energies

MDPI

Article

A Robust Conic Programming Approximation to Design an EMS in Monopolar DC Networks with a High Penetration of PV Plants

Oscar Danilo Montoya [1,*], Federico Martin Serra [2,*] and Walter Gil-González [3]

[1] Grupo de Compatibilidad e Interferencia Electromagnética (GCEM), Facultad de Ingeniería, Universidad Distrital Francisco José de Caldas, Bogotá 110231, Colombia
[2] Laboratorio de Control Automático (LCA), Facultad de Ingeniería y Ciencias Agropecuarias, Universidad Nacional de San Luis—CONICET, Villa Mercedes, San Luis 5730, Argentina
[3] Department of Electrical Engineering, Universidad Tecnológica de Pereira, Pereira 660003, Colombia; wjgil@utp.edu.co
* Correspondence: odmontoyag@udistrital.edu.co (O.D.M.); fmserra@unsl.edu.ar (F.M.S.)

Abstract: This research addresses the problem regarding the efficient operation of photovoltaic (PV) plants in monopolar direct current (DC) distribution networks from a perspective of convex optimization. PV plant operation is formulated as a nonlinear programming (NLP) problem while considering two single-objective functions: the minimization of the expected daily energy losses and the reduction in the expected CO_2 emissions at the terminals of conventional generation systems. The NLP model that represents the energy management system (EMS) design is transformed into a convex optimization problem via the second-order cone equivalent of the product between two positive variables. The main contribution of this research is that it considers the uncertain nature of solar generation and expected demand curves through robust convex optimization. Numerical results in the monopolar DC version of the IEEE 33-bus grid demonstrate the effectiveness and robustness of the proposed second-order cone programming model in defining an EMS for a monopolar DC distribution network. A comparative analysis with four different combinatorial optimizers is carried out, i.e., multiverse optimization (MVO), the salp swarm algorithm (SSA), the particle swarm optimizer (PSO), and the crow search algorithm (CSA). All this is achieved while including an iterative convex method (ICM). This analysis shows that the proposed robust model can find the global optimum for two single-objective functions. The daily energy losses are reduced by 44.0082% with respect to the benchmark case, while the CO_2 emissions (kg) are reduced by 27.3771%. As for the inclusion of uncertainties, during daily operation, the energy losses increase by 22.8157%, 0.2023%, and 23.7893% with respect to the benchmark case when considering demand uncertainty, PV generation uncertainty, and both. Similarly, CO_2 emissions increase by 11.1854%, 0.9102%, and 12.1198% with regard to the benchmark case. All simulations were carried out using the Mosek solver in the Yalmip tool of the MATLAB software.

Keywords: robust convex optimization; energy management system; photovoltaic plants; monopolar direct current networks; daily energy losses; carbon dioxide emissions

Citation: Montoya, O.D.; Serra, F.M.; Gil-González, W. A Robust Conic Programming Approximation to Design an EMS in Monopolar DC Networks with a High Penetration of PV Plants. *Energies* **2023**, *16*, 6470. https://doi.org/10.3390/en16186470

Academic Editor: Abu-Siada Ahmed

Received: 7 July 2023
Revised: 14 August 2023
Accepted: 26 August 2023
Published: 7 September 2023

1. Introduction

1.1. General Context

Government policies, multilateral organizations, and industry have strongly advocated the widespread adoption of renewable energy resources in electrical networks, with the purpose of mitigating the detrimental effects of global warming caused by the daily emission of greenhouse gases into the atmosphere [1,2]. Photovoltaic (PV) and wind technologies are the most widely used renewable energy technologies for electric systems.

These technologies are mature, with high efficiency and long useful lives [3–5]. However, the massive integration of these energy sources in electrical systems has transformed the classical passive networks with unidirectional power flows into active distribution grids. Utility companies play an active role in these grids, aiming to maximize their profits [6,7].

To maintain an efficient operation in electrical networks with a high penetration of multiple generation sources, it is necessary to design efficient energy management systems (EMSs) [8,9]. The primary purpose of an EMS is to ensure the efficient functioning of an electrical network by meeting the technical grid requirements, which pertain to voltage profiles, current magnitudes, and the power balance, while also improving economic, technical, or environmental objective functions [10].

1.2. Motivation

Designing an effective EMS typically involves formulating an optimization model that accurately represents the behavior of an electrical network and establishes a relationship between the analyzed objective function and the electrical variables [11,12]. Considering the importance of designing efficient EMS for distribution companies, this research aims to propose a convex optimization model that seeks to determine the optimal dispatch of multiple PV plants integrated into monopolar DC networks, with the objective of minimizing two possible functions: the reduction in daily CO_2 emissions or the minimization of the total daily energy losses [10]. The proposal of a convex formulation to determine the effective dispatch of PV plants arises from the need to guarantee effective solutions corresponding to the optimal global solution. This has not been achieved in studies that rely on combinatorial optimizers, as their stochastic nature makes it impossible to ensure that the global optimum is found [13–15]. In addition, the use of convex optimization allows for incorporating the stochastic nature of the demand and PV generation curves while maintaining the convexity of the solution space. This is a significant advantage of convex optimization when compared to metaheuristics [16,17].

For the sake of clarity, it is worth mentioning that a monopolar DC network implies an electrical configuration where two poles are used to provide electrical energy to all end users, where one of the poles is set with the nominal voltage value of the network, and the second one is solidly grounded at all load points [18]. These grids may be regarded as analogous to single-phase AC distribution networks, with the main advantage that no reactive power or frequency control designs are required [19]. In general, this configuration makes DC networks more efficient than their AC counterparts [20,21].

1.3. Literature Review

The problem regarding the efficient operation of PV plants in medium- and low-voltage distribution networks has been widely explored in the literature. Some of the most recent advances in this area are discussed below.

The study by [22] proposed an EMS to dispatch PV plants during the daily operation of alternating current (AC) distribution grids. This EMS employs a master–slave methodology, which employs antlion optimization for the PV plant power dispatch. In the master stage, the power flow is entrusted to the successive approximations method. This methodology was assessed in two test systems of 33 and 27 nodes while considering three PV plants installed.

The authors of [10] presented a general EMS design for monopolar DC networks in urban and rural regions, which considers three objective functions: energy losses, the costs associated with energy purchasing at the conventional sources and PV plant maintenance, and CO_2 emissions. The salp swarm algorithm was implemented to solve the nonlinear programming model that represents the studied problem. According to the numerical results obtained in two test systems composed of 27 and 33 nodes, the proposed approach demonstrated a better numerical performance than the particle swarm optimizer (PSO), the multiverse optimization approach, and the crow search algorithm in terms of the final solution and repeatability.

The work by [23] addressed the optimal PV plant location and sizing problem with the aim to reduce the expected grid power losses and improve voltage profiles. The k-means clustering method was selected to determine the best nodal locations for the PV plants. Numerical results in the IEEE 33- and 69-bus systems demonstrated the effectiveness of this approach when considering different PV penetration levels. However, the authors did not present a daily analysis, and they only focused on the peak load operating scenario, which is not realistic for installing PV plants in electrical networks.

The study by [24] presented a methodology based on the multi-period optimal power flow solution for operating battery energy storage systems (BESSs), PV plants, and wind and conventional generation sources. The main contribution of this research was the possibility of using the converter interfacing with the battery systems to independently control the active and reactive power. The proposed optimization model was solved with the help of the modeling language for mathematical programming while considering the two test feeders composed of 33 and 141 nodes.

In [8], the authors applied a convex-based optimization algorithm to operate PV plants in monopolar DC networks while considering three objective functions (technical, economic, and environmental indices). The proposed convex reformulation belongs to the family of recursive approximations. Here, Taylor's series expansion is used to linearly approximate the power flow equations, which are recursively solved until the desired convergence is reached. Numerical results in a DC version of the IEEE 33-bus grid demonstrated the proposed convex model's effectiveness when compared to four combinatorial optimizers.

