ISSN 2688-8300 (Print) ISSN 2644-3368 (Online)

JMSCM, Vol.5, No.1, October, 2023

A STOCHASTIC DETECTION MODELS COMPARISON IN TURBULENT FLOW EVENTS

[1,*]Maria Valeria Calandra & [2]Julio Marañón Di Leo

[1]*UIDET GAMEFI*

Departamento de Ciencias Básicas

Facultad de Ingeniería, Universidad Nacional de la Plata

Calle 48 y 116, La Plata (1900), Argentina

[2]*UIDET LaCLyFA-CTA/UNLP*

Departamento de Ingeniería Aeroespacial

Facultad de Ingeniería, Universidad Nacional de La Plata

Calle 116 entre 47 y 48, La Plata (1900), Argentina

[*]*Corresponding Author Email: maría.calandra@ing.unlp.edu.ar*

Abstract

The need to establish the downwind fluid dynamic field of aerodynamic bodies subjected to a given velocity field is well known, to verify their aerodynamic characteristics. In this context, different techniques allow us to establish the characteristics of the field. It is almost always necessary to carry out quantitative determinations to describe the field correctly, particularly when the field is made up of turbulent wakes. In this sense, in the experimental field, it is common to use hot-wire anemometry techniques, which have great capabilities to quantify high-frequency events. Previous work has analyzed the determination of changes in hot-wire anemometry signals for the detection of events in turbulent flows with different models, based on stochastic algorithms (CPM - Change Point Model).

The present work aims to compare the results obtained previously with the application of different CPM models developed. Previously applied and evaluated measurements are used, the implementation of the models is carried out and the results are compared. All the algorithms used can detect changes in data that do not have a known distribution, i.e. non-parametric distributions, which are typical for turbulent flow field signals. Measurements of the fluctuating components of the wind tunnel velocity at a specific point are considering. The signals used correspond to periodic detachments downstream of a flow control device (Gurney mini-flap) at the trailing edge of an airfoil. The results show which are the best models to use for the experimental detection of such turbulent events in the flow field.

Keywords: *Event detection, Turbulent flow, Vortex, Change point models.*

INTRODUCTION

Turbulent flow in a fluid is that in which the variables: velocity, density, pressure, temperature, etc. behave randomly at each point of the fluid and at each instant of time [1]. The study of turbulent flows is of great importance in many technological applications: Aeronautical, Naval, Mechanical and Structural Engineering, internal flow phenomena, combustion, etc. There are characteristics of the turbulent structure of a flow that change how the fluid moves in the environment of the objects, generating fluid-dynamic forces on them. Such changes can be losses in flow momentum, generated by the appearance of eddies and viscous dissipation, and these effects are commonly observed in aeronautical and naval applications, internal flows, etc. [1]. If an engineering problem involving turbulence is to be improved or optimized, it will be necessary to understand and control the set of turbulent events or structures that govern it [2]. A meticulous analysis of a global turbulent flow would allow detecting the existence of turbulent structures in it, normally hidden. Since the mid-1960s the analysis of turbulence has been revolutionized thanks to the use of sophisticated methodologies for experimental data analysis. There is a vast literature on methodologies for the analysis of turbulent flow measurements, among the most recent ones, we can mention the so-called POD-Proper Orthogonal Decomposition [3], multiresolution analysis methods [4], the use of ARMA (Auto Regressive Moving Average) [5], etc. All these methodologies require significant computational power for their implementation. From the application of these methodologies, it was realized that in many cases the turbulence, which contained an important portion of the kinetic energy, was organized in structures. Such organization occurs, for example, in eddies of very different shape and size. The modern approach to turbulence concentrates on the meticulous study of the various turbulent structures, by analyzing how the flow behaves around the objects. The determination of such behavior allows the effects on a moving object to be inferred. Of interest is the detection the occurrence of specific turbulent events (eddies) that are generated as the flow detaches from the object defining a particular pattern. These eddies allow us to infer important effects related to the forces acting on the object, which are usually immersed in the motion of the turbulent flow [1]. When measurements, particularly in wind tunnel experiments, are performed with point velocity measurement equipment, (Hot Wire Anemometry-HWA [6] or Laser Doppler Anemometry-LDA [7]), the possibility of processing the sensed random signal to detect this type of turbulent events and characterize their frequencies of occurrence and intermittency is of interest. From this, the aim is to analyze and understand how an object processes the fluid in which it is moving, and in this way to predict the behavior of the moving object. Our aim, then, is to apply stochastics techniques, for detecting changes in a sensed signal over time [8], for velocity fluctuations in a turbulent flow. The main objective is the use of these methodologies to analyze measurements made with hot-wire anemometry, incorporating them as another tool that allows us to determine the occurrence of events in a fluid-dynamic field, to carry out their analysis and study.

