PROCESS DESIGN AND CONTROL

Managing Distribution in Supply Chain Networks

Rodolfo Dondo, Carlos A. Méndez, and Jaime Cerdá*

INTEC (UNL—CONICET), Güemes 3450, 3000 Santa Fe, Argentina

This paper presents a novel optimization approach to the short-term operational planning of multiechelon multiproduct transportation networks. Distribution activities commonly arising in real-world chemical supply chains involve the shipping of a number of commodities from factories to customers directly and/or via distribution centers and regional warehouses. To optimally manage such complex distribution systems, a more general vehicle routing problem in supply chain management (VRP-SCM) has been defined. The new VRP-SCM problem better resembles the logistics activities to be planned at multisite manufacturing firms by allowing multiple events at every location. In this way, two or more vehicles can visit a given location to perform pickup and/or delivery operations, and vehicle routes may include several stops at the same site, i.e., multiple tours per route. More important, the allocation of customers to suppliers and the quantities of products shipped from each source to a particular client are additional model decisions. Both the capacitated vehicle routing problem (VRP) and the pickup-and-delivery problem (PDP) can be regarded as particular instances of the new VRP-SCM. The proposed MILP mathematical formulation for the VRP-SCM problem relies on a continuous-time representation and applies the general precedence notion to model the sequencing constraints establishing the ordering of vehicle stops on every route. The approach provides a very detailed set of optimal vehicle routes and schedules to meet all product demands at minimum total transportation cost. Several examples involving up to 26 locations, four products, and six vehicles housed in four different depots have been solved to optimality in very short CPU times.

1. Introduction

Distribution is concerned with the shipment and storage of multiple products downstream from the supplier side to the customer side in the supply chain (SC). Typically, products are manufactured in one or more factories, moved to warehouses for intermediate storage, and subsequently shipped to retailers or final consumers. Therefore, a distribution network generally includes factories, distribution centers (DCs), retailers, and end users at different levels, with the products going from the highest to the lowest level. Such levels of the distribution system are named echelons. The demand arises at the lowest echelon and is transmitted up to the higher ones. A DC can be regarded as an intermediate facility that allows the aggregation of products coming from different factories and destined for different retailers on the same arriving trucks. Such products are temporarily stored in the DC before they are sent to their destinations. In this way, customer orders can be satisfied through single deliveries and substantial savings in transportation costs are achieved. A distribution system with a central DC is a three-echelon network. Complex distribution systems may include more than a single layer of intermediate warehouses.

Logistics costs include inventory, inbound and outbound transportation, facility (DCs, warehouses), information, and handling costs.[1] Inbound transportation refers to the movement of products from factories to warehouses, while outbound transportation goes from facilities to retailers/end users. Freight is mostly made by trucks in two different modes: full truckload (TL) and less than truckload (LTL). TL is cheaper but the client is charged a full truck independent of the amount loaded, while the freight cost for LTL is based on the quantity being shipped and the distance traveled. Transportation costs generally decrease with the number of intermediate facilities because of shipment consolidation and shorter outbound distances. Shipment consolidation implies deliveries from many suppliers to the same retailer on a single truck. In turn, inventory, facility, and handling costs all grow with the number of facilities. In the process of selecting the distribution network design, other dimensions or performance measures such as response time (RT), product availability, and customer satisfaction should also be considered.[2] RT is the time between the placement and the delivery of the customer order. If enough stocks of requested products are available in the warehouse, RT is the time for transmitting the order to the warehouse, picking and packing the products, and physically transporting them to the customer site. Product availability is the probability of having the requested product in stock when an order arrives. On the other hand, customer satisfaction or experience is the ease with which a customer can place and receive an order. In this regard, the order visibility defined as the customer chance to track the order status from placement to delivery is another important dimension. Companies in the same industrial segment often adopt different network designs, mainly because their operational strategies are focused on different dimensions.

To effectively design and manage a large-scale distribution network, long-run strategic planning, medium-term tactical planning, and short-term operational planning should all be periodically developed.[3] Strategic planning deals with network structural decisions such as the location and size of new facilities, and typically covers a 2−5 year horizon. Tactical planning is concerned with resource allocation decisions and the setup of distribution channels over annual periods. It decides which products will be manufactured in a factory, and which warehouse will service a customer zone. In turn, operational

* To whom correspondence should be addressed. E-mail: jcerda@intec.unl.edu.ar.

10.1021/ie900792s CCC: $40.75
Published on Web 09/09/2009

planning is concerned with short-term production scheduling, inventory management, and transportation planning. Transportation often represents a significant component of the total logistics cost. Short-term transportation planning generates vehicle routes and schedules based on the available resources in order to minimize the transportation cost while meeting some customer-service-level requirements such as on-time deliveries. This paper introduces a novel optimization framework for the short-term operational planning of multiproduct multiechelon transportation networks. The transportation infrastructure is assumed to be given.

1.1. Literature Review. The development of effective computational tools for logistics management has attracted a great attention from both industry and academia to the field of supply chain management and, more recently, to the emerging area of enterprise-wide optimization, as pointed out by Grossmann.[4] Previous work has mostly focused on strategic and tactical planning of supply chain networks. Reviews of long- and medium-term supply chain optimization can be found in Vidal and Goetschalck,[5] Varma et al.,[6] and Papageorgiou.[7] In contrast, operational planning of multiechelon distribution networks, including detailed vehicle routing and scheduling, has recently received more attention.[8,9] Nonetheless, several supply chain management problems related to the process industry were reported in the literature. Among others, the following ones can be mentioned: (a) distribution of refined petroleum products from depots to gasoline stations through a fleet of multiparcel trucks;[10] (b) scheduling of multicompartment carriers transporting chemicals from upstream to downstream refineries and chemical manufacturers;[11] (c) dispatching of crude oil from oil fields to refineries, and shipment of refined products to industrial users through tankers and barges;[12] (d) collection of fresh milk from hundreds of dairy farms and subsequent delivery to processing plants, and distribution of dairy products from the central warehouse to retail outlets;[13] (e) redesign and assessment of the worldwide formulation and U.S. distribution networks of a real agrochemical supply chain.[14]

In supply chain strategic planning, the well-known location–allocation problem was introduced by Geoffrion and Graves[15] to optimally choosing *m* facilities among *n* (>*m*) locations and simultaneously assigning product demands to the open facilities. During the past decade, research interest has mainly focused on integrated logistics models for locating production and distribution facilities in a multiechelon supply chain. In addition to such strategic decisions, integrated approaches simultaneously determine the product mix to be manufactured at production facilities, and the product flow pattern from factories to customer zones via a set of warehouses.[16] The value of coordinating production and distribution planning in a two-echelon supply chain using direct shipping was studied by Chandra and Fisher.[17] They found that the coordination of production and distribution produces savings in operating costs ranging from 3 to 20%. Jayaraman and Pirkul[18] presented a heuristic procedure for the combined production–distribution, location–allocation problem, including raw-material procurement. On the other hand, Tsiakis et al.[19] proposed a mixed-integer linear programming (MILP) formulation for the design of multiproduct, multiechelon supply networks. The number, location, and capacity of warehouses to be open, the transportation infrastructure, and the flows of products throughout the system were all determined. By assuming that manufacturing sites already exist and operate, design decisions are just confined to product distribution. Jayaraman and Ross[20] developed a two-level planning approach for distribution problems involving a central manufacturing site, multiple DCs, and cross-docking sites, and customer zones demanding several commodities. At the upper level, the best set of DCs and cross-dock sites is selected. At the lower level, the product flows to be shipped from the plant to distribution centers, transshipped from DCs to cross-dock sites, and distributed to retail outlets are all determined. Both steps are accomplished by sequentially solving a pair of MILP mathematical models. You and Grossmann[21] developed an integrated approach that simultaneously finds the supply chain network design, the production planning and scheduling, and the inventory management under demand uncertainty. Location and capacity of production plants are model decisions. The trade-off between economics and responsiveness is resolved by using a bicriterion optimization model. To identify the best SC-network design, a Pareto-optimal curve is generated by solving a multiperiod mixed-integer nonlinear programming (MINLP) formulation that simultaneously maximizes the net present value and minimizes the expected response time.

Other approaches have intended to optimally coordinate short-term production and distribution, assuming a given supply chain structure. Verderame et al.[22] studied the operational planning of a given multisite network involving several batch production facilities and distribution centers. Based on customer demands, products are shipped from factories to DCs, where consumers should go to pick up the orders. The problem has been modeled through an MILP discrete-time formulation that provides the daily production mix at each factory, and the product flows from manufacturing sites to DCs. Bonfill et al.[23] introduced a mathematical framework for coordinating production and distribution planning in order to properly manage inventory profiles and material flows between sites. Production and transportation problems are defined as detailed scheduling problems, and different mathematical/heuristic algorithms were proposed. A rather simple two-tier transportation infrastructure with a single production facility was tackled. Amaro and Barbosa-Póvoa[24] presented an integrated time–discrete MILP formulation for the optimal scheduling of supply chain networks. The model provides a detailed operational plan at the production, storage, and transportation levels, by considering the supply chain topology, different operational conditions, and market opportunities. However, the resulting MILP may become hard to solve because of the huge number of binary variables and constraints to be satisfied.

This paper has focused on the day-to-day planning of distribution activities in a supply chain. It presents an optimization approach for the short-term operational planning of multiechelon, multiproduct distribution systems, providing detailed vehicle routing and scheduling. This new formulation of the vehicle routing problem in supply chain management, called VRP-SCM, has been modeled as a single-level mixed-integer linear programming problem (MILP). It is a tailor-made formulation for SCM in the process industry. The widely known vehicle routing problem (VRP) and pickup-and-delivery problem (PDP) can be regarded as particular instances of the new VRP-SCM problem formulation. The supply chain structure, product inventory levels at network facilities, and transport resources are assumed to be given. The optimal solution to the VRP-SCM problem provides not only the product flows from factories and warehouses to customer zones but also detailed routing and scheduling for the vehicle fleet executing the required pickup and delivery operations. Time windows (TW) within which pickup/delivery services should be done can be easily handled through the proposed framework.

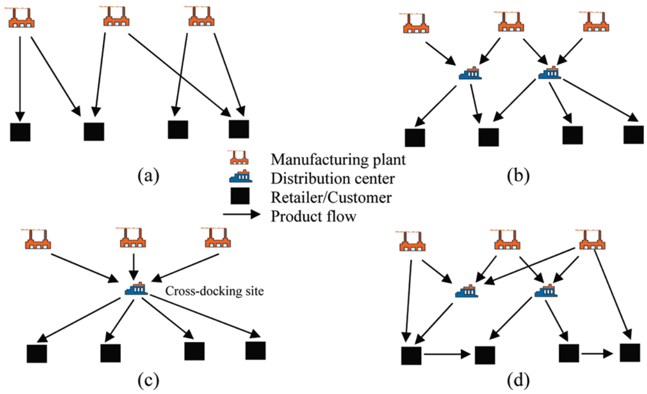

Figure 1. Alternative distribution network design options.

2. Distribution Network Design Options

The network design strongly affects the supply chain performance by setting the framework within which operational decisions such as vehicle routing and scheduling should be taken. Good designs allow achievement of a variety of distribution goals from low logistics costs to high responsiveness. Interesting surveys on supply chain design can be found in papers by Beamon,[25] Chopra and Meindl,[1] Chropra,[2] Simchi-Levi et al.,[3] and Shah.[26] The major network design options shown in Figure 1 are the following: (1) *Manufacturer storage and direct shipping*, where all inventories are stored at the factories and all shipments go directly from manufacturing sites to customer zones (see Figure 1a). It is also known as drop shipping. (2) *Distributor storage and shipping via a central distribution center*, where product stocks are located at the distribution center and all shipments go from the DC to customers (see Figure 1b). (3) *Cross-docking* with manufacturer storage and all shipments via a central DC, where products received from suppliers are just crossed from one to another loading dock in 24−48 h and shipped to consumer zones (see Figure 1c). (4) *Hybrid infrastructure*, which results from the combination of the previous options (see Figure 1d).

