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Multi-period design and planning of closed-loop supply chains with uncertain supply and demand

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ABSTRACT

A design and planning approach is proposed for addressing general multi-period, multi-product closed-loop supply chains (CLSCs), structured as a 10-layer network (5 forward plus 5 reverse flows), with uncertain levels in the amount of raw material supplies and customer demands. The consideration of a multi-period setting leads to a multi-stage stochastic programming problem, which is handled by a mixed-integer linear programming (MILP) formulation. The effects of uncertain demand and supply on the network are considered by means of multiple scenarios, whose occurrence probabilities are assumed to be known. Several realistic supply chain requirements are taken into account, such as those related to the operational and environmental costs of different transportation modes, as well as capacity limits on production, distribution and storage. Moreover, multiple products are considered, which are grouped according to their recovery grade. The objective function minimizes the expected cost (that includes facilities, purchasing, storage, transport and emissions costs) minus the expected revenue due to the amount of products returned, from repairing and decomposition centers to the forward network. Finally, computational results are discussed and analyzed in order to demonstrate the effectiveness of the proposed approach. Due to the large size of the addressed optimization problem containing all possible scenarios for the two uncertain parameters, scenario reduction algorithms are applied to generate a representative, albeit smaller, subset of scenarios.

1. Introduction

Nowadays modern industrial enterprises are operating in a business environment undergoing significant transformations that introduce new and important challenges. These involve market changes (e.g. higher competition, changeable product specifications and shorter product life cycles), new regulations for the recovery and recycling of end-use products, and the need of an increasing sustainability of the whole operation, including a reduction of environmental and social impacts. Thus, in order to ensure a profitable operation of their supply chains, enterprises need to address the ensuing challenges.

The constantly shifting and increasing customer requirements are the major challenges due to their direct effect on production systems performance (Gupta & Maranas, 2003). Thus, as pointed out by Papageorgiou (2009), the need to account for this source of uncertainty has widely been recognized as an increasingly important issue. In general, different sources can be identified, such as product price and demand, production and transport costs, raw material accessibility, etc. Optimization approaches handling uncertainty considerations are then very advantageous. Nevertheless, such approaches lead to very large-scale models due to parameters with large uncertainty spaces. Reviews on optimization techniques to deal with uncertainty in the structuring and managing of corporations and their processes, can be found in Sahinidis (2004) and Li and Ierapetritou (2008).

The increasing need for remanufacturing due to resources shortage, environmental deterioration and new regulations, requires companies to organize their activities in order to explore and take full advantage of the coordination of forward and reverse material flows. Closed-loop supply chains (CLSC) extend the traditional definition of supply chains by explicitly exploring the synergy between the two flows. Thus, CLSC involves issues associated to new, end-use and remanufactured products, creating an added challenge for the design and planning problem. Therefore, practitioners and academics are paying an increasing interest to CLSCs, aiming to collect and recycle used products, with the objective of linking together environmental issues and business opportunities (Guide & Van

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Wassenhowe, 2009). In recent years, the sustainability of supply chains (SCs) along with their associated environmental and social impacts have received increasing attention (Srivastava, 2007), giving rise to a more comprehensive concept of sustainability, which now integrates economic, environmental and societal issues. Thus, the development of integrated frameworks for supply chain management (SCM) is a necessary condition for achieving sustainable supply chains. These frameworks should also account for an important economic facet of sustainable SCs, that is the capability of offsetting disturbances, such as in the present case the supply and demand fluctuations, by reducing readily the impact they cause on their performance. As a result, the number of works dealing with aspects of the sustainable SCM problem and with the systematic incorporation of environmental aspects in more traditional approaches, has started to grow (Côté, Lopez, Marche, Perron, & Wright, 2008; Duque, Barbosa-Povoa, & Novais, 2010; Georgiadis & Besiou, 2008; Hugo & Pistikopoulos, 2005; Ilbery & Maye, 2005; Pinto-Varela, Barbosa-Póvoa, & Novais, 2011).

The importance of environmental factors is also reflected in the explicit and increasing consideration given to the improvement of methodologies and indicators for environmental impact assessment (Corbett & Kleindorfer, 2003). Environmental indicators have been used for explicit inclusion in the SCs of such factors as waste from all network processes and greenhouse gas emissions. Paksoy, Bektas, and Özceylan (2011) state that greenhouse gas emissions, and CO_2 in particular, are by far the most prominent factors with respect to hazardous consequences on human health. In addition, these authors remark that one important source of greenhouse gas emissions in SCs come from the transportation activities between network entities.

The features that distinguish this paper from the existing bibliography include the investigation of the impact of supply and demand uncertainties on the design and planning problem of a generic multi-product multi-period CLSC, structured as a 10-layer network. In particular, the formulation considers multi-period multi-commodity problems and uncertain conditions varying during the planning horizon. In addition, the approach takes into account environmental impact, mainly in the form of CO_2 emissions, deriving from the transportation of material/products in both the forward and reverse networks. All these features, considered as part of the addressed problem, have been traditionally reported in the literature as being handled separately and few attempts have been made to comprehensively integrate these aspects simultaneously.

The rest of this article is organized as follows. In the next section, relevant literature on CLSC design and planning under uncertainty is reviewed. Section 3 states both the underlying problem assumptions and the details of the approach in order to make it flexible in representing a wide variety of network configurations. In Section 4 a mathematical model is proposed for the multi-period, multi-product CLSC design and planning problem with uncertain levels in the amount of raw material and customer demands. In Section 5, to highlight the benefits of such a formulation, a case-study is presented based on the example introduced by Paksoy et al. (2011), which is modified in order to illustrate the application of the multi-period multi-product stochastic formulation. In Section 6, a parametric and scenario analysis is performed to show the benefits of a stochastic model based on scenarios, as opposed to its static counterpart. The computational experiments also allow deriving a number of managerial insights about the network configuration with respect to changes in relevant SC parameters.

2. Overview of the literature

In this section a selective summary of relevant papers related to the approach proposed in this work is presented. A comprehensive review of a wide variety of models and case studies in reverse and closed-loop logistics network design can be found in Aras, Boyaci, and Verter (2010).

In the last decade, many researchers have been working in order to gradually obtain more comprehensive and computationally tractable approaches that can better capture the essence of many CLSC networks. This can be seen in some papers such as Shapiro (2004), Papageorgiou (2009) and Melo, Nickel, and Saldanha da Gama (2009). It becomes clear that many characteristics of real-world relevance for CLSC management are still distant from being fully incorporated in the models available in the literature.

Few papers have been proposed considering stochastic programming approaches applied to CLSCs configurations under uncertainty. Some of the most relevant papers are: Salema, Barbosa-Póvoa, and Novais (2007), Listeş (2007), Francas and Minner (2009), Pishvaee, Jolai, and Razmi (2009), Lee and Dong (2009), Wang and Hsu (2010), Pishvaee, Rabbani, and Torabi (2011), Vahdani, Tavakkoli-Moghaddam, Modarres, and Baboli (2012), Zeballos, Gomes, Barbosa-Povoa, and Novais (2012), Amin and Zhang (2013) as well as Cardoso, Barbosa-Póvoa, and Relvas (2013).

Salema et al. (2007) proposed a MILP model to deal with the design of a reverse logistic network with capacity limits, one-period planning horizon, multi-product management and scenarios to deal with the uncertainty on product demand and return. Listeş (2007) proposed a similar model but with a solution algorithm based on Benders decomposition. Francas and Minner (2009) accounted for uncertainty in demand and return considering two different fixed network structures and two different market structures. The authors studied capacity investment from a network perspective in a single-period problem. Pishvaee et al. (2009) proposed a scenario-based stochastic optimization model for a single-product, single-period forward and reverse logistics networks considering production/recovery, hybrid distribution/collection centers, customers, and disposal centers. In their formulation, the demand, quantity and quality of return flows and variable costs are assumed to be uncertain. Lee and Dong (2009) proposed a two-stage stochastic programming model for the design of a multi-period CLSC network. Uncertainty is considered in the demand of forward products and in the supply of returned products at customers. Since optimization mathematical techniques are computationally incapable to obtain solutions due to the problem size and complexity, those authors developed a heuristic algorithm based on simulated annealing. Wang and Hsu (2010) developed an interval programming model where the uncertainty was expressed by fuzzy numbers. Results were obtained considering that customer demand and recovery rate are uncertain parameters. Pishvaee et al. (2011) proposed a robust optimization model to determine the number and location of collection/inspection, recovery and redistribution centers, and the quantity of flows between each pair of network entities, in a single-product setting. In addition, the problem considered in the paper includes customers at a first and second market, and the following uncertain parameters: quantity of returned products from the first market customers, second market customers' demands and transportation costs. Vahdani et al. (2012) developed a fuzzy multi-objective robust formulation which minimizes the total costs and the expected transportation costs after failure of facilities in a logistics network. Zeballos et al. (2012) introduced a two-stage scenario-based modeling approach in order to deal with the design and planning decisions in multi-period, multi-product CLSCs subject to uncertain conditions. In their paper, uncertainty is associated to the quantity and quality of the flow of products of the reverse network. Amin and Zhang (2013) proposed a mixed-integer linear stochastic programming model (scenario-based) for a single-period multi-product CLSC location problem including multiple plants, collection centers and demand markets. The model considers demand and returns as uncertain parameters, and includes environmental factors on the objective function. Cardoso et al. (2013) developed an optimization model for the design and planning of generic CLSCs, where capacity expansion and dynamic transportation links were explored under an uncertain products demand context. Uncertainty was considered using a scenario tree approach. An integrated approach of both forward and reverse flows was explored, where a closed link between all the involved operations was performed.

