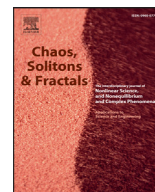




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## Network topology and interbank credit risk



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### ABSTRACT

Modern financial systems are greatly entangled. They exhibit a complex interdependence, including a network of bilateral exposures in the interbank market. The most frequent interaction consists in operations where institutions with surplus liquidity lend to those with a liquidity shortage. These loans may be interpreted as links between the banks and the links display features in some way representative of scale-free networks. While the interbank market is responsible for efficient liquidity allocation, it also introduces the possibility for systemic risk via financial contagion. Insolvency of one bank can propagate through links leading to insolvency of other banks. In this paper, we explore the characteristics of financial contagion in interbank networks whose distribution of links approaches a power law, as well as we improve previous models by introducing a simple mechanism to describe banks' balance sheets, that are obtained from information on network connectivity. By varying the parameters for the creation of the network, several interbank networks are built, in which the concentration of debt and credit comes from the distribution of links. The results suggest that more connected networks that have a high concentration of credit are more resilient to contagion than other types of networks analyzed.

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### 1. Introduction

The financial crisis during 2007–2008 highlighted, once again, the high degree of interdependence of financial systems. A combination of excessive borrowing, risky investments, lack of transparency and high interdependence led the financial system to the worst financial meltdown since the Great Depression. An increasing interest in financial contagion, partially motivated by the crisis, gave rise to

several works in this field in the last years (see, for example, [1–4]).

The interdependence of financial systems is manifested in multiple ways. Financial institutions are connected through mutual exposure in the interbank market, through which institutions with surplus liquidity can lend to those with a liquidity shortage. Equally important, financial institutions are indirectly connected because they are exposed in the same assets and share the same depositors.

With respect to the direct connection from having mutual exposure, the structure of interdependence can be easily illustrated in a visual representation of a network, in which the nodes of the network are financial institutions, while the links are the loans–debts between nodes. The direction of the link indicates the cash flow at the time of debt repayment (from debtor to creditor) as well as the direction

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of the impact or financial loss if borrowers default on their repayment. Theoretical works [5,6] have shown that the possibility of contagion via mutual exposure depends on the precise structure of the interbank market. In recent studies, different models have been used to generate artificial interbank networks, in order to identify whether a given network is more or less prone to contagion.

Nier et al. [7] simulate contagion from the initial failure of a bank in an Erdős–Rényi random network, finding a negative nonlinear relationship between contagion and bank connectivity. An increase in the amount of interbank exposure initially has no effect on contagion, since the losses are absorbed by each affected node. However, as the number of connections rises, contagion increases to the point that a further increase in connectivity causes contagion to decline. Studying a similar model on a power-law network, Cont and Moussa [8] find results similar to those of Nier et al. [7] regarding the relation between connectivity, the level of capitalization, and contagion. Battiston et al. [9] simulate contagion in a regular network and find a nonlinear relationship between connectivity and contagion, but with the opposite effect: initially, the increase in the number of connections decreases network contagion, while later additions cause contagion to increase. Ladley [10] evaluates the relation between connectivity and contagion in a partial equilibrium model of heterogeneous banks interacting in the interbank market. The author shows that, under small systemic shocks, higher connectivity increases resilience against contagion, while larger shocks have the opposite effect. The differences in the results indicate that the possibility and extent of contagion depend considerably on the structure of the network and the specific assumptions of each model.

Empirical studies reveal that some interbank networks have features of scale-free networks: this means that the distribution of connections among banks follows a power law [11–14]. However, it is worth to note that other interbank networks do not present scale-free characteristics (on the e-MID electronic money market, see, e.g., ref. [15]). Based on this stylized fact, Montagna and Lux [1] simulate networks whose links distribution follows power laws in order to evaluate the relevance of some known quantities (like the size of the banks) for contagion measures. More recent studies have emphasized core-periphery structures as relevant mechanisms in interbank network formation [2,15]. In such models, the idea is that banks organize themselves around a core of intermediaries, giving rise to a hierarchical structure (interbank tiering).

In general terms, some of the most significant features reported in the literature can be summarized as follows:

1. Networks have a low density of links, that is, they are far from complete.
2. They exhibit asymmetrical in-degree and out-degree distributions.
3. They exhibit approximate power law distributions for in- and out-degree distributions whose exponent varies between 2 and 3.

A characteristic reported in ref. [12] in a study of the Brazilian network is also worth noting: there is a positive association between the size of the exposure (assets) and the number of debtors (in-degree) of an institution and a positive

association between the size of liabilities and the number of creditors (out-degree) of an institution. More (less) connected financial institutions have a larger (smaller) exposure.