CHANGE POINT DETECTION PROBLEM

The problem of change detection has been a vast area of research since the 1950s [9-11]. Because the problem is very general, the literature is very diverse and takes place in very different fields. In particular, many of the methods have their origin in the quality control community, where the main objective is to monitor the results of an industrial manufacturing process, aiming to detect faults in the process as early as possible [8]. However, there are many other applications where change detection techniques are important, for example, in the study of genetic sequences, climatological studies, bioinformatics applications, intrusion in computer networks, financial market, etc. There is a lot of literature on all these topics, but their application to the analysis of fluid velocity signals is not well known. In our case, the sensed signal corresponds to the fluctuating values of the air velocity in a turbulent flow. In recent years [12], extensive work has begun on the subject of change detection in a process and certain basic criteria have been defined. Many statistical problems require the identification of change points in a data sequence. Statistical Process Control (SPC) refers to the monitoring of processes due to a change in their distribution. Traditional methods assume that the distribution of the process is fully known before any change, including all its parameters, in which case the process is said to be "in control", and "out of control" if a change occurs that causes the process to correspond to a different distribution. The aim is to design control charts that can detect deviations from the baseline distribution. Usually, in control charts, the Average Run Length function (ARL) is employed, where ARL_0 indicates the average number of observations between false positive detection assuming no change has occurred, and ARL_δ indicates the average delay before a change in size δ is detected. This is analogous to the classical idea applied in hypothesis test design of having a bounded Type I error and a controlled Type II error. Historically control charts were developed for monitoring changes in the mean value of a process, but variations have now been developed that also allow changes in standard deviation to be monitored in both Gaussian and non-Gaussian distributions, which prompted us to investigate the applicability of these new methodologies to the detection of changes in a turbulent random signal. Control charts traditionally require full knowledge of the process "in control", but this is not a problem if there is a large reference sample of observations that are known to generate the distribution "in control". In the case of fixed sample sizes, this is called Phase I analysis, while sequential monitoring of the process when observations are received over time is called Phase II analysis [12]. In some cases, the reference sample may be small or non-existent. In these cases, it would be impossible to accurately estimate the parameters "in control". This has important implications; it was found that even small deviations from the actual values can cause the charts to show a significantly different ARL_0 from the desired value [13]. A worse situation can occur when the distribution "in control" is incorrectly specified, such as the use of a Gaussian distribution for processes exhibiting skewness. In these circumstances, non-parametric control charts are needed that assume no knowledge of the "in-control" distribution ("free distribution" charts), maintaining a desired value of ARL_0 regardless of the true distribution of the process under study. In previous work [14, 15], studies were initiated to analyse the application of

Change Point Models (CPM), used to detect deviations in the sensed signal. In the present work, non-parametric tests for the implementation of CPM models are considered again, using applications of the detection algorithm, with routines coded in R language (https://cran.r-project.org/web/packages/cpm/index.html - http://CRAN.R-project.org/package=ecp - http://CRAN.R-project.org/package=changepoint), employing the Kolmogorov-Smirnov (CPM-KS) [12][16], James-Matteson (CPM-JM) [21], and Killick- Eckley (CPM-KE) [23] tests.

This work presents results obtained from the analyses carried out for the device indicated and comparing the different models used. From these results it is expected to continue with other configurations of the device, to evaluate its behaviour in all the established study cases.

