linear part, represented in state space, describes the behavior of the electrical variables for a given speed. This block is linear when the speed is regarded constant. The non-linear part is linked to the mechanical behavior. The speed is computed in the non-linear part of the model, and it is updated in the linear part.

This model splitting is made possible because there are two or three orders of magnitude in the ratio between the mechanical and electrical time constants. Therefore, this model is valid only if it is evaluated in a time interval much smaller than the mechanical time constant of the motor.

Fig. 1 shows the block diagram of the analyzed IM model. The linear block, composed of differential equations of the electrical circuits, has the stator voltage as the input, and the stator and rotor current as the outputs. Speed is a parameter of the linear system that is being steadily updated. Using the values of these two currents, the non-linear block computes the instant motor torque and, by means of a differential equation for the rotating motion, it also calculates the motor speed, which is then used to feed back the linear block.

The electrical behavior of the stator and rotor circuits is described by (1) and (2), respectively.

$$\mathbf{u}_S = R_S\ \mathbf{i}_S + L_S \frac{d\,\mathbf{i}_S}{dt} + L_0 \frac{d\left(\mathbf{i}_R e^{j\varepsilon}\right)}{dt} \qquad (1)$$

$$0 = R_R\ \mathbf{i}_R + L_R \frac{d\,\mathbf{i}_R}{dt} + L_0 \frac{d\left(\mathbf{i}_S e^{-j\varepsilon}\right)}{dt} \qquad (2)$$

where:

$\mathbf{u}_S = u_{Sa} + j\ u_{Sb}$ is the stator voltage, and $\mathbf{i}_R\, e^{j\varepsilon} = i_{Ra} + j\ i_{Rb}$ and $\mathbf{i}_S = i_{Sa} + j\ i_{Sb}$ are rotor and stator currents, respectively. R_S, L_S are the stator's resistance and inductance; R_R, L_R are the same parameters but for the rotor. L_0 is the mutual inductance between stator and rotor. The angular position of the rotor is ε. The terms $e^{j\varepsilon}$ and $e^{-j\varepsilon}$ represent the changes in the rotor-stator coordinate systems, respectively, that must be applied to the currents.

The kinematics of the model is defined as:

$$\omega = \frac{d\varepsilon}{dt} \qquad (3)$$

where:

ω: angular speed of the motor shaft

The torque generated by induction motor [4] must balance the inertial torque, the load torque T_L and the friction torque T_f. This fact is described by:

$$\frac{J}{p}\frac{d\omega}{dt} + T_f + T_L = \frac{2}{3} L_0\ p\ \Im\left[\mathbf{i}_S \left(\mathbf{i}_R e^{j\varepsilon}\right)^*\right] \qquad (4)$$

where:

p : number of pole pairs of the electrical machine

J : moment of inertia

$\Im$: denotes imaginary part

$()^*$: denotes conjugate complex

In (2), the rotor current is referred to the coordinate system fixed to the rotor. However, the system can be represented in a stationary reference frame (axes *a* and *b*) [4]. In such a case, and by separating the real from the imaginary part, the following description is obtained,

$$\begin{cases} u_{Sa} = R_S i_{Sa} + L_S \dfrac{di_{Sa}}{dt} + L_0 \dfrac{di_{Ra}}{dt} \\ u_{Sb} = R_S i_{Sb} + L_S \dfrac{di_{Sb}}{dt} + L_0 \dfrac{di_{Rb}}{dt} \\ 0 = R_R i_{Ra} + L_R \dfrac{di_{Ra}}{dt} + L_R i_{Rb}\omega + L_0 \dfrac{di_{Sa}}{dt} + L_0 i_{Sb}\omega \\ 0 = R_R i_{Rb} - L_R i_{Ra}\omega + L_R \dfrac{di_{Rb}}{dt} - L_0 i_{Sa}\omega + L_0 \dfrac{di_{Sb}}{dt} \end{cases} \qquad (5)$$

With similar substitutions into (4), expression (6) can be obtained.

$$\frac{J}{p}\frac{d\omega}{dt} + T_f + T_L = \frac{2}{3} L_0 p \left(i_{Sb} i_{Ra} - i_{Sa} i_{Rb}\right) \qquad (6)$$