The goal of this paper is to identify, through simulations of networks whose distributions approach power laws, how scale-free networks behave with regard to financial contagion via mutual exposure and which characteristics make a given network more or less prone to propagate crises. Our particular interest is in evaluating the role of the exponents that characterize a scale-free network, because these exponents determine the concentration of debt (out-degree) and credit (in-degree) in the financial network. We construct networks whose connectivity distribution approaches a power law using the algorithm introduced by Bollobás et al. [16]. In addition to consider the network structure, we have also developed a simple method to determine the banks' balance sheets from the information of connectivity of the interbank network. By varying the parameters for the creation of the network, several interbank networks are built, in which the concentration of debt and credit comes from the distribution of links. Three main types of interbank network are analyzed for their resilience to contagion: (1) those where the concentration of debt is greater than the concentration of credit, (2) those where the concentration of credit is greater than the concentration of debt, and (3) those with similar concentrations of debt and credit. For all the networks that we have generated, the financial contagion starts with the failure of a single node, which affects neighboring nodes by defaulting on its obligations in the interbank lending market. Thus, this work focuses on the problem of credit risk, disregarding other equally important sources of contagion, as the risk of adverse shocks spreads to several institutions at the same time.

The paper is structured as follows: Section 2 describes the model used in the simulation of financial networks and the balance sheet of each node. Section 3 introduces the method used to simulate financial contagion and presents impact indices, by which we evaluate the nodes with respect to their default effect. Section 4 presents the results of the various simulations performed, and Section 5 summarizes the main conclusions.

## 2. Generating scale free bank networks

In their study on scale free networks, Barabasi and Albert (BA) [17] propose a preferential attachment mechanism to explain the emergence of the power-law degree distribution in nondirected graphs. The algorithm proposed by Bollobás et al. [16] is a generalization for directed networks of the model developed by Barabasi and Albert [17]. The network is formed by preferential attachment that depends on the distribution of in-degree,  $k_{in}$ , and out-degree,  $k_{out}$ . This algorithm has the advantage of producing different exponents for the in and out degrees, which are necessary to reproduce the characteristics of real networks. The following procedure describes the steps for generating the network according to [16].

Let  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta_{in}$  and  $\delta_{out}$  be non-negative real numbers such that  $\alpha + \beta + \gamma = 1$ . Let  $G_0$  be an initial network, that we assume as two nodes connected through two directed links, and let  $t_0$  be the number of links of  $G_0$ . At each step,  $t$ ,

starting with  $t = t_0 + 1$ , we add a new link to the network, so that in step  $t$  the network has  $t$  links and a random number of nodes,  $n(t)$ . At each step the addition of the new link may be accompanied or not by adding a new node, according to the following method [16]:

- Adding a debtor node: With probability  $\alpha$ , we create a new node  $v$  with a link from  $v$  to an existing node,  $u$ , selected with probability:

$$p(u = u_i) = \frac{k_{in}(u_i) + \delta_{in}}{t + n(t)\delta_{in}} \tag{1}$$

- Adding a link, with no new nodes: With probability  $\beta$ , we select an existing node  $v$  with probability:

$$p(v = v_i) = \frac{k_{out}(v_i) + \delta_{out}}{t + n(t)\delta_{out}} \tag{2}$$

and add a link from  $v$  to an existing node  $u$ , chosen with probability:

$$p(u = u_i) = \frac{k_{in}(u_i) + \delta_{in}}{t + n(t)\delta_{in}} \tag{3}$$

- Adding a creditor node: With probability  $\gamma$ , we add a new node  $u$  with a link from an existing node  $v$  to  $u$ , where  $v$  is selected with probability:

$$p(v = v_i) = \frac{k_{out}(v_i) + \delta_{out}}{t + n(t)\delta_{out}} \tag{4}$$

where  $k_{in}(u_i)$  is the in-degree of node  $u_i$  and  $k_{out}(v_i)$  is the out-degree of node  $v_i$ . Since the probability  $\beta$  refers to the addition of a new link without the creation of a node, increasing the value of  $\beta$  implies increasing the average network connectivity. In turn, the parameters  $\alpha$  and  $\gamma$  are related to the addition of new nodes while increasing the connectivity of existing nodes. On the other hand,  $\delta_{in}$  and  $\delta_{out}$  correspond to a fraction of the probability independent of the degree of the recipient node. So, in the limit of both  $\delta$ s going to zero and  $\beta$  going to zero one recovers the original BA algorithm.