As it can be seen, several of the above mentioned papers addressing uncertainty issues use the scenario-based two-stage stochastic methodology. The two-stage stochastic framework comprises two types of decision variables, the first stage (design) variables and second stage (control) variables representing the decisions that are determined before and after realization of uncertain parameters, respectively (Birge & Louveaux, 1997; Dantzig, 1955). This methodology is normally used in single-period settings. On the other hand, multi-period problems can lead to multi-stage stochastic programming approaches (Nickel, Saldanha-da-Gama, & Ziegler, 2012). The idea behind a multi-stage stochastic programming methodology is to take some actions before the realization of uncertain parameters (stage 1) and to make decisions associated to different stages trying to fix any infeasibilities arising due to a particular revelation of uncertainty. Thus, multi-period problems can be represented as a framework with multiple stages (one stage for each time period), and therefore, the corrective actions can be made over the sequence of stages.

Multi-stage stochastic programming methodology has been used for addressing different types of problems such as the electric power generation under uncertain demand (Nowak & Römisch, 2000), the expansion of capacity in process networks (Ahmed, King, & Parija, 2003; Tarhan & Grossmann, 2008), the remanufacturing planning with uncertain quality of inputs (Denizel, Ferguson, & Souza, 2010) and the location of facilities, flow of commodities and investments required to carry out different activities in a forward network, assuming demand and interest rates as uncertain parameters (Nickel et al., 2012). The problem of design and planning of CLSCs with uncertain parameters using multi-stage stochastic programming methodology has been poorly explored. Most formulations using the mentioned methodology are based on approximating the underlying continuous probability distributions of the uncertain parameters by discrete probability measures, and specifying the problems in the form of scenario trees. In such cases, each path between the root node and a tree leaf node represents a scenario that contains the possible realization of the stochastic process for all stages over the planning horizon of the problem. The stages of the approaches reflect the way the information flows through the time periods. Since the multi-stage scenario-based stochastic approaches for practical problems are mostly large scale, whose size increases with the number of scenarios, it is desirable to reduce the scenario tree. Several theoretical developments can be found in the open literature for techniques for scenario reduction (Dupacova, Growe-Kuska, & Romisch, 2003; Growe-Kuska, Heitsch, & Romisch, 2003; Heitsch & Romisch, 2003; Karuppiah, Martin, & Grossmann, 2010).

Additionally, the inclusion of environmental issues in CLSC problems in general, and in design and planning approaches in particular, has been addressed by few authors. The focus of the emerging proposals is related to the inclusion of additional linear terms in the objective functions (Amin & Zhang, 2013; Paksoy et al., 2011). Paksoy et al. (2011) included the cost for CO_2 emissions from the transportation activities in a single-period multi-product CLSC context. These authors used the estimated CO_2 emission cost given by Forkenbrock (2001) to evaluate the operational and environmental performance of the network. On the other hand, Amin and Zhang (2013) introduced two terms in the objective function to quantify both the use of environmental friendly materials by plants and of clean technology by collection centers. The two parameters are qualitative (varying between 0 and 1) and should be determined by decision makers. It is important to note that the estimation of greenhouse gas emissions, in particular CO_2 produced by different transportation modes, results in complex and nonlinear functions of several parameters (for example, load, traffic flow, speed, road surface conditions and coefficient of drag) (Bektaş & Laporte, 2011; Kanaroglou & Buliung, 2008). Nevertheless, Paksoy et al. (2011) remarked that the use of rigorous computations within a strategic/tactical level network design problem would be impractical and it is more reasonable to apply widely available estimators of emissions (such as ton per kilometer).

3. Problem description

The problem consists of optimizing the design and planning of a CLSC considering an uncertain environment. Thus, the problem objective is to minimize the total cost of the network, while trying to maximize the revenue of recycled products, and to determine the facilities to be opened, and the products and quantities to be manufactured, transported, stored and recycled so as guarantee costumers' demands. It is important to note that the revenue of recycled products represents the economic incentive of the firm to choose and use the recyclable products.

The CLSC includes raw material suppliers (I_s), factories (I_f), warehouses (I_w), distribution centers (I_{dc}), customers (I_c), collection centers (I_{cc}), dismantlers (I_d), repairing centers (I_{rc}), final disposal locations (I_{fd}) and decomposition centers (I_{dp}) (see Fig. 1). While suppliers and plants receive reusable parts of used products from decomposition centers, the warehouses and distribution centers get repaired products from repairing centers.

It is also assumed that used products flow through the reverse network at different rates, which depend on their origin and destination. End-life products are collected from customers and transported to the next entities on the network (collection centers), with a return rate of α. One part of the used products in collection centers is sent to repairing centers with the objective of being refurbished (β). Thus, $(1-\beta)$ denotes the recycle rate of product delivered to dismantlers from collection centers; χ specifies the rate at which the repaired products are moved to warehouses from repairing centers; $(1-\chi)$ defines the rate at which products in the repairing centers are transported to distribution centers. While the parameter δ denotes the fraction of products in dismantlers that is transported to decomposition centers, the flow $(1-\delta)$ represents the products sent to final disposal. Finally, some components of the decomposed products are transported to suppliers (ε) and the remainder $(1-\varepsilon)$ is sent to production plants. It is worth noting that the different flow rates, dependent on the type of product, are taken as suggested in Paksoy et al. (2011) for the single-period planning problem of a CLSC.

Therefore, the main problem features can be described as follows:

- the whole planning horizon is divided into several time periods,
- multiple products flow through the network,
- products in the network are grouped according to their salvage grade (e.g. fully reusable, partially reusable and non-reusable),
- recycled products are treated equally to new ones,
- transport operations between any pair of entities can be performed using different transportation modes (TR). Each transportation mode has a minimum and maximum load capacity,
- uncertainty is associated with customer demand and raw material supply, with the latter being subject to minimum and maximum constraints,

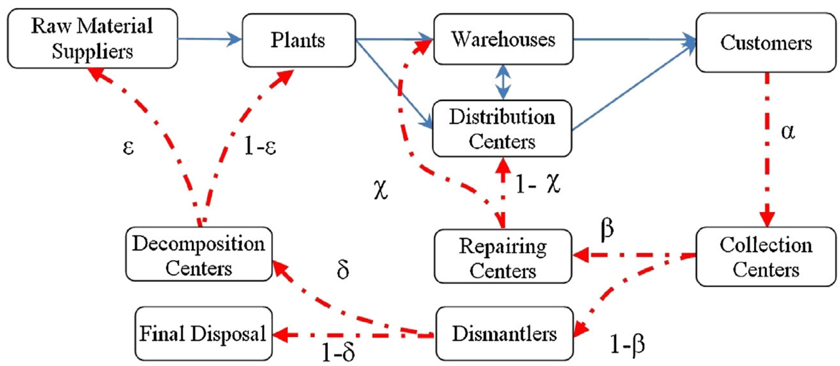

Fig. 1. Closed-loop supply chain structure.

- storage capacities of network entities have minimum and maximum limits,
- maximum and minimum processing levels of plants are imposed,
- the unit costs of transport, purchasing, storage, facilities and estimated CO_2 emissions, as well as the revenue of recycled products, are known and fixed.
- the problem objective is to determine the design and planning of a closed-loop supply chain minimizing the total cost of the network, while guaranteeing costumers' demands and maximizing the amount of recycled products.
- the opening of the network entities is decided at the beginning of the planning horizon.

4. Formulation

The design and planning problem of a CLSC under uncertainty is formulated as a mixed-integer multi-period multi-product linear programming model. A multi-stage stochastic approach is used in order to deal with the uncertainty arising from demand and supply. It is worth remarking that since the model is focused on strategic/tactical decision making levels, model parameters such as the storage, emissions and transport costs are based on average levels over the time period. In the mixed-integer linear programming (MILP) formulation, the network design decisions at the beginning of the planning horizon (before the resolution of the underlying uncertainty) are adopted as the first-stage variables. The rest of variables (production, distribution and storage variables) are selected as control variables, which are associated to different stages and which allow to fix any infeasibilities arising due to a particular revelation of the demand and supply uncertainty. Thus, there is a sub-set of control variables in each stage that allows revising decisions during the planning horizon considering the uncertainty previously realized. The formulation is able to represent the dynamic nature of the network, which is an important advantage with respect to deterministic formulations and also to two-stage models with single-period planning considerations.