METHODS

KOLMOGORV-SMIRNOV CPM (CPM-KS)

We will consider the problem of detecting a change point in a fixed sequence of observations. By identifying the observations as $\{X_1, ... X_t\}$, the aim is to test whether they have been generated by the same probability distribution. We assume that the distribution is not known a priori. Using the language of statistical hypothesis testing, the null hypothesis is that there is no change point and all observations come from the same distribution, while the alternative hypothesis is that there is a change point τ in the sequence that partitions it into two sets, with $X_1, ... X_\tau$ coming from the pre-change F_0 distribution, and $X_{\tau+1}, ... X_t$ coming from a different F_1 distribution after the change [11],

$$\begin{aligned} H_0&: X_i \sim F_0 \quad \text{for} \quad i = 1, \dots, t \\ H_1&: X_1, \dots, X_\tau \sim F_0, \quad X_{\tau+1}, \dots, X_t \sim F_1 \end{aligned} \tag{1}$$

You can test the existence of a change point immediately after any observation, X_k,, by partitioning the observations into two samples $S_1 = \{X_1, ..., X_k\}$ and $S_2 = \{ X_{k+1} ..., X_t\}$ of sizes $n_1 = k$ and $n_2 = t - k$, respectively, and then applying a hypothesis test for two samples. We will use the Kolmogorov-Smirnov (KS) test for this, which is based on comparing the empirical distribution function of the two samples, as defined,

$$\begin{aligned} \hat{F}s_1(x) &= \frac{1}{k}\sum_{i=1}^{k} I(X_i \le x) \\ \hat{F}s_2(x) &= \frac{1}{t-k}\sum_{i=k+1}^{t} I(X_i \le x) \end{aligned} \tag{2}$$

Where $I(X_i < x)$ is the indicator function

$$I(X_i < x) = \begin{cases} 1 & \text{si} \quad X_i < x \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

For the KS test the statistic is defined as the maximum difference between the empirical distributions seen above (Eq. 2 and 3) where,

$$D_{k,t} = \sup_{x} \left| \hat{F}_{S_1}(x) - \hat{F}_{S_2}(x) \right|, \tag{4}$$

We reject the null hypothesis H_0 if $D_{k,t} > h_{k,t}$ for some threshold $h_{k,t}$
As it is not known where the change point will be located, we do not know which value of k to use for partitioning the sample. That is why we specify a more general hypothesis H_0, i.e. there is no change in the sequence of values. The alternative hypothesis is then that there is a change point for some nonspecific value of k. Then we can make this test by calculating $D_{k,t}$, for each value $1 < k < t$ and take the maximum value. However, the statistical variance $D_{k,t}$ depends on the value of k. Because of this, we standardize the $D_{k,t}$ statistics so that they have equal mean and variance for all values of k.
The standardisation of the KS statistic is complex. This is due to the fact that there are no closed expressions for mean and variance of $D_{k,t}$, except asymptotically when t is large. Instead of considering the statistic $D_{k,t}$ the *p-value* $p_{k,t}$ is used, defined as the probability of observing a value more extreme than $D_{k,t}$. This value can be considered already standardised with respect to the sample size and easier to correct than the mean or variance for small samples.
We will have $q_{k,t} = 1 - p_{k,t}$ and define by,

$$q_t = \underset{k}{máx}\, q_{k,t}, \tag{5}$$

In this case we will have $q_t > h_t$, where h_t is some possible chosen threshold, then the null hypothesis H_0 is discarded and we conclude that a change has occurred at some point in the data sequence. The above formulation of the nonparametric CPMs rely on the calculation of ranks.
One of the most important issues in the implementation of this CPM is the number of pre-change observations; this has a great impact on the performance of the model. As the prior distribution change is unknown, it will be easy to detect changes when the number of previous observations is large, making it possible to obtain a better estimated distribution and a more accurate empirical function distribution.

JAMES-MATTESON CPM (CPM-JM)

It assumes that at most only one change point exists [19]. A natural way to proceed is to choose τ as the most likely location for a change point, based on some criterion. Here, τ is chosen from some subset of $\{1, 2, \dots, T-1\}$, then a test for homogeneity is performed. This should necessarily incorporate the fact that τ is unknown.
Now, suppose there is a known number of change points k in the series, but with unknown locations. Thus, there exist change points $0 < \tau_1 < \dots < \tau_k < T$, that partition the sequence into $k+1$ clusters, such that observations within clusters are identically distributed, and observations between adjacent clusters are not. Then it maximizes the objective function using dynamic programming.
This is a nonparametric technique, which they call *E-Divisive* [21], for performing multiple change point analysis of a sequence of multivariate observations. The *E-Divisive* method combines bisection [20] with a multivariate divergence measure from Székely and Rizzo [22].
For random variables $X; Y \in R^d$; let ϕ_x and ϕ_y denote the characteristic functions of X and Y, respectively. A divergence measure between multivariate distributions may be defined as

$$\int_{\mathbb{R}^d} \left|\phi_x(t) - \phi_y(t)\right|^2 w(t)dt \tag{6}$$

in which *w(t)* denotes an arbitrary positive weight function, for which the above integral exists. Using the following weight function [22]

$$w(t;\alpha) = \left(\frac{2\pi^{d/2}\Gamma(1-\alpha/2)}{\alpha 2^\alpha \Gamma((d+\alpha)/2)}|t|^{d+\alpha}\right)^{-1} \tag{7}$$