Bollobás et al. [16] show that, when the number of nodes goes to infinity and the connectivity grows, one obtains:

$$p(k_{in}) \sim C_{IN} k_{in}^{-X_{IN}} \tag{5}$$

$$p(k_{out}) \sim C_{OUT} k_{out}^{-X_{OUT}} \tag{6}$$

where:

$$X_{IN} = 1 + \frac{1 + \delta_{in}(\alpha + \gamma)}{\alpha + \beta} \tag{7}$$

$$X_{OUT} = 1 + \frac{1 + \delta_{out}(\alpha + \gamma)}{\beta + \gamma} \tag{8}$$

The limit  $N \rightarrow \infty$  obviously can not be achieved, but the result is valid when the number of nodes grows and we consider the more connected ones, i.e., power laws for  $k_{in}$  and  $k_{out}$  will emerge in the tail of the distribution of large networks.

We want to compare networks with different values for  $X_{IN}$  and  $X_{OUT}$  (featuring different concentrations of  $k_{in}$  and  $k_{out}$ ), while keeping other characteristics similar, such as average connectivity and total concentration of links distribution. We are particularly interested in networks with values of  $X_{IN}$  and  $X_{OUT}$  around 2 and 3, in agreement with estimated

empirical values (for example, [11–13]). We also restrict the degrees of freedom of the model, imposing the following constraints on the parameters:

$$\alpha + \gamma = 0.75 \quad \text{and} \quad \delta_{in} + \delta_{out} = 4 \tag{9}$$

We consider  $\delta_{in}$  and  $\delta_{out}$  with ratios 1:3 or 3:1, in order to accentuate the asymmetry of the network. In addition to these constraints, we will cover the spaces of parameters  $\alpha \times \gamma$  and  $\delta_{out} \times \delta_{in}$  by sweeping the following radial lines:

$$\alpha = \frac{\delta_{out}}{\delta_{in}} \gamma \rightarrow \alpha = \frac{4 - \delta_{in}}{\delta_{in}} \gamma \tag{10}$$

The intersection points of Eq. 10 with constraints 9 give us the set  $(\alpha, \gamma, \delta_{in}, \delta_{out})$ , which in turn define pairs  $(X_{IN}, X_{OUT})$  as shown in Fig. 1.

Using the Eqs. 9 and 10 we restate the parameters  $\alpha, \beta, \gamma$  and  $\delta_{out}$  as functions of  $\delta_{in}$ , and replacing such expressions in the equations for  $X_{IN}$  and  $X_{OUT}$  respectively, we obtain two parametric equations:

$$X_{IN} = 1 + \frac{16 + 12\delta_{in}}{16 - 3\delta_{in}} \quad \text{and} \quad X_{OUT} = 1 + \frac{68 - 9\delta_{in}}{4 + 3\delta_{in}} \tag{11}$$

from which we finally have:

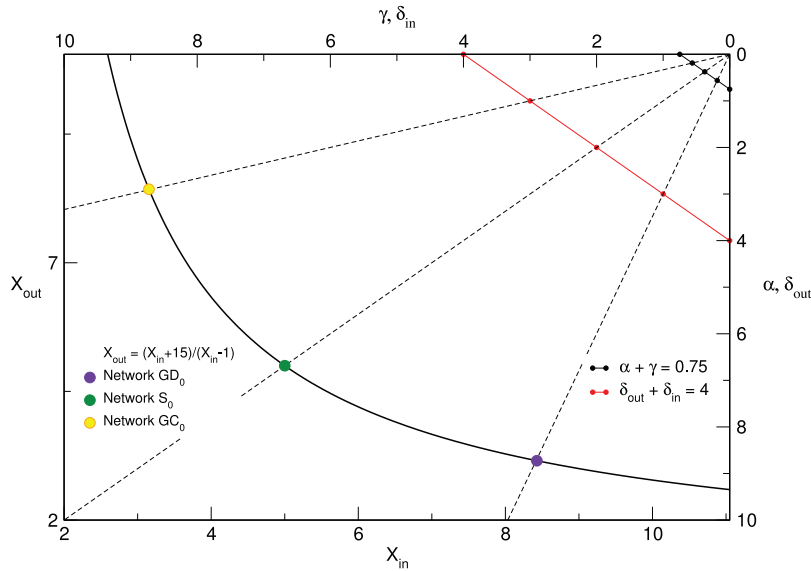
$$X_{OUT} = \frac{X_{IN} + 15}{X_{IN} - 1} \tag{12}$$

The networks constructed using relationship 12 are therefore generated through variation of a single degree of freedom, having similar average connectivity and link concentration (limited by the constraints 9), differing in the value of pairs  $(X_{IN}, X_{OUT})$ . The exponent of a power law distribution reflects the concentration of the distribution: a smaller absolute value of the exponent corresponds to a more concentrated distribution. Therefore, differences between exponents  $X_{IN}$  and  $X_{OUT}$  represent differences between the concentrations of the *in* and *out* degree distributions.