In the proposed approach, the uncertainty is described by a set of discrete events, each one with a given probability that represents its expected occurrence. The uncertainty is modeled as a multi-layered scenario tree, since demand and supply are considered as dynamic parameters changing during each time period. Because demand and supply are considered as independent stochastic processes, they are addressed separately. Nevertheless, the effects of both parameters are taken into account in the tree nodes. While a node represents a given level of demand and supply at a given time, a scenario is formed by the sequence of nodes from the root node until a particular leaf node at the last time period. Thus, each node in the scenario tree represents a possible business state, associated with specific values of demand and supply. The formulation aims at generating the CLSC design, while trying to protect the company against all the possible outcomes characterized in the scenario tree. The formulation objective is to determine the CLSC configuration taking into account the probable demand and supply states due to future events.

The definition of sets, variables and parameters of the model are defined as follows:

Sets

I	supply chain entities,
I_s	suppliers,
I_f	factories,
I_w	warehouses,
I_{dc}	distribution centers,
I_c	customers,
I_{cc}	collection centers,
I_d	dismantlers,
I_{rc}	repairing centers,
I_{fd}	final disposal locations,
I_{dp}	decomposition centers,
I^f	set of entities belonging to the forward network, $\{I_s \vee I_f \vee I_w \vee I_{dc} \vee I_c\}$
I^r	set of entities belonging to the reverse network, $\{I_c \vee I_{cc} \vee I_d \vee I_{rc} \vee I_{fd} \vee I_{dp}\}$
I^{rf}	set of entities that make the link between the reverse and forward networks $\{I_{rc} \vee I_{dp}\}$,
A^f	set of arcs between entities belonging to the forward network, $\{(i,j): (i \in I_s, j \in I_f) \vee (i \in I_f, j \in I_w) \vee (i \in I_f, j \in I_{dc}) \vee (i \in I_w, j \in I_{dc}) \vee (i \in I_{dc}, j \in I_w) \vee (i \in I_{dc}, j \in I_c) \vee (i \in I_w, j \in I_c) \vee (i \in I_c, j \in I_{cc})\}$
A^{sf}	set of arcs between suppliers and factories, $\{(i,j): (i \in I_s, j \in I_f)\}$
A^r	set of arcs between entities belonging to the reverse network, $\{(i,j): (i \in I_c, j \in I_{cc}) \vee (i \in I_{cc}, j \in I_d) \vee (i \in I_{cc}, j \in I_{rc}) \vee (i \in I_d, j \in I_{fd}) \vee (i \in I_d, j \in I_{dp}) \vee (i \in I_{rc}, j \in I_w) \vee (i \in I_{rc}, j \in I_{dc}) \vee (i \in I_{dp}, j \in I_s) \vee (i \in I_{dp}, j \in I_f)\}$,
A^{rf}	$\{(i,j): (i \in I_{rc}, j \in I_w) \vee (i \in I_{rc}, j \in I_{dc}) \vee (i \in I_{dp}, j \in I_s) \vee (i \in I_{dp}, j \in I_f)\}$,
A^{rc}	$\{(i \in I_{cc}, j \in I_{rc}) \vee (i \in I_d, j \in I_{dp}) \vee (i \in I_{rc}, j \in I_{dc}) \vee (i \in I_{dp}, j \in I_s)\}$,
A^{cc}	$\{(i,j): (i \in I_c, j \in I_{cc})\}$,
A	$\{A^f \vee A^r\}$,
PR	products,
TP	transportation modes,
T	time units,
Es	discrete events related to the supply levels of raw material,
Ed	discrete events related to the demand levels of customers,
Ω	combination of events belonging to Es and Ed $\{(es, ed): es \in Es, ed \in Ed\}$,
SC	$SC = \Omega_{t1} \times \Omega_{t2} \times \Omega_{t3} \times \ldots \Omega_T$, set of scenarios. Each scenario is formed by a given sequence of events from the root node until a particular leaf node at the last time period.
Ω_{st}	combination of events of scenario $s \in SC$ at time period t,

Parameters

rl_{ipt}	fraction of product p that is returned from entity i to the following entities in the network at time t. This parameter is equal to α when considering the product amount that is recovered from the end-users and sent to collection centers. If there is no loss of products in a given entity, the parameter equals one.

ff_{ipt} specifies the rate of product p that is moved from entity i to another entity at time t. The parameter takes values for reverse supply chain entities I_{cc}, I_{rc}, I_d and I_{dp}. Thus, this parameter is equal to β, χ, δ and ε considering the arcs A^{rc}.
Rmn_{iptes} minimum supply limit of product p, of entity $i \in I_s$, when occurring event $es \in Es$ at time t.
Rmx_{iptes} maximum supply limit of product p, of entity $i \in I_s$, when occurring event $es \in Es$ at time t.
d_{ipted} product p demand for customer i, at time t, considering event $ed \in Ed$.
Pmn_{ipt} minimum processing capacity of product p, of entity $i \in I_f$, at time t.
Pmx_{ipt} maximum processing capacity of product p, of entity $i \in I_f$, at time t.
Imn_i minimum storage capacity of entity $i \in I$.
Imx_i maximum storage capacity of entity $i \in I$.
$trmn_{ijr}$ minimum load capacity of transportation mode r, between entities i and j.
$trmx_{ijr}$ maximum load capacity of transportation mode r, between entities i and j.
dst_{ij} distance between entities i and j
fcc_i cost for opening/use of entity $i \in I$
c_{ijrt} unit transport cost of product p from entity i to entity j, at time t.
em_{ijrt} estimated emission cost per unit of product p transported between entities i and j, at time t.
u_{ipt} unit raw material purchasing cost of product p at entity $i \in I_s$, at time t.
stc_{ipt} unit storage cost of product p at entity $i \in I$, at time t.
pc_{pt} unit revenue of product p obtained by recycling materials, at time t.
M Large Number
P_{es} occurrence probability of the supply level $es \in Es$
P_{ed} occurrence probability of the demand level $ed \in Ed$
Pb_s occurrence probability of scenario s. $Pb_s = (P_{es}P_{ed})\Omega_{st1}\,(P_{es}P_{ed})\Omega_{st2}\,(P_{es}P_{ed})\Omega_{st3}\ldots(P_{es}P_{ed})\Omega_{sT}$

Continuous variables
x_{ijprts} amount of product p transported from entity i to entity j, at time t, in scenario s
z_{irts} amount of products stored in entity i, at time t, in scenario s

Binary variables
y_i entity i included in the network,
e_{ijrts} transport from entity i to j using the transportation mode r, at time t, for each scenario s.

The resulting mixed integer linear programming formulation of the multi-period multi-product design and planning problem is presented in this section.

$$\sum_{i':(ii')\in A^r}\sum_{r\in TR} rl_{ipts} + x_{i'iprts} - z_{ipts} = \sum_{j:(i,j)\in A^r}\sum_{r\in TR} x_{ijprts} \quad \forall i \in I^r, \forall p \in PR, \forall s \in SC, \forall t \in T/\{t_1\} \tag{7}$$

$$Z_{ip(t-1)s} + \sum_{i':(i',i)\in A^r}\sum_{r\in TR} rl_{ipt}x_{i'iprts} - Z_{ipts} = \sum_{j:(i,j)\in A^r}\sum_{r\in TR} x_{ijprts} \quad \forall i \in I^r, \forall p \in PR, \forall sc \in SC, \forall t \in T\backslash\{t_1\} \tag{8}$$

$$ff_{ipt}\sum_{i':(i,i')\in A^r}\sum_{r\in TR} x_{ii'prts} = \sum_{j:(i,j)\in A^{rc}} j:(i,j)\in A^{rc}\sum_{r\in TR} x_{ijprts} \quad \forall i \in I^r\backslash\{I_{fd}I_c\}, \forall p \in PR, \forall s \in SC, \forall t \in T \tag{9}$$

$$\sum_{(i,i')\in A}\sum_{p\in PR}\sum_{r\in TR}\sum_{t\in T}\sum_{s\in SC} x_{ii'prts} \le My_i \quad \forall i \in I \tag{10}$$

$$\sum_{(i,i')\in A}\sum_{p\in PR}\sum_{r\in TR}\sum_{t\in T}\sum_{s\in SC} x_{i'iprts} \le My_i \quad \forall i \in I \tag{11}$$

$$e_{ii'rts} \le My_i \quad \forall (i,i') \in A, \forall r \in TR, \forall s \in SC, \forall t \in T \tag{12}$$

$$e_{i'rts} \le My_i \quad \forall (i',i) \in A, \forall r \in TR, \forall s \in SC, \forall t \in T \tag{13}$$