Other optimization methodologies have been applied to the optimal placement and sizing/operation of PV plants in electrical distribution networks, including the krill herd algorithm [25], the horse herd optimization algorithm [26,27], PSO [28,29], genetic algorithms [30,31], and the gravitational search algorithm [32], among others. Table 1 summarizes the most important works on EMS in monopolar DC grids with high PV penetration.

Table 1. Summary of the methodologies implemented in the literature for EMS in monopolar DC grids.

Method/Algorithm	Objective Function	Robust	Year	Ref.
Perturb and observe algorithm	Minimization of operating costs	✗	2020	[33]
Antlion optimizer	Minimization of operating costs or energy losses and reduction of CO_2 emissions	✗	2022	[22]
Salp swarm algorithm	Minimization of operating costs or energy losses and reduction of CO_2 emissions	✗	2022	[10]
Loss sensitivity factor and k-means clustering	Minimization of power losses and voltage regulation improvement	✗	2022	[23]
Weight-based method	Minimization of operating costs or energy losses and reduction of CO_2 emissions	✗	2023	[34]
Antlion optimizer	Minimization of operating costs or reduction of CO_2 emissions	✗	2023	[8]
Vortex search algorithm	Minimization of operating and maintenance costs	✗	2023	[35]
Crow search algorithm	Minimization of operating and maintenance costs	✗	2023	[36]
Robust conic programming approximation	Minimization of operating costs or energy losses and reduction of CO_2 emissions	✓	2023	This study

The main characteristics of the works listed in Table 1 are as follows: (i) most of the optimization algorithms are based on the application of the combinatorial optimization methods to locate/operate distributed energy resources (DERs) in electrical networks; (ii) the most common objective functions include voltage profile improvements, energy loss reduction, greenhouse gas emissions reduction, and energy purchasing and operating cost minimization; and (iii) the demand and PV generation curves are assumed to be deterministic (no stochastic analyses are included). These aspects constitute a clear opportunity to contribute to the current literature on monopolar DC networks and designing an efficient EMS system for PV plants integrated in these systems.

1.4. Contributions and Scope

Considering the above, the main contributions of this research article are the following:

i. The application of convex optimization to obtain an approximated second-order cone programming model that represents the problem regarding the efficient design of an EMS for dispatching PV plants in monopolar DC networks.
ii. The fact that the proposed convex model includes uncertainty in the power available from PV generators and the load demand makes it a robust approach.
iii. The fact that deterministic and uncertain scenarios are evaluated with regard to the expected daily behavior of the constant power loads and the PV generation curves.

Note that, within the scope of this research, the following considerations are made: (i) the distribution company provides the parametric information regarding branches and peak load consumption, which therefore have no uncertainties; (ii) the average generation and demand curves are also provided by the utility, which, in the case of the stochastic scenario, have expected variations of about $\pm 10\%$ with respect to the deterministic curves; and (iii) the load nodes are modeled while considering only constant power consumption, i.e., no resistive or current loads are considered, given that this is the worst-case scenario with regard to energy losses when constant power loads are connected.

1.5. Document Structure

The remainder of this document is structured as follows. Section 2 reveals the general nonlinear programming model that represents the problem of operating PV plants in monopolar DC networks via an EMS. Section 3 describes the proposed convexification approach, which is based on the hyperbolic representation of the product between two positive variables as a l_2-norm. Section 4 presents the main characteristics of the test feeder, which corresponds to the DC version of the IEEE 33-bus system, considering different operating scenarios that include deterministic and stochastic analyses. Section 5 shows the main numerical results for the stochastic and deterministic scenarios, as well as a comparative analysis with multiple combinatorial optimizers for the latter. Finally, Section 6 presents the main concluding remarks derived from this work, in addition to some future lines of research.

2. General Problem Formulation

The problem regarding the optimal operation of distributed energy resources (DERs) in monopolar DC networks corresponds to a nonlinear programming model that belongs to the family of non-convex optimization, given the product between voltage variables in the power balance constraints. This research considers two objective functions: the minimization of (i) energy losses and the total greenhouse gas emissions from conventional sources, i.e., the CO_2 emissions to the atmosphere by conventional generators. In addition, the set of constraints includes the power balance per node and time, the energy storage behavior, and voltage regulation, among others. The complete optimization model is detailed below.

2.1. Objective Functions

The first objective function aims to minimize the expected energy losses in a daily operation scenario. This objective function is formulated below.

$$\min E_{\text{loss}} = \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{N}} \sum_{m \in \mathcal{N}} G_{km} v_{kh} v_{mh} \Delta_h, \tag{1}$$

where E_{loss} quantifies the expected daily energy losses in all the branches of the monopolar DC network, as a function of the conductance effects between nodes k and m (i.e., G_{km}) and the voltage magnitudes in these nodes per period (i.e., v_{kh} and v_{mh}, respectively). Note that Δ_h is the fraction of time defined for the daily period, typically one hour or fractions of one. In addition, $\mathcal{N}$ and $\mathcal{H}$ are the sets containing the nodes of the network and the number of periods under analysis.

The second objective function seeks to minimize the expected CO_2 emissions (kg) generated by conventional sources (diesel plants) or the equivalent emissions at the terminals of the substation nodes. This objective function is presented below.

$$\min E_{\mathrm{CO2}} = \sum_{h \in \mathcal{H}} \sum_{k \in \mathcal{N}} \left(C_{\mathrm{CO2}}^{\mathrm{sub}} p_{kh}^{\mathrm{sub}} + C_{\mathrm{CO2}}^{\mathrm{diesel}} p_{kh}^{\mathrm{diesel}} \right) \Delta_h, \tag{2}$$

where E_{CO2} denotes the expected daily CO_2 emissions of the network, which are calculated with the emission coefficients at the terminals of the substation ($C_{\mathrm{CO2}}^{\mathrm{sub}}$) and the diesel sources ($C_{\mathrm{CO2}}^{\mathrm{diesel}}$). These are associated with the generation power outputs p_{kh}^{sub} and p_{kh}^{diesel}, respectively.

Remark 1. *The main characteristic of the objective functions (1) and (2) is that both are convex, as the components G_{km} in Equation (1) are part of a positive semi-definite matrix and Equation (2) is a linear function.*

2.2. Set of Constraints

The efficient operation of PV plants in monopolar DC networks is subject to different technical constraints associated with Kirchhoff's laws applied to DC circuits [37]. These constraints include the power equilibrium per node, the power generation capabilities of the conventional and renewable sources, the voltage regulation bounds, and the current capacities of the distribution lines, among others. This set of constraints is listed from Equations (3)–(8)

$$p_{kh}^{\mathrm{sub}} + p_{kh}^{\mathrm{diesel}} + p_{kh}^{pv} - p_{kh}^{d} = v_{kh} \sum_{m \in \mathcal{N}} G_{km} v_{mh}, \quad \{\forall k \in \mathcal{N}, \forall h \in \mathcal{H}\} \tag{3}$$

$$i_{kmh} = g_{km}(v_{kh} - v_{mh}) \quad \{\forall km \in \mathcal{L}, \forall h \in \mathcal{H}\} \tag{4}$$

$$p_{k}^{\mathrm{min,sub}} \leq p_{k}^{\mathrm{sub}} \leq p_{kh}^{\mathrm{max,sub}}, \quad \{\forall k \in \mathcal{N}, \forall h \in \mathcal{H}\} \tag{5}$$

$$p_{k}^{\mathrm{min,diesel}} \leq p_{kh}^{\mathrm{diesel}} \leq p_{k}^{\mathrm{max,diesel}}, \quad \{\forall k \in \mathcal{N}, \forall h \in \mathcal{H}\} \tag{6}$$

$$p_{kh}^{\mathrm{min},pv} \leq p_{kh}^{pv} \leq p_{kh}^{\mathrm{max},pv}, \quad \{\forall k \in \mathcal{N}, \forall h \in \mathcal{H}\} \tag{7}$$