For some fixed constant $\alpha \in (0,2)$. Then, if $E/X/^{\alpha}$, $E/Y/^{\alpha} < \infty$, a characteristic function-based divergence measure may be defined as

$$D(X,Y;\alpha) = \int_{\mathbb{R}^d} \left|\phi_x(t) - \phi_y(t)\right|^2 \left(\frac{2\pi^{d/2}\Gamma(1-\alpha/2)}{\alpha 2^\alpha \Gamma((d+\alpha)/2)}|t|^{d+\alpha}\right)^{-1} dt \tag{8}$$

then we may employ an alternative divergence measure based on Euclidean distances, defined in [22]

$$\mathcal{E}(X,Y;\alpha) = 2E|X-Y|^\alpha - E|X-X'|^\alpha - E|Y-Y'|^\alpha \tag{9}$$

An empirical divergence measure analogous to previous Eq. 9 may be defined as

$$\hat{\mathcal{E}}(X_n,Y_m;\alpha) = \frac{2}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m}\left|X_i - Y_j\right|^\alpha - \binom{n}{2}^{-1}\sum_{1\le i<k\le n}|X_i - Y_k|^\alpha - \binom{m}{2}^{-1}\sum_{1\le j<k\le m}\left|X_j - Y_k\right|^\alpha \tag{10}$$

Let

$$\hat{\mathcal{Q}}(X_n,Y_m;\alpha) = \frac{mn}{m+n}\hat{\mathcal{E}}(X_n,Y_m;\alpha) \tag{11}$$

denote the scaled sample measure of divergence discussed above. This statistic leads to a consistent approach for estimating change point locations. Let $Z_1, \ldots, Z_T \in R^d$ be an independent sequence of observations and let $1 \le \tau < k \le T$ be constants. Now define the following sets, $X\tau = \{Z_1, Z_2, \ldots, Z_\tau\}$ and $Y\tau = \{Z_{\tau+1}, Z_{\tau+2}, \ldots, Z_k\}$. A change point location $\hat{\tau}$ is then estimated as

$$(\hat{\tau},\hat{\kappa}) = \underset{(\tau,\kappa)}{\operatorname{argmax}}\, \hat{\mathcal{Q}}(X_\tau, Y_\tau(\kappa);\alpha) \tag{12}$$

To estimate multiple change points, we iteratively apply the above technique as follows. Suppose that $k - 1$ change points have been estimated at locations $0 < \hat{\tau}_1 < \ldots < \hat{\tau}_{k-1} < T$. This partitions the observations into k clusters $\hat{C}_1, \hat{C}_2, \ldots, \hat{C}_k$, such that $\hat{C}_i = \{Z_{\hat{\tau}_{i-1}+1}, \ldots, Z_{\hat{\tau}_i}\}$, in which $\hat{\tau}_0 = 0$ and $\hat{\tau}_k = T$. Given these clusters, we then apply the procedure for finding a single change point to the observations within each of the k clusters. Specifically, for the *ith* cluster $\hat{C}_i$ denote a proposed change point location as $\hat{\tau}(i)$ and the associated constant $\hat{\kappa}(i)$, as defined by Eq. 12. Now, let

$$i^* = \underset{i\in\{1,\ldots,k\}}{\operatorname{argmax}}\, \hat{\mathcal{Q}}\left(X_{\hat{\tau}(i)}, Y_{\hat{\tau}(i)}\left(\hat{\kappa}(i)\right);\alpha\right) \tag{13}$$

in which $\hat{\tau}_k = \hat{\tau}(i^*)$ denotes the *kth* estimated change point, located within the cluster $\hat{C}_{i^*}$, and $\hat{\kappa}_k = \hat{\kappa}(i^*)$ the corresponding constant. This iterative procedure has running time $O(kT^2)$, in which *k* is the unknown number of change points.