$$\sum_{p\in PR} x_{ii'prts} \ge trmn_{ii'r}e_{ii,rts} \quad \forall (i,i') \in A, \forall r \in TR, \forall s \in SC, \forall t \in T \tag{14}$$

$$\sum_{p\in PR} x_{ii'prts} \le trmn_{ii'r}e_{ii,rts} \quad \forall (i,i') \in A, \forall r \in TR, \forall s \in SC, \forall t \in T \tag{15}$$

$$\sum_{p\in PR} z_{irts} \ge Imx_iy_i \quad \forall i \in I, \forall s \in SC, \forall t \in T \tag{16}$$

$$\sum_{p\in PR} z_{irts} \le Imx_iy_i \quad \forall i \in I, \forall s \in SC, \forall t \in T \tag{17}$$

$$\text{Minimize} \sum_{i\in I} fcc_iy_i + \sum_{s\in SC} Pb_s\left(\sum_{t\in T}\sum_{i,i'\in A}\sum_{p\in PR}\sum_{r\in TR} dst_{ii'}(c_{ii'rt} + em_{ii'rt})x_{ii'prts} + \sum_{t\in T}\sum_{i,i'\in A^{sf}}\sum_{p\in PR}\sum_{r\in TR} u_{ipt}x_{ii'prts} + \sum_{t\in T}\sum_{i\in I}\sum_{p\in PR}\sum_{r\in TR} stc_{ipt}z_{ipts} \right.$$
$$\left. \sum_{t\in T}\sum_{i,i'\in A^{sf}}\sum_{p\in PR}\sum_{r\in TR} pc_{pt}x_{ii'prts}\right) \tag{1}$$

$$\sum_{i'\in(I_w\cup I_{dc})}\sum_{r\in TR} x_{i'iprts} \ge d_{ipted}, \quad \forall i \in I_c, \forall \in PR, \forall se \in SC, \forall t \in T, \forall (ed, es) \in \Omega_{st} \tag{2}$$

$$\sum_{i':(ii')\in A^{sf}}\sum_{r\in TR} x_{ii'iprts} \ge Rmn_{iptes}y_i, \quad \forall i \in I_s, \forall p \in PR, \forall s \in SC, \forall t \in T, \forall (ed, es) \in \Omega_{st} \tag{3}$$

$$\sum_{i':(ii')\in A^{sf}}\sum_{r\in TR} x_{ii'iprts} \ge Rmx_{iptes}y_i, \quad \forall i \in I_s, \forall p \in PR, \forall s \in SC, \forall t \in T, \forall (ed, es) \in \Omega_{st} \tag{4}$$

$$\sum_{i':(ii')\in A^f}\sum_{r\in TR} x_{i'iiprts} - z_{ipts} = \sum_{i':(ii')\in A^f}\sum_{r\in TR} x_{ijprts} \quad \forall i \in I^f, \forall p \in PR, \forall s \in SC, \forall t \in T/\{t_1\} \tag{5}$$

$$\sum_{i':(ii')\in A^{rf}}\sum_{r\in TR} x_{i'iipr(t-1)s} + z_{ip(t-1)s} + \sum_{i':(i'i)\in A^f}\sum_{r\in TR} x_{i'iprts} - z_{ipts} = \sum_{j:(i,j)\in A^f}\sum_{r\in TR} x_{ijprts} \quad \forall i \in I^f, \forall p \in PR, \forall s \in SC, \forall t \in T\backslash\{t_1\} \tag{6}$$

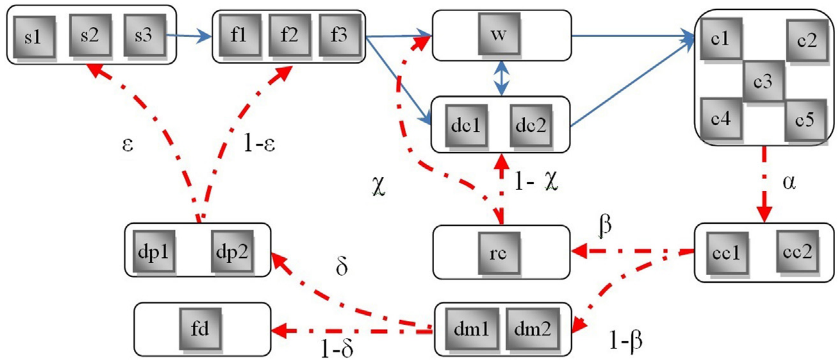

Fig. 2. Closed-loop supply chain super-structure.

$$\sum_{i':(i,i')\in A}\sum_{r\in TR} x_{ii'prts} \geq pmn_{ip} y_i \quad \forall i \in I, \forall p \in PR, \forall s \in SC, \forall t \in T \tag{18}$$

$$\sum_{i':(i,i')\in A}\sum_{r\in TR} x_{ii'prts} \leq pmx_{ip} y_i \quad \forall i \in I, \forall p \in PR, \forall s \in SC, \forall t \in T \tag{19}$$

The objective function of the stochastic formulation (1) is to minimize the total cost considering the network design cost and the expected cost and revenue of decisions related to raw material consumption, storage, transport, emissions, as well as to the recycled products. Since the uncertainty is modeled through multiple scenarios, which follow discrete levels of demand and supply, the expected second stage cost and revenue are equal to the sum over all the scenarios of the scenario probability, multiplied by the resulting scenario cost and revenue. The objective function includes five terms. The first term denotes the cost for opening/use of facilities, which is independent of the scenarios. The second represents the transport and emissions costs on each arc A of the forward and reverse chains, for each transportation mode and type of product, considering all time periods. The third represents the purchasing cost of raw material transported from suppliers to plants. The fourth denotes the storage cost over all entities, products and time periods. Finally, the last term represents the gain obtained by introducing recovered products/materials into the forward network. The costs and revenue are weighted by the probability (Pb_s) of the scenario $s \in SC$. It is worth noting that each scenario s is made up by a given sequence of events Ω_{st} from the root node until a particular leaf node at the last time period. Ω_{st} denotes the events $es \in Es$ and $ed \in Ed$ that occur at scenario s at time period t.

Eq. (2) imposes the minimum level of demand satisfaction for a given period considering a particular event $ed \in Ed$. Constraints (3) and (4) limit the maximum and minimum supply capacity of raw material when occurring the event $es \in Es$ at time t. Constraints (5) and (6) are the balance equations for the forward network. While constraint (5) represents the balance at time period $t1$, constraint (6) characterizes the material balance at any time period after $t1$, considering the flow of products returned by the reverse network entities directly linked to the forward network. Thus, constraint (6) states that the products refurbished in a given period $(t-1)$ will be incorporated to the forward network in the next period (t). It is worth noting that the terms $x_i^* ipr(t-1)s$ and $Z_{ip}(t-1)s$ in constraint (6) refer to the flow of product and the storage level at a previous time period $(t-1)$ considering the scenario s.

Constraints (7) and (8) are the balance equations for the reverse network at time period $t1$ and the rest of time periods (distinct from $t1$), respectively. The parameter rl_{ipt} is the fraction of product p that is sent from entity i to the subsequent entities in the network at time t. This parameter is equal to the value of α when the relationship of the end-users and collection centers is taken into account.

Constraint (9) describes the flow of products in the reverse network. For example, when considering the collection centers, the ff_{ipt} parameter takes the value of β and the constraint specifies the fraction of used products that must be send to repairing centers. The rest of products $(1-ff_{ipt})$ are sent to the dismantlers by default. Depending on the entity i taken into account, the parameter ff_{ipt} takes the value of $\chi, \varepsilon, \delta$ (see Fig. 1). Constraints (10) and (11) allow the existence of incoming and outgoing flows of products to a given entity only if the entity is part of the network. Constraints (12)–(15) bound the flow of products between two entities. In particular, constraints (12) and (13) allow the existence of incoming and outgoing transportation moves if a given entity is part of the network at time period t. Constraints (14) and (15) limit the maximum and minimum transport capacity between two entities i and $i\acute{\imath}$, at a particular time period and using a specific transportation mode r. Constraints (16) and (17) limit the minimum and maximum

Table 1
Minimum and maximum supply levels.

Suppliers	Product types	Supply outcomes/ occurrence probability	Minimum supply for time period $t1$ and $t2$ [tons]			Maximum supply for time period $t1$ and $t2$ [tons]		
			$es1/0.2$	$es2/0.3$	$es3/0.5$	$es1/0.2$	$es2/0.3$	$es3/0.5$
	Frcy		4000	8467	11,600	13,358	17,429	21,500
s1	Prcy		3934	8400	11,534	13,286	17,358	21,429
	Nrcy		3734	8200	11,334	13,072	17,143	21,215
	Frcy		5067	7734	10,400	8286	11,143	14,000
s2	Prcy		6400	8800	11,467	9429	12,286	15,143
	Nrcy		7334	10,000	12,667	10,715	13,572	16,429
	Frcy		5267	8600	12,600	10,643	14,929	19,215
s3	Prcy		5334	8667	12,667	10,715	15,000	19,286
	Nrcy		5467	8800	12,800	10,858	15,143	19,429

Table 2
Customer demand depending on the scenario and the associated scenario probabilities.