The major advantage of *drop shipping* is the ability to centralize inventories at the manufacturer site, thus guaranteeing a high level of product availability with a lower amount of inventory. Its biggest disadvantage is the larger response time and the higher transportation cost due to disaggregate shipping and longer outbound distances. Many partial shipments may be received at the destination, and more expensive package carrier services will be required. An improved version of drop shipping is obtained by incorporating the so-called milk runs, i.e., *direct shipment with milk runs*. A milk run is a route followed by a truck collecting lots of products ordered by the same customer from different suppliers. In this way, shipments from multiple suppliers are consolidated on a single truck and TL shipments can be made, thus lowering outbound transportation costs. In a distribution network involving milk runs, therefore, a truck may sequentially stop at several supplying or demanding sites. The well-known vehicle routing problem (VRP) deals with a two-tier distribution network where the cargo moves from a central facility to multiple customer locations through a vehicle fleet using direct shipment with milk runs; i.e., several customers are serviced by the same vehicle. Besides, each customer should be serviced by only one vehicle. Another problem receiving a great attention in operations research is the pickup-and-delivery problem (PDP). PDP also involves a two-echelon distribution network where some loads picked up at some given sites (multiple sources) are to be delivered to some given destinations using direct shipping with milk runs. PDP assumes predefined suppliers for one or multiple delivery sites and a prefixed cargo to be loaded/unloaded at every pickup/delivery location. The overall cargo to be collected at pickup sites should be equal to the total amount unloaded at delivery locations. In both VRP and PDP, partial shipments to the same customer site and the execution of both loading and unloading operations at a given location are not permitted. Ropke et al.[27] proposed very efficient MILP formulations for one-to-one PDP with time windows (PDPTW), where every request involves a single pickup node and a single delivery node. Using branch-and-cut algorithms, instances with up to eight vehicles and 96 requests were solved to optimality.

In the second design option involving a central DC, production at manufacturing sites is driven by replenishment orders placed by the distributor to meet forecasted customer demands. Suppliers send the requested products to the DC, from which appropriate shipments are subsequently forwarded to every retailer. The DC is an intermediate layer with four major functions: receiving, storing, picking, and shipping products. It is the place where inventory is mostly held and the items ordered by a customer are loaded onto a truck to make a single delivery. Then, multiple vehicles arrive at and leave the DC every working day. From the modeling viewpoint, there will be multiple vehicle stops at the same facility to perform loading and unloading operations. Major advantages of warehousing are the consolidation of shipments from multiple suppliers and the postponement of product customization until receipt of customer orders at the warehouse. As a result, there is a significant saving in transportation costs and a better response time. However, higher amounts of inventories should be carried because of the demand uncertainty and a narrower range of products is generally available at the distributor storage. Cross-docking is between the two previous distribution strategies. The transportation infrastructure now includes a DC or transfer location where products from factories are cross-docked and sent to customers on a daily basis. Cross-docking aims to improve warehousing by reducing inventory costs through demand aggregation while keeping transportation costs lower by postponing product customization. A combination of manufacturer and distributor storage is generally used. Manufacturer storage is planned for high-value products whose demands are hard to forecast, while some stocks of fast-moving items are still kept in inventory at the DC to get a better responsiveness. Most companies combine all the above options into a distribution network. The selected design is tailored to meet the items to be distributed and the needs of customers to be serviced. For these reasons, hybrid networks are usually favored. The proposed mathematical framework for short-term distribution planning can handle any type of transportation infrastructure with the exception of cross-docking sites to be considered in a future paper. Tracking of product inventory levels with time at cross-docking points should be additionally considered.

3. VRP-SCM Problem Definition

Let us consider a general distribution network in a supply chain represented by a graph G (I, A) shown in Figure 2. Nodes $i \in I$ stand for factories, warehouses, distribution centers, and customer zones, and A is the set of minimum-cost arcs interconnecting nodes in the network. Besides, there is a set P including the products to move along the arcs from factories and warehouses to customers, and a set V comprising the vehicles transporting products from sources to destinations. Three types of nodes are included in a distribution network:

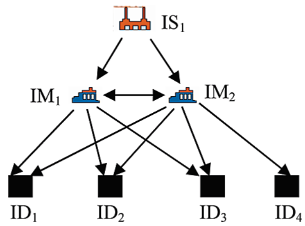

Figure 2. Three-echelon supply chain distribution network.

(1) "Pure" source nodes (*IS*), usually manufacturer storages, from which products are sent to DCs, warehouses and customer zones. Then, trucks stopping by a node $i \in IS$ just carry out pickup activities. (2) Mixed nodes (*IM*), such as distribution centers or regional warehouses that receive and store products from manufacturers, and ship them to customer zones. Trucks visiting mixed nodes can carry out loading and/or unloading operations, i.e., accomplish pickup and/or delivery services. (3) Destination nodes (*ID*), such as consumer zones, where the visiting trucks just accomplish delivery operations. The elements of *IS* and *IM* can be regarded as source nodes or suppliers because they provide products to downstream demanding locations in the supply chain. On the other hand, the elements of *IM* and *ID* are destination nodes for product shipments. Let $SS = IS \cup IM$ denote the set of product suppliers, while $DS = IM \cup ID$ represents the set of product destinations.

On the other hand, the set A includes routes connecting manufacturers to warehouses, and warehouses with customer zones. Then, the proposed transportation infrastructure may consider (i) direct shipping networks, (ii) shipping via DC or regional warehouses networks, or (iii) a combination of both strategies, i.e., hybrid distribution networks. There are also some routes in A interconnecting manufacturing sites or linking warehouses among themselves. By so doing, transportation networks with "milk runs" can also be considered. Associated to every route $a \in A$, there is a distance-based traveling cost c_{ij} and a travel time t_{ij} between the interconnected nodes $i, j \in I$. Besides, the product set P stands for the range of products available at manufacturer and warehouse storages. A replenishment order from warehouses usually includes multiple products often available at different production sites. Consolidation of shipments from multiple suppliers to a single destination may imply transport of several different products on the same truck. Since the total shipment size must never exceed the volume/weight capacity of the truck, two important product properties for truck loading are the weight (uw_p) and the volume (uv_p) of a single unit of product p. Furthermore, vehicles moving products from manufacturer and distributor storages to consumer zones are the elements of set V. Each vehicle has a given weight (qw_v) and volume (qv_v) capacity, and a base from which it starts and ends the journey. A vehicle base can be a manufacturing site or a warehouse. Let $B \subset (IS \cup IM)$ be the set of potential bases for the vehicle fleet transporting the products and B_v ($\subset B$) the alternative bases for vehicle v. After completing the assigned tour, the vehicle should return to its base. However, different start and end bases for a vehicle can also be handled. Moreover, it usually happens that a customer zone should be serviced by vehicles based on some predefined sites (factories or warehouses). Then, the set V_i ($\subset V$) is defined to denote the set of vehicles that can visit node $i \in I$.

Usually, several vehicles can stop by manufacturing sites or warehouses to carry out pickup or delivery tasks. Besides, a vehicle may be visiting a source node several times during the same tour. In addition, product requests at a given destination can be satisfied through several partial shipments using more than one vehicle. Therefore, several loading and/or unloading operations may sequentially occur at every location and a vehicle stop cannot be identified by just the visiting node. In other words, several events $n \in N_i$ can occur at location i and the vehicle stop should be characterized by the visited node i and the event n taking place. However, the number of events at any node during the planning horizon is unknown before developing the vehicle routes. Then, the adopted value for $|N_i|$ (i.e., the maximum number of events at node i) should be at least as large as the optimal number of vehicle stops at location i. During a stop, a truck can load and/or unload different items. The type of tasks carried out by a vehicle on the event point n at node i, i.e., during the stop (n,i), is defined by other model variables. The proposed mathematical model assumes that the elements of N_i, i.e., the potential events at site i, have been preordered. Therefore, the nth-event at site i, if accomplished, will occur before the vehicle stop $(n + 1,i)$. To better illustrate the idea of allowing multiple visits to a single node by associating multiple events with it, let us analyze the vehicle routes depicted in Figure 3. In Figure 3a, the demand at destination node ID_4 is satisfied through a pair of partial shipments coming from source nodes IS_1 and IM_1 and transported by two different vehicles. Therefore, a pair of events N_1 and N_2 both involving delivery operations have been associated with node ID_4. The delivery of goods from V1 at time event N_1 occurs earlier. In turn, Figure 3b shows the routes for a pair of vehicles (V1, V2) based on source nodes IS_1 and IM_1, respectively. Vehicle V1 makes an intermediate stop on its base IS_1 at time event N_2 to reload goods before serving destination nodes ID_3 and ID_4. Then, two different events N_1 and N_2 occur at node IS_1 and a double-loop route is thus generated for vehicle V1. In short, each vehicle stop at a particular location stands for a completely different event taking place at a different time and involving a specific set of pickup/delivery operations.

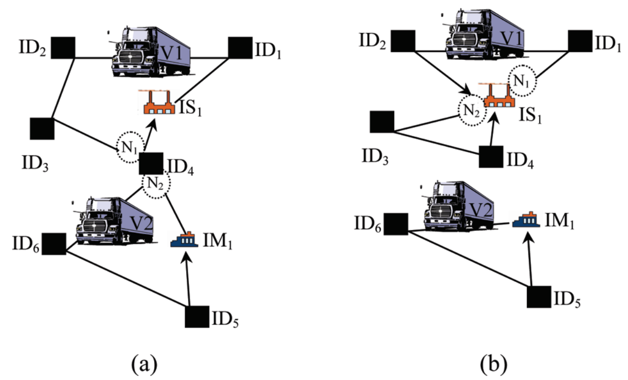

Figure 3. Multiple events at some locations: (a) two partial deliveries at destination node ID_4; (b) two stops of vehicle V1 for pickup operations at source node IS_1.

By considering multiple events at every location, the new VRP-SCM problem formulation better resembles the operations in real-world multiechelon distribution networks. The proposed model can account for the following: (a) customer requests without predefined suppliers, i.e., product flow pattern can be a model decision; (b) multiple partial shipments to a given location, i.e., load splitting; (c) transportation of multiple products from different suppliers to the same destination, or from the same supplier to different locations using the same truck, i.e., milk runs; (d) non-predefined amounts of products to pick up at source nodes; (e) multiple tours for a vehicle route with intermediate stops at source nodes to reload lots of products, provided that the total travel time does not exceed the specified service time; (f) alternative bases for vehicles, i.e., multiple depots; (g) customer time windows and maximum time service, i.e., the VRP-SCM problem with time windows (VRPTW-SCM); (h) initial inventories at manufacturer storages and DCs, but cross-docking is not permitted.

4. VRP-SCM Problem Formulation

The proposed mathematical formulation for the vehicle routing problem in supply chain management (the VRP-SCM problem) uses a continuous time-domain representation and applies the general precedence concept to define the set of 0−1 variables sequencing the vehicle stops on a given route. The problem goal is to minimize the total transportation cost while satisfying the customer-service-level requirements. Transportation costs include (i) fixed expenses incurred by used vehicles; (ii) distance-based variable costs, mainly fuel costs; and (iii) time-based variable costs, mainly driver wages. If considered, the customer satisfaction is measured through the total penalty to be paid for failing to provide delivery services within the specified time windows. The selected objective function is a weighted combination of such cost contributions.

Model variables can be grouped into two categories: (a) 0−1 decision variables and (b) continuous variables. In order to construct vehicle routes, 0−1 allocation and sequencing variables are to be defined. A vehicle route can be regarded as a sequence of vehicle stops at different locations. The assignment variable Y_{nv} denotes that event $n \in N_i$ at location $i \in I$ has been allocated to vehicle $v \in V_i$ whenever $Y_{nv} = 1$. In other words, vehicle v will be visiting node i at time event n, i.e., the stop (n,i), to perform pickup and/or delivery tasks. If instead $Y_{nv} = 0$ for any vehicle $v \in V_i$, assuming $n \in N_i$, then the events $\{n, n + 1, ..., |N_i|\}$ never occur at node i. They will stand for fictitious events. On the other hand, the 0−1 sequencing variable $X_{n,n'}$ indicates that the vehicle stop (n,i) at node i will occur earlier than the event n' at site i' whenever $X_{nn'} = 1$, assuming that $n \in N_i$, $n' \in N_{i'}$ and both sites $i, i' \in I$ are visited by the same vehicle $v \in V_{ii'}$ $(=V_i \cap V_{i'})$. A single variable $X_{n,n'}$ is enough to sequence a pair of events $n \in N_i$ and $n' \in N_{i'}$. Then, the variable $X_{n,n'}$ with $i < i'$ (or $n < n'$ if $i = i'$) is just included in the model. The separate handling of allocation and sequencing decisions permits getting a substantial saving in binary variables.