Then, operating algebraically with (5) and (6), the following expressions are found,

$$\begin{cases} \dfrac{di_{Sa}}{dt} = k\left(R_R i_{Ra} L_0 + L_R u_{Sa} - L_R R_S i_{Sa} + L_R i_{Rb}\omega L_0 + L_0^2 i_{Sb}\omega\right) \\ \dfrac{di_{Sb}}{dt} = k\left(R_R i_{Rb} L_0 + L_R u_{Sb} - L_R R_S i_{Sab} - L_R i_{Ra}\omega L_0 + L_0^2 i_{Sa}\omega\right) \\ \dfrac{di_{Ra}}{dt} = k\left(-u_{Sa} L_0 + R_S i_{Sa} L_0 - L_S R_R i_{Ra} - L_S L_R i_{Rb}\omega - L_S L_0 i_{Sb}\omega\right) \\ \dfrac{di_{Rb}}{dt} = k\left(-u_{Sb} L_0 + R_S i_{Sb} L_0 - L_S R_R i_{Rb} + L_S L_R i_{Ra}\omega + L_S L_0 i_{Sa}\omega\right) \end{cases} \qquad (7)$$

$$\frac{d\omega}{dt} = \frac{\left(T_f + T_L\right)p}{J} + \frac{2L_0 p^2}{3J}\left(i_{Sb} i_{Ra} - i_{Sa} i_{Rb}\right) \qquad (8)$$

where:

$$k = \frac{1}{L_R L_S - L_0^2}$$

For a three-phase IM, the input, output and state vectors are stated as:

$$\mathbf{u} = [u_1\ \ u_2\ \ u_3]^T \qquad (9)$$

$$\mathbf{y} = \left[i_1\ \ i_2\ \ i_3\right]^T \qquad (10)$$

$$\mathbf{x} = [i_{Sa}\ \ i_{Sb}\ \ i_{Ra}\ \ i_{Rb}]^T \qquad (11)$$

Applying now the Clarke transformations to the inputs and outputs [4], expressions (12) and (13) are attained, which allows relating the voltages and currents on IM terminals with the ones of the previous model. Using these relations and substituting into (7), equation (14) is finally obtained.

$$\begin{bmatrix} u_{Sa} \\ u_{Sb} \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad (12)$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & 0 \\ -\frac{1}{3} & \frac{1}{\sqrt{3}} \\ -\frac{1}{3} & -\frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} i_{Sa} \\ i_{Sb} \end{bmatrix} \qquad (13)$$

$$\begin{bmatrix} \frac{di_{Sa}}{dt} \\ \frac{di_{Sb}}{dt} \\ \frac{di_{Ra}}{dt} \\ \frac{di_{Rb}}{dt} \end{bmatrix} = k \begin{bmatrix} -L_R R_S & L_0^2 \omega & R_R L_0 & L_R L_0 \omega \\ -L_0^2 \omega & -L_R R_S & -L_R L_0 \omega & R_R L_0 \\ R_S L_0 & -L_S L_0 \omega & -L_S R_R & -L_S L_R \omega \\ L_S L_0 \omega & R_S L_0 & L_S L_R \omega & -L_S R_R \end{bmatrix} \begin{bmatrix} i_{Sa} \\ i_{Sb} \\ i_{Ra} \\ i_{Rb} \end{bmatrix}$$

$$+ k \begin{bmatrix} \frac{3}{2} L_R & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} L_R & -\frac{\sqrt{3}}{2} L_R \\ -\frac{3}{2} L_0 & 0 & 0 \\ 0 & -\frac{\sqrt{3}}{2} L_0 & \frac{\sqrt{3}}{2} L_0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & 0 & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{\sqrt{3}} & 0 & 0 \\ -\frac{1}{3} & -\frac{1}{\sqrt{3}} & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{Sa} \\ i_{Sb} \\ i_{Ra} \\ i_{Rb} \end{bmatrix}$$

2.1 Characteristics of the presented model

On the model represented by (8) and (14), the following characteristics can be easily noted:

- In steady state, or in a short period of time with respect to the IM mechanical time constant, speed ω is kept approximately constant. Therefore, state matrix (14) does not vary. Under these conditions, the model of the electrical part of the IM is linear.
- In Fig. 1, equation (14) corresponds to the block 'Linear Model of Electrical Part', whereas equation (8) to the block 'Non Linear Model of the Electromechanical Part'.
- On the main diagonal of the state matrix in (14), the speed does not appear. Therefore, its trace depends only on the motor parameters.