$$v_{k}^{\mathrm{min}} \leq v_{kh} \leq v_{k}^{\mathrm{max}}, \quad \{\forall k \in \mathcal{N}, \forall h \in \mathcal{H}\} \tag{8}$$

$$|i_{kmh}| \leq i_{km}^{\mathrm{max}}, \quad \{\forall km \in \mathcal{L}, \forall h \in \mathcal{H}\} \tag{9}$$

where p_{kh}^{pv} represents the power injected by a PV source connected at node k and time h; p_{kh}^{d} denotes the constant power load connected at node k in period h; g_{km} corresponds to the conductive effect of the distribution line that connects nodes k and m (note that $g_{km} = -G_{km}$); i_{kmh} stands for the current flowing from node k to node m in period h; $p_{k}^{\mathrm{min,sub}}$ and $p_{k}^{\mathrm{max,sub}}$ are the lower and upper power generation bounds for the substation bus; $p_{k}^{\mathrm{min,diesel}}$ and $p_{k}^{\mathrm{max,diesel}}$ represent the minimum and maximum bounds for diesel power generation; $p_{kh}^{\mathrm{min},pv}$ and $p_{kh}^{\mathrm{max},pv}$ correspond to the lower and upper generation bounds associated with the PV plant connected to node k at time h (note that $p_{kh}^{\mathrm{max},pv}$ depends on the solar availability in the grid's area of influence); and v_{k}^{min} and v_{k}^{max} correspond to the lower and upper regulation bounds applicable to all the network nodes at any period. Note that $\mathcal{L}$ is the set containing all the distribution lines of the grid.

The set of constraints (3)–(9) can be interpreted as follows: Equation (3) is the power equilibrium constraint at each node and period; Equation (4) corresponds to the application of Ohm's law at each distribution branch; inequality constraints (5)–(7) express the lower and upper generation constraints associated with the substation, diesel, and PV generators, respectively; box-type constraint (8) is related to the voltage regulation constraints applicable to all the nodes of the monopolar DC network; and l_1-norm in (9) imposes the thermal operating conditions regarding the current flow of each distribution line.

Remark 2. *85% of the constraints in (3)–(9) are linear, i.e., convex (see constraints (4)–(9)). However, the power balance constraint in (3) is non-convex due to the product between voltages on its right-hand side [37].*

3. Formulating the Conic Programming Approximation

A conic formulation can be elaborated in a set belonging to $\mathcal{R}^{n+1}$ with the following expression:

$$\mathcal{C} = \left\{ (x, z) \in \mathcal{R}^{n+1} : \|x\| \leq z \right\}, \tag{10}$$

where $x \in \mathcal{R}^n$ is a vector of decision variables, $z \in \mathcal{R}$ is a real variable, and $\|\cdot\|$ denotes the l_2-norm of argument. Figure 1 depicts an example of a second-order cone, i.e., $x \in \mathcal{R}^2$ and $z \in \mathcal{R}$.

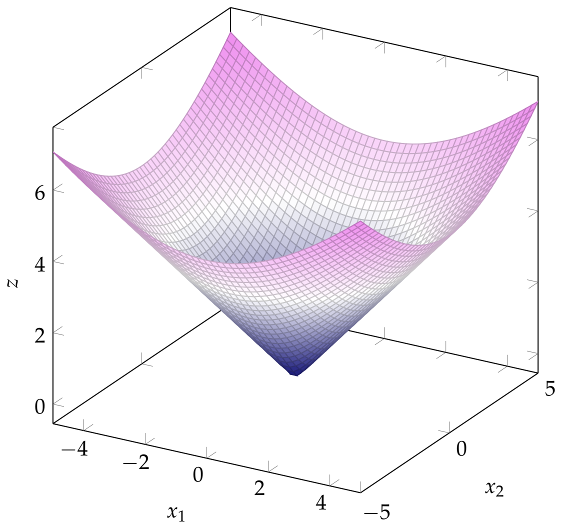

Figure 1. Representation of the second-order cone $\mathcal{C}$.

The cone $\mathcal{C}$ can integrate an affine space on both sides of the inequality, as follows:

$$\|Ax + b\| \leq c^\top x + d, \tag{11}$$

where $A \in \mathcal{R}^{n \times n}$ is a matrix, $b,\ c \in \mathcal{R}^n$ are vectors, and $d \in \mathcal{R}$ is a scalar. Therefore, an optimization problem using a conic formulation can be generally represented as follows:

$$\begin{aligned} &\min h^\top x \\ &\text{subject to :} \\ &\|Ax + b\| \leq c^\top x + d \end{aligned} \tag{12}$$

where $h \in \mathcal{R}^n$ is the costs vector of the objective function.

It is important to mention that the conic formulation (12) can include another type of constraint, e.g., linear constraints, which are a particular case of a conic constraint, as it defines $A = 0$ in (11) (see [38,39] for more details about conic formulations). The case involving quadratic constraints is presented below.

3.1. Conic Programming Application

To obtain a convex approximation that represents the problem under study, this section proposes a convexification of the power balance constraints in (3). To obtain a convex equivalent, the hyperbolic representation of the product between two variables is employed [40]. To obtain this equivalent, consider the definition of the following set of variables:

$$\omega_{kmh} = v_{kh} v_{mh}, \quad \{\forall k, m \in \mathcal{N}, \ \forall h \in \mathcal{H}\}, \tag{13}$$

where $\omega_{kkh} = v_{kh}^2$ and $\omega_{mmh} = v_{mh}^2$. Furthermore, if both sides of (13) are squared, then

$$\omega_{kmh}^2 = v_{kh}^2 v_{mh}^2 = \omega_{kkh}\omega_{mmh}, \quad \{\forall k, m \in \mathcal{N}, \ \forall h \in \mathcal{H}\}. \tag{14}$$

Now, note that (14) can be expressed using the equivalent hyperbolic representation of the product between two positive variables, as follows:

$$\begin{aligned} \omega_{kmh}^2 &= \omega_{kkh}\omega_{mmh}, \{\forall k, m \in \mathcal{N}, \ \forall h \in \mathcal{H}\} \\ \omega_{kmh}^2 &= \frac{1}{4}(\omega_{kkh} + \omega_{mmh})^2 - \frac{1}{4}(\omega_{kkh} - \omega_{mmh})^2, \{\forall k, m \in \mathcal{N}, \ \forall h \in \mathcal{H}\} \\ (2\omega_{kmh})^2 + (\omega_{kkh} - \omega_{mmh})^2 &= (\omega_{kkh} + \omega_{mmh})^2, \{\forall k, m \in \mathcal{N}, \ \forall h \in \mathcal{H}\} \\ \left\| \begin{matrix} 2\omega_{kmh} \\ \omega_{kkh} - \omega_{mmh} \end{matrix} \right\| &= \omega_{kkh} + \omega_{mmh}, \{\forall k, m \in \mathcal{N}, \ \forall h \in \mathcal{H}\} \end{aligned} \tag{15}$$

Here, Equation (15) corresponds to a non-convex constraint, as it represents the contour of a cone due to the equality imposition [41].

Remark 3. *To represent the product of two variables as an equivalent convex cone, the equality constraint in* (15) *is relaxed as an inequality constraint (see [40]), which yields*

$$\left\| \begin{matrix} 2\omega_{kmh} \\ \omega_{kkh} - \omega_{mmh} \end{matrix} \right\| \leq \omega_{kkh} + \omega_{mmh}. \{\forall k, m \in \mathcal{N}, \ \forall h \in \mathcal{H}\} \tag{16}$$

On the other hand, even though Equation (4) and the inequality constraints (8) and (9) are convex in the domain of variables $\{v_{kh}, \ v_{mh}, \ i_{kmh}\}$, they must be defined in the set of the auxiliary variables $\{\omega_{kkh}, \ \omega_{kmh}, \ \omega_{mmh}\}$, preserving their convexity properties.