Other major model variables are continuous. In order to establish distance-based transportation costs and travel times, continuous variables C_n and T_n (with $n \in N_i$, $i \in I$) are defined. C_n represents the distance-based transportation cost incurred by the visiting vehicle to move along the assigned route from the base up to stop (n,i) at location i, assuming $n \in N_i$. In turn, T_n denotes the time required by the assigned vehicle to travel from the base to the stop (n,i). The nature of the tasks carried out by vehicle v during the stop (n,i) at site i, whenever $n \in N_i$ and $y_{nv} = 1$, is established by the model variables L_{npv} and U_{npv}. If $L_{npv} > 0$, then L_{npv} units of product $p \in P$ are loaded on vehicle v during stop (n,i). If in addition $U_{np'v} > 0$, then $U_{np'v}$ units of product $p' \in P$ are also delivered to location i at event $n \in N_i$. Obviously, L_{npv} and $U_{np'v}$ are both equal to 0 if $y_{nv} = 0$. On the other hand, non-negative model variables AL_{npv} and AU_{npv} are defined to compute the units of product p transported by the visiting vehicle after completing the stop (n,i). Such variables AL_{npv} and AU_{npv} represent the accumulated amount of product p picked up and delivered by the visiting vehicle v along the route from the base to stop (n,i), respectively, assuming that $n \in N_i$ and $v \in V_i$. Therefore, the in-transit stock of product p on vehicle $v \in V_i$ after stop (n,i), assuming $y_{nv} = 1$, can be efficiently found by computing the difference between AL_{npv} and AU_{npv}.

On the other hand, the problem formulation includes six major types of restraints: (1) *Route construction constraints* assigning a particular event $n \in N_i$ on a given site $i \in I$ to at most one truck, and sequencing vehicle stops (n,i) located on the same route and, consequently, involving the same vehicle $v \in V_i$. (2) *Sequencing constraints* bounding the values of timing variables TC_n and TT_n. (3) *Product inventory constraints* restraining the overall amount of products picked up by visiting vehicles at source nodes. (4) *Product demand constraints* ensuring customer request satisfaction. (5) *Null in-transit inventory constraints* requiring that every unit of product picked up by a vehicle must be delivered to a demanding location before the end of the vehicle trip. (6) *Loading/unloading constraints* tracking the amount of every product transported by each vehicle to prevent from overloading the vehicle capacity or unloading excessive amounts of some products.

4.1. Model Assumptions. The model assumptions are as follows:

1. Problem data are known with certainty and remain invariant with time.
2. Several products can be transported on the same vehicle.
3. A customer request can include a number of products provided by either the same or different suppliers.
4. There are no predefined suppliers for some customer locations. Moreover, the amounts of products to pick up at source nodes are not given data. Then, the product flow pattern through the distribution network is a model decision.
5. A customer location can receive the same product from more than one supplier. Then, partial shipments are allowed and every location can be visited by multiple vehicles.
6. A vehicle can load/unload lots of products at source nodes different from the base where is housed. Moreover, it can provide delivery services to multiple customer locations. Then, the problem formulation is able to account for milk runs and warehousing.
7. Every location can be visited several times by the same vehicle. Then, a vehicle route may include a series of tours with intermediate stops at source nodes to load further lots of products.
8. Pickup and delivery services, if required, can both be provided by a single vehicle at mixed nodes (i.e., warehouses). Certainly, such loading and unloading operations will involve different products.
9. There are alternative bases for a vehicle. Then, the allocation of vehicles to depots is left to the model. Moreover, each vehicle route should start and end at the assigned base.
10. The vehicle stop length has two components. The fixed contribution may depend on the site, while the variable component is proportional to the amount of products to be picked up and/or delivered.
11. There is a maximum service time for each vehicle that cannot be exceeded.
12. Time-window and maximum-service-time constraints can be relaxed by including penalty cost terms in the objective function that linearly increases with the violation size.

4.2. The MILP-Based Mathematical Formulation. 4.2.1. Vehicle Routing Constraints. Allocating Vehicles to Depots. Equation 1 states that each used vehicle v must start and end its trip at node l where it is housed ($W_{lv} = 1$). The node set $B_v \subset I$ includes the alternative operational bases for vehicle v, usually including one or several nodes representing factories and/or warehouses. If vehicle v is not required, the corresponding 0−1 allocation variables W_{lv} will be equal to 0 for all possible bases $l \in B_v$.

$$\sum_{l\in B_v} W_{lv} \leq 1 \qquad \forall v \in V \tag{1}$$

Assigning the Event n at Node i to Vehicle $v \in V_i$. Equation 2 indicates that every event $n \in N_i$ at location i, here also called the vehicle stop (n,i), can at most be allocated to a single vehicle $v \in V_i$. If $Y_{nv} = 1$, then vehicle $v \in V_i$ will be visiting node i at time event n to perform a specific set of pickup and/or delivery tasks.

$$\sum_{v\in V_i} Y_{nv} \leq 1 \qquad \forall n \in N_i, i \in I \tag{2}$$

If node i is just a destination (consumer zone), N_i comprises events at which delivery activities only take place. In contrast, if node i is a pure source node, i.e., factories, N_i will just comprise events at which pickup activities may only happen. In case that, for any reason, a node i must be visited just once by only one vehicle, a single element is to be defined for N_i. For mixed nodes, $|N_i|$ is usually greater than 1. In general, the number of predefined events for node i represents the potential vehicle stops for loading/unloading operations.

Preordering of Time Events Predefined for Node i. In order to handle multiple vehicle stops at node i, several events need to be postulated for that location ($|N_i| > 1$). In that case, the events related to the same node must be allocated to vehicles in the same order that they are predefined in the set N_i. As clearly stated by eq 3, the event n' for node i can be allocated to a vehicle only if all previous events $n \in N_i$ ($n < n'$) were already assigned.

$$\sum_{v\in V_i} Y_{nv} \geq \sum_{v\in V_i} Y_{n'v} \qquad \forall (n, n') \in N_i, i \in I: n < n' \tag{3}$$

Activated Vehicle Condition. A vehicle v can be involved in loading/unloading operations only if it has been activated. As stated by eq 1, a vehicle is said to be activated only if it has a designated depot, i.e., $W_{lv} = 1$ for some operational base $l \in B_v$.

$$\sum_{i\in I_v} \sum_{n\in N_i} Y_{nv} \leq M \sum_{l\in B_v} W_{lv} \qquad \forall v \in V \tag{4}$$

In eq 4 the parameter M stands for an upper bound on the number of stops over the route traveled by vehicle v.

Traveling Cost and Time from the Vehicle Depot to the First Visited Node. Since the first node i visited by vehicle v is a model decision, the traveling cost and time from the base of vehicle v to node i (if node i is the first visited) are given by eqs 5.a and 5.b, respectively.

$$C_n \geq \sum_{l\in B_v} dc_v d_{li} W_{lv} - M_C(1 - Y_{nv}) \qquad \forall n \in N_i, i \in I, v \in V_i \tag{5.a}$$

$$T_n \geq \sum_{l\in B_v} \left(\frac{d_{li}}{sp_v}\right) W_{lv} - M_T(1 - Y_{nv}) \qquad \forall n \in N_i, i \in I, v \in V_i \tag{5.b}$$

The parameter d_{li} defines the distance between depot l and node i, whereas the values of dc_v and sp_v represent the unit distance cost and the average speed of vehicle v, respectively. M_C and M_T are upper bounds for the corresponding cost and time variables.

Note that vehicle v first visits node i only if it performs some activity on that location, i.e., $Y_{nv} = 1$ for some time event $n \in N_i$. However, it is worth noting that (a) vehicle v may stop several times at node i, i.e., $Y_{nv} = 1$ for more than one event $n \in N_i$, and (b) vehicle $v \in V_i$ may already visit other nodes before stopping at node i. Consequently, the above inequalities will provide strict lower bounds on the traveling cost and time from the vehicle base up to node i only if node i is first visited by vehicle v and the first operation carried out by vehicle v is performed on that location. In case node i is visited by vehicle v but the event $n \in N_i$ (with $Y_{nv} = 1$) is not the first operation on that node or vehicle $v \in V_i$ already stopped at some other locations before visiting node i, then nonstrict lower bounds on C_n and T_n, with $n \in N_i$, are given by inequalities 5.a and 5.b.

Traveling Cost and Time from the Base to the Vehicle Stop (i,n). If $Y_{nv} = 1$, then vehicle v will visit node i at time event n to carry out some loading/unloading tasks. Therefore, every vehicle stop is characterized by the node being visited and the event that occurs, i.e., (i,n). Based on the general precedence notion, the accumulated cost ($C_{n'}$) and time ($T_{n'}$) up to the event n' at node i' should be greater than the corresponding values up to a preceding stop (n,i) on the same route ($Y_{nv} = Y_{n'v} = 1$) whenever $X_{nn'} = 1$. As indicated by eqs 6.a and 6.c, the difference ($C_{n'} - C_n$) is bounded by the routing cost along the arc (i,i'), while ($T_{n'} - T_n$) must never be lower than the sum of the travel time along the route segment (i, i') plus the time required to perform pickup/delivery operations at the stop (n,i). If $Y_{nv} = Y_{n'v} = 1$ and $X_{nn'} = 0$, the reverse sequencing condition holds as shown by eqs 6.b and 6.d.

$$C_{n'} \geq C_n + dc_v d_{ii'} - M_C(1 - X_{nn'}) - M_C(2 - Y_{nv} - Y_{n'v}) \quad \forall n \in N_i, n' \in N_{i'}, i, i' \in I, v \in V_{ii'}: i < i' \tag{6.a}$$

$$C_n \geq C_{n'} + dc_v d_{i'i} - M_C X_{nn'} - M_C(2 - Y_{nv} - Y_{n'v}) \quad \forall n \in N_i, n' \in N_{i'}, i, i' \in I, v \in V_{ii'}: i < i' \tag{6.b}$$

$$T_{n'} \geq T_n + ft_i + \sum_{p\in P_i} vt_{ip}(L_{npv} + U_{npv}) + \frac{d_{ii'}}{sp_v} - M_T(1 - X_{nn'}) - M_T(2 - Y_{nv} - Y_{n'v}) \quad \forall n \in N_i, n' \in N_{i'}, i, i' \in I, v \in V_{ii'}: i < i' \tag{6.c}$$

$$T_n \geq T_{n'} + ft_{i'} + \sum_{p\in P_{i'}} vt_{i'p}(L_{n'pv} + U_{n'pv}) + \frac{d_{i'i}}{sp_v} - M_T X_{nn'} - M_T(2 - Y_{nv} - Y_{n'v}) \quad \forall n \in N_i, n' \in N_{i'}, i, i' \in I, v \in V_{ii'}: i < i' \tag{6.d}$$

ft_i and vt_{ip} define fixed and variable components of the stop time at node i, respectively. The variable stop time will depend on the load to pick up and/or deliver to such a location. Though eqs 6.a and 6.b assume the same variable stop time for pickup and delivery operations, different values can be easily handled by simply including different coefficients for L_{npv} and U_{npv}, respectively. The time required for traveling from node i to i' is computed based on the distance $d_{ii'}$ and the average speed of vehicle v, given by the parameter sp_v.

On the other hand, arrangement of multiple events predefined for node i and included in the event set N_i is used to define the sequencing constraints in the following simpler form:

$$C_{n'} \geq C_n - M_C(2 - Y_{nv} - Y_{n'v}) \quad \forall n \in N_i, n' \in N_i, i \in I, v \in V_i: n < n' \tag{7.a}$$

$$T_{n'} \geq T_n - M_T(2 - Y_{nv} - Y_{n'v}) \quad \forall n \in N_i, n' \in N_i, i \in I, v \in V_i: n < n' \quad (7.b)$$

It is worth remarking that, without compromising the global optimality, eqs 7.a and 7.b are very useful to reduce the computational effort when multiple vehicle stops can be performed at the same node i.