KILLICK- ECKLEY CPM (CPM-KE)

Let us assume we have an ordered sequence of data, $y_{1:n} = (y_1, \dots, y_n)$. A changepoint is said to occur within this set when there exists a time, $\tau \in \{1, \dots, n-1\}$, such that the statistical properties of $(y_1, \dots, y_\tau)$ and $(y_{\tau+1}, \dots, y_n)$ are different in some way. Extending this idea of a single changepoint to multiple changes, we will have several changepoints, *m*, together with their positions, $\tau_{1:m} = (\tau_1, \dots, \tau_m)$. Each changepoint position is an integer between *1* and *n - 1* inclusive. We define $\tau_0 = 0$ and $\tau_{m+1} = n$, and assume that the changepoints are ordered so that $\tau_i < \tau_j$ if, and only if, $i < j$. Consequently, the *m* changepoints will split the data into *m + 1* segments, with the *i*th segment containing data $y_{(\tau_{i-1}+1):\tau_i}$. Each segment will be summarized by a set of parameters. The parameters associated with the *i*th segment will be denoted $\{\theta_i, \phi_i\}$, where ϕ_i is a (possibly null) set of nuisance parameters and θ_i is the set of parameters that we believe may contain changes. Typically, we want to test how many segments are needed to represent the data, i.e., how many changepoints are present and estimate the values of the parameters associated with each segment.

We introduce the general likelihood ratio-based approach to the hypothesis test. A test statistic can be constructed which we will use to decide whether a change has occurred. The likelihood ratio method requires the calculation of the maximum log-likelihood under both null and alternative hypotheses. For the null hypothesis the maximum log-likelihood is $\log p(y_{1:n}|\hat{\theta})$, where p(▪) is the probability density function associated with the distribution of the data and $\hat{\theta}$ is the maximum likelihood estimate of the parameters.

Under the alternative hypothesis, consider a model with a changepoint at τ_1, with $\tau_1 \in \{1, 2, \dots, n-1\}$. Then the maximum log likelihood for a given τ_1 is

$$ML(\tau_1) = \log p(y_{1:\tau_1}|\hat{\theta}_1) + \log p(y_{(\tau_1+1):n}|\hat{\theta}_2) \quad (14)$$

Given the discrete nature of the changepoint location, the maximum log-likelihood value under the alternative is simply $max_{\tau_1} ML(\tau_1)$, where the maximum is taken over all possible changepoint locations. The test statistic is thus,

$$\lambda = 2\left[\max_{\tau_1} ML(\tau_1) - log\ p(y_{1:n}|\hat{\theta})\right] \quad (15)$$

The test involves choosing a threshold, *c*, such that we reject the null hypothesis if $\lambda > c$. If we reject the null hypothesis, i.e., detect a changepoint, then we estimate its position as $\hat{\tau}_1$ the value of τ_1 that maximizes $ML(\tau_1)$.

The most common approach to identify multiple changepoints in the literature is to minimize

$$\sum_{i=1}^{m+1} [C(y_{(\tau_{i-1}+1):\tau_i})] + \beta f(m) \quad (16)$$

where C is a cost function for a segment e.g., negative log-likelihood and $\beta f(m)$ is a penalty to guard against over fitting (a multiple changepoint version of the threshold c).

The changepoint package [23] implements the pruned exact linear time (PELT) [24] algorithm that minimize Eq. 16. The algorithm minimizes the expression given by Eq. 16 exactly using a dynamic programming and pruning technique to obtain the optimal segmentation for $m + 1$ changepoints reusing the information that was calculated for m changepoints.

EXPERIMENTAL SETUP

The measurements were performed in one of the close circuit boundary layer wind tunnels of our laboratory (UIDET-LaCLyFA) at the Aeronautics Department Engineering College, at the National University of La Plata, which has a test section 1 m high and 1.4 m wide. The model was a small wing with a chord length of 45 cm (C) and a wingspan of 80 cm (main wing length b), built with a NACA 4412 airfoil (shown in Fig. 1). A flow control device (Gurney mini-flap) with a length H = 2% C was added, located at the trailing edge (TE) of the airfoil at an angle of 90° to the chord axis. The airfoil was submitted to a flow with an angle of attack of 0° (incidence chord angle relative to the free stream direction) to obtain a Reynolds number of 300,000 for the tests.