Customers	Product types	Demand outcomes/ occurrence probability	Demand for time period $t1$ [tons]			Demand for time period $t2$ [tons]		
			$ed1/0.3$	$ed2/0.5$	$ed3/0.2$	$ed1/0.3$	$ed2/0.5$	$ed3/0.2$
c1	Frcy		5250	3750	6150	3750	2250	4650
	Prcy		5775	4275	6675	4275	2775	5175
	Nrcy		5175	3675	6075	3675	2175	4575
c2	Frcy		3750	2250	4650	2250	750	3150
	Prcy		5025	3525	5925	3525	2025	4425
	Nrcy		5175	3675	6075	3675	2175	4575
c3	Frcy		6000	4500	6900	4500	3000	5400
	Prcy		5775	4275	6675	4275	2775	5175
	Nrcy		5925	4425	6825	4425	2925	5325
c4	Frcy		4500	3000	5400	3000	2250	3900
	Prcy		5025	3525	5925	3525	2025	4425
	Nrcy		4425	2925	5325	2925	1425	3825
c5	Frcy		5250	3750	6150	3750	2250	4650
	Prcy		6525	5025	7425	3525	3525	4425
	Nrcy		5175	3675	6075	3675	2175	4575

amount of products stored in the network entities. Constraints (18) and (19) bound the minimum and maximum processing capacity of product p, of entity i belonging to the forward network, at time t.

5. Case study

In this section, the information of the case study is presented. In this work, the example introduced by Paksoy et al. (2011) has been adopted as reference and modified in order to illustrate the application of the multi-period multi-product stochastic model. In the case study, the design and planning of a closed loop supply chain must be determined. The network super-structure is composed of 3 suppliers (s1 to s3), 3 factories (f1 to f3), 1 warehouse (w), 2 distribution centers (dc1 and dc2), 5 customers (c1 to c5), 2 collection centers (cc1 and cc2), 2 dismantlers (dm1 and dm2), 1 repairing center (rc), 1 final disposal (fd) and 2 decomposition centers (dp1 and dp2) (see Fig. 2).

The planning horizon is equal to ten years, which is subdivided into two time periods of five years. The flow of products in the network is grouped into 3 sets: products with recycle rate of 100% (Fully recyclable, Frcy), 50% (Partially recyclable, Prcy) and 0% (non-reusable, Nrcy). The product flow through the reverse network is described by the following parameters: $\alpha = 0.7$, $\beta = 0.4$, $\chi = 0.7$, $\varepsilon = 0.7$, $\delta = \{(\delta_{Frcy}, \delta_{Prcy}, \delta_{Nrcy}) = (1, 0.7, 0)\}$. In this case, the variation of raw material supply and demand due to the uncertainty is represented by three possible levels for each parameter: es_1, es_2 and es_3, and ed_1, ed_2 and ed_3, respectively. Table 1 shows the minimum and maximum supply levels and their occurrence probabilities, while Table 2 shows the equivalent values for demand. Three types of trucks compose the transport system (truck1, truck2 and truck3). Each mode has specific unit CO_2 emission and transportation cost. More details about the example can be found in Appendix A. Tables A1–A4 show investment and storage costs, as well as storage, transport and production/processing capacities, along with revenue.

6. Results for the stochastic programming model

The application of the proposed formulation is illustrated by solving several instances of the case study. The formulation was implemented in GAMS 23.6.3 and solved with CPLEX 12.2 on a HP Z800 workstation, with Intel Xeon x5650 2.66 GHz and 16 GB RAM memory, for a 0.01% gap tolerance.

The advantages of the proposed stochastic programming approach (SPA) are drawn by comparison with the deterministic approach (DA), which assumes an expected demand and raw material supply. Since parameter values change during the 10-year horizon considered and the raw material supply and demand variations are represented by three possible levels for each parameter, the model can be characterized as a multi-layered tree with 81 scenarios. The first layer includes 9 nodes, each one with a different combination of events Es and Ed. The second layer comprises 81 nodes, which are related to the ones of the previous time period.

It is important to note that for an approach with 81 scenarios, optimal solutions are hardly available. Therefore, taking into account that the computational effort for solving the scenario-based approach directly depends on the number of scenarios, a reduction algorithm is used in order to get a reasonably good approximation of the original problem. The reduction algorithms take advantage of the probability distance between the original and the reduced probability measure (Dupacova et al., 2003; Growe-Kuska et al., 2003; Heitsch & Romisch, 2003). The probability distance depends on scenario probabilities and distances between scenario values. Thus, scenarios are deleted when they are close

Table 3
Results for solving the stochastic approach with different scenario tree structures.

Reduction parameter (rp)	Scenarios	Tree nodes	Variables		Constraints	OF	Time (s)
			Discrete	Continuous			
1 (DA_{rp1})	1	3	485	2022	1947	1,814,145,743	8.1
0.6 ($STA_{rp0.6}$)	4	9	1866	8013	7644	2,087,601,189	508.9
0.4 ($STA_{rp0.4}$)	8	15	3252	14,005	13,342	2,050,995,777	964.7
0.2 ($STA_{rp0.2}$)	21	28	6255	26,992	25,692	2,050,300,427	54,041.1
0.14 ($STA_{rp0.14}$)	28	35	7877	33,985	32,342	2,046,332,711	96,574.1
0 ($STA_{FullTree}$)	81	90	20,813	89,926	85,539	2,112,681,549[a]	100,000.0

[a] Solution with a 0.02% of gap.

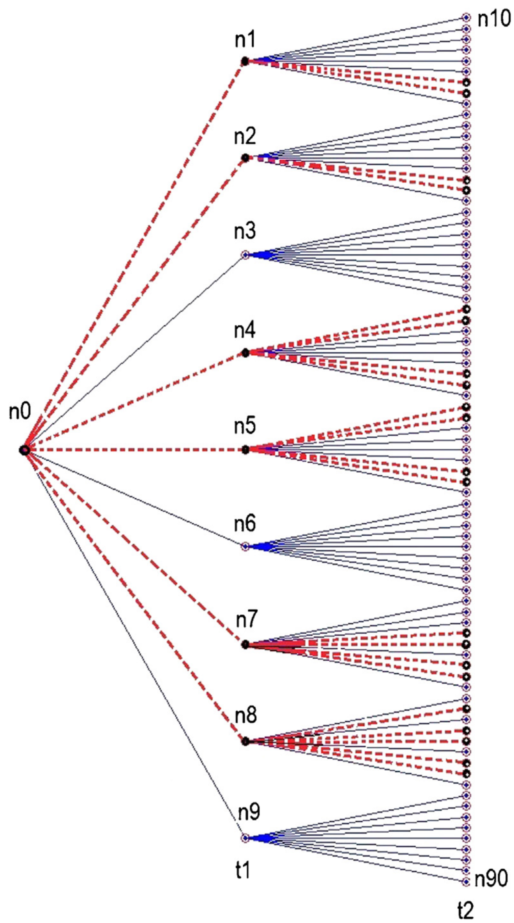

Fig. 3. Scenario tree with two periods and three events for two uncertain parameters (supply, demand).

or have small probabilities. After the reduction method is applied, a sub-set of the initial scenarios is obtained and each preserved scenario has a new probability. A probability zero is assigned to the set of eliminated scenarios. Several algorithms for reducing scenarios are available in the library SCENRED of GAMS (GAMS/SCENRED Documentation, 2013). In all cases, the algorithms take an original scenario set and, using control parameters that guide the reduction, return a reduced set of scenarios to be employed in subsequent runs. In this work, a reduced mathematical formulation is obtained using a mix of fast backward and forward algorithms (GAMS/SCENRED Documentation, 2013). To obtain a reduced tree that is computationally manageable and, at the same time, is representative of the original tree, the effects of different degrees of reduction are shown in Table 3. The table reports the statistics for solving the stochastic model based on different values of the reduction parameter (*rp*), which specifies the desired reduction in terms of the relative distance between the initial and reduced scenario trees. A value of the reduction parameter equal to 0.6 means that the reduced tree maintains 40% of the original information contained in the tree. Thus, Table 3 shows results for solving the model considering the full scenario tree ($rp = 0$), four reduced scenario trees ($rp = 0.14$, 0.20, 0.40, 0.60) and the deterministic tree ($rp = 1$). It is important to note that the case with the full scenario tree ($STA_{FullTree}$) is extremely computationally intensive and fails to converge to the optimal solution within the time limit of 100,000 CPU seconds.

