



An iterative MILP-based approach for the maritime logistics and transportation of multi-parcel chemical tankers



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ABSTRACT

The cost-effective routing and scheduling of a fleet of multi-parcel chemical tankers represents a central decision making process in both chemical and shipping industry. Ships designed for the transport of liquid or gas in bulk are called tankers. Shippers seek to choose the cargos to transport and determine the optimal route that the ship should follow to maximize its profit. Due to determining the optimal assignment and routing decisions of a large set of cargos transported by a ship fleet is inherently NP-hard, real-world problems are either intractable or result in poor solutions when solved with pure optimization approaches. To overcome this limitation, this work introduces a new continuous time precedence-based MILP mathematical formulation that is then embedded within a heuristic-based algorithm in order to obtain near-optimal solutions to large-scale problems. The applicability and efficiency of the proposed approach is illustrated by solving a real case of study corresponding to a sea-cargo shipping company operating in South-East Asia. Computational results show notable improvements and better performance when compared to other alternative reported solution techniques.

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1. Introduction

Maritime transportation logistics is concerned with the movement of a set of cargos between seaports by means of a heterogeneous ship fleet. It is fundamental to international trade as it provides a cost-effective means to transport large volumes of cargo around the world. It is estimated that over 80% of world trade is carried by sea. The review of Maritime Transport, UNCTAD (2011), details that during the first decade of the new millennium the cargo carrying capacity of oil tankers grew by 60%, that of dry bulk carriers grew by 65%, and containerships capacity more than double (up to 164%). The growth of fleet capacity facilitates the fast expansion of seaborne international trade that has increased by 40% during the same decade.

A fundamental topic in both chemical and shipping industry is the cost-efficient management of heterogeneous ship fleets. According to Christiansen, Fagerholt, Nygreen, and Ronen (2007), a ship involves a major capital investment (usually millions of US dollars, tens of millions for larger ships) and the daily operating cost of a ship may easily reach thousands of dollars and tens of thousands for the larger ships. Proper planning of fleets and their

operations has the potential of improving their economic performance and reducing shipping costs.

Broadly speaking, sea transport can be divided into tramp and liner shipping. On the one hand, the purpose of tramp shipping is to provide convenient and economical transport of bulk cargos that require cross-ocean movement. The tramp ship can be any vessel that does not have a fixed itinerary and go from place to place depending upon where they can find cargos. Bulk cargos can be classified into dry bulk and liquid bulk. Demand for the transport of liquid bulk by sea is served mainly by the sector of tanker shipping. Ships designed for the transport of liquid in bulk are called tankers. The main cargos carried in tankers are liquid and gas. On the other hand, a main function of liner shipping is to satisfy the demand for regular transport under which cargos are transported through regular routes and with regular schedules.

This paper focuses on tramp shipping that operates with a fleet of heterogeneous multi-parcel chemical tankers. The challenge is to spot the bulk cargos to serve and construct routes and schedules that maximize the ship profit. A cargo consists of a specified quantity of a given product that must be picked up at its port of loading, transported, and then delivered to its port of discharge. The bulk cargos shipped in large quantities are most oil tankers, but there are also tankers carrying chemicals, liquid food products and other commodities. In practical situations, transshipment cargos have a time interval within pickup service must begin. Generally, a fleet

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Nomenclature

<i>Sets</i>		<i>tcc_s</i>	time-charter cost per unit time for ship <i>s</i> (in USD/day)
<i>I</i>	cargos	<i>ti_s</i>	arrival time of ship <i>s</i> to the first port visited
<i>DP_p</i>	cargos that must be discharged in port <i>p</i>	<i>v_s</i>	average speed (knots) of ship <i>s</i>
<i>IP_s</i>	first port visited by ship <i>s</i>	<i>vmax_s</i>	maximum carrying capacity of ship <i>s</i> (in tonnes)
<i>LP_p</i>	cargos that must be loaded in port <i>p</i>	<i>volume_i</i>	size of cargo <i>i</i> (in tonnes)
<i>OB_s</i>	cargos that are on-board ship <i>s</i> at beginning of planning horizon		
<i>P</i>	ports	Binary variables	
<i>S</i>	ships	<i>PR_{pp's}</i>	denoting that port <i>p</i> is visited before (<i>PR_{pp's}</i> = 1) or after (<i>PR_{pp's}</i> = 0) port <i>p'</i> whenever both nodes are serviced by the same ship <i>s</i>
Parameters		<i>X_{ps}</i>	denoting that port <i>p</i> is visited by ship <i>s</i>
<i>diff</i>	maximum triangle inequality violation (in nm)	<i>Y_{is}</i>	assigning cargo <i>i</i> to ship <i>s</i>
<i>dist_{pp'}</i>	distance in nautical miles between ports <i>p</i> and <i>p'</i>		
<i>dr_i</i>	unloading rate of cargo <i>i</i> (tonnes/day)	Continuous variables	
<i>ept_i</i>	earliest time for pickup of cargo <i>i</i>	<i>LOAD_{ps}</i>	total cargo loaded on ship <i>s</i> after completing the service at port <i>p</i>
<i>fc_s</i>	cost of fuel per unit distance for ship <i>s</i> (in USD/nm)	<i>OV_{ps}</i>	visiting order of port <i>p</i> in the route of ship <i>s</i>
<i>lpt_i</i>	latest time for pickup of cargo <i>i</i>	<i>TD_{ps}</i>	accumulated travel distance of ship <i>s</i> to reach port <i>p</i>
<i>lr_i</i>	loading rate of cargo <i>i</i> (tonnes/day)	<i>TV_{ps}</i>	accumulated travel time of ship <i>s</i> to reach port <i>p</i>
<i>M_c</i>	big-M for ship capacity constraints	<i>TTD_s</i>	total travel distance for ship <i>s</i>
<i>M_d</i>	big-M for routing distance constraints	<i>TTV_s</i>	total travel time for ship <i>s</i>
<i>M_o</i>	big-M for visiting order constraints	<i>UNLOAD_{ps}</i>	total cargo unloaded from ship <i>s</i> after completing the service at port <i>p</i>
<i>M_t</i>	big-M for routing time constraints		
<i>pc_{ps}</i>	port cost for ship <i>s</i> at port <i>p</i>		
<i>sr_i</i>	shipping rate or revenue for cargo <i>i</i> (in USD)		
<i>tad</i>	time for inspections for each port visit		

of ships is utilized for moving the cargos. The ship fleet involves a fixed number of heterogeneous ships with different properties (travel costs, travel time, and capacity).

A clear trend in the research literature is that the transportation operations are devoted to road distribution by trucks. Widely known as a NP-hard problem (Laporte & Semet, 2002; Prins, 2004), the basic VRP has been studied for decades. Different variants of this problem, usually referred to the pickup and delivery activities (PDP), have been explored as well. Surveys on PDP problems can be found in Bodin, Golden, Assad, and Ball (1983), Savelsbergh and Sol (1995), Fisher (1995), and Desaulniers, Desrosiers, Erdmann, Solomon, and Soumis (2002). In particular, most of the contributions have been devoted to the pickup and delivery problem with time windows (PDPTW). Two classes of PDPTW have usually been tackled by the researchers. One of them is the so-called single-vehicle with time windows (1-PDPTW) where pickup and delivery services are all done by a single vehicle. If there are multiple vehicles available, the problem is known as the multi-vehicle pickup and delivery problem (m-PDPTW). For instance, the multiple vehicle time-window-constrained pickup and delivery problem (MVPDPTW) faced in Dondo, M  ndez, and Cerd   (2008) is capable of handling transport requests with multiple origins and/or destinations, heterogeneous vehicles, and multiple depots. In addition, the underlying ideas of the PDPTW problem have been also applied to the coordination of production and distribution activities in multi-site systems (C  ccola, Zamarripa, M  ndez, & Espu  a, 2013; Dondo & Cerd  , 2013; M  ndez, Bonfill, Espu  a, & Puigjaner, 2006). Other variant of the classical VRP is the Delivery and Pickup Problem (DPP). Despite of both PDP and DPP problems aims serving several customers from a single depot by a fleet of vehicles, the DPP take into account two types of customers: (i) “linehaul” customers, who require delivery of goods to their specific location and (ii) “backhaul” customers, who require pickups from their specific locations. According to Wang and Chen (2012), while PDP is often referred to as a mail express system, DPP can be regarded as a bi-logistic problem.

Even though the ship routing and scheduling problem is very closely related to the m-PDPTW, there are some important differences that must be considered in the development of tramp ship specific approaches. For instance, the PDP variant involves transport requests with a single origin and a single destination, and a vehicle fleet departing and returning to a central depot. However, tramp ships are moved in a continuous fashion between ports for loading and unloading, without any of the ports having a specific status as a depot. In addition, the bulk of VRP research and its variants always have been reductionist in nature, with assumptions of Euclidean distances, deterministic and static travel times, deterministic demand, hard constraints, and a single objective. In case of triangle inequality violations, researchers generally perform one of two actions: changing the network to eliminate triangle inequality violations or indicates that the network satisfies the principle and no further action is needed. However, the issue of triangle inequality is particularly relevant to certain types of routing problems, especially in maritime operations, where these violations accurately represent real world conditions. Fleming, Griffis, and Bell (2013) have demonstrated that if triangle inequality violations are ignored, the resulting solutions may not appropriately reflect the reality of the routing network, resulting in optimistic or inaccurate solutions.

In addition, since industry demands solutions that must be either optimal, or at least near-optimal, and quick to be reached, a wide variety of decomposition strategies have been extensively analyzed and solved by the communities of Operations Research and Process Systems Engineering. They claim that real-world problems can be solved robustly by maintaining the number of decisions variables at a reasonable level, even for large-scale problems. A reduce search space usually results in manageable model sizes that often guarantee a more stable and predictable optimization model behavior. Some important contributions in this direction can be found in Kopanos, M  ndez, and Puigjaner (2010), Castro, Aguirre, Zeballos, and M  ndez (2011), Aguirre,

M  ndez, Gutierrez, and De Prada (2012), and Dondo and Cerd   (2013), among many others.

Generally, practical approaches to the ship routing and scheduling problem apply heuristic approximate algorithms providing good solutions within a reasonable computer time. Both Jetlund and Karimi (2004) and Aang (2006) have proposed a MILP formulation using variable-length slots together with a set of heuristics to solve a real-world case study faced by a multi-national shipping company operating a fleet of multi-parcel chemical tankers. Specifically, three heuristic methods have been tested: (i) multi-period heuristic, which divides the time horizon into several smaller periods, solve the earlier period, carry over the solution to the next period, and so on; (ii) one-ship heuristic, which decomposes the multi-ship problem into smaller one-ship problems; (iii) set-packing heuristic that tries to generate cargo combinations to be served by each ship. A set-packing problem is solved to choose the best cargo combination and to construct one feasible solution to the multi-ship problem. Besides, Vilhelmsen, Lusby, and Larsen (2013) developed a solution method that utilizes a column generation approach in order to solve the integrated problem of routing, scheduling and bunkering of a tramp fleet. Others variants of ship routing and scheduling problems have been also studied. Br  nmo, Christiansen, and Nygreen (2007), Br  nmo, Nygreen, and Lysgaard (2010), and Korsvik and Fagerholt (2010) considered flexible cargo sizes while Fagerholt, Laporte, and Norstad (2010) and Norstad, Fagerholt, and Laporte (2011) assumed speed as a decision variable. Andersson, Duesund, and Fagerholt (2011) explore a routing and scheduling problem for a special segment within tramp shipping referred to as project shipping involving cargo coupling and synchronization constraints. Finally, split loads are taken into account in Korsvik, Fagerholt, and Laporte (2011), Andersson, Christiansen, and Fagerholt (2011) and St  lhane, Andersson, Christiansen, Cordeau, and Desaulniers (2012). Complete surveys on ship routing and scheduling can be found in Christiansen, Fagerholt, and Ronen (2004), Christiansen, Fagerholt, Nygreen, and Ronen (2013). Moreover, Hoff, Andersson, Christiansen, Hasle, and Lokketangen (2010) have presented an interesting work that gives an overview of industrial aspects of combined routing and fleet composition in maritime and road-base transportation problems, showing the importance of the field and the difficulties associated with solving these types of problems.

This paper is structured as follows. In the first part, a general mixed-integer linear (MILP) mathematical programming formulation, which is based on the general precedence notion proposed by M  ndez, Henning, and Cerd   (2001), for the management of a heterogeneous ship fleet with several ports to visit and multiples cargos to serve, is introduced. The aim is to identify the cargos that each ship should serve and determine the optimal route that each ship should follow to maximize its profit. Due to an optimal assignment of cargos and schedules to a ship fleet is inherently NP-hard, the computational efficiency of the MILP approach rapidly is deteriorated and large instances of the ship routing and scheduling problem can be rarely to solve to optimality through a pure optimization approach. In this way, an iterative algorithm, which is related to the selection of the binary variables to fix for the generation of constrained MILP models, has been done by exploiting the knowledge about the problem structure. The applicability of the proposed procedure is demonstrated by solving a challenging real-world case of study arising in chemical and shipping industry. Section 2 describes the major problem characteristics of the problem addressed. In Section 3, the MILP mathematical model is presented. Afterwards, in Section 4, the proposed iterative MILP-based strategy is explained in detail. Then, a real-world large-scale problem of a shipping company is introduced and solved in Section 5.

Finally, the article concludes with some discussion and remarks in Section 6.