To study risk propagation we have selected three points on the curve in Fig. 1 (Eq. 12), representing three distinct networks, denominated as  $GD_0, S_0$  and  $GC_0$ . The network  $GD_0$  is more concentrated in debtor side: with a higher concentration of debts than credits it is generated so that the largest banks in the network are major debtors of the system. The network  $GC_0$  has higher concentration of credits: the biggest banks are major creditors of the network. The network  $S_0$  corresponds to the symmetric case, in which the concentration of debts and credits are similar.

One remarks that the choice of the parameters generates values of the exponents between 5 and 8 which are much higher than the observed in real networks. However, one should take in consideration that we are going to simulate small networks (1000 nodes, in agreement with real financial networks, for example, [11,13,18]), and in this case the estimated exponents are well below the limit  $N \rightarrow \infty$ . Indeed, generating networks of 1000 nodes with the selected parameters, we obtain exponents between 2.2 and 3.2. The estimation of the exponents of the power law of each distribution is done using the maximum likelihood estimator for discrete power laws, according to [19].

In order to complete the information about an interbank network, it is necessary to assign weights to the links, since the weights represent the magnitude of exposures between



**Fig. 1.** Parameter space and space of exponents: spaces  $\alpha \times \gamma$  and  $\delta_{out} \times \delta_{in}$  are represented with origin in the upper right corner and the space of exponents  $X_{IN} \times X_{OUT}$  with origin at the bottom left.

banks. The sum of in-degree weights of a bank,  $i$ , represents its applications in other institutions of the financial system (loans to other banks), a variable that we define as *bank assets*,  $BA_i$ . The sum of out-degrees weights represents the total obligations of  $i$  to other financial institutions (loans from other banks), which we call *bank liabilities*,  $BL_i$ . If there is a link from bank  $j$  to bank  $i$ , we define the exposure of bank  $i$  to  $j$  by  $w_{ji}$ , such that:

$$BA_i = \sum_{j \in \{k_{in}^i\}} w_{ji} \tag{13}$$

where  $\{k_{in}^i\}$  is the set of banks having obligations to the bank  $i$ . Similarly, if there is a link from bank  $i$  to bank  $j$ , we define the obligation of bank  $i$  to bank  $j$  by  $w_{ij}$ , such that:

$$BL_i = \sum_{j \in \{k_{out}^i\}} w_{ij} \tag{14}$$

where  $\{k_{out}^i\}$  is the set of banks to which bank  $i$  has obligations to pay.

In a study on the Brazilian interbank network, [12] highlight the non-linear positive relationship between link weights and connectivity of nodes in line with the widespread notion that the size of balance sheets and connectivity of banks are positively related [20]. From this assumption we define the following expression for the weight of a link from  $i$  to  $j$ :

$$w_{ij} = \frac{k_{out}^i \cdot k_{in}^j}{k_{out}^{max} \cdot k_{in}^{max}} \tag{15}$$

In Eq. 15,  $k_{in}^{max}$  and  $k_{out}^{max}$  denote the maximum values of  $k_{in}$  and  $k_{out}$  found in the network.

Once established the values of bank assets and bank liabilities,  $BA_i$  and  $BL_i$ , we define the other elements of the balance sheet: nonbank assets,  $NBA_i$  (all applications except interbank ones), nonbank liabilities,  $NBL_i$  (funding from outside the system) and equity,  $E_i$ .

For each bank,  $i$ , the balance sheet obey the identity:

$$BA_i + NBA_i = BL_i + NBL_i + E_i \tag{16}$$

Reflecting the minimum capital regulations of Basel Accords we set equity of each bank as a proportion of its assets:

$$E_i = \lambda_i (BA_i + NBA_i) \tag{17}$$

where  $\lambda_i$  represents the capital/assets ratio.

For the simulations in this work we will adopt three values of capital/assets ratio: the undercapitalized case, with  $\lambda = 0.01$ , and values  $\lambda = 0.05$  and  $\lambda = 0.1$ , consistent with the empirical values observed [21]. For each bank the capital/asset ratio is extracted from a normal distribution  $\lambda_i \sim N(\lambda, \sigma)$  subject to the constraint  $\lambda_i > \lambda$ , i.e.,  $\sigma$  is a stochastic positive deviation from the minimum  $\lambda$ , characterizing the heterogeneity of banks as regard to capitalization. The simulations are performed using  $\sigma = 0.01$ .

To represent the ratio of nonbank assets to total assets, we introduce the following relation that defines the nonbank assets for each bank,  $i$ , as:

$$NBA_i = \xi (BA_i + BL_i) \tag{18}$$

Defined this way, nonbank assets are a function of bank connectivity (via  $BA_i$  and  $BL_i$ ), being consistent with the assumption that the balance size is related to connectivity. Let's use  $\xi$  as calibration factor to control the  $NBA_i$  to total assets ratio.