In dashed lines, Fig. 3 shows the scenario tree structure for the value of the reduction parameter equal to 0.2 ($STA_{rp0.2}$). The reduced tree is marked over the full scenario structure, where *n*0 is the root node, *n*1 to *n*9 are the nodes associated to the first time period (*t*1) and *n*10 to *n*90 are the leaves nodes connected with the second time period (*t*2). In this paper, the structure of $STA_{rp0.2}$ is selected as the reference scenario since it represents a good balance between a reasonable representation of the original tree and the computational effort to solve it. Since in a real CLSC combined design and planning problem the number of scenarios could be very large, the scenario reduction method becomes a key tool to reduce the inherent combinatorial complexity of the problem, maintaining the probability information at the desired level. Thus, by properly choosing the minimum subset of scenarios that satisfies a pre-defined level of information, the model will be able to generate alternative solutions which are appropriate for the level of cautiousness of the decision maker. Thus, as shown in Table 3, a very conservative decision maker will choose the full scenario tree ($rp = 0$), generating a solution with the highest computational and economical cost. In a medium level, a riskier decision maker may choose a reduced tree, for example one obtained with a value of *rp* equal to 0.2 meaning that the reduced tree maintains 80% of the original information contained in the original tree. Thus, lower cost solutions may be generated with a reduced computational effort. In addition, as it can be seen from the results in Table 3, the objective value of $STA_{rp0.2}$ is only 0.03% smaller than the objective value of $STA_{rp0.4}$ and 0.19% greater than the objective value of $STA_{rp0.14}$. On the other hand, it is important to note that the solution of $STA_{rp0.2}$ is found to exceed by 13.0% the one obtained for deterministic tree (DA_{rp1}). This is because the $STA_{rp0.2}$ obtains an optimal solution for all combined scenarios, while DA_{rp1} achieves the solution for one single scenario.

Table 4
Optimal network structure for solving the ST approach with different values of the reduction parameter.

Entity	DA_{rp1}	$STA_{rp0.6}$	$STA_{rp0.4}$	$STA_{rp0.2}$	$STA_{rp0.14}$	$STA_{FullTree}$
s1		*	*	*	*	*
s2	*	*	*	*	*	*
s3	*	*	*	*	*	*
f1	*					*
f2		*	*	*	*	*
f3	*	*	*	*	*	
w	*	*	*	*	*	*
dc1	*	*	*	*	*	*
dc2						
cc1	*	*	*	*	*	*
cc2						
Rc	*	*	*	*	*	*
dm1						*
dm2	*	*	*	*	*	*
dp1	*	*	*	*	*	*
dp2						
fd	*	*	*	*	*	*
Total Number of Entities	11	12	12	12	12	13

Gray sectors show entities not employed.

Not only objective function values, but also the design of the CLSC is considered of importance when different scenario trees are taken into account. Table 4 lists the network structure for different degrees of tree reduction. The number of network entities increases as the value of reduction parameter changes from $rp = 1$ to $rp = 0.6$ (cases DA_{rp1} and $STA_{rp0.6}$, respectively). In addition, for cases $STA_{rp0.6}$, $STA_{rp0.4}$, $STA_{rp0.2}$ and $STA_{rp0.14}$, the network design is identical.

By considering the structures obtained in cases $STA_{rp0.2}$ and DA_{rp1}, it can be easily observed the advantage of employing a stochastic approach. The network structure obtained by the

Table 5
Results for solving the stochastic approach with different uncertain conditions.

Case	Min F	Costs					Revenue
		Transport	Emissions	Purchasing	Facility [$\times E^5$]	Storage	
DA_{rp1}	1,814,145,743	614,936,167	72,463,739	491,279	15,788	112,159	452,657,604
$STA_{rp0.2}$	2,050,300,427	779,289,695	89,760,895	674,034	16,588	233,004	478,457,203
STA_d	2,058,867,861	672,211,456	77,554,638	602,070	18,438	140,988	535,441,293
STA_s	1,956,981,383	757,816,354	88,162,965	669,011	15,788	229,496	468,696,446

All values are expressed in currency units [c.u.].

deterministic formulation is not able to be easily adapted to changes in demand and supply, since when uncertainty is present the structure is found to change. Thus, an extra supplier, s1, is drawn into the network, and plant f1 is replaced by f2, which offers a much higher production capacity (see Table 4). Therefore the inclusion of the supplier s1, and the strategic selection of the production plant f2 in association with f3 (a choice which holds for all uncertain cases with the exception of $STA_{FullTree}$), reveal the key aspects of this CLSC that ensure resilience toward uncertain supply and demand scenarios. It is important to note that while f2 is the plant with the highest production capacity, f3 is the one with the lowest (see Table A4).

It is important to remark that the joint consideration of the uncertainty of both parameters, supply and demand, is also an important factor when the problem is addressed. The results obtained when both uncertain parameters are treated separately are different to the solution generated when both uncertain parameters are simultaneously taken into account (see Tables 5 and 6). The differences involve both design and planning decisions, which impact on the value of the objective function. While the solution obtained considering uncertain supply (case STA_s) excludes entities f2, dc2, cc2, dm1 and dp2 of the network, the solution obtained for uncertain demand (case STA_d) leaves out entities f3, dc2, cc2 and dp2. As it can be seen, the solution obtained for cases STA_s and STA_d does not consider simultaneously factories f2 and f3. On the contrary, these plants are simultaneously used in the solution obtained for case $STA_{rp0.2}$, which is a key aspect of the CLSC to obtain a more resilient configuration since supply and demand fluctuations occur at the same time.

Table 6
Optimal network structure for solving the ST approach when both uncertain parameters are treated separately.

Entity	STA_d	STA_s
s1	*	*
s2	*	*
s3	*	*
f1	*	*
f2	*	
f3		*
W	*	*
dc1	*	*
dc2		
cc1	*	*
cc2		
Rc	*	*
dm1	*	
dm2	*	*
dp1	*	*
dp2		
Fd	*	*
Total Number of Entities	13	12

Gray sectors show entities not employed.

6.1. Results for managing the flow of returned products

The computational tests performed using the parameters of the reference case ($\alpha = 0.7$, $\beta = 0.4$, $\chi = 0.7$, $\varepsilon = 0.7$, $\delta = (0, 0.7, 1)$) presented in the previous section are extended in order to determine how changes in these parameters affect the network. The most relevant results for $STA_{rp0.2}$ with different values for parameters α, β, χ, δ and ε are depicted in Table 7 and Figs. 4–8. It is important to note that the figures show the percentage of increase or decrease of the objective function, costs and revenue for each solution with respect to the reference case.

In comparison with the reference case, as the return quantity α adopts the value 1 (meaning that all the supplied products are returned), the network structure obtained incorporates f1, cc2 and dm1, in order to deal with the new return level, but drops f3. For this case, as it can be seen in Fig. 4, the objective function (F) value improves. This reduction is the result of a smaller storage cost and greater revenue. The improvement is achieved even though most of the operational costs rise. On the other hand, the network structure does not change when parameter α decreases from 0.7 to 0.4 (see Table 7). Nevertheless, the objective function increases due to the diminished revenue margin.

As shown in Table 7 and Fig. 5, when $\beta = 0.2$, the objective function deteriorates due to transport, emissions and facility costs. For this case, as it can be seen in Table 7, the network structure obtained incorporates a new dismantler center (dm1) in order to handle the increase in product flow directed to these centers. When $\beta = 0.8$, the number of entities in the network also increases. However, the new entity (one collection center) increases the capacity of sending material to repairing centers, causing a reduction in the storage costs. Therefore, for $\beta = 0.8$ the objective function improvement is mainly due to a reduction in the storage cost and an increment in revenues.

As it can be seen in Fig. 6, changes in parameter χ produce minor modifications in the cost structure and these do not make alterations in the network structure. For case $\chi = 0.9$ (meaning that only 10% of the products are sent to the distribution centers), the objective function improvement is mainly due to the reduction in the storage cost and the increment in revenues. On the other hand, when parameter χ adopts the value 0.5, the objective function deterioration is mainly due to the increase in the purchasing and storage costs and the decline in revenues.

The chart in Fig. 7 shows the relationship between the reference instance and case studies with different values of δ for the fraction of partially recyclable products (δ_{Prcy}). As it can be seen, the objective function improves when their amount sent to decomposition centers raises. This decrease is a result of a greater revenue and less storage cost. Modifications in δ do not generate changes on the network structure (see Table 7).

Finally, as shown in Table 7 and Fig. 8, when ε falls below the reference value of 0.7 (meaning a decline in the rate of products sent to the suppliers), the objective function improves due to the reduction in the purchasing cost and the increment of revenues. When parameter ε adopts the value 0.9, the objective function

Table 7
Optimal network structure for case studies with the most relevant values for parameters α, β, χ, δ and ε.