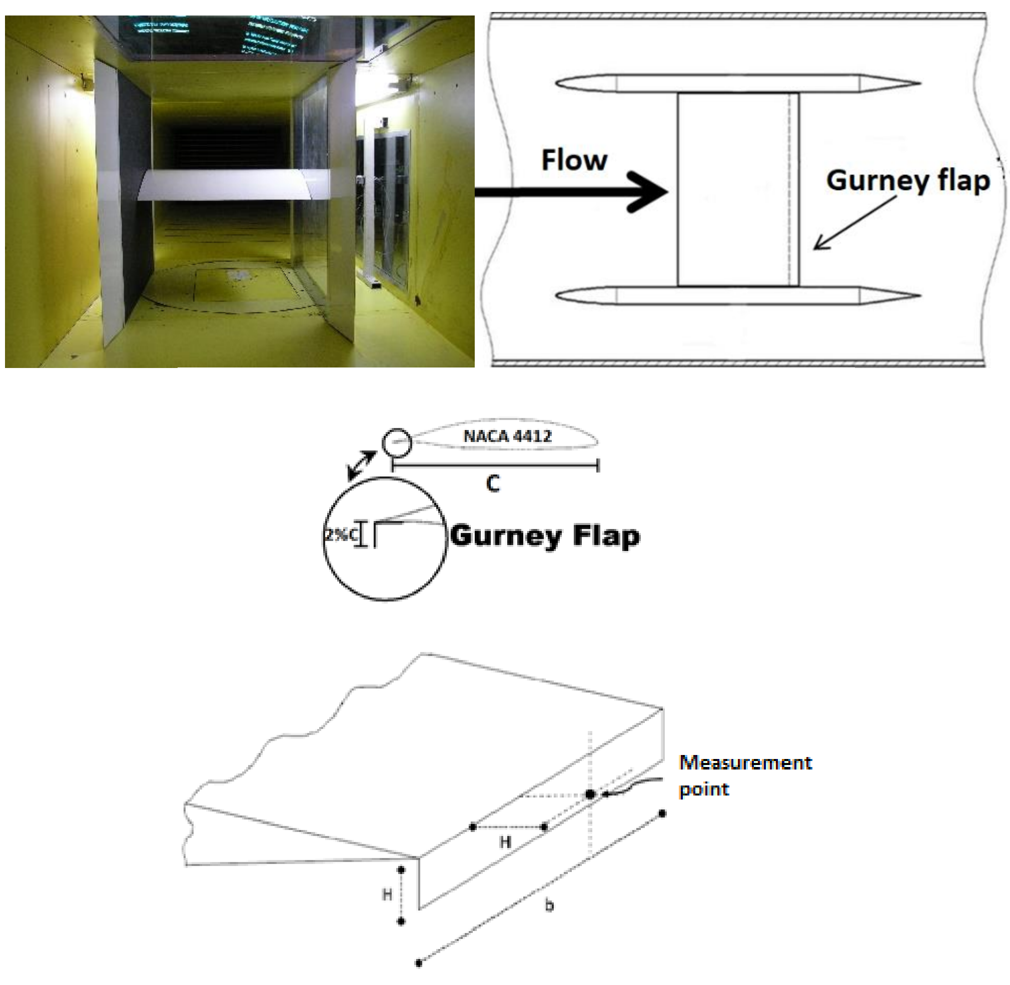

Fig. 1: Wind tunnel setup images and trailing edge measurement point.

The flow velocities components were measured using a constant temperature hot-wire anemometer and a double wire sensor. Signal acquisition was at 4,000 Hz, with a low-pass filter of 2,000 Hz and 8,192 samples. The measurements presented correspond to a point in the wake generated by the profile at a distance of 1H downstream from the trailing edge at the height of the chord, with the passive flow control device at the TE (Gurney mini-flap).

With the knowledge of the flow field generated by the presence of this device and the knowledge that it generates periodic vortex structures, periodic counter-rotating vortices (see Fig. 2), previous work [25] evaluated the possibility of using this methodology to detect the expected wake events. These events were identified by applying the wavelet transform to the signal. Wavelets are localized in both space and frequency; therefore, the wavelet transform analyses a signal locally in the frequency domain and in space or time [3]. The characteristic frequency localization in time of the wavelet transform provides a great opportunity to discover the positions of singularities and discontinuities in a signal, which is not possible with ordinary Fourier analysis [4]. The results of this methodology and the change point models were compared.

