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Spin observables in an atomic CQE system

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Abstract

A cavity quantum electrodynamical Hamiltonian is solved to study spin observables, such as the orientation and fluctuations of the total spin and entropy, in a system of N two-level atoms placed in a resonant cavity. The eigenstates of the Hamiltonian are calculated, both exactly and approximately. We calculate the time evolution of spin observables such as spin and entropy squeezing. Conclusions are drawn about the validity of the approximations in the limit of a large number of atoms.

1. Introduction

In this work, we shall look at the definition of spin squeezing [1–3] in a system composed by two-level atoms interacting with a cavity mode, as an indicator of the persistence of the spin orientation and spin fluctuations in the device. As discussed in the literature, an array of two-level atoms placed in a cavity may be realized by an array of Josephson junctions [4, 5]. Recently, interest in this problem has been renewed, in connection with phase relaxation and decoherence [6] and with coherence and entanglement [7] in bosonic systems. In this paper, we shall focus on some of the mathematical aspects of the problem, from the perspective of quantum many-body techniques, to test the validity of approximations which may be adopted to avoid the numerical complications inherent to high dimensionality, as we shall discuss later on. Cavity quantum electrodynamics (CQE) [8–10] allows for the formulation of the problem at the Hamiltonian level. Since the Hamiltonian of such a system should include the interactions between the atoms and those of the atoms with the cavity mode, one may expect to find a set of different solutions, depending on the relative strength of the interactions. Thus, one open question is related to the choice of the proper quantity which may be used to identify these solutions. We have chosen the notion of spin squeezing as a possible indicator of the persistence of the spin properties of the system [1, 11, 12]. Since the number of atoms in the array may be rather large, and therefore making the exact treatment unfeasible, we shall also focus on the applicability of approximations to construct the eigenvectors of the system. Among the approximations which may be adopted to find

the eigenvalues and eigenvectors of the Hamiltonian, we have chosen the boson mapping (BM) method [13, 14] in view of the many-body nature of the system. In previous works [15, 16], we have presented some considerations, based on the use of a deformed quantum algebra, about fermion–boson interactions, with reference to the excitations of Josephson junctions [15, 16]. In order to complete the analysis of spin observables, we have calculated the entropy squeezing [17, 18] of the system to relate it with the spin squeezing since, in principle, both quantities may signal the presence of entanglement.

This paper is organized as follows. The formalism is introduced in section 2. Therein, we define the Hamiltonian of the system and solve the eigenvalue equation exactly. The matrix elements of the Hamiltonian are also written in terms of a collective representation, in terms of the BM scheme. The observables, i.e. the spin- and entropy-squeezing parameters, are defined at the end of the section. The results of the calculations are presented and discussed in section 3. We have performed the calculations by varying the number of atoms and the average number of photons. As an initial state, we have adopted coherent states for the atomic sector (a coherent spin state (CSS)) and for the photon sector (a coherent boson state with a fixed average number of photons). The conclusions are drawn in section 4.

2. Formalism

In this section, we shall define the Hamiltonian of the system and construct the exact eigenstates in a basis which is the direct

product of Dicke states and photon-number states. The basis is labelled by the eigenvalues of an operator which commutes with the Hamiltonian. Next, we introduce the essentials of the BM methods. Finally, we define the atomic population of the states and relate it with the spin-squeezing parameter.

2.1. The Hamiltonian

We shall consider a system of N atoms, each of them having two states. The energy gap between the states of a given atom is the quantity ω_{at} . The system of N atoms may interact with a photon field of energy ω_{ph} . We write the Hamiltonian of the system as

$$H = \omega_{\text{at}} S_z + \omega_{\text{ph}} \left(a^\dagger a + \frac{1}{2} \right) + \eta (a^\dagger S_- + S_+ a) + \lambda \sum_{\substack{i, j=1 \\ i \neq j}}^N (S_+^{(j)} S_-^{(i)} + S_+^{(i)} S_-^{(j)}), \quad (1)$$

which is a CQE-type Hamiltonian [8, 9].