The identities 16–18 form a system of equations by which the value of  $NBL_i$  can be determined:

$$NBL_i = (1 - \lambda_i)(1 + \xi)BA_i + [(1 - \lambda_i)\xi - 1]BL_i \tag{19}$$

For the simulations in this work we fix  $\xi = 2$  in order to obtain balance sheets in which nonbank assets and nonbank liabilities represent on average more than 50% of total assets and liabilities.

The banks' size, measured as the magnitude of their total assets ( $NBA_i + BA_i$ ), presents a distribution with characteristics similar to the distribution of links, but with estimated exponents ranging between 1.2 and 1.5, thereby having higher concentration. In fact, the Gini coefficient for total asset concentration (Gini for the distribution of  $NBA_i + BA_i$ ) is 0.83 for the network  $GC_0$ , 0.80 for  $GD_0$  and 0.78 for  $S_0$ <sup>1</sup>.

From the method described in this section we are able to represent the balance sheet of each bank by using only information from the network and the parameters  $\lambda$  and  $\xi$ . In the following section we will describe the cascade of failures following the initial default of one bank of the network.

### 3. Contagion in interbank networks: default cascade and default impact

In this section we present the methodology used to evaluate the propagation of losses in the interbank network. We simulate the insolvency of a single bank, exposed to an external shock represented by the total loss of value of its nonbank assets. Each bank is tested independently and the impact of its default on the system evaluated.

In a hypothetical scenario a bank,  $i$ , becomes insolvent, being unable to completely fulfill its obligations. If at time  $t$ , bank  $j$  realizes that its counterparty  $i$  is unable to repay its interbank liability  $w_{ij}$  in full, then bank  $j$  must re-evaluate its application in bank  $i$ , from  $w_{ij}$  to  $w'_{ij}$ :  $(w'_{ij} - w_{ij}) < 0$ . This process adversely affect the capital of  $j$ , since the variation  $(w'_{ij} - w_{ij})$  is incorporated as a loss. It happens that the smaller value,  $w'_{ij}$ , the defaulting bank  $i$  can effectively afford, depends on the financial conditions of other banks, banks for which  $i$  had granted loans. Any further failure reduces the value of assets, increasing the losses of banks that have already defaulted.

Eisenberg and Noe [24] study the problem of calculating the values  $w'_{ij}$  that banks would be able to pay at the time of settlement of its multilateral obligations. Given the array of mutual exposures,  $W$ , the problem is to determine the vector of payments,  $p = (p_1, p_2, \dots, p_n)$ , where:

$$p_i = \sum_{j=1}^n w'_{ij} \tag{20}$$

The authors show that under mild regularity conditions, there is a unique payment vector that settle the system, and develop an iterative algorithm to solve the problem. In the context of our work, where we have a single node initially insolvent, the algorithm can be described as follows:

1. Compute the losses to all banks resulting from the failure of bank  $i$  assuming that all other banks are able to repay their liabilities. Stop if no other bank fails, otherwise:
2. Let  $j$  denote the bank or group of banks whose losses exceed their equity. Compute the losses to all banks resulting from the failure of banks  $i$  and  $j$ . Repeat step 2 until no further bank fails.

<sup>1</sup> The concentration of assets in real networks is also quite high, as reported in the literature. For example, [22] report a Gini of 0.88 for the Austrian network in 2002 and [23] reports a Gini of 0.90 for the United States in 2000.

The algorithm described above allows us to calculate two important measures to assess the impact of a bank failure on the network: the *Default Impact* and *Default Cascade*. For a bank  $i$ , the *Default Impact*,  $DI_i$ , refers to the reduction in total assets of the financial system as a result of losses incurred via contagion, as a proportion of total initial assets. If we denote the total assets of the system at the initial time as  $A_0$  and at final time (after the external shock) as  $A_t$ , the *Default Impact* is given by:

$$DI_i = \frac{A_0 - A_t - NBA_i}{A_0} \tag{21}$$

The measure *Default Cascade*,  $DC_i$ , refers to the number of insolvent banks due to the failure of bank  $i$ , as a proportion of the total number of banks of the network. Both *Default Impact* and *Default Cascade* of a bank reveal how the network would be affected by its failure, taking into account only the direct effects of loss propagation through interbank exposures.

### 4. Results

In this section we present the results obtained in contagion simulations for networks produced according to the prescription presented in Section 2. We consider three types of networks as above defined: network  $GD_0$  presenting a higher concentration of debtors than creditors, network  $GC_0$  with higher concentration of creditors and the symmetric  $S_0$ , generated with equal concentrations.