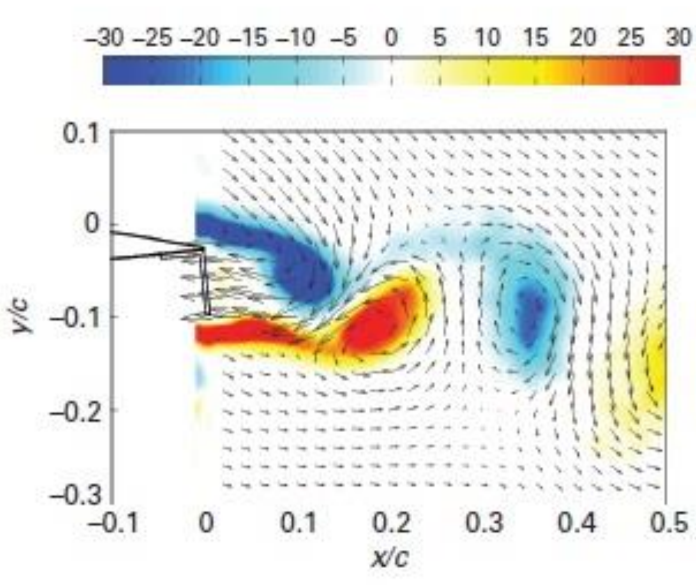

Fig. 2: Counterrotating vortices scheme downstream the Gurney mini-flap (velocity and vorticity field). [26].

To compare, we present the result analysis found in the calculations for the vertical velocity component (v) of the analysed signal. Fig. 3 presents the wavelet map applying wavelet transform to the signal using a Mexican Hat wave type (Ricker wavelet), in which a maximum can be traced in the signal [4]. Hence, the occurrence of a turbulent periodic event associated with one of the counter-rotating vortices that are released downstream of the device is observed. In Fig. 3, the value corresponding to the scales ordinate is defined with the following expression:

$$Scale = \frac{ln(\Delta t)}{\ln(10)} \tag{17}$$

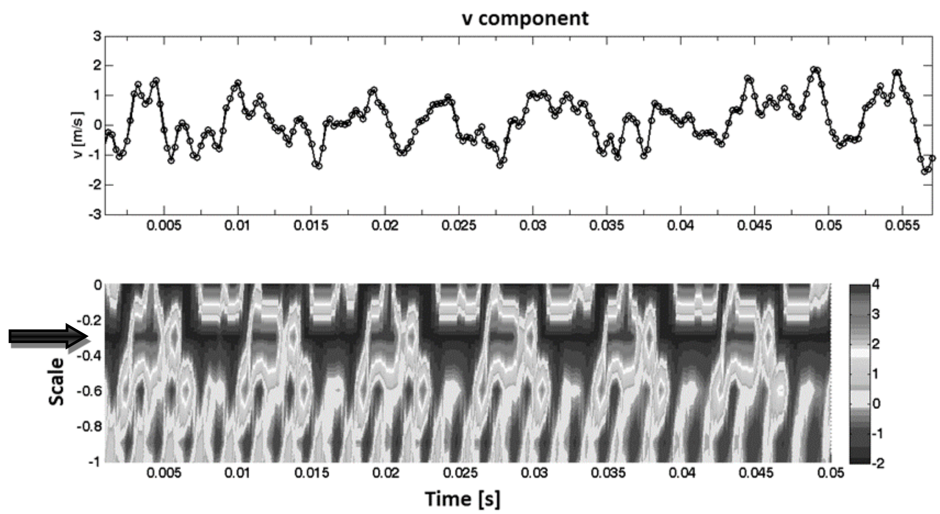

Fig. 3: Wavelets map and time series of v component velocity fluctuations for the first 0,05 seconds (Arrow indicates vortex scale: -0.3) [25]

DISCUSION AND CONCLUSION

Based on the measurements made and the implementation of the different CPM models, a comparison of the results between the models was made. It was found that, in general, the different models used are very accurate in detecting the expected events obtained by the methods usually used. As mentioned earlier, these models allow the determination of changes in a signal for which no probability distribution is known, so they can be applied to any flow case to evaluate the possible events contained therein. As can be seen in Table 1, the comparison of the results does not show a preference for any of the models. However, in this table, as in previous works, the model CPM-KS is the one with the best comparison results. This may be due to the fact that in these case, corresponding to the mini-flap in TE and as shown in Fig. 3, is that in which the predominance of counter-rotating vortices that periodically and alternately detach from the device with a characteristic frequency is clear. In Fig. 4 we shown the original data signal and the change points in vertical lines for wavelet transform and for the CPM-KS model.