The operators

$$\begin{aligned} S_+ &= \sum_{j=1}^N S_+^{(j)}, \\ S_- &= S_+^\dagger, \\ S_z &= \sum_{j=1}^N S_z^{(j)} \end{aligned} \quad (2)$$

are the collective ladder operators which rise (S_+) or lower (S_-) the states of the atoms, and S_z is the number operator for the pseudo-spin excitations. They obey the commutation rules of the $su(2)$ algebra. The second term of the Hamiltonian is the photon field of frequency ω_{ph} . The third term is the interaction of the photon with the atoms. The last term represents an effective atom–atom interaction [19, 20].

The correspondence between the set of parameters ω_{at} , ω_{ph} , η , λ and those of the central current, the activation energy, the capacitance and the frequency of the cavity mode can be found in [19–21]. For the present discussion, their values are arbitrarily fixed ($\omega_{\text{at}} \approx \omega_{\text{ph}} > \lambda \approx \eta$) since we are interested in the determination of the possible correspondence between spin observables and the population of atomic levels.

2.2. Exact solution

The operators $S_\pm^{(j)}$ and $S_z^{(j)}$ are the generators of the j th copy $su(2)_j$ of the pseudo-spin algebra, where j is the atomic index. We take the tensor product $\prod_{j=1}^N su(2)_j$ as the carrier space for the representations of the fermionic part of the Hamiltonian. The collective state with $k \leq N$ atoms in the excited state is represented by a Dicke state [22]:

$$\begin{aligned} |k\rangle_{\text{at}} &= \binom{N}{k}^{-1/2} \sum_P |k_1^P \dots k_N^P\rangle, \\ |k_1^P \dots k_N^P\rangle &= N_{k^P} \prod_{j=1}^N S_+^{(j)k_j^P} |0\rangle. \end{aligned} \quad (3)$$

The internal degeneracy of each of the two available atomic states is included in the definition of the basis $|k_1^P \dots k_N^P\rangle$. The state which represents l photons is written as the number state

$$|l\rangle_{\text{ph}} = \frac{1}{\sqrt{l!}} a^{\dagger l} |0\rangle. \quad (4)$$

We shall then express the wavefunction of the photons and atoms as the direct product

$$|l, k\rangle = |l\rangle_{\text{ph}} \otimes |k\rangle_{\text{at}}. \quad (5)$$

The basis of equation (6) is labelled by the eigenvalues $L = l + k$ of the operator O ,

$$O = a^\dagger a + S_z + \frac{1}{2}N, \quad (6)$$

which commutes with the Hamiltonian of equation (1). Thus, the matrix elements of the Hamiltonian are written in finite-dimensional subspaces, each of them associated with a fixed value of L [15].

2.3. Approximate solution: the boson mapping

In order to obtain approximate solutions of the Hamiltonian of equation (1), we have expanded the spin operators in terms of bosonic ones. We also request that the algebraic structure of operators entering in the Hamiltonian remains invariant after performing the mapping to the bosonic representation. Since the algebra obeyed by S_\pm and S_z is a $su(2)$ algebra, we shall look at the expression of these operators in terms of boson operators b^\dagger and b , such that (i) $[b, b^\dagger] = 1$ and (ii) the transformed operators $S_\pm(b, b^\dagger)$ and $S_z(b, b^\dagger)$ obey the same algebra. There are several possible schemes for a BM [14]. We have adopted the Holstein–Primakoff BM

$$\begin{aligned} S_+ &\rightarrow b^\dagger \sqrt{N - b^\dagger b}, \\ S_- &\rightarrow \sqrt{N - b^\dagger b} b, \\ S_z &\rightarrow b^\dagger b - \frac{N}{2}. \end{aligned} \quad (7)$$

In the limit of a large number of atoms (N), compared with the average number of bosons ($\langle b^\dagger b \rangle$), the operators S_+ and S_- scale as \sqrt{N} and S_z tends to the limiting value $-\frac{N}{2}$.