#### 4.1. Default impact

For each set of parameters that defines a network category ( $GD_0$ ,  $S_0$  and  $GC_0$ ) we performed 20 simulations, so that the analysis is based on 20 realizations of networks of type  $GD_0$ , 20 realizations of  $S_0$  networks and 20 realizations of  $GC_0$  networks. For each generated network and for each bank,  $i$ , the *Default Impact*,  $DI_i$  and *Default Cascade*,  $DC_i$ , were calculated. The results presented in this section are for networks with 1000 nodes, with capital level  $\lambda = 0.05$ .

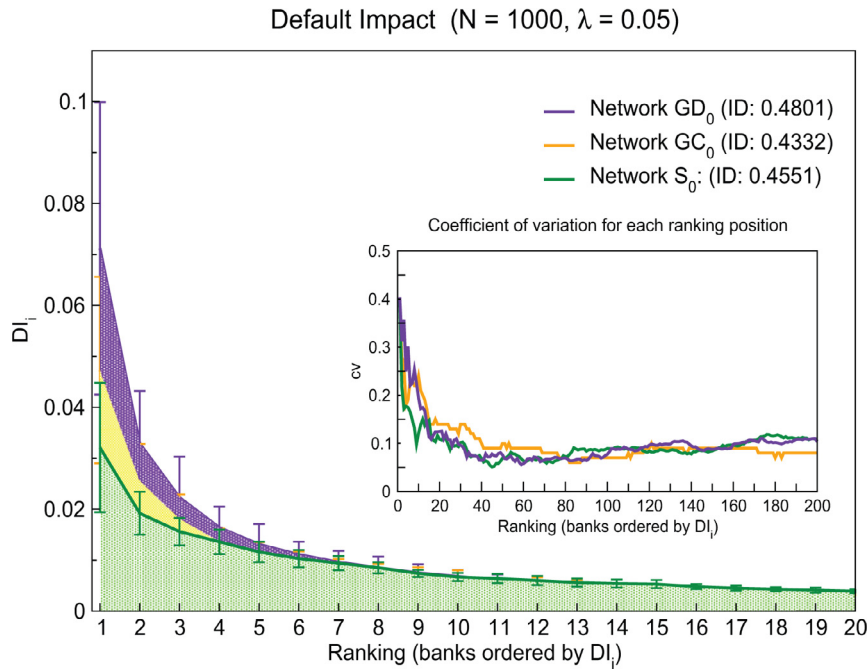
Fig. 2 shows the ranking of banks for the three networks, in decreasing order of  $DI_i$ . The values are average values for each ranking position, for example, for each network type the greater *Default Impact* (first ranking position) is the average of greater impacts for 20 simulations. Equivalently, the subsequent positions of the ranking are average values.

Its possible to see that the difference between the three types of networks is more pronounced in the first ranking positions, although these positions show greater dispersion around the mean value.

The area under the ranking curve, which corresponds to the sum of individual impacts, should be considered as a measure of the network systemic risk. We then have for each network an aggregate measure,  $DI$ , given by:

$$DI = \sum_{i=1}^n DI_i \tag{22}$$

The measure  $DI$  corresponds to a measure of central tendency: in fact, if  $DI$  is divided by  $N$  (number of nodes) we have the average value of individual default impact. Ordering the three networks by the aggregate index,  $DI$ , we have  $GD_0$  network with greater impact ( $DI = 0.48$ ), followed by  $S_0$  ( $DI =$



**Fig. 2.** Ranking of banks in decreasing order of  $DI_i$ : The figure shows the values for the 20 banks causing major impact. Recall that, for each bank,  $i$ , the *Default Impact*,  $DI_i$  refers to the losses suffered by the system via contagion (from the default of  $i$ ) as a proportion of total assets of the network. The inset graph presents the values of the coefficient of variation ( $cv = \sigma / \mu$ ) at each ranking position for the 200 banks with greater impact.

0.46), and finally the  $GC_0$  network ( $DI = 0.43$ ) (see Fig 2). That means that a higher concentration of debt links corresponds to a greater impact.

However, if the three networks are evaluated according to their principal banks (first banks in the ranking), the network  $GD_0$  remains in the first position of impact, but the other two switch positions: banks of network  $GC_0$  with higher  $DI_i$  presents greater effect over the system than the big banks of network  $S_0$ . This fact can be explained by differences in the asset concentrations of networks  $GD_0$ ,  $GC_0$  and  $S_0$ . It happens that banks with large balance sheets have a stronger effect on the system and although we have constructed the networks in a way that they present similar concentrations of connectivity, interbank exposures as defined by the model accentuates assets concentrations, also increasing the differences between them. With the highest Gini, the network  $GC_0$  has a large bank whose total asset is 122 times greater than the average assets of the network, while the largest bank in the symmetric network  $S_0$  possess a total asset 55 times greater than average.