To obtain an equivalent function of the current flow per line in Equation (4), both sides of this equation are squared, which yields

$$\begin{aligned} i_{kmh}^2 &= g_{km}^2 (v_{kh} - v_{mh})^2 \quad \{\forall km \in \mathcal{L}, \ \forall h \in \mathcal{H}\} \\ i_{kmh}^2 &= g_{km}^2 \left(v_{kh}^2 - 2v_{kh}v_{mh} + v_{mh}^2\right) \quad \{\forall km \in \mathcal{L}, \ \forall h \in \mathcal{H}\} \end{aligned} \tag{17}$$

if a new auxiliary variable $l_{kmh} = i_{kmh}^2$ is defined and the definition in (13) is employed, the following convex constraint in the domain of the new variables is reached:

$$l_{kmh} = g_{km}^2 (\omega_{kkh} - 2\omega_{kmh} + \omega_{mmh}) \quad \{\forall km \in \mathcal{L}, \ \forall h \in \mathcal{H}\} \tag{18}$$

Considering the definitions of the variables l_{kmh} and ω_{kkh}, the voltage regulation constraint (8) and the thermal limitations of the conductors in (9) can be rewritten as follows:

$$\left(v_k^{\min}\right)^2 \leq \omega_{kkh} \leq (v_k^{\max})^2, \quad \{\forall k \in \mathcal{N}, \ \forall h \in \mathcal{H}\} \tag{19}$$

$$|l_{kmh}| \leq (i_{km}^{\max})^2, \quad \{\forall km \in \mathcal{L}. \forall h \in \mathcal{H}\} \tag{20}$$

The compact form of the general optimization problem is a nonlinear programming model with a non-convex structure, as defined from (1) to (9). Next, the complete second-order cone approximation for this problem is presented.
Objective functions:

$$\min E_{\text{loss}} = \sum_{h\in\mathcal{H}} \sum_{k\in\mathcal{N}} \sum_{m\in\mathcal{N}} G_{km}\omega_{kmh}\Delta_h, \tag{21}$$

$$\min E_{\text{CO2}} = \sum_{h\in\mathcal{H}} \sum_{k\in\mathcal{N}} \left(C_{\text{CO2}}^{\text{sub}} p_{kh}^{\text{sub}} + C_{\text{CO2}}^{\text{diesel}} p_{kh}^{\text{diesel}} \right) \Delta_h, \tag{22}$$

Set of constraints:

$$p_{kh}^{\text{sub}} + p_{kh}^{\text{diesel}} + p_{kh}^{pv} - p_{kh}^{d} = \sum_{m\in\mathcal{N}} G_{km}\omega_{kmh}, \quad \{\forall k \in \mathcal{N},\ \forall h \in \mathcal{H}\} \tag{23}$$

$$l_{kmh} = g_{km}^2(\omega_{kkh} - 2\omega_{kmh} + \omega_{mmh}) \quad \{\forall km \in \mathcal{L},\ \forall h \in \mathcal{H}\} \tag{24}$$

$$p_k^{\text{min,sub}} \le p_k^{\text{sub}} \le p_{kh}^{\text{max,sub}}, \quad \{\forall k \in \mathcal{N},\ \forall h \in \mathcal{H}\} \tag{25}$$

$$p_k^{\text{min,diesel}} \le p_{kh}^{\text{diesel}} \le p_k^{\text{max,diesel}}, \quad \{\forall k \in \mathcal{N},\ \forall h \in \mathcal{H}\} \tag{26}$$

$$p_{kh}^{\text{min},pv} \le p_{kh}^{pv} \le p_{kh}^{\text{max},pv}, \quad \{\forall k \in \mathcal{N},\ \forall h \in \mathcal{H}\} \tag{27}$$

$$\left(v_k^{\min}\right)^2 \le \omega_{kkh} \le (v_k^{\max})^2, \quad \{\forall k \in \mathcal{N},\ \forall h \in \mathcal{H}\} \tag{28}$$

$$|l_{kmh}| \le (i_{km}^{\max})^2, \quad \{\forall km \in \mathcal{L}.\ \forall h \in \mathcal{H}\} \tag{29}$$

$$\left\| \begin{matrix} 2\omega_{kmh} \\ \omega_{kkh} - \omega_{mmh} \end{matrix} \right\| \le \omega_{kkh} + \omega_{mmh}. \quad \{\forall k, m \in \mathcal{N},\ \forall h \in \mathcal{H}\} \tag{30}$$

Remark 4. *The optimization model* (21)–(30) *represents is convex and belongs to the family of the second-order cone approximations, given the hyperbolic relaxation of the product between two variables, with the main advantage that, under good operating conditions (loads far from the voltage collapse point), it converges to the global optimal solution [40].*

To illustrate the general workflow of the proposed EMS using a conic approximation, Algorithm 1 shows the solution flow applied to the optimization model (21)–(30), which depends on the selected objective function and all the potential operation scenarios associated with the demand behavior and the PV generation forecast.

Algorithm 1: EMS operation for PV systems in monopolar DC networks

Data: Power generation output in PV plants
Select the monopolar DC network under analysis;
Compute the per-unit equivalent representation of the network;
Define the expected power demand curve and PV generation curves with their uncertainties;
for *Each expected demand or generation input variations* **do**
  Implement the conic approximation in (21)–(30) using a convex optimization tool;
  Select the objective function under minimization, i.e., E_{loss} or E_{CO2};
  Solve the proposed conic optimization model;
  Obtain the generation profile of the PV plants for a day-ahead operation;
  Define the expected objective function value;
Construct different operation plants for each possible demand and PV generation forecasts;
Result: Publish the day-ahead operation plan

It is worth mentioning that, in order to avoid a repetitive solution for each possible demand behavior and PV generation projection, this research proposes a robust convex analysis involving the worst possible operation scenario, with the aim to define the optimal day-ahead PV generation plan. This analysis is presented in the next subsection.

3.2. Robust Formulation

For its implementation, the EMS requires information such as the power output of the PV generators and the load demand. However, this information is not 100% accurate, as it is a forecast. Therefore, uncertainty must be incorporated in order to obtain a more accurate EMS. One way to account for uncertainty in a model is through robust optimization, which aims to identify the worst-case scenario for the problem. First, robust optimization defines the set of decision variables (x) of the model. Secondly, it includes the set of uncertainties (w) in order to achieve the minimum of the model under the worst-case cost while observing all constraints. A robust optimization model can be defined as follows:

$$\begin{aligned} \min_{x} \max_{w} \;& f(x,w), \\ \text{subject to } \;& h(x,w) = 0 \; \forall \, w \in \mathcal{W}, \\ & g(x,w) \leq 0 \; \forall \, w \in \mathcal{W}, \end{aligned} \tag{31}$$

where $f(x,w)$, $h(x,w)$, and $g(x,w)$ are the objective function, the set of equality constraints, and the set of inequality constraints that include the set of uncertainties in the original optimization model, respectively. $\mathcal{W}$ denotes the set of uncertainties.

Now, it is necessary to include the set of uncertainties in the conic programming approximation described in (21)–(30). Initially, the following variables are defined:

$$\gamma_{kh}^{p+} + \gamma_{kh}^{p-} \leq 1, \; \{\forall \, k \in \mathcal{N}, \; \forall \, h \in \mathcal{H}\} \tag{32}$$

$$\gamma_{kh}^{d+} + \gamma_{kh}^{d-} \leq 1, \; \{\forall \, k \in \mathcal{N}, \; \forall \, h \in \mathcal{H}\} \tag{33}$$

$$p_{kh}^{pv} = \bar{p}_{kh}^{pv} + \hat{p}_{kh}^{pv}\gamma_{kh}^{p+} - \hat{p}_{kh}^{pv}\gamma_{kh}^{p-}, \; \{\forall \, k \in \mathcal{N}, \; \forall \, h \in \mathcal{H}\} \tag{34}$$

$$p_{kh}^{d} = \bar{p}_{kh}^{d} + \hat{p}_{kh}^{d}\gamma_{kh}^{d+} - \hat{p}_{kh}^{d}\gamma_{kh}^{d-}, \; \{\forall \, k \in \mathcal{N}, \; \forall \, h \in \mathcal{H}\} \tag{35}$$

where $\gamma \in \{0,1\}$ defines the set of uncertainties; $\bar{p}_{kh}^{pv}$ and $\hat{p}_{kh}^{pv}$ are the power available from the PV generator and its deviation, respectively; $\bar{p}_{kh}^{d}$ is the power consumption of the load, and $\hat{p}_{kh}^{d}$ is its deviation.