2. Problem definition

The ship routing and scheduling problem aims at generating the optimal routes for a ship fleet in order to carry multiples cargos with maximum profit while satisfying all problem constraints. The ships operate between ports. These ports are used for loading and unloading cargos as well as for loading fuel, fresh water, and supplies, and discharging waste. Let us define a shipping network, represented by the directed graph $G = \{P, A\}$ comprising sea lanes $A = \{(p_1, p_2) | p_1, p_2 \in P\}$ that link up ports $P = \{p_1, p_2, \dots, p_n\}$. The edge $(p_1, p_2) \in A$ is supposed to be the lowest distance route connecting port p_1 to port p_2 . At each port $p \in P$, LP_p denotes the set of cargos to be picked up at such port while the set DP_p defines the cargos to be delivered at port p . The set I contains the cargos that can be transported during the planning horizon. A cargo consists of a specified quantity of a given product that must be picked up at its port of loading, transported, and then delivered to its port of discharge. The data set associated to any cargo $i \in I$ includes the quantity (volume in tonnes) to be transported, the revenue obtained for transporting it and its pickup and discharge port. There is also a service time for loading and discharging and a time windows defined by $[ept_i, lpt_i]$, where ept_i and lpt_i are the earliest and latest time within pickup service must begin, respectively.

A fleet of ships $S = \{s_1, s_2, \dots, s_n\}$ is utilized for moving the cargos. The ship fleet involves a fixed number of heterogeneous ships with different properties (travel costs, travel time, and capacity). Ships designed for the transport of liquid or gas in bulk are called tankers. The average sailing speed is assumed to be constant at v (nm/h) for all ships. The carrying capacity stands for a finite load capacity in tonnes or m^3 that cannot be exceeded. Generally, the ship capacity and the cargo quantities are such that ships can carry multiples cargos simultaneously. This means that new loading ports can still be visited with some cargos on-board. However, in some cases the controlled fleet may have insufficient capacity to serve all cargos during the planning horizon. Consequently, no all ships can visit all ports and take all cargos. A cargo that is not served can be transferred to a third party carrier in the spot market (Jetlund & Karimi, 2004). The set of cargos on-board of ship s at time zero is defined by OB_s . Ship capacities, cargos properties, and port locations are all problem data.

At the beginning of planning horizon, each ship knows the next port to visit and estimates the time of arrival at that location. Geographically, the origin can be either a port or a point at sea, while the final point will be determined by the solution process and corresponds to the last delivery port. Thus, the condition of return to assigned central depot as assumed in the conventional PDPTW problem is not enforced. Each route, regarded as a sequence of visits at different ports, is an open path ending with the last schedule delivery. Every ship can perform pickup and delivery tasks in multiples ports but the number of visited ports must never exceed a maximum amount K defined for every ship. The scheduler planner decides the value of K . The ships are charged when visiting ports and passing channels. Such costs depend largely on the size/capacity of the ship and the numbers of berths visited. When a ship arrives at a given port, it does not necessarily pick up and drop off a cargo. In some ports, it will only do one activity but not the other. The duration of the tour assigned to ship s is computed by traveling and service times. Traveling time between two ports can be determined by route length (nautical miles) and the sailing speed of the ship (knots) while the time needed for carrying out loading and discharge operations at each port comprises a fixed inspection time plus a variable time period

that directly increases with the total cargo to be load/unloaded. For each cargo, the loading lr_i and discharge rate dr_i (pump capacity) is known a priori. Generally, ships spend 50–70% of their route time in ports for cargo handling operations.

Therefore, the problem goal aims at identifying the cargos that each ship should serve and determines the optimal route that it should follow to maximize its profit. The total profit is determined by the revenues earned for transporting cargos minus operation costs. Three types of cost are usually considered. First, the time-charter cost. Second, the distance-based transportation cost accounting for the fuel oil consumption. Third, the port charge depending on the capacity of ship and the number of visited ports. While the net profit is maximized, the following constraints must be satisfied: (i) each cargo can be serviced by just a single ship; (ii) the capacity of each ship cannot be exceeded; (iii) the number of visited ports should be lower than the maximum allowed K ; (iv) the pickup service at each cargo must start within the time windows.

3. The MILP mathematical model

Having introduced the major problem characteristics, the specific model equations and variables involved in the mathematical representation of the ship routing and scheduling problem are presented in this section. To define pickup and delivery routes, several operational decisions are to be made concerning: (i) the allocation of cargos to ships, (ii) the assignment of ports to ships, and (iii) the sequencing of ports in each tour. Such decisions are defined by three types of 0–1 variables. Binary variable Y_{is} is equal to one if cargo i is serviced by ship s . If port p is visited by ship s , then 0–1 variable X_{ps} becomes equal to one. In turn, sequencing variable $PR_{pp's}$ is equal to one whenever the pair of ports (p, p') are on the same route of ship s and port p is visited earlier. In addition, several nonnegative continuous variables are incorporated into the proposed formulation to define: (i) the accumulated travel distance to reach port p along the route assigned to ship s , given by TD_{ps} , (ii) the travel time to go from the starting point to port p along the route of s , TV_{ps} , and the overall traveling time (TTV_s) and traveling distance (TTD_s) for the pickup/delivery route assigned to ship s . The other variables included in the model are related to the ship capacity constraints. Continuous variables $LOAD_{p,s}$ and $UNLOAD_{p,s}$ indicate the accumulated amount of volume loaded and delivered by ship s , including initial cargos, after visit port p . Consequently, the difference ($LOAD_{ps} - UNLOAD_{ps}$) computes the current load transported by s after stop at p , which allows avoiding overcapacity or product shortage.

The proposed MILP formulation is based on the notion of general precedence extending the immediate precedence concept for routing problems presented in C  ccola et al. (2013). If a pair of ports (p, p') are visited by the same ship s ($X_{ps} = X_{p's} = 1$), the sequencing variable $PR_{pp's}$ denotes that port p is visited before ($PR_{pp's} = 1$) or after ($PR_{pp's} = 0$) port p' in the route of ship s . Consequently, the general precedence sequencing variable is only defined for each pair (p, p') , with $p < p'$. This generalized concept simplifies the mathematical model and reduces by half the number of sequencing variables when compared, for instance, to the immediate precedence formulation.

Despite global precedence formulations are computationally faster on average, such models have a serious drawback since they cannot optimize objectives when triangle inequality violations are introduced in distance matrixes. Let $dist_{pp'}$ be the traveling distance from port p to port p' , then the triangle inequality is defined as: $dist_{pp'} + dist_{p'p''} \geq dist_{pp''} \forall p, p', p'' \in P$. Sometimes, such inequality is not fulfilled, that is to say that the direct link from port p to port p'' is not always the shortest path. If some maritime restriction

applies, an indirect link (p, p', p'') may become shorter to travel from port p to port p'' via node p' . Therefore, the precedence-based MILP mathematical formulation cannot always assures that the solution found is the optimal one, although the feasibility is always ensured. To demonstrate the above affirmation, a reduced model is developed. This is defined as follows:

- (i) Each ship must deliver the cargos that have on-board at time zero ($i \in OB_s$ and $Y_{is} = 1$)

$$X_{ps} = 1 \quad \forall i \in I, \quad p \in P, \quad s \in S : ((i \in OB_s) \cap (i \in DP_p))$$

- (ii) New cargos cannot be served

$$Y_{is} = 0 \quad \forall i \in I, \quad s \in S : i \notin OB_s$$

- (iii) Accumulated travel distance by s from its starting point to the first visited port. Parameter t_{is} stands for the estimated time of arrival at port p while v_s denotes the average speed in knots of s .

$$TD_{ps} \geq t_{is} 24 v_s \quad \forall p \in P, \quad s \in S : p \in IP_s$$

- (iv) Distance-based sequencing constraints. M_d is a large positive value.

$$TD_{p's} \geq TD_{ps} + dist_{pp'} - M_d(1 - PR_{pp's}) - M_d(2 - X_{ps} - X_{p's}) \\ \forall (p, p') \in P, \quad s \in S : p < p'$$

$$TD_{ps} \geq TD_{p's} + dist_{p'p} - M_d PR_{pp's} - M_d(2 - X_{ps} - X_{p's}) \\ \forall (p, p') \in P, \quad s \in S : p < p'$$

- (v) Overall travel distance along the route assigned to ship s

$$TTD_s \geq TD_{ps} \quad \forall p \in P, \quad s \in S$$

- (vi) Objective function: Minimize the overall travel distance in order to reduce fuel costs. Parameter fc_s stands for the cost of fuel per unit distance for ship s .

$$\text{Min} \left[\sum_{s \in S} fc_s TTD_s \right]$$

In order to illustrate the proposed formulation, a simplification of the original real-world case of study presented by Jetlund and Karimi (2004) has been tackled. According to the problem data given in Fig. 1, the best routing cost for ship S6 has to be equal to USD 29224.8 and its optimal schedule has to be as that one detailed in Table 1. However, if the example is solved by using the precedence-based formulation presented above, the model reports the schedule given in Table 2 whose final routing cost is equal to USD 29310.9. The difference between both results is originated because the distance matrix (see Tables A.4 and A.5 of Appendix A) does not satisfy the triangle inequality. The problem data allows to derive that $dist_{Yinkou, Karimun} + dist_{Karimun, Paradip} \leq dist_{Yinkou, Paradip}$.

To avoid that the mathematical model presented in this paper can generate non-optimal solutions when the triangle inequality is not satisfied, a new positive variable OV_{ps} , which determines the visiting order of port p in the route of ship s , is used. In addition, a positive constant $diff$ had to be defined to compute the maximum difference between a direct connection and an indirect route. If the triangle inequality is satisfied, $diff = 0$. The description of the complete mathematical formulation used in this work is presented in the following subsections.

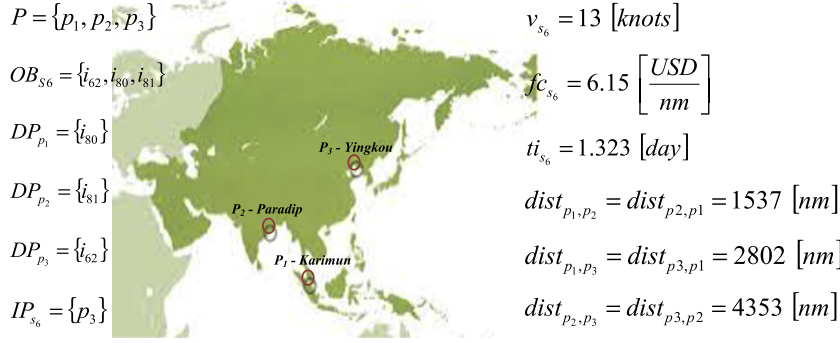


Fig. 1. A real-world example for the reduced problem.

Table 1
Optimal schedule for S6.

Port	Traveled distance (nm)	Discharges
Yingkou	413	i62
Karimun	3215	i80
Paradip	4752	i81

Table 2
Solution for S6 reported by the reduced precedence-based MILP model.

Port	Traveled distance (nm)	Discharges
Yingkou	413	i62
Karimun	3215	i80
Paradip	4766	i81

3.1. Assignment constraints

Eq. (1) determines that each cargo i can be serviced by just a single ship $s \in S$. Splitting a cargo to be picked-up by two or more ships is a forbidden option.

$$\sum_{s \in S} Y_{is} \leq 1 \quad \forall i \in I \quad (1)$$

From Eq. (1), it can be noted that all potential cargos are not necessarily assigned to ships. Besides, if some cargo $i \in OB_s$, then $Y_{is} = 1$.

If a cargo i is serviced by ship s ($Y_{is} = 1$), then both its loading and discharging port must be visited by s . On the other hand, for each $i \in OB_s$ only its unloading port will be visited by s during the planning horizon. These conditions are ensured by Eq. (2).

$$Y_{is} \leq X_{ps} \quad \forall i \in I, p \in P, s \in S: ((i \notin OB_s) \cap (i \in LP_p)) \cup (i \in DP_p) \quad (2)$$

At the beginning of planning horizon, each ship s knows the first port p to visit ($p \in IP_s$) and estimates the time of arrival at that location. In this way, if $p \in IP_s$, $X_{ps} = 1$.