Comparison is more straightforward when the networks are evaluated by the *default cascade* of their nodes, because in this case the difference is more pronounced. For *default cascade* we also define as aggregate measure the area under the ranking curve:

$$DC = \sum_{i=1}^n DC_i \tag{23}$$

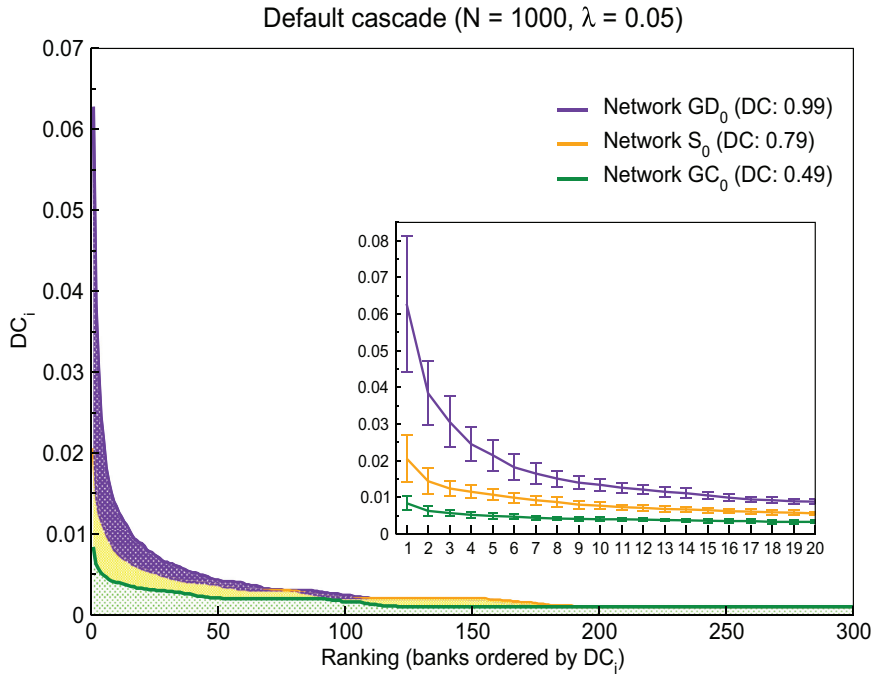
Here (see Fig. 3) the ordering of networks is clear: the network  $GD_0$  has greater potential to generate contagion in case of default of its nodes ( $DC = 0.99$ ). Secondly we have the

symmetric network  $S_0$  ( $DC = 0.79$ ), and finally the network  $GC_0$  ( $DC = 0.49$ ). The size of the balance sheet has less influence on the *Default Cascade* than over the *Default Impact*, and the effect of concentrations of debt and credit links becomes more apparent. In fact, as expected, the *Default Cascade* increase towards increased concentration of debts. Although having similar sizes, the bank that leads to higher cascade in network  $GD_0$  reaches about 6% nodes of the network, compared with less than 1% for  $GC_0$  network.

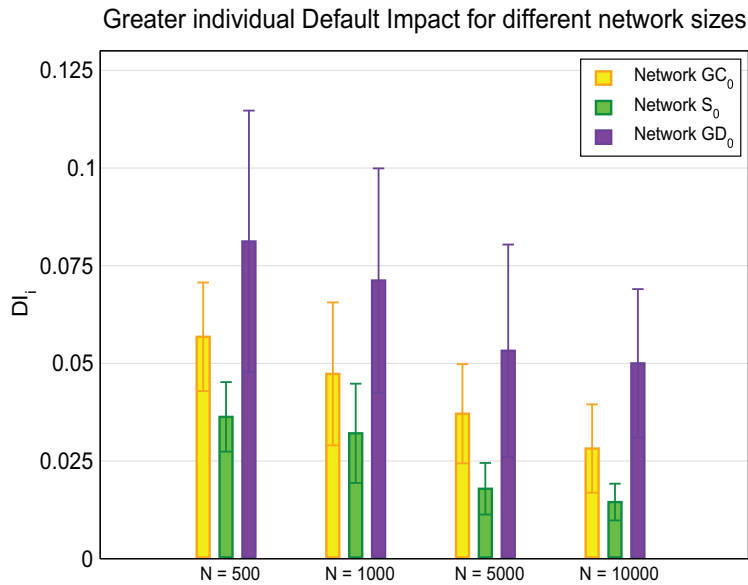
#### 4.2. Effect of system size and capital level $\lambda$

In order to evaluate the effect of the size of the system on the Default Impact,  $DI$ , we have studied different system sizes:  $N = 500$ ,  $N = 1000$ ,  $N = 5000$  and  $N = 10,000$ . We have verified (see Fig. 4) that the variation of size in the interbank network does not affect the global behavior in the values of indexes  $DI$ . For all system sizes, we observe that in network  $GD_0$  the impact is higher than  $GC_0$  and  $S_0$  networks. However, results also show that the average values of individual impacts,  $DI$ , decrease when the networks size increases, minimizing the importance of each node as the system size increases. Thus, one can conclude that in large networks, nodes are less prone to contagion and then risk is “diluted”.