Note that in (34) and (35), only the values in the following intervals are considered:

$$p_{kh}^{pv} \in [\bar{p}_{kh}^{pv} - \hat{p}_{kh}^{pv}, \; \bar{p}_{kh}^{pv} + \hat{p}_{kh}^{pv}], \tag{36}$$

$$p_{kh}^{d} \in [\bar{p}_{kh}^{d} - \hat{p}_{kh}^{d}, \; \bar{p}_{kh}^{d} + \hat{p}_{kh}^{d}]. \tag{37}$$

The robust conic programming approximation (RCPA) model yields the following formulation:
Objective functions:

$$\min_{W} \left(E_{\text{loss}} \max_{w \in \mathcal{W}} \left(\|\bar{p}^{pv}\| + \left\|\bar{P}^{d}\right\| \right) \right), \tag{38}$$

$$\min_{W} \left(E_{\text{CO2}} \max_{w \in \mathcal{W}} \left(\|\bar{p}^{pv}\| + \left\|\bar{P}^{d}\right\| \right) \right), \tag{39}$$

$$\text{subject to (23)–(26), (28)–(30), (32)–(35).} \tag{40}$$

Algorithm 2 illustrates the flowchart of the proposed RCPA model.

Algorithm 2: Robust conic programming approximation

Data: Select the monopolar DC grid with the PV generators to be analyzed

Compute the per-unit equivalent representation of the network;

Define LB $= -\infty$, UB $= +\infty$.

for $h = 1 : 24$ **do**

  while $(UB - LB < \epsilon)$ **do**

    Solve (23)–(30). Obtain the optimal solution and the objective function value, $x^* = [w^*_{km}, \; p^*_{pv}, \; p^*_{\text{sub}}]$ and E_{loss} (or E_{CO2}), respectively.

    LB $\leftarrow \max\{\text{LB}, E_{\text{loss}} \text{ (or } E_{\text{CO2}})\}$.

    Solve $f_s(x^*, w) = \max\left(\|\bar{p}^{pv}\| + \left\|\bar{p}^{d}\right\| \right)$, subject to (23)–(26), (28)–(30), (32)–(35), with $x = x^*$.

    Obtain the worst-case scenario regarding the uncertainty and the objective function, $w^* = [\hat{p}_h^{pv^*}, \; \hat{p}_h^{d^*}]$ and f_s.

    UB $\leftarrow \min\{\text{UB}, f_s\}$.

    $p_h^{pv\,\max} \leftarrow p_h^{pv\,\max} + w^*$.

    $p_h^{pv\,\min} \leftarrow p_h^{pv\,\min} - w^*$.

    $p_h^{d\,\max} \leftarrow p_h^{d\,\max} + w^*$.

    $p_h^{d\,\min} \leftarrow p_h^{d\,\min} - w^*$.

  $h \leftarrow h + 1$.

Result: Return x^* and E_{loss} (or E_{CO2})

4. Test Feeder Information

To validate the proposed second-order cone optimization model, the monopolar DC of the IEEE 33-bus grid was considered with information on the demand and generation profiles in Medellín (Colombia) [8]. The electrical configuration of this test feeder is depicted in Figure 2. The main characteristic of this system is that three PV plants, with installed capacities of 2.4 MW, are allocated in nodes 12, 15, and 31. This grid has a PV penetration level of around 43.72%, which is computed as the total energy demand and the total generation available from the PV systems, using the integrals of the demand and generation curves. This is a medium-voltage distribution grid that operates with 12,660 V at the terminals of the substation bus, which is located at node 1.

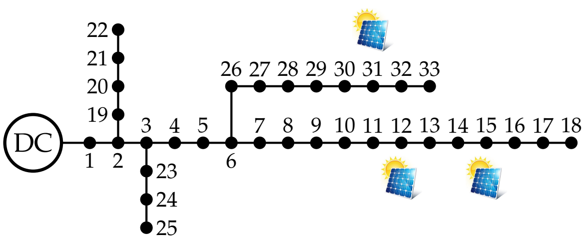

Figure 2. Urban distribution network composed of 33 nodes.

The parametric information regarding the loads and branches of the 33-bus grid is presented in Table 2.

The daily expected demand behavior measured at the terminals of the substation bus and the daily renewable generation profile depicted in Figure 3 were taken from the work by [10], who presented a complete characterization of Medellín (Colombia).

Table 2. Parametric information of the urban distribution network used for validation.

Line l	Node i	Node j	R_{ij} (Ω)	P_j (kW)	I_l^{max} (A)
1	1	2	0.0922	100	320
2	2	3	0.4930	90	280
3	3	4	0.3660	120	195
4	4	5	0.3811	60	195
5	5	6	0.8190	60	195
6	6	7	0.1872	200	95
7	7	8	1.7114	200	85
8	8	9	1.0300	60	70
9	9	10	1.0400	60	55
10	10	11	0.1966	45	55
11	11	12	0.3744	60	55
12	12	13	1.4680	60	40
13	13	14	0.5416	120	40
14	14	15	0.5910	60	25
15	15	16	0.7463	60	20
16	16	17	1.2890	60	20
17	17	18	0.7320	90	20
18	2	19	0.1640	90	30
19	19	20	1.5042	90	25
20	20	21	0.4095	90	20
21	21	22	0.7089	90	20
22	3	23	0.4512	90	85
23	23	24	0.8980	420	70
24	24	25	0.8900	420	40
25	6	26	0.2030	60	85
26	26	27	0.2842	60	85
27	27	28	1.0590	60	70
28	28	29	0.8042	120	70
29	29	30	0.5075	200	55
30	30	31	0.9744	150	40
31	31	32	0.3105	210	25
32	32	33	0.3410	60	20

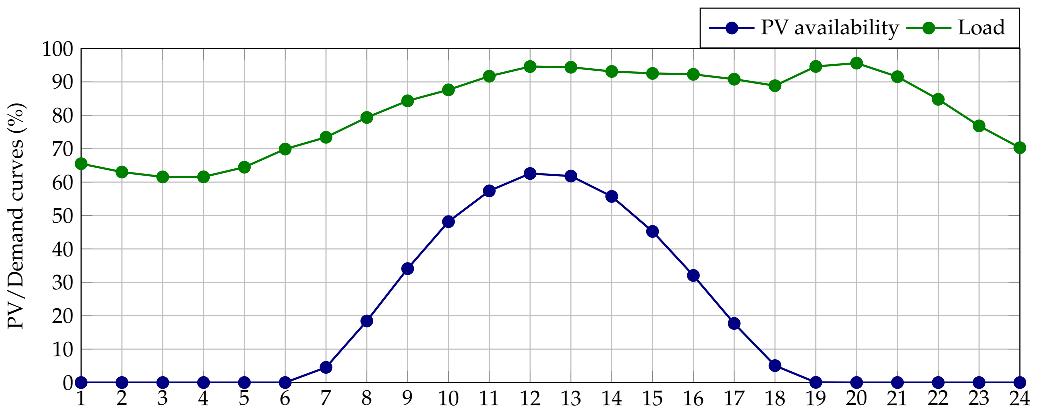

Figure 3. Expected daily behavior of the generation and demand curves for Medellín (Colombia).