3.2. Route sequencing constraints

A ship route can be regarded as a sequence of ship stops at different ports. Positive variable OV_{ps} defines the absolute position of port p in the route of ship s . If $p \in IP_s$, $OV_{ps} = 1$. Besides, Eq. (3) states that if two ports (p, p') are on the same route ($X_{ps} = X_{p's} = 1$) and port p is visited before ($PR_{pp's} = 1$), then $OV_{p's}$ must always be greater than OV_{ps} by at least 1. In case p' is visited

earlier ($PR_{pp's} = 0$), the reverse statement holds (see Eq. (4)). Constraints (3) and (4) both become redundant whenever ports (p, p') are serviced by different vehicles ($X_{ps} + X_{p's} < 2$, for any s). By definition, M_o is a large positive number.

$$OV_{p's} \geq OV_{ps} + 1 - M_o(1 - PR_{pp's}) - M_o(2 - X_{ps} - X_{p's}) \quad \forall (p, p') \in P, s \in S: p < p' \quad (3)$$

$$OV_{ps} \geq OV_{p's} + 1 - M_o PR_{pp's} - M_o(2 - X_{ps} - X_{p's}) \quad \forall (p, p') \in P, s \in S: p < p' \quad (4)$$

Finally, Eq. (5) establishes an upper bound for the value of OV_{ps} .

$$OV_{ps} \leq \sum_{p' \in P} X_{p's} \quad \forall p \in P, s \in S \quad (5)$$

3.3. Traveling distance constraints

Eq. (6) determines the traveled distance by ship s up to the first port visited $p \in IP_s$. Parameter ti_s stands for the estimated time of arrival at port p while v_s denotes the average speed in knots of s .

$$TD_{ps} = ti_s 24 v_s \quad \forall p \in P, s \in S: p \in IP_s \quad (6)$$

Let $dist_{pp'}$ stands for the minimum distance between ports (p, p'), which are both visited by the same ship s ($X_{ps} = X_{p's} = 1$), then if p is visited before ($PR_{pp's} = 1$), the traveled total distance by s to reach port p' ($TD_{p's}$) must be always greater than TD_{ps} by at least $dist_{pp'}$. If node p is visited later ($PR_{pp's} = 0$), the reverse statement holds. Such conditions are enforced by the pair of Eqs. (7) and (8). M_d is a positive large number.

$$TD_{p's} \geq TD_{ps} + dist_{pp'} - M_d(1 - PR_{pp's}) - M_d(2 - X_{ps} - X_{p's}) \quad \forall (p, p') \in P, s \in S: p < p' \quad (7)$$

$$TD_{ps} \geq TD_{p's} + dist_{p'p} - M_d PR_{pp's} - M_d(2 - X_{ps} - X_{p's}) \quad \forall (p, p') \in P, s \in S: p < p' \quad (8)$$

Considering the fact that the proposed model is able to allow triangle inequality violations in distances matrixes, Eqs. (7) and (8) must be rewritten as given by Eqs. (7') and (8'). Parameter $diff$ is a positive number defining the maximum difference between a direct connection and an indirect route.

$$TD_{p's} \geq TD_{ps} + dist_{pp'} + diff - M_d(1 - PR_{pp's}) - M_d(2 - X_{ps} - X_{p's}) \quad \forall (p, p') \in P, s \in S: p < p' \quad (7')$$

$$TD_{ps} \geq TD_{p's} + dist_{p'p} + diff - M_d PR_{pp's} - M_d(2 - X_{ps} - X_{p's}) \quad \forall (p, p') \in P, s \in S: p < p' \quad (8')$$

Finally, the overall traveling distance for the route assigned to ship s is computed by Eq. (9). However, if Eqs. (7') and (8') are used by the model ($\text{diff} > 0$), Eq. (9) must be replaced by Eq. (9').

$$TTD_s \geq TD_{ps} \quad \forall p \in P, \quad s \in S \quad (9)$$

$$TTD_s \geq TD_{ps} - (OV_{ps} - 1)\text{diff} \quad \forall p \in P, \quad s \in S \quad (9')$$

3.4. Traveling time constraints

Eq. (10) states that the estimated time of arrival of ship s at the first port visited $p \in IP_s$ is equal to ti_s .

$$TV_{ps} = ti_s \quad \forall p \in P, \quad s \in S : p \in IP_s \quad (10)$$

Moreover, let us assume that port p and p' are both visited by the same ship s ($X_{ps} = X_{p's} = 1$). If port p is visited before ($PR_{pp's} = 1$), then Eq. (11) states that the service starting time at port p' ($TV_{p's}$) should be greater than TV_{ps} by at least the inspection time at port (tad), plus the variable time for loading/discharging cargos, plus the traveling time from p to p' . If not, ($PR_{pp's} = 0$), the reverse statement holds and Eq. (12) will become active. If ports p, p' are not visited by the same ship s ($X_{ps} + X_{p's} < 2$, for any s), Eqs. (11) and (12) both become redundant. The quantity of tonnes of cargo i is defined by parameter volume_i while the loading and discharge rates are determined by lr_i and dr_i , respectively. By definition, M_t is a positive large number.

$$TV_{p's} \geq TV_{ps} + \sum_{i \in I: (i \notin OB_s) \cap (i \in LP_p)} (Y_{is} \text{volume}_i) / lr_i + \sum_{i \in I: i \in DP_p} (Y_{is} \text{volume}_i) / dr_i + tad + \text{dist}_{pp'} / (24v_s) - M_t(1 - PR_{pp's}) - M_t(2 - X_{ps} - X_{p's}) \quad \forall (p, p') \in P, \quad s \in S : p < p' \quad (11)$$

$$TV_{ps} \geq TV_{p's} + \sum_{i \in I: (i \notin OB_s) \cap (i \in LP_{p'})} (Y_{is} \text{volume}_i) / lr_i + \sum_{i \in I: i \in DP_{p'}} (Y_{is} \text{volume}_i) / dr_i + tad + \text{dist}_{p'p} / (24v_s) - M_t PR_{pp's} - M_t(2 - X_{ps} - X_{p's}) \quad \forall (p, p') \in P, \quad s \in S : p < p' \quad (12)$$

In case the matrix of distances does not satisfy the triangle inequality condition, then the above time-based sequencing constraints must be replaced by Eqs. (11') and (12'). As the parameter diff is expressed in distance units (nm), it is necessary to change it to time units (day).

$$TV_{p's} \geq TV_{ps} + \sum_{i \in I: (i \notin OB_s) \cap (i \in LP_p)} (Y_{is} \text{volume}_i) / lr_i + \sum_{i \in I: i \in DP_p} (Y_{is} \text{volume}_i) / dr_i + tad + (\text{dist}_{pp'} + \text{diff}) / (24v_s) - M_t(1 - PR_{pp's}) - M_t(2 - X_{ps} - X_{p's}) \quad \forall (p, p') \in P, \quad s \in S : p < p' \quad (11')$$

$$TV_{ps} \geq TV_{p's} + \sum_{i \in I: (i \notin OB_s) \cap (i \in LP_{p'})} (Y_{is} \text{volume}_i) / lr_i + \sum_{i \in I: i \in DP_{p'}} (Y_{is} \text{volume}_i) / dr_i + tad + (\text{dist}_{p'p} + \text{diff}) / (24v_s) - M_t PR_{pp's} - M_t(2 - X_{ps} - X_{p's}) \quad \forall (p, p') \in P, \quad s \in S : p < p' \quad (12')$$

Since the first visited port $p \in IP_s$ is known beforehand, all port $p' (\neq p)$ that is on the route of ship s should be visited after. Thus, time-based sequencing constraints ((11) and (12)) and ((11') and (12')) are reduced to the following pairing conditions, respectively:

$$TV_{p's} \geq TV_{ps} + \sum_{i \in I: (i \notin OB_s) \cap (i \in LP_p)} (Y_{is} \text{volume}_i) / lr_i + \sum_{i \in I: i \in DP_p} (Y_{is} \text{volume}_i) / dr_i + tad - M_t(1 - X_{p's}) \quad \forall s \in S, \quad (p, p') \in P : (p \neq p') \cap (p \in IP_s) \quad (13)$$

$$TV_{p's} \geq TV_{ps} + \sum_{i \in I: (i \notin OB_s) \cap (i \in LP_p)} (Y_{is} \text{volume}_i) / lr_i + \sum_{i \in I: i \in DP_p} (Y_{is} \text{volume}_i) / dr_i + tad + \text{diff} / (24v_s) - M_t(1 - X_{p's}) \quad \forall s \in S, \quad (p, p') \in P : (p \neq p') \cap (p \in IP_s) \quad (13')$$

In case cargo i is served by ship s ($Y_{is} = 1$), then its loading port p should be visited before its unloading port p' , but other ports can be visited in between. Consequently, the time-based sequencing constraints can be expressed as one of the following pairing conditions:

$$TV_{p's} \geq TV_{ps} + \sum_{i' \in I: (i' \notin OB_s) \cap (i' \in LP_p)} (Y_{i's} \text{volume}_{i'}) / lr_{i'} + \sum_{i' \in I: i' \in DP_p} (Y_{i's} \text{volume}_{i'}) / dr_{i'} + tad + \text{dist}_{pp'} / (24v_s) - M_t(1 - Y_{is}) \quad \forall i \in I, \quad s \in S, \quad (p, p') \in P : (i \notin OB_s) \cap (i \in LP_p) \cap (i \in DP_{p'}) \quad (14)$$

$$TV_{p's} \geq TV_{ps} + \sum_{i' \in I: (i' \notin OB_s) \cap (i' \in LP_p)} (Y_{i's} \text{volume}_{i'}) / lr_{i'} + \sum_{i' \in I: i' \in DP_p} (Y_{i's} \text{volume}_{i'}) / dr_{i'} + tad + (\text{dist}_{pp'} + \text{diff}) / (24v_s) - M_t(1 - Y_{is}) \quad \forall i \in I, \quad s \in S, \quad (p, p') \in P : (i \notin OB_s) \cap (i \in LP_p) \cap (i \in DP_{p'}) \quad (14')$$

Finally, the overall traveling time for the tour assigned to ship s is computed by Eq. (15). The duration of each trip is computed by adding the sum of both the inspection time and the time required to perform discharged activities to the service initial time at the port last visited.

$$TTV_s \geq TV_{ps} + tad + \sum_{i \in I: i \in DP_p} (Y_{is} \text{volume}_i) / dr_i \quad \forall p \in P, \quad s \in S \quad (15)$$

If the parameter diff takes a value greater than 0, Eq. (15) should be replaced by Eq. (15').

$$TTV_s \geq TV_{ps} + tad + \sum_{i \in I: i \in DP_p} (Y_{is} \text{volume}_i) / dr_i - (OV_{ps} - 1)\text{diff} / (24v_s) \quad \forall p \in P, \quad s \in S \quad (15')$$

3.5. Time-window constraints

The pickup of a cargo i must be started within the specified time windows $[ept_i, lpt_i]$. In this way, Eq. (16) prohibits starting the load of cargo i after the allowed latest time lpt_i . It is assumed that the ship requires half of the total administrative time, before it can serve the cargo (Jetlund & Karimi, 2004).

$$TV_{ps} \leq lpt_i - 0.5tad + M_t(1 - Y_{is}) \quad \forall i \in I, \quad s \in S, \quad p \in P : (i \notin OB) \cap (i \in LP_p) \quad (16)$$

Moreover, Eqs. (17) and (18) state that a ship s cannot start the service of an assigned cargo i before its earliest time window ept_i . Then, the arrival time at the next port must exceed the earliest time that the cargo is available for pickup, plus the cargo loading time, plus half administrative time, plus time for sailing to the next port.

$$TV_{p's} \geq ept_i + 0.5tad + dist_{pp'}/(24v_s) + Y_{is} volume_i / lr_i - M_t(1 - PR_{pp's}) - M_t(2 - X_{ps} - X_{p's}) - M_t(1 - Y_{is}) \quad \forall (p, p') \in P, \quad s \in S, \quad i \in I : (p < p') \cap (i \notin OB) \cap (i \in LP_p) \quad (17)$$

$$TV_{ps} \geq ept_i + 0.5tad + dist_{p'p}/(24v_s) + Y_{is} volume_i / lr_i - M_t PR_{pp's} - M_t(2 - X_{ps} - X_{p's}) - M_t(1 - Y_{is}) \quad \forall (p, p') \in P, \quad s \in S, \quad i \in I : ((p < p') \cap (i \notin OB_s) \cap (i \in LP_{p'})) \quad (18)$$

In case the matrix of distances does not satisfy the triangle inequality, then the above time windows constraints should be rewritten as follows:

$$TV_{ps} \leq lpt_i + (OV_{ps} - 1)diff/(24v_s) - 0.5tad + M_t(1 - Y_{is}) \quad \forall i \in I, \quad s \in S, \quad p \in P : (i \notin OB_s) \cap (i \in LP_p) \quad (16')$$

$$TV_{p's} \geq ept_i + OV_{ps}diff/(24v_s) + 0.5tad + dist_{pp'}/(24v_s) + Y_{is} volume_i / lr_i - M_t(1 - PR_{pp's}) - M_t(2 - X_{ps} - X_{p's}) - M_t(1 - Y_{is}) \quad \forall (p, p') \in P, \quad s \in S, \quad i \in I : ((p < p') \cap (i \notin OB_s) \cap (i \in LP_p)) \quad (17')$$

$$TV_{ps} \geq ept_i + OV_{p's}diff/(24v_s) + 0.5tad + dist_{p'p}/(24v_s) + Y_{is} volume_i / lr_i - M_t PR_{pp's} - M_t(2 - X_{ps} - X_{p's}) - M_t(1 - Y_{is}) \quad \forall (p, p') \in P, \quad s \in S, \quad i \in I : ((p < p') \cap (i \notin OB_s) \cap (i \in LP_{p'})) \quad (18')$$

3.6. Ship capacity constraints

Eq. (19) states that the tonnes collected by ship s after visiting the first port $p \in IP_s$ must be greater than or equal to the initial cargos plus the cargos $i \in LP_p$ served by s ($Y_{is} = 1$). Besides, Eq. (20) enforces that the total tonnes discharged from ships s after visiting port $p \in IP_s$ has as lower bound the total amount of cargos served by s that are discharged in p ($i \in DP_p$ and $Y_{is} = 1$).

$$LOAD_{ps} \geq \sum_{i \in I : i \in OB_s} Y_{is} volume_i + \sum_{i \in I : (i \notin OB_s) \cap (i \in LP_p)} Y_{is} volume_i \quad \forall p \in P, \quad s \in S : p \in IP_s \quad (19)$$

$$UNLOAD_{ps} \geq \sum_{i \in DP_p} Y_{is} volume_i \quad \forall p \in P, \quad s \in S : p \in IP_s \quad (20)$$