Upper Bound on the Routing Cost and Time for the Whole Tour Assigned to Vehicle v. Constraint 8.a states that the overall traveling cost for vehicle v (CV_v) must always be greater than the routing expenses up to any stop (i,n) by at least the travel cost along the arc $i-l$ connecting the node i with the vehicle base l. In a similar way, eq 8.b computes the total time required by vehicle v (TV_v) by adding both the stop time at the visited node i and the travel time along the edge (i,l) to the starting service time T_n. The largest right-hand side of constraint 8.a will set the strictest bound on the value of TV_v. Since the last visited node is unknown beforehand, eqs 8.a and 8.b must be written for every event n at node $i \in I$. Parameters M_C and M_T represent upper bounds for variables CV_n and TV_n, respectively.

$$CV_v \geq C_n + \sum_{l \in B_v} dc_v d_{il} W_{lv} - M_C(1 - Y_{nv}) \quad \forall n \in N_i, i \in I, v \in V_i \quad (8.a)$$

$$TV_v \geq T_n + ft_i + \sum_{p \in P_i} vt_{ip}(L_{npv} + U_{npv}) + \sum_{l \in B_v} \frac{d_{il}}{sp_v} W_{lv} - M_T(1 - Y_{nv}) \quad \forall n \in N_i, i \in I, v \in V_i \quad (8.b)$$

Time-Window and Maximum Service Time Constraints. The service time for any event $n \in N_i$ at customer node i should be started at a time T_n within the interval [a_i, b_i]. Also, the total routing time TV_v for vehicle $v \in V$ must be smaller than the maximum service time t_v^{max}. These constraints are explicitly stated by eqs 9 and 10, respectively.

$$a_i \leq T_n \leq b_i \qquad n \in N_i, i \in ID \quad (9)$$

$$TV_v \leq t_v^{max} \qquad v \in V \quad (10)$$

Time-window constraints can be softened by introducing a penalty term in the objective function to punish late start of vehicle services ($T_n > b_i$, $i \in N_i$). The penalty to be paid increases with the tardiness level TD_n, where TD_n is a non-negative variable given by $TD_n \geq T_n - b_i$. If there are several product deliveries with different time windows at the same customer site, then a similar number of different nodes associated with the same location are to be defined.

4.2.2. Vehicle Cargo Constraints. Product Availability Constraints. The overall shipment of product p from source node i (L_{npv}) can never exceed the available inventory INV_{ip}.

$$\sum_{v \in V_i} \sum_{n \in N_i} L_{npv} \leq INV_{ip} \qquad \forall i \in (IS_p \cap IM_p), p \in P \quad (11)$$

Product Demand Constraints. As stated by eq 12, the overall requirement of product p at destination i should be satisfied. In order to fulfill large product requirements, multiple vehicle visits represented by multiple events n taking place at node i can be handled.

$$\sum_{v \in V_i} \sum_{n \in N_i} U_{npv} = DEM_{ip} - BL_{ip} \qquad \forall i \in (ID_p \cap IM_p), p \in P \quad (12)$$

Sometimes, a non-negative continuous variable BL_{ip} can also be included in eq 12 to represent the unsatisfied demand of product p at node i. In this case, positive values for this variable should be penalized in the objective function.

Null In-Transit Inventory Constraints. Every unit of product p picked up by a vehicle v must be delivered to some destination before the end of the vehicle trip.

$$\sum_{n \in N_i} \sum_{i \in IS \cup IM} L_{npv} = \sum_{n \in N_i} \sum_{i \in IM \cup ID} U_{npv} \qquad p \in P, v \in V \quad (13)$$

Vehicle Loading/Unloading Operation Constraints. If vehicle v makes a stop during event n at node i, i.e., $Y_{nv} = 1$, the cargo of product p being loaded from node i must not be larger than the available inventory of product p (INV_{ip}). In addition, the cargo of product p unloaded at node i from the vehicle must not exceed the quantity of product p required at node i (DEM_{ip}).

$$L_{npv} \leq INV_{ip} Y_{nv} \qquad \forall n \in N_i, i \in (IS_p \cap IM_p), p \in P, v \in V_i \quad (14.a)$$

$$U_{npv} \leq DEM_{ip} Y_{nv} \qquad \forall n \in N_i, i \in (ID_p \cap IM_p), p \in P, v \in V_i \quad (14.b)$$

Accumulated Amount of Product p Picked up by Vehicle v up to the Stop (i,n). Equations 15.a and 15.b provide a lower bound on the accumulated amount of product p picked up by vehicle v after the event n at node i ($Y_{nv} = 1$) in terms of the accumulated load of p collected up to a prior stop plus the amount loaded at stop (i,n).

$$AL_{n'pv} \geq AL_{npv} + L_{n'pv} - M_L(1 - X_{nn'}) - M_L(2 - Y_{nv} - Y_{n'v}) \quad \forall n \in N_i, n' \in N_{i'}, i, i' \in I, p \in P_{ii'}, v \in V_{ii'}(i,n) < (i',n') \quad (15.a)$$

$$AL_{npv} \geq AL_{n'pv} + L_{npv} - M_L X_{nn'} - M_L(2 - Y_{nv} - Y_{n'v}) \quad \forall n \in N_i, n' \in N_{i'}, i, i' \in I, p \in P_{ii'}, v \in V_{ii'}(i,n) < (i',n') \quad (15.b)$$

Accumulated Amount of Product p Delivered by Vehicle v up to the Stop (i,n). Equations 16.a and 16.b provide a lower bound on the accumulated amount of product p delivered by vehicle v after event n at node i ($Y_{nv} = 1$) in terms of the accumulated quantity of p delivered up to a prior stop plus the amount unloaded during stop (i,n).

$$AU_{n'pv} \geq AU_{npv} + U_{n'pv} - M_L(1 - X_{nn'}) - M_L(2 - Y_{nv} - Y_{n'v}) \quad \forall n \in N_i, n' \in N_{i'}, i, i' \in I, p \in P_{ii'}, v \in V_{ii'}: (i,n) < (i',n') \quad (16.a)$$

$$AU_{npv} \geq AU_{n'pv} + U_{npv} - M_L X_{nn'} - M_L(2 - Y_{nv} - Y_{n'v}) \quad \forall n \in N_i, n' \in N_{i'}, i, i' \in I, p \in P_{ii'}, v \in V_{ii'}: (i,n) < (i',n') \quad (16.b)$$

Maximum Volumetric and Weight Vehicle Capacity Constraints. The total cargo transported by vehicle v immediately after the stop (i,n) can be computed in terms of the variables AL_{npv} and AU_{npv}.

$$\sum_{p\in P} uv_p(AL_{npv} - AU_{npv}) \le vq_v Y_{nv} \quad \forall n \in N_i, i \in I, p \in P, v \in V_i \quad (17.a)$$

$$\sum_{p\in P} uw_p(AL_{npv} - AU_{npv}) \le wq_v Y_{nv} \quad \forall n \in N_i, i \in I, p \in P, v \in V_i \quad (17.b)$$

where uv_p and uw_p define the volume and weight per unit of product p. In turn, vq_v and wq_v denote the maximum volumetric and weight capacity of vehicle v, respectively. Here, it should be noted that products characterized by dissimilar volume and weight per unit can be effectively handled to optimize the vehicle capacity usage.

Lower Bounds on the Cargo Transported by Vehicle v after Stop (i,n). Equation 18.a states that the total quantity of product p delivered by vehicle v from the assigned base to stop (i,n), should never exceed the total amount of p picked up by v on that route segment. In turn, constraints 18.b and 18.c enforce lower bounds on the accumulated amount of product p picked up or delivered by vehicle v up to any stop (n,i). Such lower bounds are strict for the first pickup/delivery operation (n,i) assigned to vehicle v.

$$AL_{npv} \ge AU_{npv} \quad \forall n \in N_i, i \in I, p \in P, v \in V_i \quad (18.a)$$

$$AL_{npv} \ge L_{npv} \quad \forall n \in N_i, i \in I, p \in P, v \in V_i \quad (18.b)$$

$$AU_{npv} \ge U_{npv} \quad \forall n \in N_i, i \in I, p \in P, v \in V_i \quad (18.c)$$

Upper Bounds on the Accumulated Amount of Product p Loaded/Unloaded by Vehicle v after Stop (i,n). Equation 19.a states that the total amount of product p collected by vehicle v after leaving stop (i,n), i.e., AL_{npv}, must never be greater than the total quantity of p loaded in the vehicle along the entire tour. In a similar form, eq 19.b indicates that the total cargo of product p delivered by vehicle v after the service stop (i,n), AU_{nvp}, should never exceed the overall amount of product p unloaded from the vehicle along the whole tour.

$$AL_{npv} \le \sum_{i'\in I} \sum_{n'\in N_{i'}} L_{n'pv} \quad \forall n \in N_i, i \in I, p \in P, v \in V_i \quad (19.a)$$

$$AU_{npv} \le \sum_{i'\in I} \sum_{n'\in N_{i'}} U_{n'pv} \quad \forall n \in N_i, i \in I, p \in P, v \in V_i \quad (19.b)$$

4.2.3. Objective Function. Depending on the relative relevance of the major costs involved in the supply chain distribution problem, alternative objective functions can be used. Some of them used in this work are given below.

(a) The weighted sum of distance-based travel costs and vehicle fixed costs:

$$\min \sum_{v\in V} CV_v + \sum_{v\in V} \sum_{l\in B_v} fc_v W_{lv} \quad (20.a)$$

(b) The weighted sum of distance-based and time-based travel costs plus vehicle fixed costs:

$$\min \sum_{v\in V} CV_v + \sum_{v\in V} \sum_{l\in B_v} fc_v W_{lv} + \sum_{v\in V} utc_v TV_v \quad (20.b)$$

where utc_v is the time-based unit traveling cost for vehicle v (in \$/h).

Table 1. Product Inventories (+) and Demands (−) for Examples I−V

location	P1	P2	P3	P4
		Inventory		
Barcelona (examples I−III)	+1750	+1000	+1000	+500
examples IV and V	+1250	+1000	+1250	+500
Madrid (examples I−III)	+1500	+1500	+1500	+1500
example IV	+1250	+1750	+1250	+1500
example V	+3000	+2000	+2000	+2000
Bilbao (example III)	+1500	+1500	+1500	+1500
examples IV and V	+500	+500	+500	+500
Málaga (example V)	+1500	+1500	+1500	+1500
		Demand		
Barcelona (examples III−IV)	−250	−500	−250	−500
example V	−500	−	−500	−
Girona (all examples)	−120	−	−150	−
Lérida (all examples)	−	−75	−75	−
Tarragona (all examples)	−50	−200	−	−100
Valencia (all examples)	−120	−120	−	−
Zaragoza (all examples)	−200	−	−250	−150
Andorra (all examples)	−800	−	−200	−
Santander (all examples)	−	−150	−100	−50
Bilbao (examples I and II)	−120	−	−120	−120
example IV	−250	−500	−250	−500
example V	−500	−	−500	−
Valladolid (all examples)	−50	−150	−	−200
Teruel (all examples)	−200	−100	−	−
San Sebastián (all examples)	−100	−50	−	−
Soria (all examples)	−	−200	−50	−100
Burgos (all examples)	−	−100	−150	−
Vic (all examples)	−100	−	−100	−
Perpignan (all examples)	−150	−150	−	−
Lugo (examples II−V)	−	−100	−	−100
La Coruña (examples II−V)	−100	−	−100	−
Málaga (example V)	−800	−	−	−
Badajoz (example V)	−	−220	−430	−
Sevilla (example V)	−	−	−200	−450
Granada (example V)	−300	−250	−	−370
Murcia (example V)	−	−380	−200	−
Cádiz (example V)	−	−	−340	−
Córdoba (example V)	−	−420	−	−430

(c) Fixed and variable transportation costs plus the penalties for unsatisfied demands, late services, and overtime journeys:

$$\min \sum_{v\in V} CV_v + \sum_{v\in V} \sum_{l\in B_v} fc_v W_{lv} + \sum_{v\in V} utc_v TV_v + \sum_{v\in V} (co_v OVT_v + \sum_{i\in ID_v} \sum_{n\in N_i} cl_i TD_n) + \sum_{p\in P} \sum_{i\in I} c_{B,i} B_{ip} \quad (20.c)$$

where co_v represents the penalty cost per unit overtime for vehicle v, cl_i is the penalty cost per unit tardiness at destination node i, and $c_{B,i}$ is the penalty cost per unit unsatisfied demand at the receiving node i.