Transforming the operators of equation (2) by their BMs (7) in the Hamiltonian of equation (1), and after expressing the interaction between fermions as a scalar product, we obtain

$$\begin{aligned} H_{\text{BM}} &\approx (\omega_{\text{at}} + \lambda(2N - 3)) b^\dagger b + \omega_{\text{ph}} \left(a^\dagger a + \frac{1}{2} \right) - \frac{N}{2} \omega_{\text{at}} \\ &\quad - 2\lambda b^{\dagger 2} b^2 \\ &\quad + \eta (a^\dagger \sqrt{N - b^\dagger b} b + b^\dagger \sqrt{N - b^\dagger b} a). \end{aligned} \quad (8)$$

In the above expression, the terms which are proportional to λ originate in the rearrangement (normal ordering) of the fermionic interactions of the Hamiltonian of equation (1). After performing the BM, the original spin–spin interaction in H manifests, in H_{BM} , as a Kerr nonlinearity of the effective boson field, thus producing terms of the form $b^{\dagger 2} b^2$. We recall that some features of the Kerr nonlinearity in $su(2)$ systems have been studied in [23], and a nonlinear Kerr Hamiltonian

has been related to a mesoscopic Josephson junction model in [24]. Note also that the initial photon–atom interaction in H is now replaced by an effective boson–boson term with couplings of the type $\sqrt{N - b^\dagger b}$; furthermore, its expansion in powers of the average boson number yields the scale factor \sqrt{N} [23].

All these features strongly simplified the numerical problems associated with a large number of atoms. We shall then compare the predictions of both Hamiltonians, the original one and its boson-mapped version, for the spin observables of the system.

2.4. Spin squeezing and atomic population parameters

Atomic spin-squeezed states are quantum-correlated systems with reduced fluctuations in one of the collective spin components. For a component of the spin vector \vec{S} along a general unit vector $\vec{n} \equiv (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$, \vec{S}_n , we shall define the squeezing factor as

$$\zeta^2 = \frac{2(\Delta S_n)^2}{|\langle \vec{S}_\perp \rangle|}, \quad (9)$$

where \vec{S}_\perp is the spin component in the direction perpendicular to \vec{n} and ΔS_n is the deviation of the spin in the direction specified by \vec{n} . Consequently, S_n is squeezed if $\zeta^2 < 1$ [1]. As pointed out in [25], the definition given in equation (9) assumes $SU(2)$ invariance, i.e. $\langle \vec{S}_\perp \rangle = \langle \vec{S} \rangle$.

Since we are interested in the study of the inversion of the population from the ground state of the atoms to the excited level, we shall analyse also the time evolution of the quadratic deviation of the z -component of the total spin, $\Delta^2 S_z$. In this scheme, the optimal squeezing is achieved when the quantum fluctuations of the z -component of the spin are minimal.

2.5. Entropy squeezing

The information entropies of the operator S_σ , ($\sigma = x', y', z'$), for a two-level atom system are [17, 18]

$$H(S_\sigma) = \sum_j P_j(\sigma) \log(P_j(\sigma)), \quad (10)$$

where

$$P_j(\sigma) = \langle \sigma, j | \rho_A(t) | \sigma, j \rangle \quad (11)$$

are the probability measures of the operator S_σ (and $|\sigma, j\rangle$ are the corresponding eigenvectors). The quantities $H(S'_x)$, $H(S'_y)$, $H(S'_z)$, when \mathbf{n}_σ is along the direction of $\langle \vec{S} \rangle$, satisfy the condition

$$H_{x'} + H_{y'} + H_{z'} \geq 2 \log(2^N) - \frac{2}{2^N} \sum_k \binom{N}{k} \log \binom{N}{k}. \quad (12)$$

This condition may also be written as

$$\delta H(S_{x'}) \delta H(S_{y'}) \geq \frac{2^{2N} \prod_k \binom{N}{k}^{-\frac{2}{2^N}}}{\delta H(S_{z'})}, \quad (13)$$

with $\delta H(S_\sigma) = \exp(H(S_\sigma))$. The atomic squeezing of the system is determined by using the entropy uncertainty relation of equation (13). The fluctuation in the component S_σ of

the spin of the atomic system is said to be squeezed if the information entropy $H(S_\sigma)$ satisfies

$$E(S_\sigma) = \delta H(S_\sigma) - \frac{2^N \prod_k \binom{N}{k}^{-\frac{1}{2^N}}}{\sqrt{\delta H(S_{z'})}} < 0. \quad (14)$$

Thus, in what follows, we shall denote with $E(S_\sigma)$ the entropy-squeezing parameter.