If two ports (p, p') are on the same route ($X_{ps} = X_{p's} = 1$, for some ship s) and port p is earlier visited ($PR_{pp's} = 1$), then the total cargo collected up to port p' ($LOAD_{p's}$) must be always greater than $LOAD_{ps}$ by at least the tonnes load in port p' . If node p is visited later ($PR_{pp's} = 0$), the reverse statement holds. Such conditions are enforced by the pair of Eqs. (21) and (22). By definition, M_c is an upper bound on the total tonnes collected or discharged along the assigned route. Similarly, Eq. (23) states that the tonnes discharged up to port p' ($UNLOAD_{p's}$) must be always greater than $UNLOAD_{ps}$ by at least the cargos discharged in p' . In case port p' is visited earlier, Eq. (23) becomes redundant and Eq. (24) is activated.

$$LOAD_{p's} \geq LOAD_{ps} + \sum_{i \in I : (i \notin OB_s) \cap (i \in LP_{p'})} (Y_{is} volume_i) - M_c(1 - PR_{pp's}) - M_c(2 - X_{ps} - X_{p's}) \quad \forall (p, p') \in P, \quad s \in S : p < p' \quad (21)$$

$$LOAD_{ps} \geq LOAD_{p's} + \sum_{i \in I : (i \notin OB_s) \cap (i \in LP_p)} (Y_{is} volume_i) - M_c PR_{pp's} - M_c(2 - X_{ps} - X_{p's}) \quad \forall (p, p') \in P, \quad s \in S : p < p' \quad (22)$$

$$UNLOAD_{p's} \geq UNLOAD_{ps} + \sum_{i \in DP_{p'}} (Y_{is} volume_i) - M_c(1 - PR_{pp's}) - M_c(2 - X_{ps} - X_{p's}) \quad \forall (p, p') \in P, \quad s \in S : p < p' \quad (23)$$

$$UNLOAD_{ps} \geq UNLOAD_{p's} + \sum_{i \in DP_p} (Y_{is} volume_i) - M_c PR_{pp's} - M_c(2 - X_{ps} - X_{p's}) \quad \forall (p, p') \in P, \quad s \in S : p < p' \quad (24)$$

The tonnes transported by ship s just after leaving a port p can be computed as the difference between $LOAD_{ps}$ and $UNLOAD_{ps}$. Eq. (25) specifies that the maximum carrying capacity of ship s , given by parameter $vmax_s$, cannot be exceeded. In turn, Eq. (26) establishes a lower bound on the load transported.

$$LOAD_{ps} - UNLOAD_{ps} \leq vmax_s X_{ps} \quad \forall p \in P, \quad s \in S \quad (25)$$

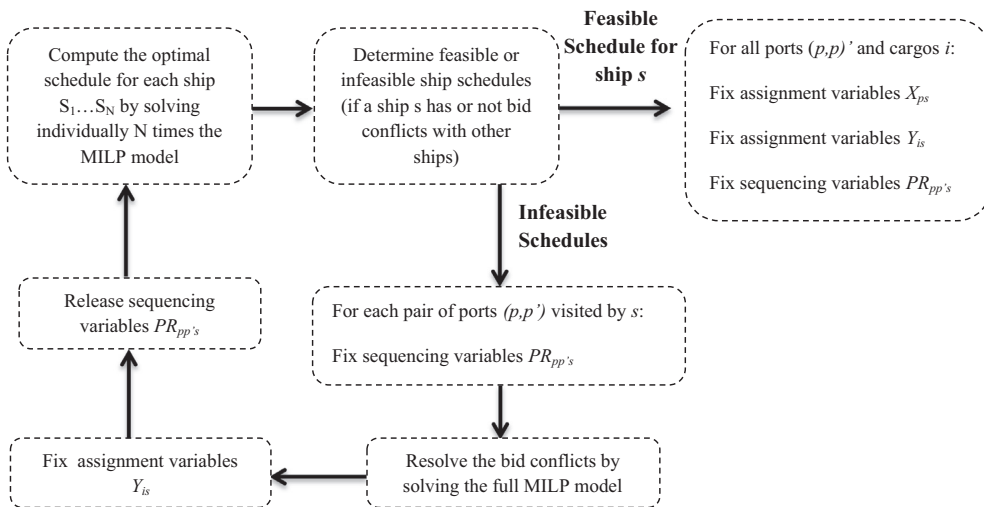


Fig. 2. Representative scheme of the iterative MILP-based algorithm proposed for the ship routing and scheduling problem.

```

01 feasible solution condition <- false
02 WHILE feasible solution condition not meet DO

// 1. Determine the best schedule for each open ship by individually solving the MILP
model. At first iteration, all ship tours are open and all cargos are mobile.
03   FOR each open ship s DO
04     solve the MILP model for s
05     FOR each mobile cargo i DO
06       set auxiliary variable carryis <- optimal value of assignment variable Yis
07     END FOR
08     FOR each pair of ports (p, p') that are visited by s DO
09       fix sequencing variable PRpp's <- optimal value of sequencing variable PRpp's
10     END FOR
11   END FOR

//2. Determine the ship schedules that can be closed.
12   FOR each open ship s DO
13     according to the values of carryis, determine if there are no ship conflicts
14     IF ship s has a feasible schedule THEN
15       close the tour of ship s
16       FOR each cargo i DO
17         fix assignment variable Yis <- carryis
18         IF Yis is equal to one THEN
19           categorize cargo i as closed
20         END IF
21       END FOR
22     END IF
23   END FOR

//3. Determine if a feasible solution has been found or the procedure must follow
iterating to solve bid conflicts.
24   IF all ship schedules are closed THEN
25     feasible solution condition <- true
26   ELSE
27     FOR each mobile cargo i not served by any ship DO
28       FOR each open ship s DO
29         fix assignment variable Yis <- 0
30       END FOR
31     END FOR
32     Solve the MILP model for all open ships
33     FOR each mobile cargo I DO
34       FOR each open ship s DO
35         IF carryis is equal to 1 and Yis is equal to 0 THEN
36           fix assignment variable Yis <- 0
37         END IF
38       END FOR
39     END FOR

//4. Release sequencing and assignment variables
40     FOR each mobile cargo i not served by any ship DO
41       FOR each open ship s DO
42         Release assignment variable Yis
43       END FOR
44     END FOR

45     FOR each pair of ports (p, p') DO
46       FOR each open ship s DO
47         Release sequencing variable PRpp's
48       END FOR
49     END FOR
50   END IF
51 END WHILE

```

Fig. 3. Pseudo-code of the iterative MILP-based algorithm proposed for the ship routing and scheduling problem.

$$LOAD_{ps} - UNLOAD_{ps} \geq 0 \quad \forall p \in P, \quad s \in S \quad (26)$$

In addition, the total tonnes loaded on ship s after visiting a port p can never be greater than the total cargos collected by ship s along the tour plus the initial cargos. Similarly, the total cargo unloaded from s after visiting port p can never be greater than the total amount discharged along the whole tour. Thus, Eqs. (27) and (28) establish upper bounds on the values of $LOAD_{ps}$ and $UNLOAD_{ps}$.

$$LOAD_{ps} - \sum_{i \in I} Y_{is} \text{volume}_i \leq v_{\max}(1 - X_{ps}) \quad \forall p \in P, \quad s \in S \quad (27)$$

$$UNLOAD_{ps} - \sum_{i \in I} Y_{is} \text{volume}_i \leq v_{\max}(1 - X_{ps}) \quad \forall p \in P, \quad s \in S \quad (28)$$

3.7. Objective function

The objective function is to maximize the net profit. This is determined by the revenues from all assigned cargos minus operation expenses, including all port charges, the time-charter cost, and the fuel cost. In Eq. (29), parameter sr_i stands for the shipping rate for cargo i . Since the route duration includes all travel times, waiting times and service times, the parameter tcc_s refers to the time-charter cost. In addition fc_s denotes the fuel cost per unit distance while pc_{ps} stands for the fixed cost that ship s pays when it visits port p .

$$\text{Max} \left[\sum_{s \in S} \sum_{i \in I} sr_i Y_{is} - \sum_{s \in S} tcc_s TTV_s - \sum_{s \in S} fc_s TTD_s - \sum_{s \in S} \sum_{p \in P} pc_{ps} X_{ps} \right] \quad (29)$$

4. The iterative MILP-based algorithm

Real-world ship routing and scheduling problem are commonly characterized by a high combinatorial complexity that easily

exceeds the capabilities of current optimization codes. Consequently, the mathematical model presented in Section 3 could not find the optimal solutions to these hard problems in a reasonable amount of time when a large number of ports, cargos, and ships is considered. To overcome such a limitation, effective modeling techniques and solution strategies are required. In this way, this section describes a new iterative MILP-based algorithm that combines the robustness of the traditional MILP formulations with the inherent benefits of heuristic rules.

Basically, the procedure is based on a systematic decomposition strategy that, by solving highly constrained versions of the model in each iteration, allows that the number of decisions be maintained at a reasonable level, even for large-scale problems. The idea is that the computational efficiency of the MILP branch-and-bound solution procedures can be improved by fixing the value of a set of binary variables. The general structure of the proposed algorithm is shown in Fig. 2. At first, the solution approach is decomposed by individually scheduling, in few seconds of CPU time, each ship tour. Since such strategy can return infeasible solutions for the whole problem because a cargo can be served by more than one ship, the developed algorithm aims gradually to build a feasible solution by solving a highly constrained version of the full-space model, generated by fixing a subset of the binary variables. Otherwise, if some ship s has a feasible schedule, it is assumed by the algorithm as the best solution for s . The procedure continues until all ship schedules become feasible.

The detailed pseudo-code of the iterative algorithm is depicted in Fig. 3 and described next as a four-step procedure. First (lines 3–11), the optimal schedule for every “open trip” is determined by individually solving the corresponding one-ship model. An “open trip” is one whose final schedule has not been defined. In addition, potential cargos to be served can be classified into two types: fixed and mobile depending on whether they should be stay on a determined trip or can be transferred to other ones. At beginning of procedure, all trips are open and all cargos are mobile. As several iterations are executed in this step (one for every ship or open trip) and the value of binary variable Y_{is} can be rewritten, a 0–1 auxiliary variable $carry_{is}$ is used to save the information

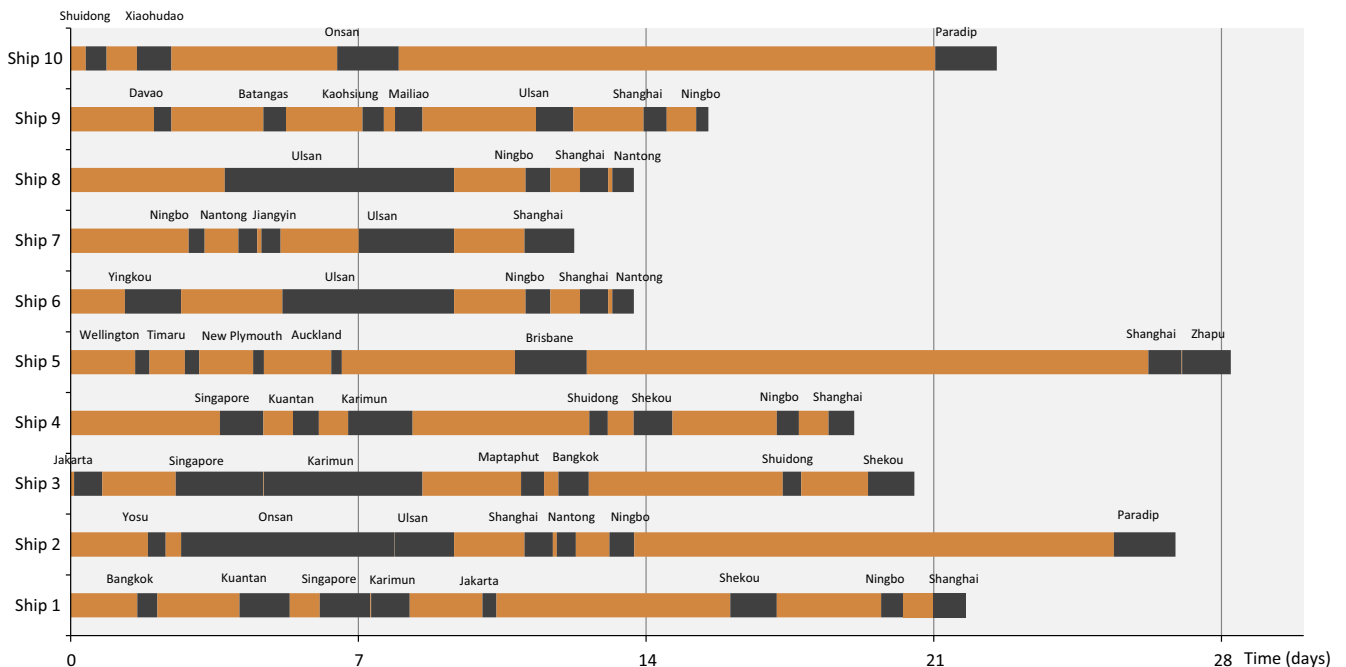


Fig. 4. Optimal schedules found by solving individually the exact formulation for every ship.

Table 3

Detailed optimal schedules found by solving individually the exact formulation for every ship.