5. Results and Discussion

This section shows all the numerical results obtained with the RCPA approach in the modified DC 33-bus system. The computational implementation was carried out in YALMIP (R20230622 version), a toolbox for convex optimization of the MATLAB 2021a software [42]. The RCPA model shown in (38)–(40) was executed on a PC with an Intel Quad-Core i7-7700HQ processor @2.80 GHz, 16 GB RAM (Dell Inc., Round Rock, TX, USA; Intel Corporation, Santa Clara, CA, USA, and 64-bit Windows 10 Home Single Language. The YALMIP solver used to solve the proposed model was Mosek [43].

This research considered the following simulation scenarios to validate the proposed deterministic and robust convex models.

i. The minimization of the daily energy losses in the deterministic case.
ii. The minimization of the daily energy losses while considering uncertainties in the demand and PV curves.
iii. The minimization of the daily CO_2 emissions in the deterministic case.
iv. The minimization of the daily CO_2 emissions while considering uncertainties in the demand and PV curves.

5.1. Minimization of Daily Energy Losses

This subsection compares the proposed RCPA approach against methods such as the iterative convex model (ICM) [8], multi-verse optimization (MVO) [10], the particle swarm optimizer (PSO) [10], the crow search algorithm (CSA) [10], and the salp swarm algorithm (SSA) [10]. This comparison sought to evaluate the performance of the proposed RCPA method only with regard to the objective function (21). Table 3 presents the numerical results obtained by all approaches for the minimization of the daily energy losses. These losses are about 2186.2799 kWh/day without PV generation (benchmark case).

Table 3. Numerical results for all approaches used in this study.

Method	E_{loss} (kWh/day)	Reduction (%)
Benchmark case	2186.2799	—
CSA	1270.1562	41.9033
PSO	1268.5973	41.9746
MVO	1231.2531	43.6827
SSA	1225.3323	43.9536
ICM	1224.8548	43.9754
RCPA	1224.8548	43.9754

According to the results shown in Table 3, the following conclusions can be drawn:

i. The RCPA approach achieves the best solution, with a daily energy loss reduction of 44.0082% with respect to the benchmark case. The SSA and MVO approaches are close to the best solution, with reductions of 43.9536% and 43.9536%, respectively. This demonstrates that the RCPA approach finds the best solution for the problem, unlike the random-based optimization approaches, such as the SSA, MVO, PSO, and CSA. If energy loss costs of 0.1302 USD/kWh (taken from [8]) are assumed, the reductions would be USD 119.2793, 119.4823, 124.3445, 125.1154, and 125.1775 per day for the CSA, PSO, MVO, SSA, and RCPA approaches, respectively, indicating that the RCPA can reach the best solution with the lowest energy losses costs.
ii. The CSA and PSO approaches yield the worst solutions to the problem, with expected reductions of less than 42%. This demonstrates that these random-based optimization approaches only find local solutions. The proposed RCPA outperforms these methods by 2.1049% and 2.0336%.
iii. The ICM approach is a convex model that finds the same solution as the proposed RCPA approach. However, this approach does not have a robust formulation that allows including uncertainties in demand and PV generation.

5.2. Minimization of Daily Energy Losses While Considering Uncertainties

This subsection examines the performance of the proposed RCPA approach (see model (38) and (40)) while considering demand and PV generation uncertainties, whose values are assumed to be $\pm10\%$ of the nominal values. Figure 4 illustrates the PV generation and demand curves with these values, and Table 4 lists the numerical results obtained for this case. The benchmark case included the dispatch of PV generators.

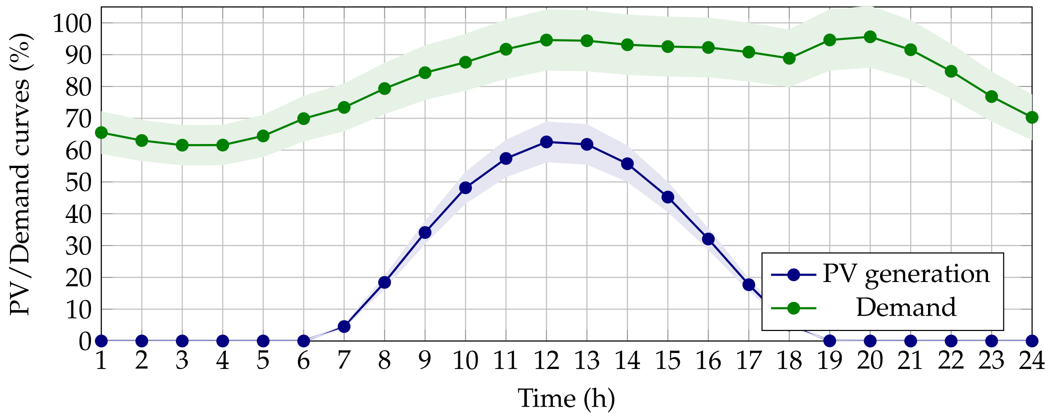

Figure 4. PV generation and demand curves considering uncertainties of ±10% with respect to the nominal values.

Table 4. Numerical results obtained when including demand and PV generation uncertainties for E_{loss}.

Uncertainty	E_{loss} (kWh/day)	Increase (%)
Benchmark case	1224.8548	—
Demand	1504.3148	22.8157
PV generation	1227.3335	0.2023
Demand/PV generation	1516.2396	23.7893

From Table 4, note that:

i. The daily energy losses increase to 1504.3148 kWh/day when a demand uncertainty of ±10% is included. This increase is around 22.8157% when compared to the benchmark case. This result is expected, as the daily energy losses are calculated while considering an increase in the power demanded.
ii. PV generation uncertainty does not entail a significant change in the daily energy losses; they increase by 0.2023% with respect to the benchmark case. However, this increase is explained by the fact that the PV generators are programmed for the worst-case scenario.
iii. The daily energy losses for this case are 1516.2396 kWh/day. This is the highest result, with increments of 23.7893% (without uncertainty), 0.7927% (only demand uncertainty), and 23.5393% (only PV generation uncertainty). This is expected and logical, given that this scenario considers a more significant uncertainty, and all dispatched powers correspond to the worst possible case.

5.3. Minimization of the Environmental Objective Function

This subsection presents an analysis of the numerical results obtained by applying the RCPA to minimize the CO_2 emissions caused by the generation from conventional sources. Table 5 presents a comparison with the CSA, the PSO, the MVO, the SSA, and the ICM reported in [8]. The total CO_2 emissions for the benchmark case (without PV generation) are about 12,345.1497 kg/day.

Table 5. Numerical results obtained by all approaches for E_{CO_2}

Method	E_{CO_2} (kg/day)	Reduction (%)
Benchmark case	12,345.1497	–
CSA	9328.7685	24.4337
PSO	9282.4081	24.8093
MVO	9187.9682	25.5743
SSA	9166.6746	25.7568
ICM	8965.4072	27.3771
RCPA	8965.4072	27.3771

The numerical data in Table 5 show that:

i. The proposed RCPA finds the best possible solution for the deterministic case, with total CO_2 emissions reductions of about 27.3771% in comparison with the benchmark case. This solution is the same as that reported by the ICM. However, all of the combinatorial optimizers, i.e., the CSA, the PSO, the MVO, and the SSA, get stuck in local optima, thus confirming that the RCPA approach, together with the ICM, are the best options for operating PV systems in monopolar DC networks.
ii. For the deterministic reduction in energy losses, the CSA and PSO approaches are the worst combinatorial methods, with reductions lower than 25%, i.e., differences greater than 2% when compared to the convex approaches.
iii. The proposed RCPA outperforms the best combinatorial optimization method (SSA) by about 1.62%, which corresponds to an additional improvement of about 201.2674 kg/day.

The main result in Table 5 is that convex optimization methods are the best options in the design of EMS for renewable energy applications in monopolar DC grids. This is due to the fact that the convexity of the solution space and the objective functions ensure a 100% solution repeatability, unlike random-based algorithms, which can reach different local optima in each evaluation.