5. Computational Results and Discussion

The proposed MILP formulation has been used to optimally perform the distribution activities commonly arising in the operation of real-world chemical supply chains. Five illustrative examples involving different logistics features were effectively addressed through the proposed approach. They can be regarded as modified versions of a case study first introduced by Méndez et al.[28] and subsequently tackled by Bonfill et al.[23] All the examples involve the management of a chemical supply chain network comprising up to 26 locations with at most four of them behaving like product suppliers (factories and distribution centers) and the remain-

Table 2. Distances between Locations for Examples I−V (in km)

Examples I−IV

	Barcelona	Girona	Lérida	Tarragona	Vic	Valencia	Zaragoza	Perpignan	Andorra	Madrid	Bilbao	Valladolid	San Sebastián	Teruel	Soria	Burgos	Coruña	Lugo	Santander
Barcelona		103	178	101	70	351	311	192	198	640	606	663	620	409	453	583	1043	1020	693
Girona	103		226	194	68	444	375	96	215	705	678	703	618	505	523	581	1091	1018	737
Lérida	178	226		107	158	348	149	316	183	479	454	507	464	319	297	427	865	864	537
Tarragona	101	194	107		162	260	240	283	260	560	535	598	555	311	388	518	972	955	628
Vic	70	68	158	162		411	307	158	151	637	610	635	550	473	455	513	1023	950	669
Valencia	351	444	348	260	411		328	535	501	370	607	580	605	167	376	517	961	863	673
Zaragoza	311	375	149	240	307	328		465	322	330	305	367	324	185	157	287	822	724	397
Perpignan	192	96	316	283	158	535	465		163	788	645	793	668	594	613	671	1181	1108	753
Andorra	198	215	183	260	151	501	322	163		625	545	660	505	472	450	580	1018	1017	653
Madrid	640	705	479	560	637	370	330	788	625		395	215	395	302	231	237	609	511	393
Bilbao	606	678	454	535	610	607	305	645	545	395		280	119	462	231	158	644	546	108
Valladolid	663	703	507	598	635	580	367	793	660	215	280		354	441	210	122	455	357	248
San Sebastián	620	618	464	555	550	605	324	668	505	395	119	354		449	268	232	735	637	227
Teruel	409	505	319	311	473	167	185	594	472	302	462	441	449		231	372	896	798	528
Soria	453	523	297	388	455	376	157	613	450	231	231	210	268	231		141	665	567	297
Burgos	583	581	427	518	513	517	287	671	580	237	158	122	232	372	141		535	437	156
Coruña	1043	1091	865	972	1023	961	822	1181	1018	609	644	455	735	896	665	535		98	547
Lugo	1020	1018	864	955	950	863	724	1108	1017	511	546	357	637	798	567	437	98		449
Santander	693	737	537	628	669	673	397	753	653	393	108	248	227	528	297	156	547	449	

Example V

	Madrid	Valencia	Badajoz	Granada	Málaga	Murcia	Sevilla	Cádiz	Córdoba
Madrid		370	401	434	544	401	538	663	400
Valencia	370		716	519	648	241	697	808	545
Badajoz	401	716		438	436	675	217	342	272
Granada	434	519	438		129	278	256	335	166
Málaga	544	648	436	129		407	219	265	187
Murcia	401	241	675	278	407		534	613	444
Sevilla	538	697	217	256	219	534		125	138
Cádiz	663	808	342	335	265	613	125		263
Córdoba	400	545	272	166	187	444	138	263	

Table 3. Vehicle Features

vehicles	operative cost fixed (euros)	operative cost variable (euros/km)	loading/unloading time fixed (h)	loading/unloading time variable (units/h)	capacity weight (kg)	capacity volume (m^3)	average speed (km/h)	maximum routing time (h)
V1 and V2 (all examples)	5000	3	1	250	15 000	25	70	72
V3 and V4 (examples III and IV)	5000	3	1	250	15 000	25	70	72
V3−V6 (example V)	4000	2.5	1	250	10 000	20	70	72

Table 4. Product Characteristics

	P1	P2	P3	P4
weight (kg/unit)	3	6	5	5
volume (m^3/unit)	0.005	0.015	0.010	0.005

ing ones as demanding sites (customer zones). Up to six vehicles (V1−V6) are available to carry out the required distribution activities of four different products (P1, P2, P3, P4) in a cost-effective way. The node−route allocation and the sequence of visits on every vehicle route are problem decisions properly made by the model in order to minimize a cost function involving fixed and variable transportation costs. Based on their geographical vicinity, a partial assignment of demanding points to supply sites has been defined and such node−route preallocations were incorporated in the model as additional problem data. By doing that, the proposed formulation not only allows obtaining a more realistic representation of the real-world problem but also permits reducing the search space and, consequently, the computational effort to reach the optimal solution. Available stocks (+) and given demands (−) of products (P1, P2, P3, and P4) at problem nodes for all five examples are reported in Table 1. In turn, distances between locations and vehicle characteristics are summarized in Tables 2 and 3, respectively, whereas product features are given in Table 4. On every stop, lots of several products can be sequentially loaded on and/or unloaded from the truck. The stop time at each site for accomplishing pickup and/or delivery operations comprises a fixed time of 1 h and a variable time period that directly increases with the total cargo to be loaded/unloaded at a rate of 250 units/h. Though pickup and delivery rates are assumed to be equal, the approach can easily handle nonequal rate values. Therefore, servicing times at source/sink nodes are additional variables to be determined by solving the problem formulation. A modified instance of example IV using different rates for pickup and delivery operations and considering delivery time windows at some destinations has also been solved. Finally, each vehicle tour is bounded by a maximum time horizon $t_v^{max} = 72$ h, i.e., a tour starting at Monday 8:00 a.m. from the allocated vehicle base should be returning to that base before Thursday 8:00 a.m. All the examples were solved to global optimality with a modest computational effort using a 64 bit four-processor (3.0 GHz) Pentium IV PC with GAMS as the modeling language and CPLEX as the MILP solver.

5.1. Example I. In example I, two vehicles (V1 and V2) with similar features are available to move some amounts of four products P1−P4 from a pair of sources to a set of fifteen retailers in order to meet their specified product demands (see Table 1). As shown in Figure 4, customer locations are geographically dispersed over the Iberian Peninsula. Limited stocks (+) of products P1−P4 from a manufacturing plant located in Madrid and a distribution center placed in Barcelona need to be efficiently delivered to the clients in order to minimize the total transportation cost. Cities that can be visited from the vehicle based on either Madrid or Barcelona are listed at the top of Table 5. Through such node preassignments, each supply site satisfies product demands of customer located within its "sphere of influence", i.e., the closer markets. For instance, cities such as Burgos, Soria, San Sebastián, Bilbao, Santander, and Valladolid can be only visited by vehicle V2 transporting products from Madrid while Barcelona's distribution center is responsible for supplying products to the neighboring markets of Girona, Perpignan, Vic, Andorra, Lérida, and Tarragona. Nodes placed on the "borderline" of the suppliers' influence zones, such as Zaragoza, Teruel, and Valencia, are modeled as free allocation decisions. It is worth remarking that such node−route assignment decisions are mainly driven by vehicle capacities and geographical considerations. Because truck capacities are large enough, it was assumed that a single event (i.e., only one vehicle stop) can at most occur at every node. The optimal solution for example I was found in just 14 s of CPU time. The optimal schedule of the required logistics activities is shown in Table 5, while the best vehicle routes are depicted in Figure 4. In addition, Table 6 reports the problem size in terms of binary and continuous variables, and linear constraints. For this particular case, Teruel and Valencia are served from the plant in Madrid through vehicle V1 whereas Zaragoza is allocated to Barcelona and visited by V2. Note that a high fraction of the volumetric vehicle capacity (89.9% for V1 and 90.2% for V2) is occupied by the cargo at the start of the trip, while the routing time for both vehicles (46.2 h for V1 and 52.9 h for V2) is substantially lower than the maximum allowed service time. Further pickup/delivery tasks may be added. No change in the optimal solution occurs when a pair of events is allowed at the supply sites. From Figure 4, one can conclude that example 1 looks like a multidepot multicommodity vehicle routing problem (m-VRP) with finite inventories at the depots.

5.2. Example II. The second example revisits example I, but this time two new demanding locations (Lugo and La Coruña) are incorporated in the list of customers to be served by Madrid. As expected, the addition of two further locations in the influence region of Madrid's factory raises the product demands to be satisfied by vehicle V2, thus producing significant changes on the distribution activities performed by the two vehicles (V1, V2). Because of the maximum tour duration constraint, customers from Teruel and Valencia can no longer

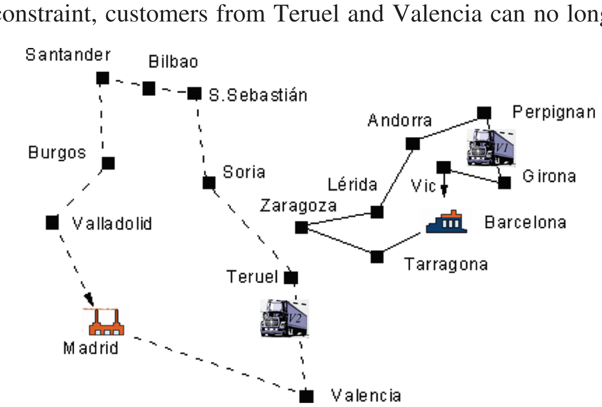

Figure 4. Optimal solution for example I.

Table 5. Optimal Vehicle Routing and Schedule for Example I

Allowed Supplying-Site and Demanding-Site Allocations	
supplying site	demanding sites
Barcelona	Girona, Lérida, Tarragona, Valencia, Zaragoza, Andorra, Teruel, Vic, Perpignan
Madrid	Valencia, Zaragoza, Santander, Bilbao, Valladolid, San Sebastián, Teruel, Soria, Burgos

Detailed Schedule of Vehicle Activities

vehicle	site	arrival time	P1	P2	P3	P4	used capacity wt %	used capacity vol %
V1 (—)	Barcelona	0.0	+1420	+425	+775	+250	79.6	89.9
	Tarragona	13.9	−50	−200		−100	67.2	74.9
	Zaragoza	19.7	−200		−250	−150	49.9	57.9
	Lérida	25.3		−75	−75		44.4	50.4
	Andorra	29.5	−800		−200		21.7	26.4
	Perpignan	36.8	−150	−150			12.7	14.4
	Girona	40.4	−120		−150		5.3	6.0
	Vic	43.4	−100		−100		0.0	0.0
1070 km	Barcelona	46.2						
V2 (− −)	Madrid	0.0	+590	+870	+420	+470	76.3	90.2
	Valencia	15.7	−120	−120			69.1	80.6
	Teruel	20.0	−200	−100			61.1	70.6
	Soria	25.5		−200	−50	−100	48.1	54.6
	San Sebastián	31.8	−100	−50			44.1	49.6
	Bilbao	35.1	−120		−120	−120	33.7	40.0
	Santander	39.0		−150	−100	−50	22.7	26.0
	Burgos	43.5		−100	−150		13.7	14.0
	Valladolid	47.2	−50	−150		−200	0.0	0.0
1756 km	Madrid	52.9						

traveled distance	2826 km
routing cost	8478 euros
total cost	18478 euros
CPU time	14 s

Table 6. Model Sizes

example	binary variables	continuous variables	linear constraints
I	104	353	2547
II	134	406	3282
III	111	445	3172
IV	148	533	4133
V	224	788	6356

be served by Madrid. Their product demands need to be satisfied from Barcelona. As a result, vehicle capacity constraints for V1 become active, forcing such a vehicle to perform two consecutive tours in order to meet the total demand of the assigned locations. In contrast to the solution reported for example I, the new vehicle route departing from Barcelona now includes Teruel and Valencia. Then, the event set N_{BAR} should include two elements to allow an intermediate stop of V1 for replenishing products in Barcelona. Otherwise, the problem has no feasible solution. The solution time for this extended example rises to 64 s, and the new schedule of pickup/delivery tasks is reported in Table 7. In turn, the problem size is given in Table 6. As shown in Figure 5, the route traveled by V1 comprises a pair of tours starting and ending at Barcelona, with the first one including five cities and four more in the second. Example II shows the ability of the proposed formulation to generate vehicle routes with multiple stops at some location such as Barcelona. As a consequence of the double-tour trip, the capacity constraints for V1 become more relaxed at the expense of increasing the total routing time, which is now closer to its maximum value. The time margin up to t_v^{max} drops to just 10.5 h. Moreover, the traveled distance is increased by almost 42%, from 2826 to 3671 km, with regard to example I mainly due to the additional two cities visited by V2.