	Port	Arrival time (day)	Departure time (day)	Cargoes loaded	Cargoes unloaded	Used capacity (% tonnes)
S1	Bangkok	1.62	2.11	–	43–46	27.08
USD 156237	Kuantan	4.11	5.33	21–22, 25	47	32.62
	Singapore	6.06	7.29	17–19	48–50	56.09
	Karimun	7.31	8.25	2, 5, 9	–	78.79
	Tanjung Priok	10.02	10.36	–	51–52	74.97
	Shekou	16.05	17.18	–	2, 17–19	36.37
	Ningbo	19.72	20.26	–	9	23.64
	Shanghai	20.98	–	–	5, 21–22, 25	0.00
S2	Yosu	1.88	2.31	–	53	5.45
USD 191287	Onsan	2.69	7.87	36	–	60.00
	Ulsan	7.88	9.33	11–15, 38	54, 55	97.98
	Shanghai	11.04	11.73	–	13, 14, 38	78.85
	Nantong	11.83	12.29	–	15	69.75
	Ningbo	13.11	13.71	–	11, 12	54.54
	Paradip	25.38	–	–	36	0.00
S3	Tanjung Priok	0.083	0.768	–	56	0.00
USD 143261	Singapore	2.556	4.687	17–19	–	33.18
	Karimun	4.703	8.561	1, 2, 6, 30–32	–	82.16
	Maptaphut	10.958	11.520	–	30, 31	68.53
	Bangkok	11.869	12.607	–	6, 32	47.24
	Shuidong	17.325	17.773	–	1	38.6
	Shekou	18.398	–	–	2, 17–19	0.00
S4	Singapore	3.63	4.69	17	–	69.51
USD 193108	Kuantan	5.41	6.04	21, 25	–	91.46
	Karimun	6.75	8.32	1, 2, 5, 9	57	96.91
	Shuidong	12.62	13.07	–	1	85.26
	Shekou	13.70	14.64	–	2, 17	45.13
	Ningbo	17.18	17.72	–	9	28.06
	Shanghai	18.44	–	–	5, 21, 25	0.00
S5	Wellington	1.57	1.92	–	58	7.31
USD 171609	Timaru	2.78	3.13	–	59	1.22
	New Plymouth	4.44	4.70	–	60	0.61
	Auckland	6.34	6.60	–	61	0.00
	Brisbane	10.81	12.56	40, 41	–	87.80
	Shanghai	26.22	27.03	–	40	54.88
	Zhapu	27.04	–	–	41	0.00
S6	Yingkou	1.32	2.69	–	62	0.00
USD 138912	Ulsan	5.15	9.33	11–16, 38	–	87.81
	Ningbo	11.07	11.67	–	11, 12	58.97
	Shanghai	12.39	13.08	–	13, 14, 38	22.67
	Nantong	13.18	–	–	15, 16	0.00
S7	Ningbo	2.87	3.26	–	64	78.47
USD 115221	Nantong	4.08	4.54	–	63	61.21
	Jiangyin	4.64	5.11	–	65	43.10
	Ulsan	7.01	9.33	13, 14, 38	–	79.40
	Shanghai	11.04	–	–	13, 14, 38, 66, 67	0.00
S8	Ulsan	3.75	9.33	11–16, 38	–	87.81
USD 46841	Ningbo	11.07	11.67	–	11, 12	58.97
	Shanghai	12.39	13.08	–	13, 14, 38	22.67
	Nantong	13.18	–	–	15, 16	0.00
S9	Davao	2.03	2.45	–	68	81.72
USD 158610	Batangas	4.69	5.24	–	69–71, 74	56.90
	Kaohsiung	7.10	7.62	–	73	34.48
	Mailiao	7.89	8.56	–	72	0.00
	Ulsan	11.32	12.23	11–14	–	54.79
	Shanghai	13.94	14.50	–	13, 14	28.84
	Ningbo	15.22	–	–	11, 12	0.00
S10	Shuidong	0.37	0.87	–	75, 76	47.5
USD 216097	Xiaohudao	1.61	2.45	–	77–79	0.00
	Onsan	6.48	7.98	36	–	100.00
	Paradip	21.03	–	–	36	0.00

defining that a cargo i is served by ship s . After selecting the cargoes to be collected by every ship, the values of binary sequencing variable $PR_{pp's}$ are fixed to their current ones. In other words, the ship routes cannot be modified in the following algorithm steps.

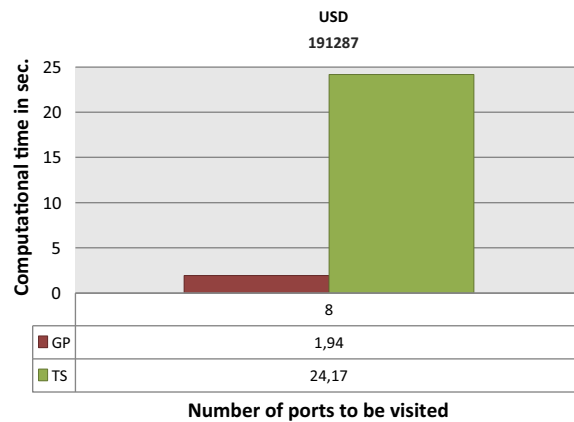
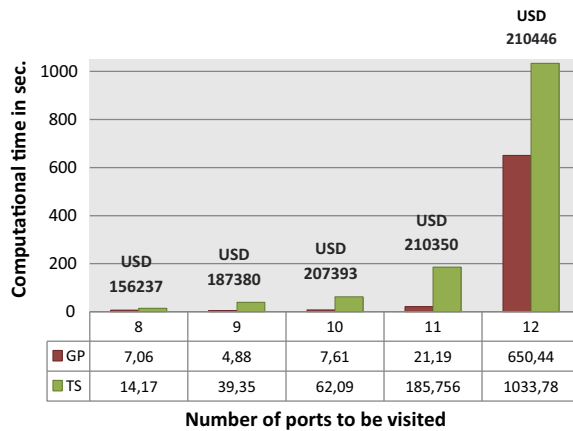
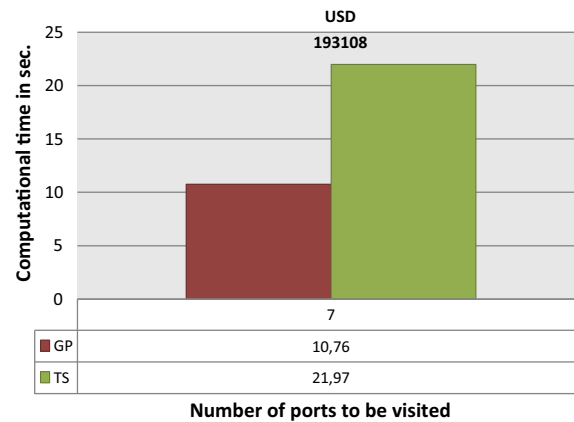
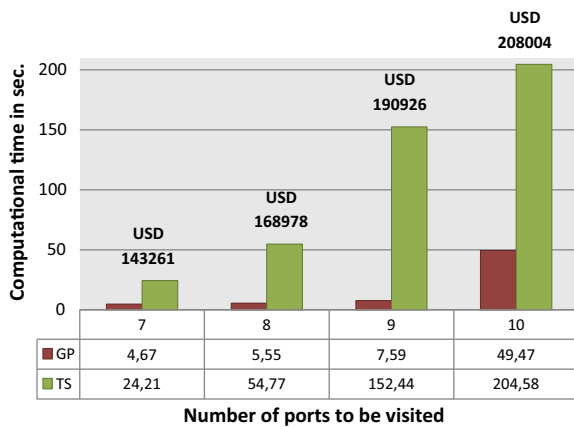
In the second step (lines 12–23), every trip will be classified as open or closed. According to the values of variable $carry_{is}$, it is

possible to determine if a ship s has bid conflicts with the other ones (for some cargo i : $carry_{is} = 1$ and $carry_{is'} = 1$, $s \neq s'$). If such condition is false, the best feasible schedule for ship s has been achieved by the procedure. Consequently, the trip must be categorized as closed and all cargoes served by s are classified as fixed. It allows us to remove a significant number of binary variables from

Table 4

Computational statistics for the exact optimization models.

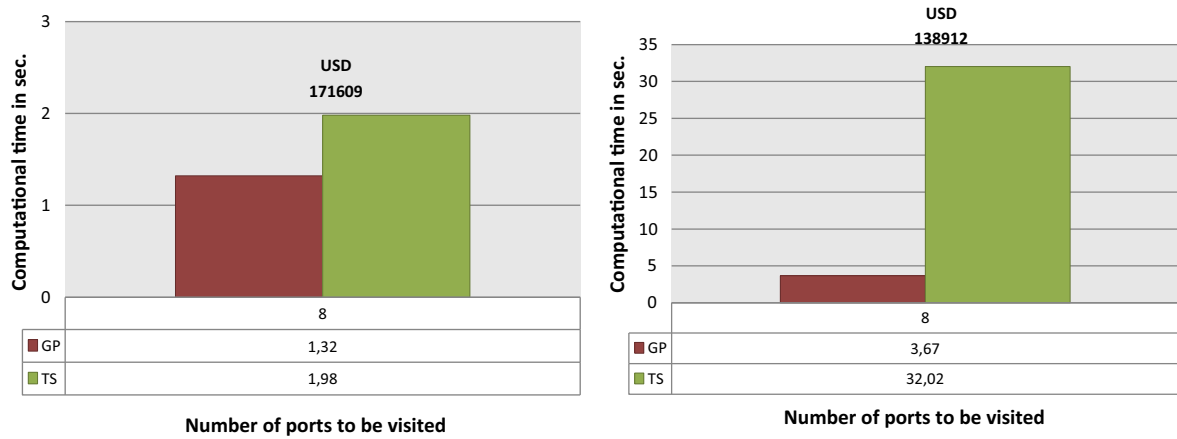
	CPU time (s)	Integrality GAP (%)	Objective function	Binary variables	Continuous variables	Linear constraints
Ship 1	7.06	–	156237.45	928	183	9379
Ship 2	1.94	–	191287.65	928	183	9375
Ship 3	4.67	–	143261.45	928	183	9375
Ship 4	10.76	–	193108.06	928	183	9375
Ship 5	1.32	–	171609.13	928	183	9375
Ship 6	3.67	–	138911.88	928	183	9375
Ship 7	1.99	–	115211.13	928	183	9378
Ship 8	1.59	–	46841.15	928	183	9377
Ship 9	5.74	–	158610.35	928	183	9375
Ship 10	1.59	–	216096.52	928	183	9376
All ships	34683.70 ^a	41.45	1041923.34	7450	1821	93,373

^a MIP solver terminated because memory capacity was exceeded.**Figs. 5 and 6.** Comparison of computational requirements for solving the problem instances for S1 and S2.**Figs. 7 and 8.** Comparison of computational requirements for solving the problem instances for S3 and S4.

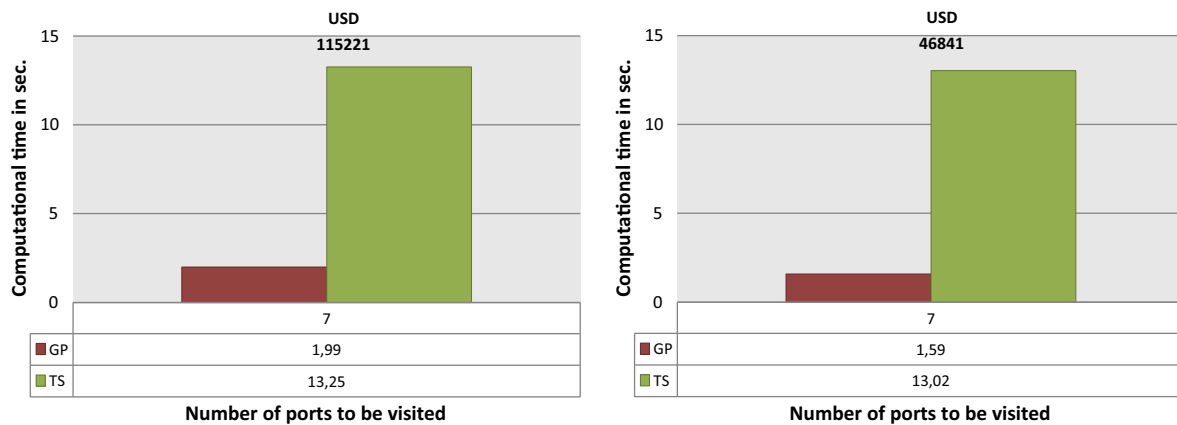
the original model because the values of assignment and sequencing variables Y_{is} , X_{ps} , and PR_{pps} are fixed and will not be considered for next iterations. By repeating this step at every iteration k , the proposed approach generates the best sequence of feasible schedules. When all trip schedules becomes feasible and, no cargos are served by more than one ship, the procedure is stopped and the best routing and scheduling of every ship is given as solution.