5.4. Minimization of the Environmental Objective Function While Considering Uncertainties

This subsection analyzes the performance of the proposed RCPA approach (see models (39) and (40)) in minimizing the expected CO_2 emissions generated by conventional sources while considering demand and PV generation uncertainty, whose assumed level is $\pm$10% of the nominal values (see Figure 4). Table 6 presents the numerical results obtained for this scenario.

Table 6. Numerical results for E_{CO_2} when considering uncertainties in the demand and PV generators.

Uncertainty	E_{CO_2} (kg/day)	Increase (%)
Benchmark case	8965.4072	—
Demand	9968.2305	11.1854
PV generation	9047.0174	0.9102
Demand/PV generation	10,051.9989	12.1198

From Table 4, it can be stated that:

i. The demand uncertainty increases the CO_2 emissions to 9968.2305 kg/day, which constitutes an increment of 11.1854% in comparison with the benchmark case. This is expected, as the system's generators are programmed for the worst-case scenario.
ii. PV generation uncertainty does not significantly change the CO_2 emissions, which only increase by 0.9102% with respect to the benchmark case. However, this value continues to increase because the worst-case scenario is considered.
iii. The expected CO_2 emissions from conventional sources amount to 10,051.9989 kg/day when taking uncertainty into account. This result surpasses those of other cases by 12.1198% (without uncertainty), 0.8403% (only demand uncertainty), and 11.1084% (only PV generation uncertainty).

6. Conclusions

This paper described a robust conic convex model for efficiently managing energy in monopolar DC networks which considers uncertainties in demand and PV generation. The EMS model belongs to the NLP family and was transformed into a convex optimization problem via the second-order cone equivalent of the product between two positive variables. Furthermore, two single-objective functions were considered to evaluate the performance of the proposed RCPA approach, i.e., the minimization of the expected daily energy losses and the CO_2 emissions from conventional generation sources. Each objective function included demand and PV generation uncertainties, which were solved using a min–max strategy. This strategy involved solving the model under the worst-case possible configuration while observing the set of constraints. Numerical simulations in the monopolar DC version of the IEEE 33-bus grid showed the effectiveness and robustness of the proposed approach. This was supported by a comprehensive comparison of four different combinatorial optimizers: the SSA, the MVO, the PSO, and the CSA. The global optimum was found for two single-objective functions; the first one (daily energy losses) was reduced by 44.0082% with respect to the benchmark case, while the other one (CO_2 emissions) was reduced by 27.3771%. When uncertainties were included in the model, each objective function obtained a worse result, as the worst-case scenario was taken into account. In all the cases considered, the daily energy losses increased by 22.8157% (only demand uncertainty), 0.2023% (only PV generation uncertainty), and 23.7893% (both PV generation and demand uncertainty). Similarly, with respect to the benchmark case, the expected CO_2 emissions showed increments of 11.1854%, 0.9102%, and 12.1198%, respectively.

In future works, the following works could be carried out: (i) extending the proposed optimization model to the integration of battery energy storage systems and wind generation and (ii) designing an EMS for monopolar DC networks that is based on a semi-definite programming model and includes a robust analysis.

Author Contributions: Conceptualization, methodology, software, and writing (review and editing): O.D.M., F.M.S. and W.G.-G. All authors have read and agreed to the published version of the manuscript.

Funding: This research received support from the Ibero-American Science and Technology Development Program (CYTED) through thematic network 723RT0150, Red para la integración a gran escala de energías renovables en sistemas eléctricos (RIBIERSE-CYTED). This research was also supported by project no. 6-23-7, titled Desarrollo de una metodología para la compensación óptima de potencia reactiva en sistemas eléctricos de distribución from at Universidad Tecnológica de Pereira.

Data Availability Statement: No new data were created or analyzed in this study. Data sharing does not apply to this article.

Acknowledgments: To God, who opens the doors of scientific knowledge and enlightens us to achieve our goals. The authors also want to thank Vicerrectoria de Investigación, Innovación y Extensión from Universidad Tecnológica de Pereira for the support provided in this investigation.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Karimi-Arpanahi, S.; Jooshaki, M.; Moeini-Aghtaie, M.; Abbaspour, A.; Fotuhi-Firuzabad, M. Incorporating flexibility requirements into distribution system expansion planning studies based on regulatory policies. *Int. J. Electr. Power Energy Syst.* **2020**, *118*, 105769. [CrossRef]
2. Iweh, C.D.; Gyamfi, S.; Tanyi, E.; Effah-Donyina, E. Distributed Generation and Renewable Energy Integration into the Grid: Prerequisites, Push Factors, Practical Options, Issues and Merits. *Energies* **2021**, *14*, 5375. [CrossRef]
3. Kayal, P.; Chanda, C. Placement of wind and solar based DGs in distribution system for power loss minimization and voltage stability improvement. *Int. J. Electr. Power Energy Syst.* **2013**, *53*, 795–809. [CrossRef]
4. Sun, W.; Harrison, G.P. Wind-solar complementarity and effective use of distribution network capacity. *Appl. Energy* **2019**, *247*, 89–101. [CrossRef]
5. Strielkowski, W.; Civín, L.; Tarkhanova, E.; Tvaronavičienė, M.; Petrenko, Y. Renewable Energy in the Sustainable Development of Electrical Power Sector: A Review. *Energies* **2021**, *14*, 8240. [CrossRef]