As shown in Table 1, only for the CPM-JM and CPM-EK models are there cases in which differences of more than 9.5% are found compared to the wavelet determination. Smaller differences is found for CPM-KS model, and also always correlated to the wavelet transform.

Table 1: Change point detection with differents models.

Wavelet	Nonparametric distribution CPM models					
	CPM-KS		CPM-JM		CPM-EK	
Change [s]	Change [s]	Relative Difference [%]	Change [s]	Relative Difference [%]	Change [s]	Relative Difference [%]
0.00225	0.00225	0	0.00250	11.11	0.00250	11.11
0.00425	0.0045	5.88	0.00500	17.65	0.00450	5.88
0.00825	0.00875	6.06	0.00900	9.09	0.00875	6.06
0.01225	0.01175	4.08	0.01200	2.04	0.01175	4.08
0.01575	0.01550	1.59	0.01725	9.52	0.01725	9.52
0.02025	0.02000	1.23	0.02000	1.23	0.02000	1.23
0.02375	0.02150	9.47	N/A	N/A	0.02175	8.42
0.02550	0.02450	3.92	0.02475	2.94	0.02450	3.92
0.02825	0.02800	0.88	0.02950	4.42	0.02800	0.88
0.03350	0.03350	0.00	0.03375	0.75	0.03350	0.00
0.03625	0.03775	4.14	0.03775	4.14	N/A	N/A
0.04025	0.04050	0.62	0.04050	0.62	N/A	N/A
0.04425	0.04300	2.82	0.04375	1.13	0.04300	2.82
0.04825	0.04800	0.52	0.05025	4.14	N/A	N/A

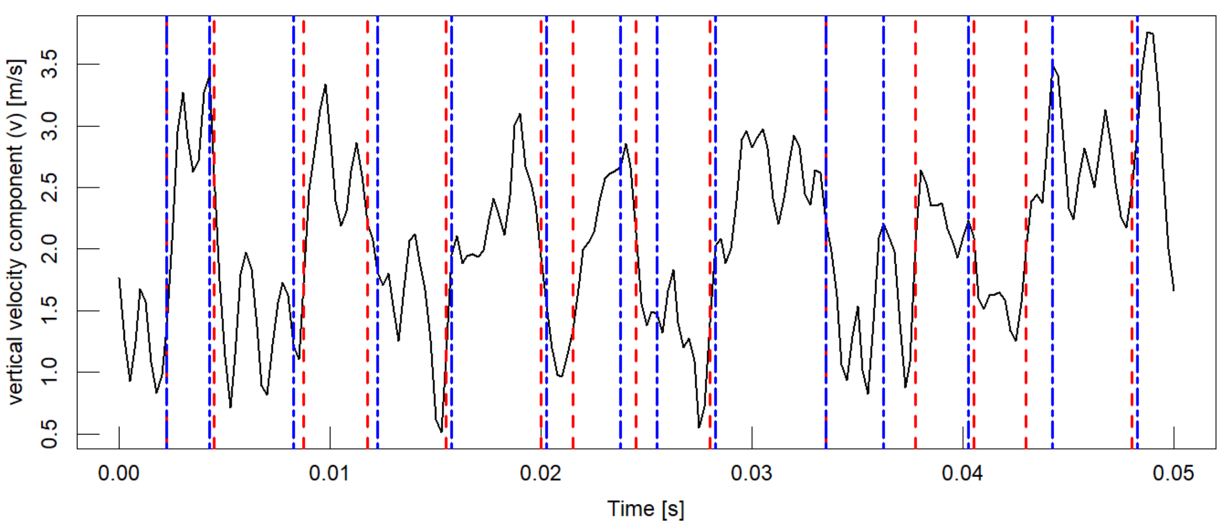

Fig. 4: Original signal with change points indicated for wavelet transform (blue lines) and CPM-KS model (red lines).

It was also found that the CPM-JM and CPM-EK models do not detect some of the changes predicted by the application of the wavelet transform. Nevertheless, all the models shown in this work are considered useful for evaluating turbulent events under different conditions.
We think that the reviews made [14] [15] [25] of the most suitable models for evaluation will primarily allow us to determine and analyse the downstream fluid dynamic behaviour in such flow control devices.

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