Entity	Reference Case	α		β			χ		δ		ε		
		0.4	1	0.2	0.6	0.8	0.5	0.9	0.4	1	0.3	0.5	0.9
s1	*	*	*	*	*	*	*	*	*	*	*	*	*
s2	*	*	*	*	*	*	*	*	*	*	*	*	*
s3	*	*	*	*	*	*	*	*	*	*	*	*	*
f1			*										
f2	*	*	*	*	*	*	*	*	*	*	*	*	*
f3	*	*		*	*	*	*	*	*	*	*	*	*
w1	*	*	*	*	*	*	*	*	*	*	*	*	*
dc1	*	*	*	*	*	*	*	*	*	*	*	*	*
dc2													
cc1	*	*	*	*	*	*	*	*	*	*	*	*	*
cc2			*			*							
rc1	*	*	*	*	*	*	*	*	*	*	*	*	*
dm1			*	*									
dm2	*	*	*	*	*	*	*	*	*	*	*	*	*
dp1	*	*	*	*	*	*	*	*	*	*	*	*	*
dp2													
fd1	*	*	*	*	*	*	*	*	*	*	*	*	*
Total Number of Entities	12	12	15	13	12	13	12	12	12	12	12	12	12

Gray sectors show entities not employed.

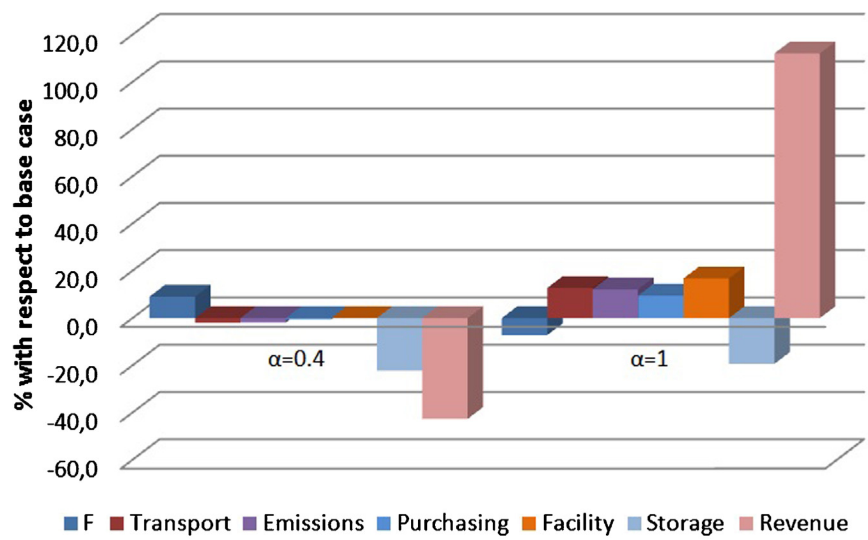

Fig. 4. Results (objective function, costs and revenue) obtained for $\alpha = 0.4$ and $\alpha = 1.0$.

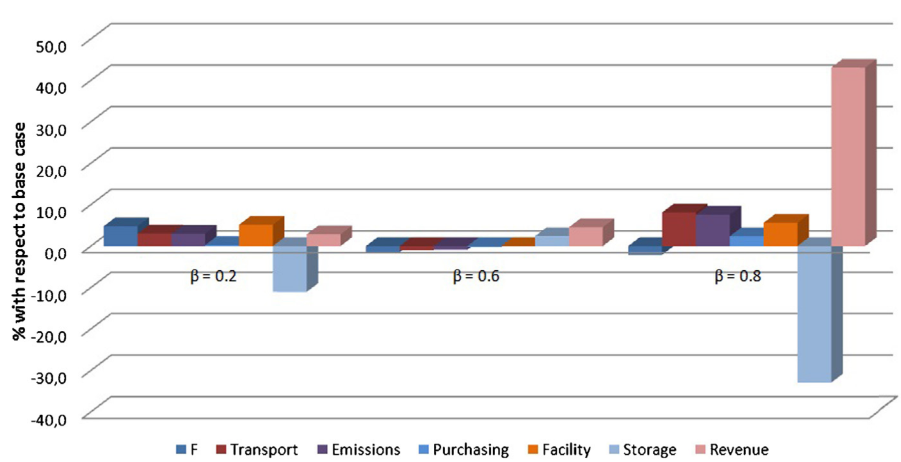

Fig. 5. Results (objective function, costs and revenue) obtained for $\beta = 0.2$, 0.6 and 0.8.

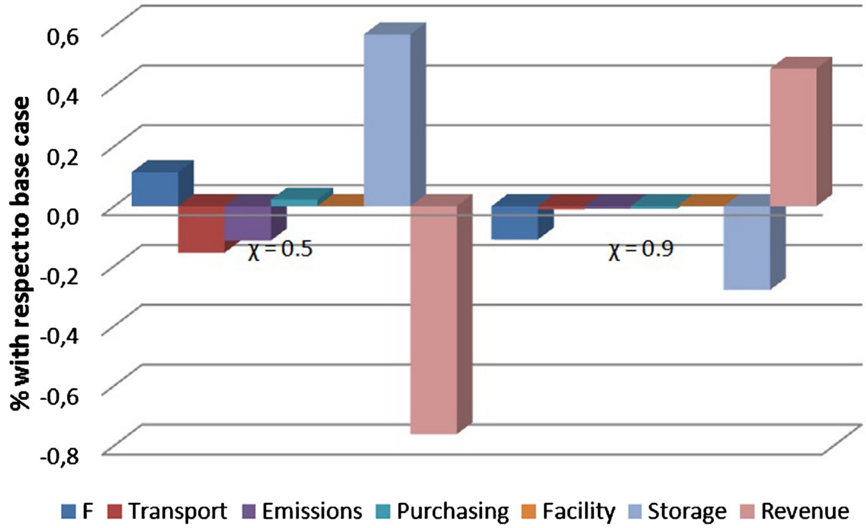

Fig. 6. Results (objective function, costs and revenue) obtained considering $\chi = 0.5$ and $\chi = 0.9$.

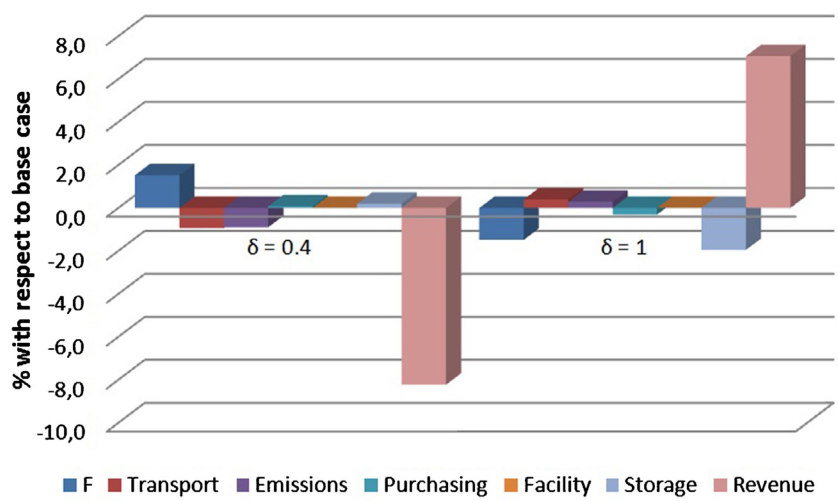

Fig. 7. Results (objective function, costs and revenue) obtained for $\delta_{Prcy} = 0.4$ and $\delta_{Prcy} = 1$.

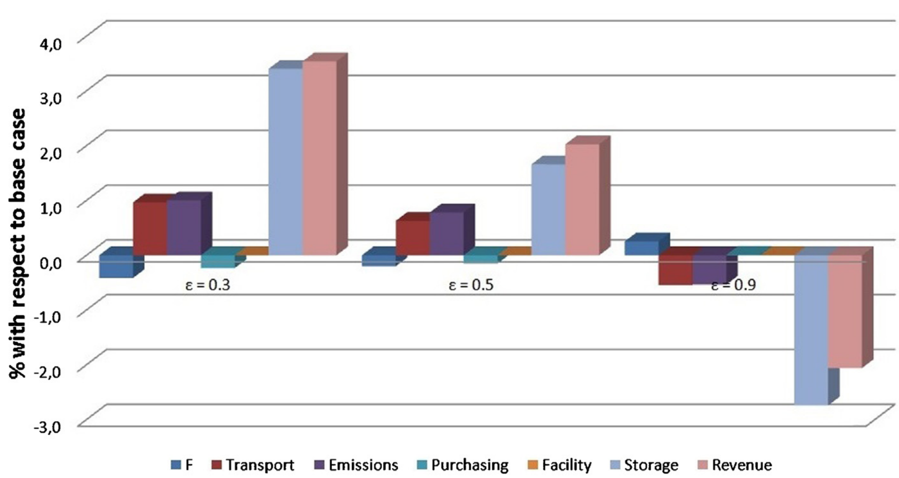

Fig. 8. Results (objective function, costs and revenue) obtained for $\varepsilon = 0.3$, 0.5 and 0.9.