6. Lakshmi, S.; Ganguly, S. Transition of Power Distribution System Planning from Passive to Active Networks: A State-of-the-Art Review and a New Proposal. In *Sustainable Energy Technology and Policies*; Springer: Singapore, 2017; pp. 87–117. [CrossRef]
7. Grisales-Noreña, L.F.; Rosales-Muñoz, A.A.; Montoya, O.D. An Effective Power Dispatch of Photovoltaic Generators in DC Networks via the Antlion Optimizer. *Energies* **2023**, *16*, 1350. [CrossRef]
8. Montoya, O.D.; Grisales-Noreña, L.F.; Hernández, J.C. Efficient Day-Ahead Dispatch of Photovoltaic Sources in Monopolar DC Networks via an Iterative Convex Approximation. *Energies* **2023**, *16*, 1105. [CrossRef]
9. Alam, M.S.; Arefifar, S.A. Energy Management in Power Distribution Systems: Review, Classification, Limitations and Challenges. *IEEE Access* **2019**, *7*, 92979–93001. [CrossRef]
10. Grisales-Noreña, L.F.; Ocampo-Toro, J.A.; Rosales-Muñoz, A.A.; Cortes-Caicedo, B.; Montoya, O.D. An Energy Management System for PV Sources in Standalone and Connected DC Networks Considering Economic, Technical, and Environmental Indices. *Sustainability* **2022**, *14*, 16429. [CrossRef]
11. Jamii, J.; Trabelsi, M.; Mansouri, M.; Mimouni, M.F.; Shatanawi, W. Non-Linear Programming-Based Energy Management for a Wind Farm Coupled with Pumped Hydro Storage System. *Sustainability* **2022**, *14*, 11287. [CrossRef]
12. Shaaban, M.; Ahmed, M.; Salama, M.; Rahimi-Kian, A. Optimization unit for real-time applications in unbalanced smart distribution networks. *J. Adv. Res.* **2019**, *20*, 51–60. [CrossRef]
13. Rodríguez, N.; Gupta, A.; Zabala, P.L.; Cabrera-Guerrero, G. Optimization Algorithms Combining (Meta)heuristics and Mathematical Programming and Its Application in Engineering. *Math. Probl. Eng.* **2018**, *2018*, 3967457. [CrossRef]
14. Pervez, I.; Antoniadis, C.; Massoud, Y. A Reduced Search Space Exploration Metaheuristic Algorithm for MPPT. *IEEE Access* **2022**, *10*, 26090–26100. [CrossRef]
15. Montoya, O.D.; Bohórquez-Bautista, K.J.; Moreno-Arias, D.A.; Gil-González, W. Comparative Methods for Solving Optimal Power Flow in Distribution Networks Considering Distributed Generators: Metaheuristics vs. Convex Optimization. *Tecnura* **2022**, *26*, 87–129. [CrossRef]
16. Mutapcic, A.; Boyd, S. Cutting-set methods for robust convex optimization with pessimizing oracles. *Optim. Methods Softw.* **2009**, *24*, 381–406. [CrossRef]
17. Chaerani, D.; Rusyaman, E.; Muslihin, K.R.A. Comprehensive survey on convex analysis in robust optimization. *J. Phys. Conf. Ser.* **2021**, *1722*, 012075. [CrossRef]
18. Bracinik, P.; Rybarik, M.; Hoger, M.; Regula, M. The influence of direct current distribution networks on the formation of stray currents. In Proceedings of the 2022 ELEKTRO (ELEKTRO), Krakow, Poland, 23–26 May 2022. [CrossRef]
19. Garces, A. Uniqueness of the power flow solutions in low voltage direct current grids. *Electr. Power Syst. Res.* **2017**, *151*, 149–153. [CrossRef]
20. Garces, A. On the Convergence of Newton's Method in Power Flow Studies for DC Microgrids. *IEEE Trans. Power Syst.* **2018**, *33*, 5770–5777. [CrossRef]
21. Montoya, O.D.; Serra, F.M.; Angelo, C.H.D. On the Efficiency in Electrical Networks with AC and DC Operation Technologies: A Comparative Study at the Distribution Stage. *Electronics* **2020**, *9*, 1352. [CrossRef]
22. Cortés-Caicedo, B.; Grisales-Noreña, L.F.; Montoya, O.D.; Rodriguez-Cabal, M.A.; Rosero, J.A. Energy Management System for the Optimal Operation of PV Generators in Distribution Systems Using the Antlion Optimizer: A Colombian Urban and Rural Case Study. *Sustainability* **2022**, *14*, 16083. [CrossRef]
23. Yunus, S.; Latif, M.; Darwison, D. Optimal PV Placement to Reduce Power Loss and Improve Voltage in Distribution Network System Using K-means Clustering Method. *Andalas J. Electr. Electron. Eng. Technol.* **2022**, *2*, 7–12. [CrossRef]
24. da Silva, D.J.; Belati, E.A.; López-Lezama, J.M. A Mathematical Programming Approach for the Optimal Operation of Storage Systems, Photovoltaic and Wind Power Generation. *Energies* **2023**, *16*, 1269. [CrossRef]
25. Ngo, V.Q.B.; Latifi, M.; Abbassi, R.; Jerbi, H.; Ohshima, K.; khaksar, M. Improved krill herd algorithm based sliding mode MPPT controller for variable step size P&O method in PV system under simultaneous change of irradiance and temperature. *J. Frankl. Inst.* **2021**, *358*, 3491–3511. [CrossRef]
26. Sarwar, S.; Hafeez, M.A.; Javed, M.Y.; Asghar, A.B.; Ejsmont, K. A Horse Herd Optimization Algorithm (HOA)-Based MPPT Technique under Partial and Complex Partial Shading Conditions. *Energies* **2022**, *15*, 1880. [CrossRef]
27. Alanazi, A.; Alanazi, M.; Nowdeh, S.A.; Abdelaziz, A.Y.; El-Shahat, A. An optimal sizing framework for autonomous photovoltaic/hydrokinetic/hydrogen energy system considering cost, reliability and forced outage rate using horse herd optimization. *Energy Rep.* **2022**, *8*, 7154–7175. [CrossRef]
28. Sood, V.K.; Ali, M.Y.; Khan, F. Energy Management System of a Microgrid Using Particle Swarm Optimization (PSO) and Communication System. In *Microgrid: Operation, Control, Monitoring and Protection*; Lecture Notes in Electrical Engineering; Springer: Singapore, 2020; pp. 263–288. [CrossRef]
29. Boujdaini, L.E.; Mezrhab, A.; Moussaoui, M.A.; Jurado, F.; Vera, D. Sizing of a stand-alone PV–wind–battery–diesel hybrid energy system and optimal combination using a particle swarm optimization algorithm. *Electr. Eng.* **2022**, *104*, 3339–3359. [CrossRef]
30. Alizadeh, S.; Mahdavian, M.; Ganji, E. Optimal placement and sizing of photovoltaic power plants in power grid considering multi-objective optimization using evolutionary algorithms. *J. Electr. Syst. Inf. Technol.* **2023**, *10*, 7. [CrossRef]
31. Katyara, S.; Shaikh, M.F.; Shaikh, S.; Khand, Z.H.; Staszewski, L.; Bhan, V.; Majeed, A.; Shah, M.A.; Zbigniew, L. Leveraging a Genetic Algorithm for the Optimal Placement of Distributed Generation and the Need for Energy Management Strategies Using a Fuzzy Inference System. *Electronics* **2021**, *10*, 172. [CrossRef]

32. Yaghoobi, Z.S.; Nazari, M.E.; Safdarian, F. Comparison between the Optimal Sizing of a Photovoltaic System in Interconnected and Isolated Modes Using Gravitational Search Algorithm. In Proceedings of the 2018 1st International Conference on Advanced Research in Engineering Sciences (ARES), Dubai, United Arab, 15 June 2018. [CrossRef]
33. Grisales-Noreña, L.F.; Ramos-Paja, C.A.; Gonzalez-Montoya, D.; Alcalá, G.; Hernandez-Escobedo, Q. Energy management in PV based microgrids designed for the Universidad Nacional de Colombia. *Sustainability* **2020**, *12*, 1219. [CrossRef]
34. Montoya, O.D.; Grisales-Noreña, L.F.; Giral-Ramírez, D.A. Multi-Objective Dispatch of PV Plants in Monopolar DC Grids Using a Weighted-Based Iterative Convex Solution Methodology. *Energies* **2023**, *16*, 976. [CrossRef]
35. Grisales-Noreña, L.F.; Rosales-Muñoz, A.A.; Cortés-Caicedo, B.; Montoya, O.D.; Andrade, F. Optimal Operation of PV Sources in DC Grids for Improving Technical, Economical, and Environmental Conditions by Using Vortex Search Algorithm and a Matrix Hourly Power Flow. *Mathematics* **2022**, *11*, 93. [CrossRef]
36. Grisales-Noreña, L.F.; Cortés-Caicedo, B.; Alcalá, G.; Montoya, O.D. Applying the Crow Search Algorithm for the Optimal Integration of PV Generation Units in DC Networks. *Mathematics* **2023**, *11*, 387. [CrossRef]
37. Li, J.; Liu, F.; Wang, Z.; Low, S.H.; Mei, S. Optimal Power Flow in Stand-Alone DC Microgrids. *IEEE Trans. Power Syst.* **2018**, *33*, 5496–5506. [CrossRef]
38. Garces, A. *Mathematical Programming for Power Systems Operation: From Theory to Applications in Python*; John Wiley & Sons: Hoboken, NJ, USA, 2021.
39. Boyd, S.P.; Vandenberghe, L. *Convex Optimization*; Cambridge University Press: Cambridge, UK, 2004.
40. Farivar, M.; Low, S.H. Branch Flow Model: Relaxations and Convexification—Part I. *IEEE Trans. Power Syst.* **2013**, *28*, 2554–2564. [CrossRef]
41. Bomze, I.M.; Peng, B. Conic formulation of QPCCs applied to truly sparse QPs. *Comput. Optim. Appl.* **2022**, *84*, 703–735. [CrossRef]
42. Lofberg, J. YALMIP: A toolbox for modeling and optimization in MATLAB. In Proceedings of the 2004 IEEE International Conference on Robotics and Automation (IEEE Cat. No. 04CH37508), Taipei, Taiwan, 2–4 September 2004; pp. 284–289.
43. *The MOSEK Optimization Toolbox for MATLAB Manual*, Version 9.0; MOSEK ApS: Copenhagen, Denmark, 2019.