2.6. Initial condition

We shall study the time evolution of the system under the action of H . The initial state, which is not an eigenstate of H , is a product state of the form

$$|I\rangle = |I\rangle_{\text{ph}} \otimes |I\rangle_{\text{at}}, \quad (15)$$

where $|I\rangle_{\text{at}}$ is the atomic initial state and $|I\rangle_{\text{ph}}$ is the initial radiation field. We adopt, as an initial condition for the radiation field, a coherent state

$$|I\rangle_{\text{ph}} = e^{-|z|^2/2} e^{za^\dagger} |0\rangle, \quad (16)$$

with $|z|^2 = \langle a^\dagger a \rangle$. For the atomic initial state, $|I\rangle_{\text{at}}$, we adopt a CSS, which is defined as an eigenstate of the spin component in the direction $\vec{n}_0 = (\sin \theta_0 \cos \phi_0, \sin \theta_0 \sin \phi_0, \cos \theta_0)$, then $\vec{S} \cdot \vec{n}_0 |I\rangle_{\text{at}} = S |I\rangle_{\text{at}}$, with eigenvalue S . The angles θ_0 and ϕ_0 are the polar and azimuth angles of the unit vector \vec{n}_0 . The initial atomic state is written

$$|I\rangle_{\text{at}} = e^{z_{\text{at}} S_{z'}} |0\rangle, \quad (17)$$

with

$$z_{\text{at}} = -e^{-i\phi_0} \tan\left(\frac{\theta_0}{2}\right). \quad (18)$$

3. Results and discussion

Hereafter, we shall show and discuss the results of the calculations, which have been performed by using the eigenvalues and eigenvector of the Hamiltonian of equation (1), obtained by exact diagonalization, and by applying the BM method. We focus our attention on the dependence of the results upon the number of atoms in the cavity (N) and the average number of photons ($\langle n_{\text{ph}} \rangle$), for two different situations, namely: (a) for a manageable number of atoms and photons (so that the exact diagonalization is easily performed), and (b) for a large number of atoms, where the use of the BM is expected to be valid, and for a large number of photons. In doing so, we shall determine the validity of the BM method in situations where the exact solution is unfeasible, due to dimensional reasons. We shall compare the time evolution of two observables, that is the spin-squeezing parameter of equation (9), and the inversion of the population of atomic levels (that is, by following the evolution of ΔS_z^2), and relate these quantities with the information entropy of equation (14). This may be relevant for experimental measurements of the spin alignment and fluctuations because the spin-squeezing parameter measures these quantities along the direction of the total spin, while the inversion of the population of the atomic levels refers to the changes in the z -component of the spin