The next step is to decide about bid conflicts between ships (lines 24–39). As the values of binary sequence variable PR_{pps} have already been fixed and the best schedules for some ships have been

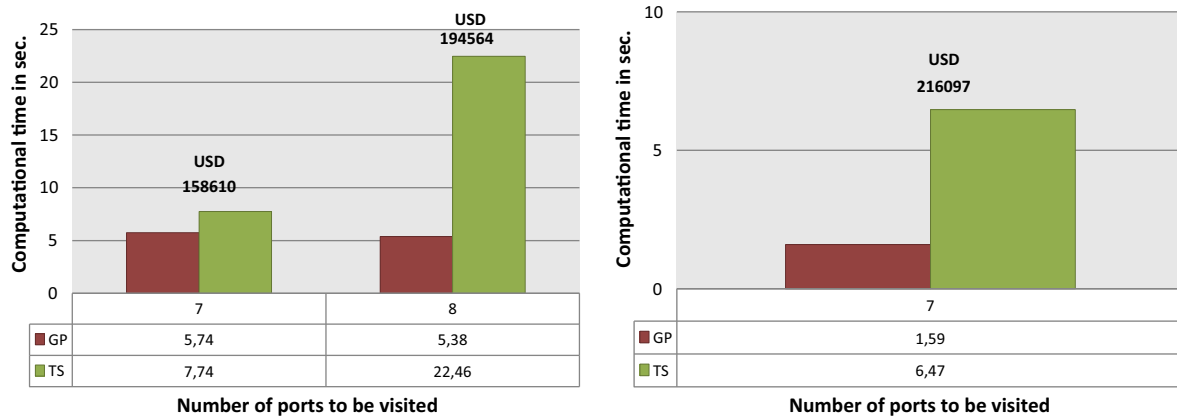
defined, a highly constrained model from original MILP formulation model has to be solved in this stage. In addition, in an attempt to maintain manageable model sizes, assignment variables Y_{is} for cargos not included in any ship schedule are all fixed to 0. Therefore, only the profitable cargos can freely be assigned to any open ship. Then, the algorithm determines to which ship is assigned each disputed cargo conserving the optimal routes fixing above. This raises a trade-off between routing cost and revenues obtained from serviced cargos. If some ship loses a cargo, its assigned route provided by step 1 should be improved by



Figs. 9 and 10. Comparison of computational requirements for solving the problem instances for S5 and S6.



Figs. 11 and 12. Comparison of computational requirements for solving the problem instances for S7 and S8.



Figs. 13 and 14. Comparison of computational requirements for solving the problem instances for S9 and S10.

modifying or reordering the ports on its individual trip. To do that, in the fourth step (lines 40–51), the sequencing variables PR_{pps} are released and the procedure is repeated beginning with the first step. The procedure is continued until every cargo is assigned at most to one ship.

5. Results and discussion

The proposed exact formulation and the iterative MILP-based algorithm have been tested by solving a real industrial case study dealing with a chemical shipping company that operates a fleet of

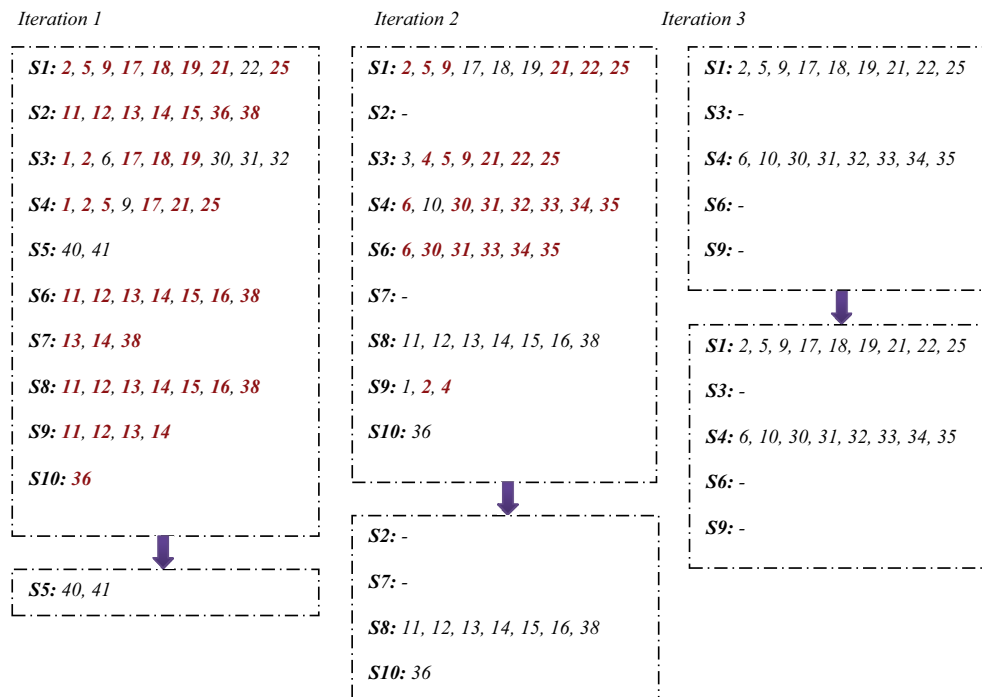


Fig. 15. The final results of each iteration executed by the MILP-based algorithm.

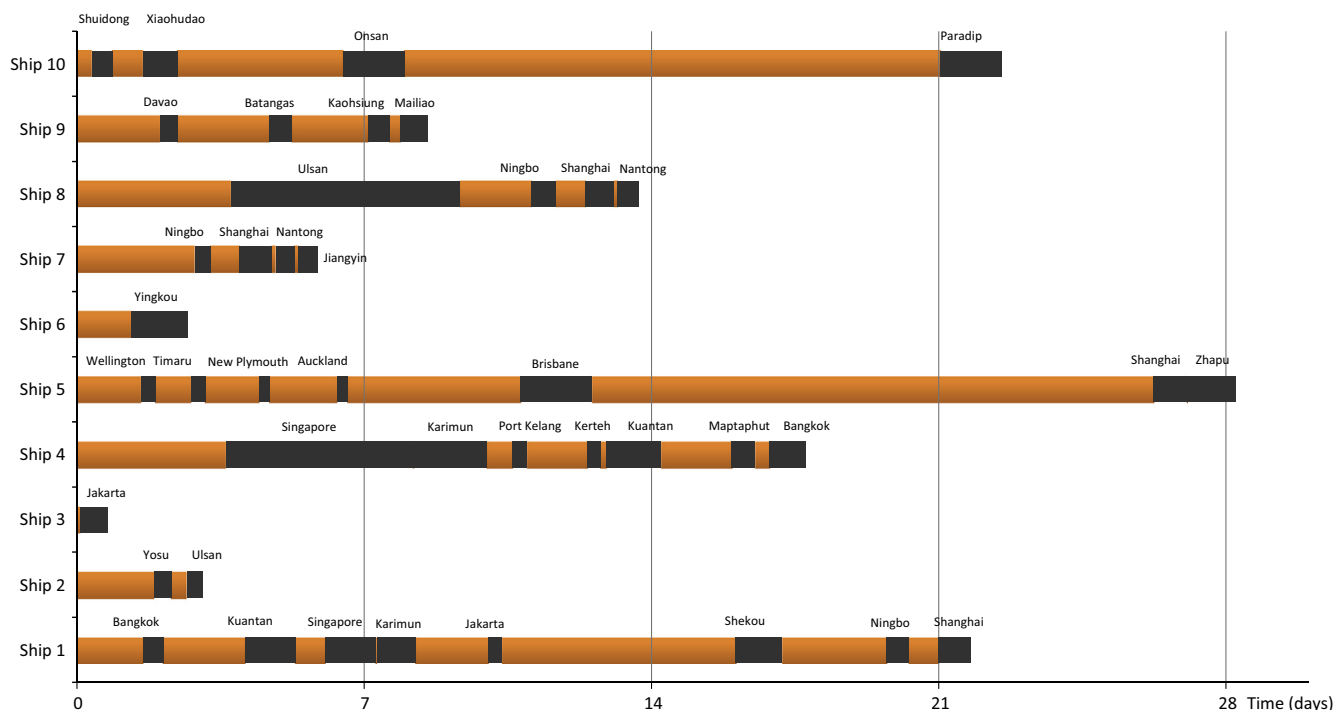


Fig. 16. Best schedules found for the shipping company.

small, multi-parcel ships in the Asia Pacific Region. Such example, previously tackled by [Jetlund and Karimi \(2004\)](#) and [Aang \(2006\)](#), comprises a sea-cargo shipping company operating with 10 ships, 36 ports, and 79 potential cargos. Among these 79 potential cargos, 37 cargos are on-board of ships at beginning of planning horizon to be delivered to their destinations, while that the remaining 42 are

new potential cargos that can be served during the planning horizon (pickup and delivery). Information related to ship characteristics is given in [Table A.1](#) of [Appendix A](#), while [Table A.2](#) provides the loading/discharging ports, volumes, service revenues, and pickup time windows for all cargos. However, if a cargo was loaded in its corresponding ship at beginning of planning horizon, only its

Table 5

Detailed best schedules found for the shipping company.

	Port	Arrival time (day)	Departure time (day)	Cargoes loaded	Cargoes unloaded	Used capacity (% tonnes)
S1	Bangkok	1.62	2.11	–	43–46	27.08
USD 156237	Kuantan	4.11	5.33	21–22, 25	47	32.62
	Singapore	6.06	7.29	17–19	48–50	56.09
	Karimun	7.31	8.25	2, 5, 9	–	78.79
	Tanjung Priok	10.02	10.36	–	51–52	74.97
	Shekou	16.05	17.18	–	2, 17–19	36.37
	Ningbo	19.72	20.26	–	9	23.64
	Shanghai	20.98	–	–	5, 21–22, 25	0.00
S2	Yosu	1.88	2.31	–	53	5.45
USD 15181	Ulsan	2.68	–	–	54, 55	0.00
S3	Tanjung Priok	0.08	–	–	56	0.00
USD 62722						
S4	Singapore	3.63	8.20	33	–	40.85
USD 131935	Karimun	8.21	9.99	6, 10, 30–32	57	57.22
	Port Kelang	10.62	10.97	–	10	51.12
	Kerteh	12.44	12.76	–	33	46.85
	Kuantan	12.90	14.23	34, 35	–	55.39
	Maptaphut	15.96	16.52	–	30, 31	37.10
	Bangkok	16.87	–	–	6, 32, 34, 35	0.00
S5	Wellington	1.57	1.92	–	58	7.31
USD 171609	Timaru	2.78	3.13	–	59	1.22
	New Plymouth	4.44	4.70	–	60	0.61
	Auckland	6.34	6.60	–	61	0.00
	Brisbane	10.81	12.56	40, 41	–	87.80
	Shanghai	26.22	27.03	–	40	54.88
	Zhapu	27.04	–	–	41	0.00
S6	Yingkou	1.32	–	–	62	0.00
USD 70772						
S7	Ningbo	2.87	3.25	–	64	78.47
USD 103312	Shanghai	3.97	4.75	–	66, 67	35.36
	Nantong	4.84	5.30	–	63	18.10
	Jiangyin	5.40	–	–	65	0.00
S8	Ulsan	3.75	9.33	11–16, 38	–	87.81
USD 46842	Ningbo	11.07	11.67	–	11, 12	58.97
	Shanghai	12.39	13.08	–	13, 14, 38	22.67
	Nantong	13.18	–	–	15, 16	0.00
S9	Davao	2.03	2.45	–	68	81.72
USD 125291	Batangas	4.69	5.24	–	69–71, 74	56.90
	Kaohsiung	7.10	7.62	–	73	34.48
	Mailiao	7.89	–	–	72	0.00
S10	Shuidong	0.37	0.87	–	75, 76	47.5
USD 216097	Xiaohudao	1.61	2.45	–	77–79	0.00
	Onsan	6.48	7.98	36	–	100.00
	Paradip	21.03	–	–	36	0.00

discharge port, volume, and shipping rate are specified in Table A.2. From this table, it follows that transshipment cargoes have a time interval within pickup service must begin. Such interval is defined by both an earliest pickup time and a latest pickup time.

The carrier operates in Asia, but the fleet also serves Australia, India, and the Middle East. Every ship can perform pickup and delivery tasks in multiples ports but the number of visited ports must never exceed a maximum amount K defined for every ship. The scheduler planner decides the value of K . Based on real data from the company, the stop time at each port for performing pickup and/or delivery operations comprises a fixed time $tad = 6$ h and a variable time period that directly increases with the total cargo to be loaded/discharged at a rate of $lr_i = dr_i = 200$ tonnes/h. At every ship stop, the carrier has to pay a port charge pc_{ps} . As shown in Table A.3, this port expense depends on the size or capacity of the ship. Traveling time between two ports can be defined by route length (nautical miles) and the sailing speed of every ship (knots). The average speed of all ships is $v_s = 13$ knots. Distances between ports in nm are given in Tables A.4 and A.5. The planning horizon is usually 3–4 weeks long.

On the one hand, the performance of the proposed MILP formulation was tested by individually solving the ship routing and scheduling problem for each of 10 ships integrating the company's fleet. Then, the results obtained were compared the ones reported by Jetlund and Karimi (2004) and Aang (2006) whose MILP formulations use variable-length slots. On the other hand, the MILP model also has been tested on the full example in order to underline the high combinatorial complexity of the problem addressed and to highlight the remarkable benefits of the proposed iterative MILP-based algorithm, which is able to provide high-quality solutions with a remarkable computational efficiency. All alternatives were solved by using a DELL PRECISION T5500 Workstation with six-core Intel Xeon Processor (2.67 GHz) and the modeling language GAMS and CPLEX 12.2 as the MILP solver.