5.3. Example III. This example deals with the short-term planning of a distribution network that, in addition to meeting specified customer demands, faces the task of restoring inventories at Barcelona's distribution center. To do that, lots of products are dispatched to that site from the factory located in Madrid where they are manufactured. Besides, the distribution network includes a new warehouse open in Bilbao to meet product demands of neighboring cities. In this way, the complex operation of a real-world three-echelon distribution network involving manufacturing plants, distribution centers, and final customers is explicitly considered. To accomplish the required logistics activities in this new scenario, an additional vehicle V3 that begins and ends its trip in Bilbao is assumed to be available. From the problem description, it becomes clear that the Madrid-based vehicle V2 must visit Barcelona's warehouse at some point of its route. Moreover, four cities (Lérida, Zaragoza, Valencia, Teruel) can be served by either Madrid or Barcelona. In this case, those cities finally lie on the optimal route of V2 departing from Madrid. Since pickup and delivery operations should be accomplished by vehicles V1 and V2 at Barcelona, respectively, a pair of events has been associated with that location (i.e., $|N_{BAR}| = 2$). The new optimal solution was found in just 23.8 s. The best vehicle routes and schedules are summarized in Table 8 and illustrated in Figure 6, while the problem size is given in Table 6. By analyzing Figure 6, it is easy to conclude that the installment of a new warehouse at Bilbao largely reduces the average length of the individual tours, since vehicles just visit neighboring markets. For instance, the Barcelona-based vehicle V1 makes a shorter tour with a total length of 33.8 h to satisfy the demands of Vic, Girona, Perpignan, Andorra, and Tarragona. In turn, vehicle V3 departing from Bilbao performs a tour lasting 47.9 h to sequentially visit Santander, Lugo, La Coruña, Valladolid, Burgos, Soria, and San Sebastián. The initial cargo of these vehicles makes partial use of the available

Table 7. Optimal Vehicle Routing and Schedule for Example II

Allowed Supplying-Site and Demanding-Site Allocations	
supplying site	demanding sites
Barcelona	Girona, Lérida, Tarragona, Valencia, Zaragoza, Andorra, Teruel, Vic, Perpignan
Madrid	Valencia, Santander, Bilbao, Valladolid, San Sebastián, Zaragoza, Teruel, Soria, Burgos, Lugo, La Coruña

Detailed Schedule of Vehicle Activities

							used capacity	
vehicle	site	arrival time	P1	P2	P3	P4	wt %	vol %
V1 (−)	Barcelona	0.0	+570	+495	+325	+250	50.4	59.1
	Tarragona	9.0	−50	−200		−100	38.0	44.1
	Valencia	15.1	−120	−120			30.8	34.5
	Teruel	19.5	−200	−100			22.8	24.5
	Zaragoza	24.3	−200		−250	−150	5.5	7.5
	Lérida	29.8		−75	−75		0.0	0.0
	Barcelona	34.0	+1170	+150	+450		44.4	50.4
	Girona	43.5	−120		−150		37.0	42.0
	Perpignan	47.0	−150	−150			28.0	30.0
	Andorra	51.5	−800		−200		5.3	6.0
	Vic	58.7	−100		−100		0.0	0.0
1623 km	Barcelona	61.5						
V2 (− −)	Madrid	0.0	+370	+750	+520	+570	73.7	84.6
	Valladolid	12.9	−50	−150		−200	60.1	70.6
	La Coruña	22.0	−100		−100		54.7	64.6
	Lugo	25.2		−100		−100	47.4	56.6
	Santander	33.4		−150	−100	−50	36.4	42.6
	Bilbao	37.2	−120		−120	−120	26.0	33.0
	San Sebastián	41.3	−100	−50			22.0	28.0
	Burgos	46.2		−100	−150		13.0	16.0
	Soria	50.2		−200	−50	−100	0.0	0.0
2048 km	Madrid	55.9						

traveled distance	3671 km
routing cost	11013 euros
total cost	21013 euros
CPU time	64 s

volumetric capacity, i.e., 65.4% and 75.0%, respectively. On the other hand, the vehicle V2 performs the longest trip with a travel time of 49.7 h and an initial cargo making almost full use of the vehicle capacity. Such a trip, however, is still shorter than the routes assigned to V2 in the two previous examples. The large cargo to be shipped from Madrid comes mainly from the replenishment order destined for Barcelona's DC. In order to reduce the high transportation cost of the trip from Madrid to Barcelona traveled by V2, other cities such as Teruel, Valencia, Zaragoza, and Lérida located along the route are also effectively served by that vehicle. In example III, vehicle V2 is just allowed to perform delivery operations at Barcelona's DC. Then, the continuous variables $L(n,p,\mathrm{V2})\ \forall\ p \in P,\ n \in N_{\mathrm{BAR}}$ have been omitted in the problem formulation. Besides, vehicle V1 can only perform pickup operations at Barcelona, i.e., $U(n,p,\mathrm{V1}) = 0,\ \forall\ p \in P,\ n \in N_{\mathrm{BAR}}$.

5.4. Example IV. The logistics problem previously tackled is now extended to consider a more general three-echelon distribution network where the plant located in Madrid must also supply some amounts of fresh products to Bilbao's warehouse for restoring inventory levels. To achieve such a goal, the vehicle fleet incorporates another vehicle V4 based on Madrid not only to meet those new demands from Bilbao but also to visiting customers located on its route to/from Bilbao. Therefore, two vehicles V2 and V4 are departing from Madrid and those markets in the vicinity of Madrid (i.e., Valladolid, Soria, Burgos, Teruel) can be assigned to either truck. Because those vehicles should stop at Barcelona and Bilbao, respectively, they can also serve some cities in the sphere of influence of the two regional warehouses (see top of Table 9). As a result, there will be a significant increase in node−route assignment variables. Besides, more than a

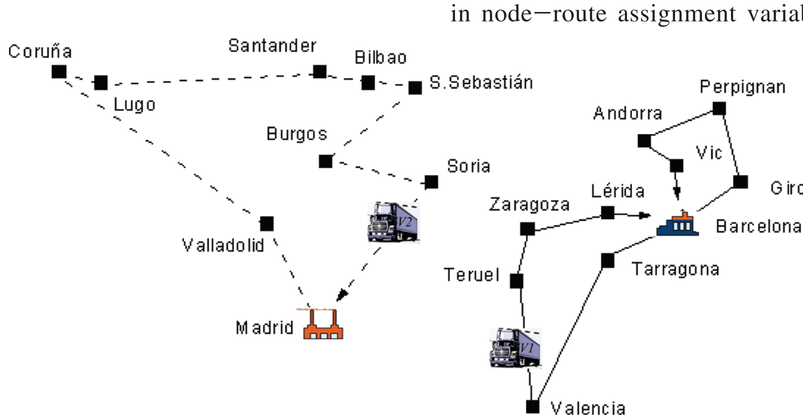

Figure 5. Optimal solution for example II.

Table 8. Optimal Vehicle Routing and Schedule for Example III

Allowed Supplying-Site and Demanding-Site Allocations

supplying site	demanding sites
Barcelona	Girona, Lérida, Tarragona, Valencia, Zaragoza, Andorra, Teruel, Vic, Perpignan
Madrid	Barcelona, Lérida, Valencia, Zaragoza, Teruel, Soria, Tarragona
Bilbao	Santander, Valladolid, San Sebastián, Burgos, Soria, La Coruña, Lugo

Detailed Schedule of Vehicle Activities

							used capacity	
vehicle	site	arrival time	P1	P2	P3	P4	wt %	vol %
V1 (—)	Barcelona	0.0	+1220	+350	+450	+100	56.7	65.4
	Vic	10.5	−100		−100		51.4	59.4
	Girona	13.2	−120		−150		44.0	51.0
	Perpignan	16.7	−150	−150			35.0	39.0
	Andorra	21.2	−800		−200		12.3	15.0
	Tarragona	29.9	−50	−200		−100	0.0	0.0
758 km	Barcelona	33.8						
V2 (− −)	Madrid	0.0	+770	+795	+575	+650	88.0	99.1
	Teruel	16.5	−200	−100			80.0	89.1
	Valencia	21.1	−120	−120			72.8	79.5
	Barcelona	28.0	−250	−500	−250	−500	22.8	24.5
	Lérida	37.6		−75	−75		17.3	17.0
	Zaragoza	41.3	−200		−250	−150	0.0	0.0
1477 km	Madrid	49.4						
V3 (---)	Bilbao	0.0	+250	+750	+400	+450	63.3	75.0
	Santander	10.1		−150	−100	−50	52.3	61.0
	Lugo	15.5		−100		−100	45.0	53.0
	La Coruña	19.9	−100		−100		39.7	47.0
	Valladolid	23.7	−50	−150		−200	26.0	33.0
	Burgos	31.4		−100	−150		17.0	21.0
	Soria	34.6		−200	−50	−100	4.0	5.0
	San Sebastián	44.2	−100	−50			0.0	0.0
1760 km	Bilbao	47.9						

traveled distance	3995 km
routing cost	11985 euros
total cost	26985 euros
CPU time	23.8 s

single vehicle stop should occur at sites such as Madrid, Barcelona, and Bilbao. For this reason, the cardinality of the event sets for such supply sites has been equaled to 2. Otherwise, there is no feasible solution. Moreover, V2 and V4 are only allowed to accomplish delivery operations at Barcelona and Bilbao, respectively. Therefore, continuous variables $L(n,p,\text{V2})\ \forall\ p \in P,\ n \in N_{\text{BAR}}$, and $L(n,p,\text{V4})\ \forall\ p \in P,\ n \in N_{\text{BIL}}$, have been omitted in the problem formulation. Similarly, vehicles V1 and V3 based in Barcelona and Bilbao, respectively, just perform pickup activities at those regional warehouses. Despite the additional problem complexity, the CPU time drops to only 13 s. The optimal distribution schedule is described in Table 9 and illustrated in Figure 7. By analyzing the best vehicle routes, one can identify the following distribution patterns. On one hand, vehicles V1 and V3 based at the regional warehouses of Barcelona and Bilbao, respectively, fulfill product requirements at their neighboring zones. Thus, Bilbao satisfies demands from Burgos, La Coruña, Lugo, and Santander, while Barcelona serves Tarragona, Andorra, Perpignan, Vic, and Girona. On the other hand, vehicles supplying products to warehouses are also used to visit "borderline" nodes. For instance, vehicle V2, in addition to Barcelona, stops at Zaragoza, Lérida, Valencia, and Teruel, whereas V4 not only delivers lots of products at Bilbao but also visits Soria, San Sebastián, and Valladolid. This information can be used to develop efficient heuristic procedures that help making some a priori logistics decisions. In this way, preassignment of locations to vehicles

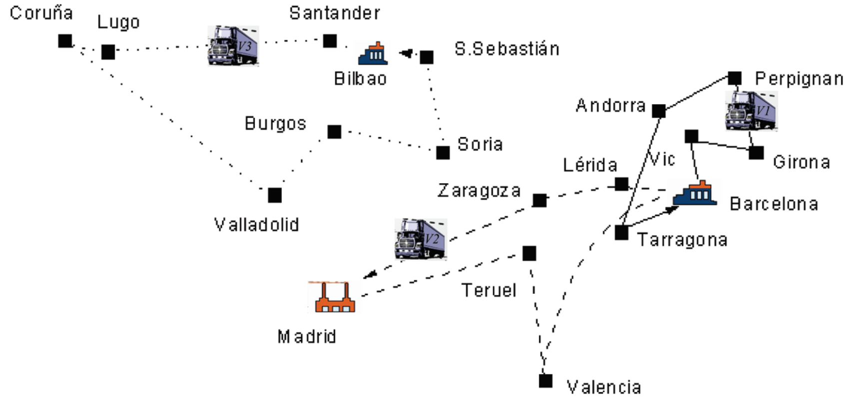

Figure 6. Optimal solution for example III.