The effect of leverage is more important as it is frequently argued that financial crises are the result of an excessively low ratio between bank capital and assets. In order to analyze the effect of capital level  $\lambda$  on the system, Fig 5 shows the value of Default Impact  $DI$  for three different values of capital level  $\lambda$ . This figure shows that when the average



**Fig. 3.** Ranking of banks in decreasing order of  $DC_i$ : The figure shows the values of the  $DC_i$ , for the 300 banks causing greater cascades. As defined earlier, the Cascade Default,  $(DC_i) DI_i$ , refers to the number of insolvent banks due to the initial failure of bank  $i$  as a proportion of total number of banks in the network. The inset graph shows in detail the top 20 banks.



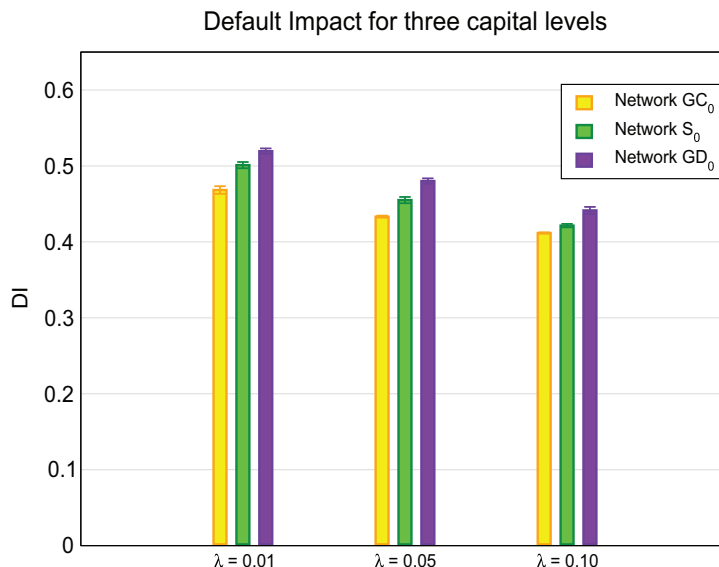
**Fig. 4.** Values of Default Impact of the first ranking position (bank with larger  $DI_i$ ), for different network sizes.

capitalization of the banks is higher, a decrease in contagion occurs for all network categories. But this lower contagion rate is not as important as expected, when  $\lambda$  goes from 0.01 to 0.1 the Default Impact decreases less than 10%. Obviously a decrease of the average capital has the reverse effect. Testing the network for low values of  $\lambda$  gives us an idea of possible amplified contagion in the event of macroeconomic stress, when much of the network may become less capitalized.

### 5. Conclusions

In this paper, we have analyzed the financial contagion via mutual exposures in the interbank market through simulations of networks whose degree distributions approach power laws.

We have seen that among the measures of systemic importance ( $DI$  and  $DC$ ), *Default Cascade* ( $DC$ ) is the one that



**Fig. 5.** Values of  $DI$  in the three categories of network for  $\lambda = 0.01$ ,  $\lambda = 0.05$  and  $\lambda = 0.10$ .

most differentiates the categories of network analyzed. We also observe that, for all categories, both the *Default Impact* and *Default Cascade* of each node alone does not reach large percentage of the network assets and number of nodes, respectively. This result is in agreement with the results of stress tests on empirical networks [25].

Comparisons of the different types of network suggest that, for networks whose distributions are close to power laws and exposure is positively related to connectivity, the best scenario is one with a more connected network with a high concentration of credits, featuring large creditors nodes, which act as stabilizers of the network. These results suggest that the asymmetry observed in distributions of certain real networks is a positive factor, as long as the network are more concentrated in the distribution of credits (*in links*). The results also suggest that the size of the balance sheet is the most important factor in determining the impact on assets resulting from the failure of a node, and should not be disregarded or replaced by topological measures that reflect only information on network connectivity. At the same time, the network structure has important consequences on the *Default Cascade*. In some cases, the banks that trigger the largest cascades are not the ones with the bigger balance sheet.

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