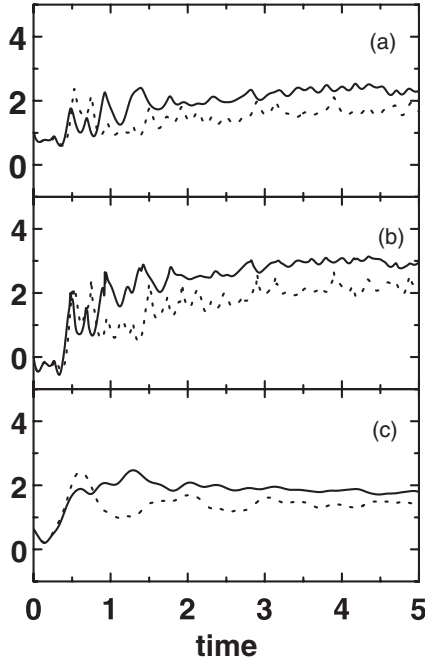


Figure 1. Squeezing parameter ζ^2 of equation (9) (inset (a)), entropy squeezing of equation (14) (inset (b)) and the quadratic deviation of the spin component S_z (inset (c)), as a function of time. (The timescale is arbitrary because we are setting $\hbar = 1$, and use dimensionless coupling constants λ and η ($\lambda = 0.002$, $\eta = 0.001$), and frequencies ω_{at} and ω_{ph} , (both of order unity), to construct the Hamiltonian. The same timescale is used in the following figures.) The system considered has $N = 5$ atoms, and the average number of photons is $\langle n_{ph} \rangle = 4$. The parameters of the CSS are $\phi_0 = \theta_0 = \pi/4$. The results obtained with the exact and BM eigenvectors are shown by solid and dotted lines, respectively.

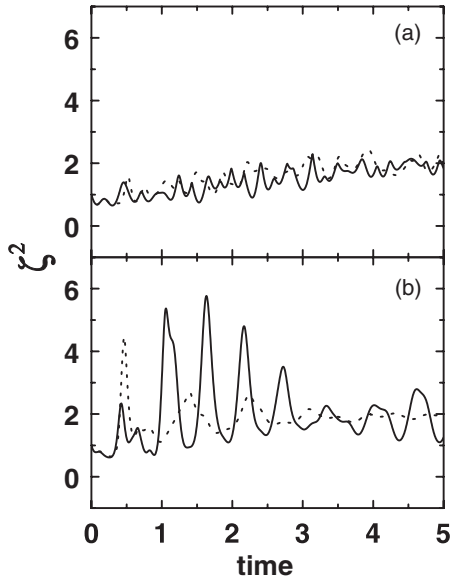


Figure 2. Squeezing parameter ζ^2 of equation (9), as a function of time, and for different values of the number of photons. Insets (a) and (b) correspond to $\langle n_{ph} \rangle = 1$ and $\langle n_{ph} \rangle = 20$, respectively. The other parameters used in the calculation are given in the caption to figure 1, and also for this case the number of atoms is $N = 5$. The results obtained with the exact and BM eigenvectors are shown by solid and dotted lines, respectively.

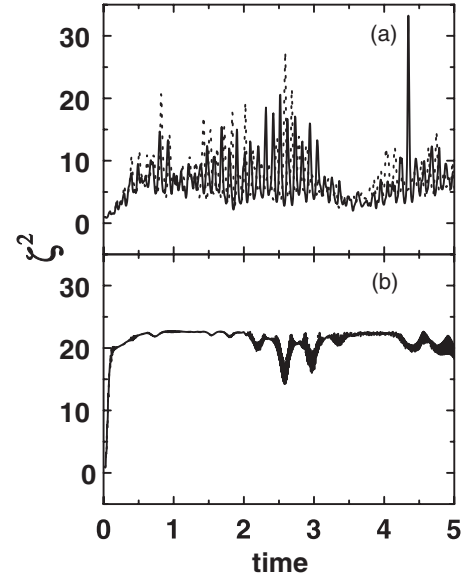


Figure 3. Squeezing parameter ζ^2 of equation (9), as a function of time, for different values of the number of atoms. Insets (a) and (b) correspond to $N = 5$ and $N = 100$, respectively. The other parameters used in the calculation are $\langle n_{ph} \rangle = 4$, $\lambda = 0.002$ and $\eta = 0.015$. The parameters of the CSS are given in the caption to figure 1. The results obtained with the exact and BM eigenvectors are shown by solid and dotted lines, respectively.

relative to the value of the spin-raising operator S_+ . The results of the calculations are shown in figures 1–3.

Figure 1 shows the results corresponding to the time dependence of the spin-squeezing parameter ζ^2 (inset (a)), the entropy-squeezing factor $E(S_\sigma)$ (inset (b)) and the quadratic deviation of the z -component of the spin ΔS_z^2 (inset (c)), for a system with $N = 5$ atoms and $\langle n_{ph} \rangle = 4$ photons. From the results obtained with the exact eigenvectors and eigenvalues, it is seen that these quantities show the same oscillatory trend, as a function of time. The agreement with the BM solution is rather good at small values of time, in the domain where the squeezing of the atomic spin shows up, while both sets of results tend to different asymptotic values for larger values of time. This discrepancy is due to the small number of atoms ($N = 5$) used in the calculations. This is a feature of the BM method which has been studied in detail in the literature [14].