Table 9. Optimal Vehicle Routing and Schedule for Example IV

Allowed Supplying-Site and Demanding-Site Allocations

supplying site	demanding sites
Barcelona	Girona, Lérida, Tarragona, Valencia, Zaragoza, Andorra, Teruel, Vic, Perpignan
Madrid	Barcelona, Lérida, Valencia, Zaragoza, Andorra, Teruel, Soria, Burgos, Bilbao, San Sebastián, La Coruña, Valladolid, Lugo
Bilbao	Santander, Valladolid, San Sebastián, Burgos, La Coruña, Lugo

Detailed Schedule of Vehicle Activities

							used capacity	
vehicle	site	arrival time	P1	P2	P3	P4	wt %	vol %
V1 (—)	Barcelona	0.0	+1220	+350	+450	+100	56.7	65.4
	Vic	10.5	−100		−100		51.4	59.4
	Girona	13.2	−120		−150		44.0	51.0
	Perpignan	16.7	−150	−150			35.0	39.0
	Andorra	21.2	−800		−200		12.3	15.0
	Tarragona	29.9	−50	−200		−100	0.0	0.0
758 km	Barcelona	33.8						
V2 (---)	Madrid	0.0	+770	+795	+575	+650	88.0	99.1
	Zaragoza	16.9	−200		−250	−150	70.7	82.1
	Lérida	22.4		−75	−75		65.2	74.6
	Barcelona	26.5	−250	−500	−250	−500	15.2	19.6
	Valencia	38.6	−120	−120			8.0	10.0
	Teruel	42.9	−200	−100			0.0	0.0
1477 km	Madrid	49.4						
V3 (— — —)	Bilbao	0.0	+100	+350	+350	+150	32.7	40.0
	Burgos	7.1		−100	−150		23.7	28.0
	Lugo	15.3		−100		−100	16.3	20.0
	La Coruña	18.5	−100		−100		11.0	14.0
	Santander	28.1		−150	−100	−50	0.0	0.0
1348 km	Bilbao	31.9						
V4 (— —)	Madrid	0.0	+400	+900	+300	+800	80.7	90.0
	Soria	13.9		−200	−50	−100	67.7	74.0
	San Sebastián	20.1	−100	−50			63.7	69.0
	Bilbao	23.4	−250	−500	−250	−500	13.7	14.0
	Valladolid	34.4	−50	−150		−200	0.0	0.0
1113 km	Madrid	40.1						

traveled distance	4696 km
routing cost	14088 euros
total cost	34088 euros
CPU time	13 s

may reduce the search space, thus improving the computational efficiency of the proposed approach.

To show the impact of considering (i) delivery time windows at some customer locations and (ii) different rate values (in unit/h) for pickup and delivery tasks, a modified instance of example IV has also been solved. Prespecified delivery time windows for 10 locations are shown in Table 10, while loading and unloading times per unit of product now take the following values: $(vt_{ip})^{L} = 0.004$ (h/unit) and $(vt_{ip})^{U} = 0.0025$ (h/unit), $\forall i, p$. The new best solution is described in Table 10 and Figure 8. Comparison of Figures 7 and 8 reveals that time window constraints produce several node reorderings on every vehicle trip and some node exchanges between routes. Burgos is now serviced by V4 rather than V3, while San Sebastián moves from the V4 route to the V3 route. Moreover, the optimal transportation cost has increased from 34088 to 35015.

5.5. Example V. This large-size example extends the previous one by considering an additional distribution center

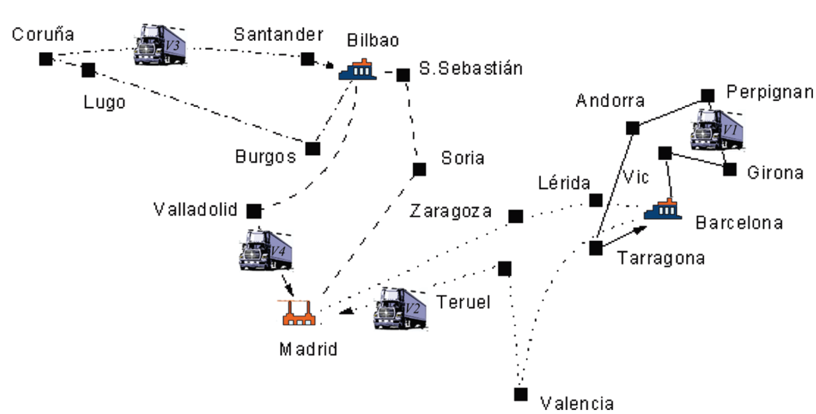

Figure 7. Optimal solution for example IV.

Table 10. Optimal Solution for a Modified Instance of Example IV

Allowed Supplying-Site and Demanding-Site Allocations	
supplying site	demanding sites
Barcelona	Girona, Lérida, Tarragona, Valencia, Zaragoza, Andorra, Teruel, Vic, Perpignan
Madrid	Barcelona, Lérida, Valencia, Zaragoza, Andorra, Teruel, Soria, Burgos, Bilbao, San Sebastián, La Coruña, Valladolid, Lugo
Bilbao	Santander, Valladolid, San Sebastián, Burgos, La Coruña, Lugo

Detailed Schedule of Vehicle Activities							
vehicle	site	arrival time	time windows	P1	P2	P3	P4
V1 (—)	Barcelona	0.0		+1220	+350	+450	+100
	Vic	10.5		−100		−100	
	Andorra	14.1	5−20	−800		−200	
	Perpignan	20.0	5−25	−150	−150		
	Girona	23.1		−120		−150	
	Tarragona	27.5	20−30	−50	−200		−100
775 km	Barcelona	30.9					
V2 (---)	Madrid	0.0		+770	+795	+575	+650
	Teruel	16.5	10−20	−200	−100		
	Valencia	20.6	15−25	−120	−120		
	Zaragoza	26.9	15−28	−200		−250	−150
	Lérida	31.5			−75	−75	
	Barcelona	35.4		−250	−500	−250	−500
1764 km	Madrid	49.3					
V3 (— — —)	Bilbao	0.0		+200	+300	+200	+150
	Santander	5.9	0−20		−150	−100	−50
	La Coruña	15.5		−100		−100	
	Lugo	18.4			−100		−100
	San Sebastián	29.0	25−40	−100	−50		
1509 km	Bilbao	32.1					
V4 (— —)	Madrid	0.0		+300	+950	+450	+800
	Valladolid	14.1	10−20	−50	−150		−200
	Burgos	17.8	10−25		−100	−150	
	Bilbao	21.7		−250	−500	−250	−500
	Soria	29.7			−200	−50	−100
957 km	Madrid	34.9					

traveled distance	5005 km
routing cost	15015 euros
total cost	35015 euros
CPU time	5.65 s

strategically located in Málaga as well as six new demanding zones situated in the south part of the Iberian Peninsula. In contrast to example IV, a heterogeneous vehicle fleet composed of two large trucks (V1 and V2) and four medium-size trucks (V3−V6) is assumed to be available to meet all product requirements, as reported in Table 3. Also, available inventories at Madrid's warehouse have been increased in order to satisfy the product demands of the new customers. An important feature of the proposed approach, properly addressed in this example, is the ability of the model to manage the execution of simultaneous pickup and delivery activities during a vehicle stop at some location, typically a distribution center or warehouse. In this way, it can easily handle real-world operations where a vehicle first replenishes

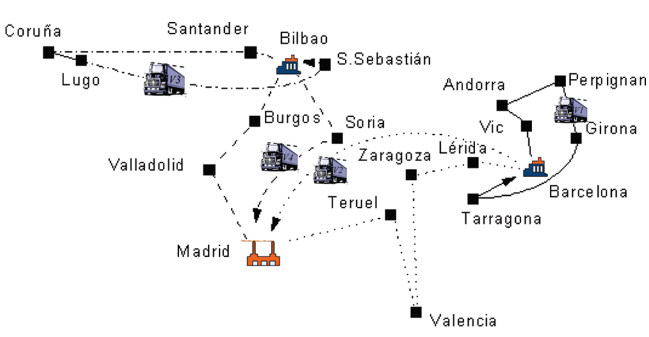

Figure 8. Optimal solution for the modified instance of example IV.

stocks at a regional warehouse by unloading its cargo and then picks up some lots of other products to meet customer demands on the route back to its base. These synchronized activities can be easily observed during the visits of V2 and V6 to the distribution centers located at Barcelona and Málaga, respectively. For instance, as clearly shown in Table 11, the pickup and delivery operations taking place during the visit of V2 to Barcelona accounts for the delivery of 500 units of both P1 and P3, and the subsequent loading of 275 units of P2 that are destined to Lérida and Soria during the return trip. Despite the inherent higher problem complexity and model size (see Table 6), the best solution is found in just 300 s. The new set of optimal routes is presented in Table 11 and illustrated in Figure 9. It is worth noting that the traveled distance rises 1956 km with regard to example IV because of the addition of new customer demands to be served from Madrid and Málaga. Such an increase that impacts directly on the distribution operational cost comes mainly from (i) the new route of vehicle V5 departing from Málaga and serving three of the new customer demands, and (ii) the new trip of V6 aiming to replenishing the stock of P1 in Málaga, serving the demands of Murcia and Granada, and later picking up some amounts of P2 and P3 at Málaga's warehouse to meet demands from Badajoz during the return trip. All these complex logistics activities can be easily handled by the proposed optimization approach.

Table 11. Optimal Vehicle Routing and Schedule for Example V

Allowed Supplying-Site and Demanding-Site Allocations

supplying site	demanding sites
Barcelona	Girona, Lérida, Tarragona, Valencia, Zaragoza, Andorra, Teruel, Vic, Perpignan
Madrid	Barcelona, Lérida, Valencia, Zaragoza, Andorra, Teruel, Soria, Burgos, Bilbao, San Sebastián, La Coruña, Valladolid, Lugo, Badajoz, Murcia, Granada, Málaga
Bilbao	Santander, Valladolid, San Sebastián, Burgos, La Coruña, Lugo
Málaga	Badajoz, Granada, Murcia, Sevilla, Cádiz, Córdoba, Valencia

Detailed Schedule of Vehicle Activities

							used capacity	
vehicle	site	arrival time	P1	P2	P3	P4	wt %	vol %
V1 (—)	Barcelona	0.0	+1220	+350	+450	+100	56.7	65.4
	Tarragona	10.9	−50	−200		−100	44.4	50.4
	Andorra	17.0	−800		−200		21.7	26.4
	Perpignan	24.4	−150	−150			12.7	14.4
	Girona	27.9	−120		−150		5.3	6.0
	Vic	31.0	−100		−100		0.0	0.0
758 km	Barcelona	33.8						
V2 (---)	Madrid	0.0	+1020	+200	+875	+250	66.7	73.6
	Teruel	14.8	−200	−100			58.7	63.6
	Valencia	19.4	−120	−120			51.5	54.0
	Barcelona	26.3	−500	+275	−500		35.8	40.5
	Lérida	35.0		−75	−75		30.3	33.0
	Zaragoza	38.7	−200		−250	−150	13.0	16.0
	Soria	44.3		−200	−50	−100	0.0	0.0
1535 km	Madrid	50.0						
V3 (— — —)	Bilbao	0.0	+100	+250	+200	+150	35.5	35.0
	Santander	5.3		−150	−100	−50	19.0	17.5
	La Coruña	15.4	−100		−100		11.0	10.0
	Lugo	18.6		−100		−100	0.0	0.0
1299 km	Bilbao	28.2						
V4 (— —)	Madrid	0.0	+650	+300	+650	+200	80.0	76.3
	Valladolid	11.3	−50	−150		−200	59.5	58.8
	Burgos	15.6		−100	−150		46.0	43.8
	Bilbao	19.9	−500		−500		6.0	6.3
	San Sebastián	26.6	−100	−50			0.0	0.0
1009 km	Madrid	33.8						
V5 (•••)	Málaga	0.0		+420	+540	+880	96.2	80.5
	Córdoba	11.0		−420		−430	49.5	38.3
	Sevilla	17.4			−200	−450	17.0	17.0
	Cádiz	22.8			−340		0.0	0.0
715 km	Málaga	28.9						
V6 (•• — ••)	Madrid	0.0	+1100	+630	+200	+370	99.3	94.0
	Murcia	15.9		−380	−200		66.5	55.5
	Granada	23.2	−300	−250		−370	24.0	20.0
	Málaga	29.7	−800	+220	+430		34.7	38.0
	Badajoz	42.8		−220	−430		0.0	0.0
1645 km	Madrid	52.1						

traveled distance	6961 km
routing cost	18549 euros
total cost	44549 euros
CPU time	320.0 s