5.1. Validation of the exact optimization approach

On the one hand, the proposed MILP formulation was tested by individually solving the ship routing and scheduling problem for each of 10 ships. The optimal routes and schedules generated by

Table 6

Comparison of different solution strategies for the real-world problem.

	Jetlund and Karimi (2004)		Aang (2006)		Exact optimization approach		Iterative MILP-based algorithm	
	Cargos served	Total profit (USD)	Cargos served	Total profit (USD)	Cargos served	Total profit (USD)	Cargos served	Total profit (USD)
S1	1, 5, 8–9, 18–19, 21, 25, 43–52	99696	2, 5, 9, 17–19, 21–22, 25, 43–52	156237	10, 43–52	51956	2, 5, 9, 17–19, 21–22, 25, 43–52	156237
S2	11, 13, 36, 53–55	113177	53–55	15181	53–55	15181	53–55	15181
S3	6, 30–32, 56	63218	56	62722	56	62722	56	62722
S4	2–4, 7, 10, 17, 22–24, 57	116228	6, 8, 10, 30–32, 34–35, 57	128518	1, 2, 6, 17, 18, 30, 31, 57	178143	6, 10, 30–35, 57	131935
S5	40–41, 58–61	171609	40–41, 58–61	171609	40–41, 58–61	171609	40–41, 58–61	171609
S6	12, 14–16, 38	73991	62	70772	62	70772	62	70772
S7	63–67	103312	63–67	103312	63–67	103312	63–67	103312
S8	–	–40887	11–16, 38	46841	11–16, 38	46841	11–16, 38	46841
S9	68–74	125291	68–74	125291	68–74	125291	68–74	125291
S10	75–79	108997	36, 75–79	216097	36, 75–79	216097	36, 75–79	216097
		934632 ^a		1096580 ^b		1041923 ^c		1099997 ^d

^a Net profit founded by executing our MILP model with the proposed solution.^a The CPU time is not reported.^b The CPU time reported is 62067 s.^c MIP solver terminated in 34684 s because memory capacity was exceeded.^d The procedure ended in 764 s.**Table A.1**

Ship characteristics.

	Size (tonnes)	Cost (USD/day)	Immediate destination	Arrival time (day)	Fuel cost (USD/nm)	Maximum number <i>K</i> of ports to be visited
S1	11000	9000	Bangkok	1.620	7.18	8
S2	11000	9000	Yosu	1.875	7.18	8
S3	11000	8000	Tanjong Priok	0.083	7.18	7
S4	8200	8000	Singapore	3.625	6.15	7
S5	8200	7000	Wellington	1.573	6.15	8
S6	5800	7000	Yingkou	1.323	6.15	8
S7	5800	7000	Ningbo	2.865	6.15	7
S8	5800	7000	Ulsan	3.750	6.15	7
S9	5800	7000	Davao	2.031	6.15	7
S10	6000	7000	Shuidong	0.367	6.15	7

the mathematical model are depicted in Fig. 4. For ship schedules, port activities are represented with black rectangles while sailing operations are showed in orange color. More details are given in Table 3, including the times at which ships arrive to the ports, together with departure times and pickup/delivery operations performed at each visited port. Moreover, the used volume ship capacity and the optimal net profits are also given in Table 3. Such solutions match exactly with the ones obtained by solving the MILP models proposed by Jetlund and Karimi (2004) and Aang (2006). From Table 3, it follows that the 10 individual schedules do not provide a feasible solution to the entire multi-ship problem. Since only one ship is considered at a time, the final solution shows that there are several cargos that are served by multiple ships. For instance, ships 2, 6, 7, and 8 select to transport cargo 38. On the other hand, cargos 3, 4, 7, 8, 10, 20, 23, 24, 26–29, 33–35, 37, 39, and 42 are not served by any ship. The required CPU time, the amount of linear constraints and the number of binary variables are given in Table 4.

On the other hand, the last row of Table 4 reports the computational information of the full problem instance. Note that this complex instance results into a MILP model of 93723 equations, 7450 binary variables, and 1821 continuous variables. The MIP solver terminated in 34684 s because memory capacity was exceeded. The best solution reported was USD 1041923 (see Table 6), with

an integrality gap of 41.45%. However, in next subsection we will show that a best solution can be quickly discovered by the proposed iterative approach with a more reasonable computational cost.

In order to test the effectiveness and current limitations of precedence-based formulation and time slots-based representations, some instances previously studied were again solved but this time increasing the number of ports that can be visited by every ship. This idea comes from Jetlund and Karimi (2004) whose proposed that a set of time slots can be postulated for every ship in order to allocate them to the sailing legs *K* that the planner wanted to schedule. The computational effort and objectives values are summarized in Figs. 5–14. The CPU times in seconds for the global-precedence formulation are represented with red columns while computational times for the time-slots based model are showed¹ in green color. The number of ports to be visited is increased until no-improvement in the net profit is achieved. From the pictures, it can be easily observed that although the usefulness and performance of continuous and time-slots models strongly depends on the particular problem and solution characteristics, the results obtained allow us to draw the following interesting conclusions: (i) the computational cost for both cases grows exponentially with the problem size, thus showing a typical behavior of NP-hard problems, (ii) the time-slots models may generate faster solutions that precedence ones whenever the number of time intervals that are postulated is a good approximation to the real data, and (iii) the precedence-based model were considerably faster than the time-slots formulation.

5.2. Validation of the iterative MILP-based algorithm

After validating the MILP formulation by comparing its results with those provided by other authors in the literature, the proposed MILP-based algorithm was applied to solve the full problem instance involving 10 ships, 79 cargos, and 36 ports. It is worth mentioning that the procedure was executed without setting a time limit to each MILP run. The algorithm converges in three iterations and each of them is detailed in Fig. 15. The complete solution is depicted in Fig. 16 and detailed in Table 5. By analyzing

¹ For interpretation of color in Figs. 5–14, the reader is referred to the web version of this article.

Table A.2

Cargos properties.

	Origin	Destination	Pickup time windows	Volume (tonnes)	Shipping rate (USD)	Status at time zero
C1	Karimun	Shuidong	25–29 April	950	45125	Not served
C2	Karimun	Shekou	25–29 April	596	29800	Not served
C3	Karimun	Taichung	25–29 April	1049	31470	Not served
C4	Karimun	Kaohsiung	25–29 April	700	28000	Not served
C5	Karimun	Shanghai	25–29 April	501	20040	Not served
C6	Karimun	Bangkok	25–29 April	2092	62760	Not served
C7	Karimun	Jasaan	25–29 April	1000	45000	Not served
C8	Karimun	Anyer	25–29 April	1011	28308	Not served
C9	Karimun	Ningbo	23–27 April	1400	50400	Not served
C10	Karimun	Port Kelang	23–27 April	500	30000	Not served
C11	Ulsan	Ningbo	26–29 April	995	33830	Not served
C12	Ulsan	Ningbo	26–29 April	678	23052	Not served
C13	Ulsan	Shanghai	26–29 April	1000	34000	Not served
C14	Ulsan	Shanghai	26–29 April	505	17170	Not served
C15	Ulsan	Nantong	26–29 April	1000	36000	Not served
C16	Ulsan	Nantong	26–29 April	315	11340	Not served
C17	Singapore	Shekou	21–25 April	2700	67500	Not served
C18	Singapore	Shekou	21–25 April	350	8750	Not served
C19	Singapore	Shekou	21–25 April	600	15000	Not served
C20	Singapore	Botany Bay	14–23 April	800	44800	Not served
C21	Kuantan	Shanghai	22–27 April	800	30000	Not served
C22	Kuantan	Shanghai	22–27 April	300	11250	Not served
C23	Kuantan	Ulsan	22–27 April	400	15000	Not served
C24	Kuantan	Ulsan	22–27 April	700	26250	Not served
C25	Kuantan	Shanghai	22–27 April	1000	37500	Not served
C26	Singapore	Kandla	18–25 April	1000	26250	Not served
C27	Singapore	Kandla	18–25 April	500	13125	Not served
C28	Singapore	Kandla	18–25 April	500	13125	Not served
C29	Singapore	Kandla	18–25 April	300	7875	Not served
C30	Karimun	Maptaphut	25–29 April	500	16000	Not served
C31	Karimun	Maptaphut	25–29 April	1000	32000	Not served
C32	Karimun	Bangkok	25–29 April	250	8000	Not served
C33	Singapore	Kerteh	25–30 April	350	23999.5	Not served
C34	Kuantan	Bangkok	01–05 May	500	20000	Not served
C35	Kuantan	Bangkok	01–05 May	200	8000	Not served
C36	Onsan	Paradip	21–25 April	6000	289800	Not served
C37	Ulsan	Lanshantao	21–25 April	300	9000	Not served
C38	Ulsan	Shanghai	21–25 April	600	18000	Not served
C39	Brisbane	Kaohsiung	24–28 April	1100	57607	Not served
C40	Brisbane	Shanghai	24–28 April	2700	141399	Not served
C41	Brisbane	Zhapu	24–28 April	4500	235665	Not served
C42	Brisbane	Taichung	24–28 April	150	7855.5	Not served
C43	Ulsan	Bangkok	–	315	12600	Loaded on ship 1
C44	Ulsan	Bangkok	–	315	12600	Loaded on ship 1
C45	Ulsan	Bangkok	–	315	12600	Loaded on ship 1
C46	Ulsan	Bangkok	–	199	7960	Loaded on ship 1
C47	Ulsan	Kuantan	–	1490	48425	Loaded on ship 1
C48	Ulsan	Singapore	–	455	13650	Loaded on ship 1
C49	Ulsan	Singapore	–	105	3150	Loaded on ship 1
C50	Ulsan	Singapore	–	509	15270	Loaded on ship 1
C51	Ulsan	Tanjung Priok	–	210	15006.6	Loaded on ship 1
C52	Ulsan	Tanjung Priok	–	210	15006.6	Loaded on ship 1
C53	Kuantan	Yosu	–	850	36125	Loaded on ship 2
C54	Kuantan	Ulsan	–	300	12000	Loaded on ship 2
C55	Kuantan	Ulsan	–	300	12000	Loaded on ship 2
C56	Abu Jubail	Tanjung Priok	–	2086	74053	Loaded on ship 3
C57	Ulsan	Karimun	–	3000	120000	Loaded on ship 4
C58	Ulsan	Wellington	–	500	30750	Loaded on ship 5
C59	Ulsan	Timaru	–	500	30750	Loaded on ship 5
C60	Ulsan	New Plymouth	–	50	3075	Loaded on ship 5
C61	Ulsan	Auckland	–	50	3075	Loaded on ship 5
C62	Yosu	Yingkou	–	5359	96462	Loaded on ship 6
C63	Taichung	Nantong	–	1001	36036	Loaded on ship 7
C64	Taichung	Ningbo	–	650	26000	Loaded on ship 7
C65	Taichung	Jiangyin	–	1050	42000	Loaded on ship 7
C66	Mailiao	Shanghai	–	2000	52000	Loaded on ship 7
C67	Mailiao	Shanghai	–	500	13000	Loaded on ship 7
C68	Karimun	Davao	–	800	60000	Loaded on ship 9
C69	Karimun	Batangas	–	350	29998.5	Loaded on ship 9
C70	Karimun	Batangas	–	300	24999	Loaded on ship 9
C71	Kerteh	Batangas	–	500	10800	Loaded on ship 9
C72	Kerteh	Mailiao	–	2000	43200	Loaded on ship 9
C73	Kerteh	Kaohsiung	–	1300	28080	Loaded on ship 9
C74	Kuantan	Batangas	–	300	24999	Loaded on ship 9

(continued on next page)

Table A.2 (continued)

	Origin	Destination	Pickup time windows	Volume (tonnes)	Shipping rate (USD)	Status at time zero
C75	Karimun	Shuidong	–	731	31067.5	Loaded on ship 10
C76	Karimun	Shuidong	–	488	20740	Loaded on ship 10
C77	Karimun	Xiaohudao	–	1000	40000	Loaded on ship 10
C78	Karimun	Xiaohudao	–	1000	26000	Loaded on ship 10
C79	Karimun	Xiaohudao	–	850	22100	Loaded on ship 10

Table A.3

Port charges.