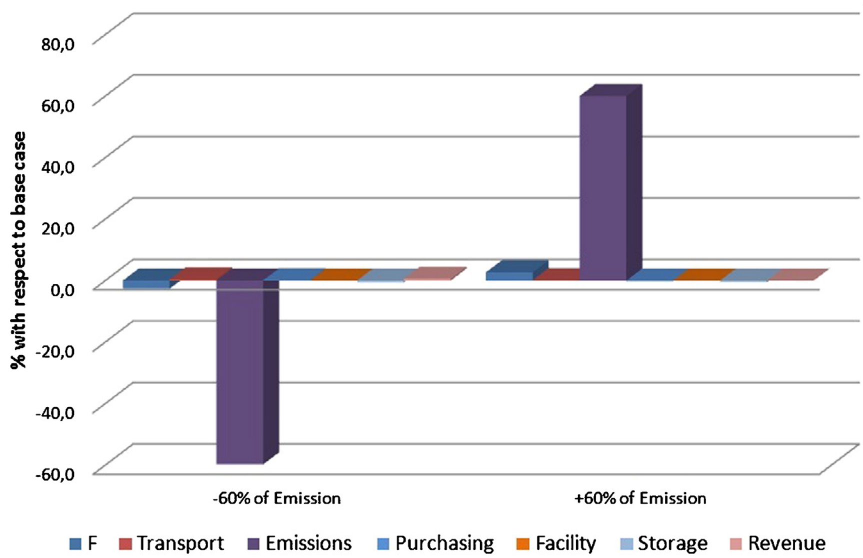

Fig. 9. Results obtained considering the increase and decrease of the emission cost.

deterioration is mainly due to the decline in revenues. This happens despite the fact that transport, emissions and storage costs decrease.

Based on the above results and unit costs and revenue assumed, it is apparent that in order to manage the flow of returned products, parameters α and β are the most critical and influential on the overall performance of the CLSC.

6.2. Results for managing the emission cost and revenue

Fig. 9 shows the relative importance of the emission cost variations over the objective function terms. Emission costs are increased and decreased with respect to the original values (emission costs per product unit are equal to 0.77, 0.86 and 0.95 cents for truck types 1, 2 and 3, respectively). In particular, when emission costs are increased by 60%, the total environmental cost raises 60% and the objective function deteriorates by an increase of 2.7%. This increase is attenuated by decreasing purchasing and storage costs (0.2% and 0.5%, respectively). The opposite situation occurs when emission costs are decreased by 60%. The network structure remains the same for both scenarios.

The relevant results obtained considering changes in the unit revenue are illustrated in Fig. 10. When the product unit revenue is increased by 60%, the objective function value decreases, improving by about 15.9%. This is due to the increased revenue per product unit that is associated to more flow of product in the network and less storage cost. The final increase of revenue is about 124.4%. In addition, the surge in products flow in the network causes changes in the network structure. Thus, while the solution obtained for the base case excludes entities f1, dc2, cc2, dm1 and dp2 of the network, the solution obtained when unit revenue is increased by 60% leaves out entities f3, dc2, cc2 and dp2. On the other hand, when the product unit revenue is decreased by 60%, the objective function value deteriorates, increasing by about 13.5%. In this case, transport, emissions and purchasing costs and the revenue intake decrease by about 3.5%, 3.5%, 0.7% and 64%, respectively, while the storage cost increases by about 4.5%. The network structure remains identical to the reference case.

The above analysis provides useful insights to CLSC managers in order to make appropriate decisions for improving the company performance. Based on the results obtained for managing the flow of returned products, it should be noted that the company will get a better economical performance by collecting more products after

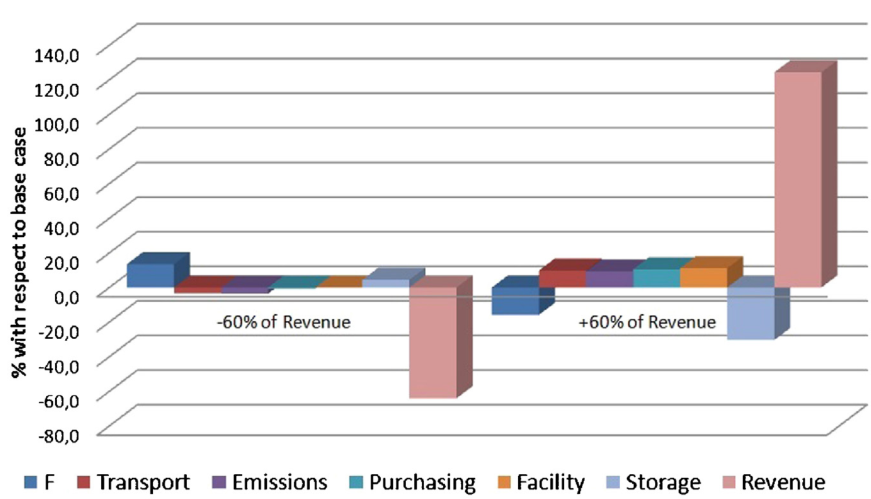

Fig. 10. Results obtained considering the increase and decrease of the unit revenue.

they are discarded by users. In addition, an increase in the amount of goods entering repair centers is also a very important factor for the company. Considering the results obtained for revenues, it is important to remark that an increase in the unit revenue for the recovered product makes a substantial improvement on the total revenue and on the decrease of storage costs, which have a positive effect on the network performance.

7. Conclusions

In this paper a MILP multi-stage stochastic model is introduced to deal with the design and planning problem of multi-period multi-product closed-loop supply chains. The MILP formulation is proposed for addressing general CLSCs, structured as a 10-layer network (5 forward plus 5 reverse flows), with uncertain levels in the amount of raw material and customer demands. The effects of uncertain demand and supply on the network are considered by means of scenarios. The goal is to minimize the expected cost of facilities, purchasing, storage, transport and emissions, minus the expected revenue due to returned products. To show the application of the mathematical formulation, several instances of an example proposed in the literature are examined. The results show the relevance of considering the multi-stage stochastic approach, instead of deterministic, when considering multi-period problems with uncertainty. Regarding the way for obtaining a reasonable representation of the original problem (able to be computationally manageable), a reduced scenario tree is used. The importance of using a reduction algorithm to decrease the size of the problem, considering several outcomes at each time period for each uncertain parameter is illustrated. A parametric analysis of a CLSC is further carried out to highlight the benefits of using the multi-stage model to derive managerial insights into its design and planning.

As future work, a more accurate stochastic model to deal with the continuous distribution functions of the uncertain parameters considered is to be developed. In addition, a specialized solution method is to be investigated to further increase the efficiency of the solution space.

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Appendix A.

Table A1
Investment cost, storage cost and storage capacities.

Entities	Investment cost [$] $\times 10^5$	Storage cost [$]	Minimum storage capacity	Maximum storage capacity
s1–3	–	–	800	24,500
f1	6000	1.6	800	22,500
f2	6800	1.6	800	22,500
f3	5000	1.6	800	22,500
W	648	1.7	800	22,500
dc1	920	1.8	500	20,500
dc2	786	1.8	800	21,500
cc1	930	2.1	1000	15,500
cc2	890	2.1	800	14,500
Cc	870	2.6	500	10,500
d1	780	2.7	100	8000
d2	850	2.7	400	9000
dp1	680	2.8	500	5000
dp2	720	2.8	500	5000

Table A2
Transport capacities [tons].

Product types	Minimum capacity	Maximum capacity
Forward network		
Truck 1	1000	7000
Truck 2	5000	10,000
Truck 3	8000	15,000
Reverse network		
Truck 1	500	5000
Truck 2	3000	8000
Truck 3	5000	13,000

Table A3
Revenue obtained when used products are returned to the forward network.

Product types	Entity	$/ton	Entity	$/ton	Entity	$/ton
Frcy		70		80		74
Prcy	s1	54	f1	58	w	50
Nrcy		0		0		38
Frcy		64		82		78
Prcy	s2	50	f2	62	dc1	56
Nrcy		0		0		36
Frcy		72		84		82
Prcy	s3	58	f3	54	dc2	58
Nrcy		0		0		38

Table A4
Production/processing capacities.

Entities	Product types	Minimum production/ processing capacity	Maximum production/ processing capacity
	Frcy	8200	16,400
f1	Prcy	7200	15,000
	Nrcy	9000	18,000
	Frcy	10,200	20,400
f2	Prcy	9500	19,000
	Nrcy	11,800	23,600
	Frcy	5500	11,000
f3	Prcy	6300	12,600
	Nrcy	8500	17,000
	Frcy		
Reverse entities	Prcy	1000	18,000
	Nrcy		

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