Another important characteristic of the model expressed in (14) is that it is a minimal realization. This feature is not evident and, therefore, it requires the following demonstration:

The simplified model of the IM can be represented by two electrical circuits that are magnetically coupled. One circuit corresponds to the stator, the other to the rotor and both have, as concentrated parameters, a resistance, an inductance and an EMF, all of them connected in series. The magnetic coupling is due to the mutual inductance and to the relative speed between the magnetic field of the stator and the rotor.

For an IM working at a certain speed, the dynamic behavior of this model can be characterized by the current through each circuit mentioned above. These two currents are independent one from the other and, hence, one of them cannot be expressed as a function of the other. The only relation existing between both circuits is the mutual induced EMF. In this model, these currents are vectors, and they can be expressed by their components according to axis a (i_{Sa}, i_{Ra}) and axis b (i_{Sb}, i_{Rb}). In the model described by (14), the space state variables vector is formed by the four components of these currents. The system's dynamics can be described with four variables as a minimum. For this reason, the system defined in (14) is a minimal realization for a given speed.

3. TRANSIENT ANALYSIS OF THE INDUCTION MACHINE CURRENTS

In steady state, the angular speed ω of the induction machine shaft operating as a motor is expressed as (15) and, when it operates as a generator, it is expressed as (16). These expressions respond to the operating principle of these machines

$$\omega = \frac{2\pi (f_S - f_R)}{p} \quad (15)$$

$$\omega = \frac{2\pi (f_S + f_R)}{p} \quad (16)$$

where:

f_S : stator current frequency

f_R : rotor current frequency

In a linear system, the transient behavior is described by the homogeneous solution of the differential equations system of the model. The homogeneous solution describes the system response when it is let evolve from any operating point, with zero input. Physically, in an IM, the homogeneous solution describes the evolution of motor variables when it is working in any operative state, and a zero input is applied to it. In the model developed here, a zero input means that the stator voltage vector is zero as well. Hence, the motor changes its operation and starts working as a generator. Consequently, expression (16) must be complied as the relationship between the stator and rotor currents. In order to verify this hypothesis, the imaginary parts of the system eigenvalues are analyzed.

By symbolically computing the eigenvalues of the state matrix of the model presented by (14), equations, (17), (18), (19) and (20) are obtained.

$$e_1 = \frac{k}{2}\left(-L_R R_S - L_S R_R + j\frac{\omega}{k} \right) + M \quad (17)$$

$$e_2 = \frac{k}{2}\left(-L_R R_S - L_S R_R - j\frac{\omega}{k} \right) + M^* \quad (18)$$

$$e_3 = \frac{k}{2}\left(-L_R R_S - L_S R_R + j\frac{\omega}{k} \right) - M \quad (19)$$

$$e_4 = \frac{k}{2}\left(-L_R R_S - L_S R_R - j\frac{\omega}{k} \right) - M^* \quad (20)$$

where:

$$M = \frac{k}{2}\sqrt{\begin{matrix} -L_S^2 L_R^2 \omega^2 + 2jL_R^2 \omega R_S L_S + L_R^2 R_S^2 - 2jL_R \omega R_R L_S^2 \\ +2L_S L_0^2 \omega^2 L_R \text{-} 2L_R R_S L_S R_R - 2jL_R \omega R_S L_0^2 \\ +L_S^2 R_R^2 + 2j\omega R_R L_0^2 L_S + 4R_S L_0^2 R_R - L_0^4 \omega^2 \end{matrix}}$$

and M^* stands for the conjugated complex of M

Eigenvalues e_1 and e_2 are conjugated complexes that describe the rotor current in transient state. Likewise the case for eigenvalues e_3 and e_4 as regards the stator current is. The imaginary part of eigenvalues e_1 and e_2 is the angular pulsation of the transient rotor current, and the imaginary part of e_3 and e_4 represents, again, the angular pulsation but for the stator current.