	Port name	Port cost (USD) 9000–11,000 tonnes	Port cost (USD) 6000–9000 tonnes
P1	Anyer	6250	4853
P2	Auckland	5000	4500
P3	Bangkok	6048	4594
P4	Batangas	9370	8230
P5	Botany Bay	10752	9000
P6	Brisbane	6846	5500
P7	Davao	7421	7085
P8	Jasaan	7500	7000
P9	Jiangyin	5224	4599
P10	Kandla	5845	5500
P11	Kaohsiung	4644	4188
P12	Karimun	1988	1629
P13	Kerteh	10957	9361
P14	Kuantan	3608	3210
P15	Lanshantao	5312	4127
P16	Maptaphut	5681	4819
P17	Mailiao	5900	5112
P18	Nantong	6302	4543
P19	New Plymouth	4839	4325
P20	Ningbo	4740	3116
P21	Onsan	5587	3688
P22	Paradip	6501	5764
P23	Port Kelang	3686	2513
P24	Shanghai	6177	5125
P25	Shuidong	5757	4840
P26	Shekou	5682	5202
P27	Singapore	7248	5348
P28	Taichung	4624	4219
P29	Tanjung Priok	5004	4724
P30	Timaru	3748	3445
P31	Ulsan	6591	5692
P32	Wellington	4044	4000
P33	Xiaohudao	7809	6760
P34	Yingkou	5000	4325
P35	Yosu	5772	5056
P36	Zhapu	5000	4000

Fig. 16, it follows that some ports are visited by more than one ship. For instance, ships S1, S5, and S8 all visit the port of Shanghai to accomplish pickup and delivery activities. In contrast, no ship visits ports of Anyer, Botany Bay, Jasaan, Kandla, Lanshantao, and Taichung. Consequently, cargos to be loaded/discharged in such ports are not served in any tour. From 79 available cargos, 15 are outsourced to a third party carrier in the spot market because the company cannot transport them. To summarize, the best solutions reported in the literature and the ones found by the proposed approaches in this paper are given in Table 6. Note that the exact optimization approach reported a net profit equal to USD 1041923, with an integrality gap of 41.45% after 34684 s of CPU time while the iterative solution approach obtained the total revenue of USD 1099997 in just 764 s. From Table 6, it follows that both solutions proposed the same schedules for all ships excepting for S1 and S4. The best solution found by the proposed approach is 17.69% better than that one achieved by the heuristic presented by Jetlund and Karimi (2004). Even though compared with the best solution reported by Aang (2006) the net profit increased only a 0.31% (S4 select cargo 33 instead of cargo 8), the

computational cost is decreased by almost 61303 s, from 62067 to 764 s. Furthermore, expected profit for the company's actual ship-routing and cargo assignment plan is US\$ 794634 (Jetlund & Karimi, 2004). Consequently, the new schedule improves profits by approximately 40% with regards to the actually used by the company.

6. Conclusions

This paper has presented a new iterative MILP-based algorithm for coping with large-scale ship routing and scheduling problems. The mathematical model, which is based on the notion of general precedence, utilizes a continuous time domain representation and is able to optimize multiple objectives when triangle inequality violations are introduced in distance matrixes. However, such exact optimization approach remains computationally efficiently only for small-to-medium size problems. In order to overcome this limitation, an iterative procedure was derived by embedding the rigorous formulation within heuristic rules to effectively find feasible and near-optimal solutions for large-scale instance of the

Table A.4

Distances between ports (in nm).

	Anyer	Auckland	Bangkok	Batangas	Botany Bay	Brisbane	Davao	Jassan	Jiangyin	Kandla	Kaohsiung	Karimun	Kerteh	Kuantan	Lanshantao	Maptaphut	Mailiao	Nantong
Anyer	0	4754	1284	1533	3827	3615	154	1564	2596	3018	1922	580	700	669	2840	1199	1980	2567
Auckland	4754	0	5664	4465	1287	1315	3980	4166	5217	7468	4798	5046	5142	5115	5383	5579	4875	5188
Bangkok	1284	5664	0	1470	5002	4575	1826	1658	2413	3637	1754	834	588	625	2656	109	1797	2384
Batangas	1533	4465	1470	0	3962	3535	699	455	1280	4091	581	1287	1183	1212	1518	1385	656	1250
Botany Bay	3827	1287	5002	3962	0	467	3416	3675	4735	6403	4368	4335	4431	4404	4901	4917	4445	4706
Brisbane	3615	1315	4575	3535	467	0	2989	3248	4322	6614	3955	3908	4004	3977	4488	4490	4032	4293
Davao	1554	3980	1826	699	3416	2989	0	412	1738	4324	1119	1521	1465	1478	1948	1740	1211	1709
Jassan	1564	4166	1658	455	3675	3248	412	0	1520	4157	904	1354	1298	1310	1754	1573	981	1491
Jiangyin	2596	5217	2413	1280	4735	4322	1738	1520	0	5083	696	2279	2128	2158	627	2329	623	30
Kandla	3018	7468	3637	4091	6403	6614	4324	4157	5083	0	4414	2790	3054	3025	5326	3552	4466	5053
Kaohsiung	1922	4798	1754	581	4368	3955	1119	904	696	4414	0	1610	1463	1503	939	1669	83	667
Karimun	580	5046	834	1287	4335	3908	1521	1354	2279	2790	1610	0	250	222	2522	748	1662	2250
Kerteh	700	5142	588	1183	4431	4004	1465	1298	2128	3054	1463	250	0	41	2371	503	1506	2099
Kuantan	669	5115	625	1212	4404	3977	1478	1310	2158	3025	1503	222	41	0	2401	539	1539	2129
Lanshantao	2840	5383	2656	1518	4901	4488	1948	1754	627	5326	939	2522	2371	2401	0	2572	866	598
Maptaphut	1199	5579	109	1385	4917	4490	1740	1573	2329	3552	1669	748	503	539	2572	0	1713	2300
Mailiao	1980	4875	1797	656	4445	4032	1211	981	623	4466	83	1662	1506	1539	866	1713	0	594
Nantong	2567	5188	2384	1250	4706	4293	1709	1491	30	5053	667	2250	2099	2129	598	2300	594	0
New Plymouth	4662	512	5720	4522	1147	1272	4037	4222	5281	7240	4854	5046	5164	5137	5448	5635	4931	5252
Ningbo	2407	5082	2224	1076	4611	4198	1581	1387	285	4894	507	2090	1939	1969	567	2140	434	256
Onsan	2862	5097	2653	1469	4620	4208	1837	1664	593	5322	936	2519	2368	2398	557	2568	863	564
Paradip	2017	6571	2384	2838	5651	5433	3072	2904	3830	2288	3161	1537	1801	1773	4073	2299	3213	3801
Port Kelang	777	5229	1042	1496	4518	4091	1729	1562	2488	2619	1818	195	459	430	2730	957	1871	2458
Shanghai	2535	5156	2352	1219	4674	4261	1677	1459	62	5022	635	2218	2067	2097	566	2268	562	33
Shuidong	1697	5204	1472	752	4672	4245	1433	1127	1127	4144	527	1341	1187	1218	1370	1388	538	1098
Shekou	1786	5095	1565	706	4636	4209	1411	1093	980	4236	389	1433	1280	1310	1223	1480	400	951
Singapore	559	5025	838	1291	4314	3887	1525	1358	2283	2809	1614	5	255	226	2526	753	1667	2254
Taichung	2016	4912	1841	687	4482	4069	1246	1014	584	4508	114	1705	1548	1588	827	1757	43	555
Tanjung Priok	85	4681	1279	1525	3899	3568	1496	1573	2587	3093	1913	553	692	662	2831	1193	1979	2558
Timaru	4558	762	5831	4900	1328	1623	4415	4601	5664	7137	5233	5128	5247	5214	5829	5746	5310	5634
Ulsan	2862	5097	2653	1469	4620	4208	1837	1664	593	5322	936	2519	2368	2398	557	2568	863	564
Wellington	4725	550	5871	4669	1241	1395	4184	4370	5432	7303	5002	5182	5300	5273	5598	5786	5079	5403
Xiaohudao	1822	5131	1601	742	4672	4245	1448	1130	1016	4273	425	1469	1316	1346	1259	1517	436	987
Yingkou	3119	5618	2936	1767	5136	4723	2226	2032	837	5606	1220	2802	2651	2681	507	2852	1146	808
Yosu	2766	5078	2579	1405	4599	4186	1788	1600	516	5242	883	2438	2318	2346	476	2493	782	486
Zhapu	2536	5157	2353	1220	4675	4262	1678	1461	61	5023	637	2219	2068	2098	567	2269	563	32

Table A.5

Distances between ports (in nm).

	New plymouth	Ningbo	Onsan	Paradip	Port Kelang	Shanghai	Shuidong	Shekou	Singapore	Taichung	Tanjung Priok	Timaru	Ulsan	Wellington	Xiaohudao	Yingkou	Yosu	Zhapu
Anyer	4662	2407	2862	2017	777	2535	1697	1786	559	2016	85	4558	2862	4725	1822	3119	2766	2536
Auckland	512	5082	5097	6571	5229	5156	5204	5095	5025	4912	4681	762	5097	550	5131	5618	5078	5157
Bangkok	5720	2224	2653	2384	1042	2352	1472	1565	838	1841	1279	5831	2653	5871	1601	2936	2579	2353
Batangas	4522	1076	1469	2838	1496	1219	752	706	1291	687	1525	4900	1469	4669	742	1767	1405	1220
Botany Bay	1147	4611	4620	5651	4518	4674	4672	4636	4314	4482	3899	1328	4620	1241	4672	5136	4599	4675
Brisbane	1272	4198	4208	5433	4091	4261	4245	4209	3887	4069	3568	1623	4208	1395	4245	4723	4186	4262
Davao	4037	1581	1837	3072	1729	1677	1433	1411	1525	1246	1496	4415	1837	4184	1448	2226	1788	1678
Jassan	4222	1387	1664	2904	1562	1459	1127	1093	1358	1014	1573	4601	1664	4370	1130	2032	1600	1461
Jiangyin	5281	285	593	3830	2488	62	1127	980	2283	584	2587	5664	593	5432	1016	837	516	61
Kandla	7240	4894	5322	2288	2619	5022	4144	4236	2809	4508	3093	7137	5322	7303	4273	5606	5242	5023
Kaohsiung	4854	507	936	3161	1818	635	527	389	1614	114	1913	5233	936	5002	425	1220	883	637
Karimun	5046	2090	2519	1537	195	2218	1341	1433	5	1705	553	5128	2519	5182	1469	2802	2438	2219
Kerteh	5164	1939	2368	1801	459	2067	1187	1280	255	1548	692	5247	2368	5300	1316	2651	2318	2068
Kuantan	5137	1969	2398	1773	430	2097	1218	1310	226	1588	662	5214	2398	5273	1346	2681	2346	2098
Lanshantao	5448	567	557	4073	2730	566	1370	1223	2526	827	2831	5829	557	5598	1259	507	476	567
Maptaphut	5635	2140	2568	2299	957	2268	1388	1480	753	1757	1193	5746	2568	5786	1517	2852	2493	2269
Mailiao	4931	434	863	3213	1871	562	538	400	1667	43	1979	5310	863	5079	436	1146	782	563
Nantong	5252	256	564	3801	2458	33	1098	951	2254	555	2558	5634	564	5403	987	808	486	32
New Plymouth	0	5138	5167	6488	5175	5220	5260	5151	5025	4969	4729	406	5167	175	5187	5683	5145	5221
Ningbo	5138	0	543	3641	2299	224	938	791	2094	395	2398	5517	543	5286	827	778	498	225
Onsan	5167	543	0	4070	2727	532	1367	1220	2523	824	2836	5549	1	5318	1256	768	119	533
Paradip	6488	3641	4070	0	1362	3769	2891	2984	1556	3255	2092	6384	4070	6551	3020	4353	3989	3770
Port Kelang	5175	2299	2727	1362	0	2426	1549	1641	214	1913	762	5324	2727	5211	1678	3011	2647	2428
Shanghai	5220	224	532	3769	2426	0	1066	919	2222	523	2526	5603	532	5371	955	776	455	2
Shuidong	5260	938	1367	2891	1549	1066	0	195	1345	565	1680	5639	1367	5408	231	1650	1286	1067
Shekou	5151	791	1220	2984	1641	919	195	0	1437	426	1777	5530	1220	5299	37	1503	1139	920
Singapore	5025	2094	2523	1556	214	2222	1345	1437	0	1709	558	5107	2523	5160	1473	2807	2443	2223
Taichung	4969	395	824	3255	1913	523	565	426	1709	0	2007	5347	824	5116	463	1107	759	524
Tanjung Priok	4729	2398	2836	2092	762	2526	1680	1777	558	2007	0	4613	2836	4800	1814	3111	2756	2527
Timaru	406	5517	5549	6384	5324	5603	5639	5530	5107	5347	4613	0	5549	267	5566	6065	5527	5604
Ulsan	5167	543	1	4070	2727	532	1367	1220	2523	824	2836	5549	0	5318	1256	768	119	533
Wellington	175	5286	5318	6551	5211	5371	5408	5299	5160	5116	4800	267	5318	0	5335	5834	3296	5372
Xiaohudao	5187	827	1256	3020	1678	955	231	37	1473	463	1814	5566	1256	5335	0	1539	1175	956
Yingkou	5683	778	768	4353	3011	776	1650	1503	2807	1107	3111	6065	768	5834	1539	0	686	777
Yosu	5145	498	119	3989	2647	455	1286	1139	2443	759	2756	5527	119	3296	1175	686	0	456
Zhapu	5221	225	533	3770	2428	2	1067	920	2223	524	2527	5604	533	5372	956	777	456	0

problem within a short computational time. The procedure is based on a systematic decomposition strategy that, by solving highly constrained versions of the model on every iteration, allows that the number of decisions be maintained at a reasonable level by fixing a set of binary variables.

The MILP model was first validated by solving a series of small instances deriving from a real-world case study faced by a multi-national shipping company operating a fleet of multi-parcel chemical tankers. The results obtained were compared with others presented by two authors from the literature. Comparison between results reveals that the general precedence based model has a better computational performance than the time-slots based model proposed to solve the same problem instances. Despite this, the exact approach has not converged and the MIP solver terminated because the memory capacity was exceeded when the full problem, involving 10 ships, 36 ports, and 79 potential cargos, is considered. After that, the iterative algorithm was applied to solve the same full problem instance. A convergence to a near-optimal solution was achieved in only 764 s of CPU time. Such computational performance significantly overcomes these ones achieved by other algorithms presented in the literature. Moreover, the new schedule improves profits by approximately 40% with regards to actually used by the company.

Appendix A

See Tables A.1–A.5.

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