6. Conclusions

An MILP mathematical framework for the vehicle routing problem in supply chain management (VRP-SCM) has been developed. The VRP-SCM problem aims at determining the best short-term operational planning of multiechelon multiproduct transportation networks comprising factories, warehouses, retailers, and end customers. In this way, the VRP-SCM can handle different types of distribution strategies such as direct shipping, shipping via DC or regional warehouses, and hybrid policies. It also accounts for transportation infrastructures with routes interconnecting factories and/or warehouses among themselves, thus allowing milk runs. By consolidating shipments from multiple suppliers to a single destination or from a single source to multiple locations, the so-called milk runs help to lower transportation costs through a better use of truck weight and volume capacities. Besides, several events can occur at any site to make feasible that a location can be visited several times by the same vehicle (i.e., multiple tours per route) or by multiple trucks to accomplish large pickup and/or delivery operations. Therefore, partial deliveries to meet a given customer demand also become a feasible option. In addition, the definition of VRP-SCM assumes customer requests without preassigned suppliers, though the opposite case is easily handled by the approach. Moreover, the amounts of products to pick up at source nodes are no longer fixed data but problem variables. In other words, the product flow pattern throughout the distribution network is a model decision. By solving the VRP-SCM problem, the optimal set of vehicle routes and the corresponding vehicle stop schedules can be discovered in a very detailed manner. The proposed MILP formulation for the VRP-SCM problem has been applied to solve five

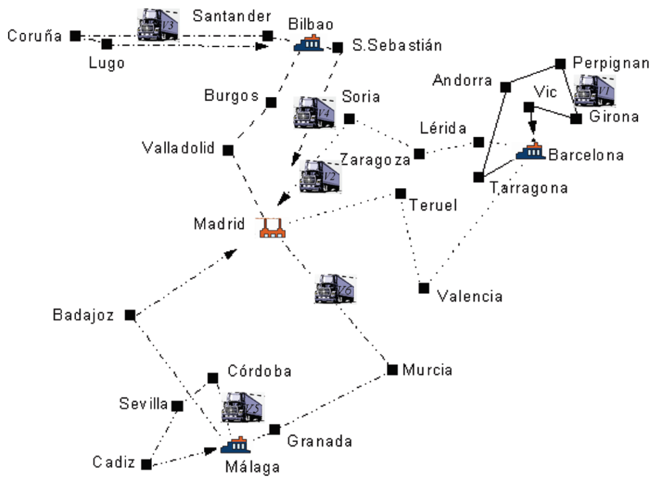

Figure 9. Optimal solution for example V.

illustrative examples dealing with distribution networks comprising up to 26 nodes with at most four of them behaving like product suppliers. Four commodities are to be transported from suppliers to customers by a heterogeneous vehicle fleet with at most six trucks housed in four depots. All the examples were solved to optimality in a reasonable CPU time. The examples permit illustration of different features of the new methodology such as the use of vehicle routes with multiple tours, and the choice of distribution policies combining shipping of products to customers directly from factories and/or via distribution centers.

Acknowledgment

Financial support received from FONCYT-ANPCyT under Grant PICT 2006-01837, from CONICET under Grant PIP-5729, and from Universidad Nacional del Litoral under Grant CAI+ID 003-13 is fully appreciated.

Nomenclature

Subscripts

i, i', j, l = nodes
n, n' = events
p = products
v = vehicles

Sets

A = set of minimum-cost arcs
I = set of nodes (factories, warehouses, distribution centers, customers)
N = set of events
P = set of products
V = set of vehicles
ID = set of destination nodes
IM = set of mixed nodes
IS = set of pure source nodes
B_v = set of operational bases for vehicle v
ID_p = set of destination nodes requiring product p
IS_p = set of pure sources for product p
IM_p = set of mixed sources for product p
N_i = set of events at node i
V_i = set of vehicles that can visit node i

Parameters

DEM_{ip} = amount of product p demanded by node i
INV_{ip} = initial inventory of product p at node i
a_i = earliest service time at node i
b_i = latest service time at node i
$c_{B,i}$ = penalty cost per unit of unsatisfied demand at node i
c_{ij} = travel cost along the arc $i-j$
co_v = penalty cost per unit overtime for vehicle v
cl_i = penalty cost per unit tardiness at destination $i \in ID$
dc_v = unit distance cost for vehicle v
d_{li} = length of the arc (l,i)
fc_v = fixed cost of using vehicle v
ft_i = fixed stop time at node i
sp_v = average travel speed of vehicle v
$t_v^{\max}$ = maximum allowed routing time for vehicle v
t_{ij} = travel time between nodes i and j
uv_p = unit volume for product p
uw_p = unit weight for product p
utc_v = time-based unit cost for vehicle v
vt_{ip} = unit load/unload time for product p at node i
M_C, M_T, M_L = upper bounds for travel cost (C), travel time (T), and load (L)

Binary Variables

W_{lv} = variable denoting that vehicle v is housed in location l
$X_{nn'}$ = variable sequencing the pair of events n and n'
Y_{nv} = variable denoting that vehicle v visits node i at event $n \in N_i$

Continuous Variables

AL_{npv} = total amount of product p loaded on vehicle v after the stop (n,i)
AU_{npv} = total amount of product p unloaded from vehicle v after stop (n,i)
BL_{ip} = unsatisfied demand of product p at node i
C_n = travel cost up to the vehicle stop (n,i), for $n \in N_i$
CV_v = overall traveling cost for vehicle v
L_{npv} = quantity of product p loaded on vehicle v during the stop (n,i) at node i
T_n = travel time up to the vehicle stop (n,i), for $n \in N_i$
TV_v = total routing time for vehicle v
U_{npv} = quantity of product p unloaded from vehicle v during stop (n,i) at node i

Literature Cited

(1) Chopra, S.; Meindl, P. *Supply chain management*; Prentice-Hall: Upper Saddle River, NJ, 2001; ISBN 0-13-026465-2.

(2) Chopra, S. Designing the distribution network in a supply chain. *Transp. Res. Part E: Logistics Transp. Rev.* **2003**, *39* (2), 123–140.

(3) Simchi-Levi, D.; Kaminsky, P.; Simchi-Levi, E. *Managing the supply chain*; McGraw-Hill: New York, 2004; ISBN 0-07-141031-7.

(4) Grossmann, I. E. Enterprise-Wide Optimization: A new frontier in process systems engineering. *AIChE J.* **2005**, *51*, 1846–1857.

(5) Vidal, C. J.; Goetschalckx, M. Strategic production-distribution models: A critical review with emphasis on global supply chain models. *Eur. J. Oper. Res.* **1997**, *98*, 1–18.

(6) Varma, V. A.; Reklaitis, G. V.; Blau, G. E.; Pekny, J. F. Enterprise-wide modeling and optimization—An overview of emerging research challenges and opportunities. *Comput. Chem. Eng.* **2007**, *31*, 692–711.

(7) Papageorgiou, L. G. Supply chain optimisation for the process industries: Advances and opportunities. *Proc. FOCAPO 2008* **2008**, 33–38.

(8) Lashine, S. H.; Fattouh, M.; Issa, A. Location/allocation and routing decisions in supply chain network design. *J. Modell. Manage.* **2006**, *1* (2), 173–183.

(9) Baldacci, B.; Toth, O.; Vigo, D. Recent advances in vehicle routing exact algorithms. *4OR: Q. J. Oper. Res.* **2007**, *5* (4), 269–298.

(10) Van der Bruggen, L.; Gruson, R.; Salomon, M. Reconsidering the distribution structure of gasoline product for a large oil company. *Eur. J. Oper. Res.* **1995**, *81*, 460–473.

(11) Jetlund, A. S.; Karimi, I. A. Improving the logistics of multi-compartment chemical tankers. *Comput. Chem. Eng.* **2004**, *28* (8), 1267–1283.

(12) Fagerholt, K. A computer-based decision support system for vessel fleet scheduling—experience and future research. *Decis. Support Syst.* **2004**, *37* (1), 35–47.

(13) Tarantilis, C. D.; Kiranoudis, C. T. A Flexible adaptive memory-based algorithm for real-life transportation operations. *Eur. J. Oper. Res.* **2007**, *179* (3), 806–822.

(14) Sousa, R.; Shah, N.; Papageorgiou, L. G. Supply chain design and multilevel planning—An industrial case. *Comput. Chem. Eng.* **2008**, *32*, 2643–2663.

(15) Geoffrion, A. M.; Graves, G. W. Multicommodity distribution system design by Benders decomposition. *Manage. Sci.* **1974**, *20*, 822–844.

(16) Sarmiento, A. M.; Nagir, R. A review of integrated analysis of production-distribution systems. *IIE Trans.* **1999**, *31* (11), 1061–1074.

(17) Chandra, P.; Fisher, M. L. Coordination of production and distribution planning. *Eur. J. Oper. Res.* **1994**, *72*, 503–517.

(18) Jayaraman, V.; Pirkul, H. Planning and coordination of production and distribution facilities for multiple commodities. *Eur. J. Oper. Res.* **2001**, *133*, 394–408.

(19) Tsiakis, P.; Shah, N.; Pantelides, C. C. Design of multi-echelon supply chain networks under demand uncertainty. *Ind. Eng. Chem. Res.* **2001**, *40*, 3585–3604.

(20) Jayaraman, V.; Ross, A. A simulated annealing methodology to distribution network design and management. *Eur. J. Oper. Res.* **2003**, *144* (3), 629–645.

(21) You, F.; Grossmann, I. E. Design of responsive supply chains under demand uncertainty. *Comput. Chem. Eng.* **2008**, *32*, 3090–3111.

(22) Verderame, P. M.; Shaik, M. A. ; Floudas, Ch. Operational planning of a multisite production and distribution network. Presented at the 2007 AIChE Annual Meeting; 2007.

(23) Bonfill, A.; Espuña, A.; Puigjaner, L. Decision support framework for coordinated production and transport scheduling in SCM. *Comput. Chem. Eng.* **2008**, *32* (6), 1206–1224.

(24) Amaro, A. C.; Barboa-Povoa, A. P. F. D. Optimal supply chain management with detailed scheduling. *Ind. Eng. Chem. Res.* **2008**, *47* (1), 116–132.

(25) Beamon, B. M. Supply chain design and analysis: models and methods. *Int. J. Prod. Econ.* **1998**, *55*, 281–294.

(26) Shah, N. Process industry supply chains: advances and challenges. *Comput. Chem. Eng.* **2005**, *29*, 1225.

(27) Ropke, S.; Cordeau, S.; Laporte, G. Models and a branch-and-cut algorithm for pickup and delivery problems with time windows. *Networks* **2007**, *49* (4), 258–272.

(28) Méndez, C. A.; Bonfill, A.; Espuña, A.; Puigjaner, L. A rigorous approach to coordinate production and transport scheduling in multi-site system. In *Proceedings of the 16th European symposium on process systems engineering and ninth international symposium on process systems engineering, Garmisch-Partenkirchen, Germany*; Elsevier: New York, 2006; pp 2171−2176.

Received for review May 15, 2009
Revised manuscript received July 28, 2009
Accepted July 31, 2009

IE900792S