From (17) and (19), it can be seen that the sum of the imaginary parts of the rotor and stator eigenvalues must be equal to the speed of the motor shaft, because the M terms are cancelled, as well as *k*. The result is equation (21). A similar procedure on (18) and (20) renders (22), thus verifying the hypothesis stated above.

$$\Im(e_1) + \Im(e_3) = \omega \quad (21)$$

$$\Im(e_2) + \Im(e_4) = -\omega \quad (22)$$

Equations (21) and (22) represent a new property of the IM model, whose demonstration is the main purpose of this work. Physically, this means that for an IM operating at a given angular speed ω in transient state, the frequencies of rotor and stator currents meet the relation (16). This was already verified with the homogenous solution of system (14).

On the other hand, it is verified that the trace of the state matrix (14) does not depend on the motor speed, as shown in (23), and it can be evaluated using the real part of the eigenvalues.

$$\begin{aligned} &\Re(e_1 + e_3) = \Re(e_2 + e_4) = \\ &= -\left(\frac{L_R R_s + L_S R_R}{L_R L_s - L_0^2} \right) = \frac{1}{2} Trace \end{aligned} \quad (23)$$

The demonstration of the property expressed in (21), (22) and (23) can be extended to a model of the transfer function matrix type due to of the equivalence between poles and eigenvalues and given that the state space model is a minimal realization. A system expressed in state space, such as (24) can be expressed as a transfer function matrix $G(s)$, (25) when the matrix D is null. Applying this transformation to system (14), matrix equation (26) is obtained.

$$\begin{cases} \dot{x}(t) = A\,x(t) + B\,u(t) \\ y = C\,x(t) + D\,u(t) \end{cases} \quad (24)$$

$$G(s) = \frac{Y(s)}{U(s)} = C\left[s\,I - A\right]^{-1} B \quad (25)$$

$$\begin{bmatrix} i_1(s) \\ i_2(s) \\ i_3(s) \end{bmatrix} = \begin{bmatrix} \frac{B_{11}(s)}{A(s)} & \frac{B_{12}(s)}{A(s)} & \frac{B_{13}(s)}{A(s)} \\ \frac{B_{21}(s)}{A(s)} & \frac{B_{22}(s)}{A(s)} & \frac{B_{23}(s)}{A(s)} \\ \frac{B_{31}(s)}{A(s)} & \frac{B_{32}(s)}{A(s)} & \frac{B_{33}(s)}{A(s)} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \\ u_3(s) \end{bmatrix} \quad (26)$$

Transfer function matrix (26) is composed of nine transfer functions that relate inputs and outputs. The roots of polynomial $A(s)$ have the same properties expressed in (21) and (22).

4. APPLICATIONS

The direct application of the property expressed by (21) and (22) is the estimation of motor speed through on-line identification of the IM model. Identification, which can be made experimentally, is realized by processing the stator currents and voltages. If a state space model is identified, a state matrix is then obtained and, after calculating its eigenvalues, speed ω can be computed as well.

Another way to estimate the speed is to identify any of the nine transference functions stated in (26) to calculate the poles and, with their imaginary part, calculate the speed.

If the state matrix is obtained, its trace can be calculated and, applying (23), it is then possible to know whether the motor parameters have changed.

5. CONCLUSIONS

A new property of the IMs models has been presented and analyzed. This property is due to the fact that IM is a reversible electrical machine and it was used to develop new methods for estimating rotor speed when it varies slowly. Note that for implementing the proposed estimation method, it is only needed to identify a linear system. In comparison with other approaches, the main advantage of our proposal is that a priori knowledge of IM parameters is not needed. In addition, it does not require any model adjustment because the parameter variations are straightforwardly reflected on the on-line identification. Another application of this property is to detect parameters variations and faults in the IMs.

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