In the region where squeezing is manifest (for time $t < 1$), the agreement between the spin-squeezing factor and the entropy squeezing is observed, as expected from the definition of both observables. This correlation indicates that they are measuring the degree of entanglement of the system.

Concerning the time dependence of ΔS_z^2 , it shows a damping of the oscillations and reaches a constant value at large t , in the interval where the atomic spin is squeezed.

Figure 2 shows the time evolution of the squeezing factor, for two values of the number of photons ($\langle n_{ph} \rangle$), for the system of $N = 5$ atoms considered before (and with the same couplings and frequencies). The rapid oscillatory behaviour, characteristic of the spin–spin interactions, dominates for the smallest number of photons, and it is modulated for the largest number of photons. The exact and BM results tend to agree, in spite of the small number of atoms considered in the

calculations. Both results reach the same asymptotic value and for $\langle n_{\text{ph}} \rangle = 20$, the BM results show a larger modulation of the oscillations.

The time dependence of the squeezing parameter, as a function of the number of atoms, for a fixed value of the number of photons is shown in figure 3. There, the increase in the number of atoms washes out the correlations leading to squeezing. The results obtained with the exact solution and with the BM expansion become almost identical for $N = 100$.

It is seen from the results depicted in figures 1–3 that the spin-squeezing parameter and the information entropy exhibit similar time dependence, which is oscillatory for values of time larger than $t = 1$ and non-oscillatory at smaller values of time³, where the atomic spin is squeezed. The conditions for spin squeezing are satisfied for $0 < t < 0.5$. Also, it is seen that the exact solution and the one obtained by applying the boson transformation do tend to agree at all times, when the number of atoms is sufficiently large, but also at small times when the number of atoms is small.

The fact that the BM approximation yields results which are comparable to the exact ones for larger values of N confirms the hand-waving notion that it should coincide with the exact solution when N tends to the thermodynamic limit, and for N much larger than the average number of photons, due to the scaling properties of the spin operators.

Concerning the dependence of the results with the number of photons, both the exact and BM results show the disappearance of the squeezing as $\langle n_{\text{ph}} \rangle$ increases. This effect is accompanied by the modulation of the oscillations induced by the spin–spin interactions.

4. Conclusions

In this work, we have calculated spin-squeezing observables, like the spin squeezing, the inversion of the atomic population and the entropy squeezing, in a system of N two-level atoms interacting with photons. We have performed the calculations in two schemes: (a) an exact diagonalization and (b) using a boson mapping (BM) method to transform the Hamiltonian. From the results obtained in the calculations of these quantities, we may conclude that the spin squeezing and the entropy squeezing are correlated, as indicators of entanglement, in the time interval where the atomic spin is squeezed. On the other hand, from the comparison between exact and BM results, one can conclude that the use of the BM technique seems to be a good alternative to treat systems with a large number of atoms. The dependence of the value of ΔS_z^2 with time seems to indicate that an array of two-level systems may indeed be a good device to keep the total spin oriented in quantum non-demolition measurements since the quadratic deviation ΔS_z^2 reaches a constant value at large values of time. In this respect, we can mention the recent work on memory effects in quantum systems, which consist essentially of two-level atoms in interaction with photons, in an optical setup [26–28].

³ To correlate this timescale (and the timescale of all figures) with the coupling constants, note that in this arbitrary scale, a unit of time is equivalent to $(1/\omega_f)$ since we are using $\hbar = 1$ in our calculations.

Finally, and concerning CQE devices [8, 9], because of the features exhibited by the present results about the spin-squeezing parameter, it could be possible to relate it with the (expected) control of the excited state population of a qubit [8] since the inversion of the population follows the evolution of the spin-squeezing factor in the time interval where the atomic spin is squeezed.

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