Instruments and schemes of in-service mathematics teachers during the design of teaching based on questioning

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ABSTRACT

This work analyzes the instruments and schemes of 55 in service teachers of mathematics, generated during an online university course where they were proposed to study the generating question of a study and research path (SRP) and design teaching from it. The theoretical framework of the research is the theory of conceptual fields (TCF) and the instrumental approach. The situation in which teachers should make a teaching proposal with this resource and what knowledge they would teach is analysed. The objective is to describe the schemes of use and the instruments generated by them, focusing on the components of the schemes and the operational invariants generating the teaching activity when teachers conceive the teaching proposal. The results allow us to understand the difficulties of teaching in the questioning paradigm.

KEYWORDS

Teacher training, mathematics teaching, scheme, instrument

RÉSUMÉ

Ce travail analyse les instruments et les schèmes de 55 professeurs de mathématiques en service, générés au cours d'un cours universitaire en ligne où il *leur a été proposé d'étudier la question génératrice d'un Parcours d'Étude et de Recherche (PER) et de concevoir un enseignement avec elle. Le cadre théorique de la recherche est la théorie des champs conceptuels (TCC) et l'approche instrumentale. Il est analysé la situation dans laquelle les enseignants devaient faire une proposition d'enseignement avec cette ressource et quelles connaissances ils enseigneraient. L'objectif est de décrire les schémas d'utilisation et les instruments générés par eux, en se concentrant sur les composantes des schèmes et sur les invariants opératoires qui engendrent l'activité enseignante lorsque les professeurs conçoivent la proposition d'enseignement. Les résultats permettent de comprendre les difficultés d'enseigner dans le paradigme du questionnement.*

MOTS-CLÉS

Formation des enseignants, enseignement des mathématiques, schème, instrument

Introduction

For more than ten years we have been researching about inquiry-based teaching using Study and Research Paths (SRP) (Chevallard, 2009) in Argentine high school. First, we carried out relatively controlled experiences in which we designed, developed and implemented SRP's. In the more than 25 school implementations that we've carried out, the teachers were researchers from the team that designed these resources, because SRP's were alien resources in Argentinian school (Costa et al., 2014; Donvito et al., 2014; Gazzola & Otero, 2021; Gazzola et al., 2013; Llanos & Otero, 2013, 2015; Otero & Corica, 2012; Otero, et al., 2012, 2014; Parra & Otero, 2017; Parra et al., 2013; Salgado & Otero, 2020). We then investigate in the initial and continuous courses of teacher training, how to teach them the Anthropological Theory of Didactics (ATD) basics and how could they teach using SRP's. The distance between the usual practices of teachers and the type of teaching intended with an SRP, designed to teach in the so-called paradigm of questioning the world (Chevallard, 2013) is very large. Our research aims at teachers develop some typical didactic gestures of teaching in the paradigm of questioning the world, at least during initial and continuing training.

In our first works (Otero & Llanos, 2019) we developed and analyzed in two cohorts of teachers in training at the university level, a codisciplinar SRP in physics and mathematics, where the generative question was *why did Movediza Stone fall down?* This question referred to the movement, the loss of balance and fall of a famous oscillating rock, located in the Argentine city of Tandil. One of the main difficulties identified was the almost non-existent experience with mathematical modelling, physics or both tasks, that faculty students of exact sciences had. This would relate to a conception of modelling more linked to the application idea than a modelling conception directed to the generation of new knowledge. Few papers refer to how teachers use and transform an SRP while they are teaching with them (Matheron, 2008; Wozniak, 2015).

In a case study (Gueudet et al., 2018) we analyzed the documentary work of a teacher of the French high school who used an SRP related to the operation of satellite dishes. This problem refers to the construction of the tangents to a curve from the analytical geometry. Involved some properties of synthetic and analytical geometry, and in addition the reflection of light on different surfaces from geometric and wave optics. Continuing with this research Parra & Otero (2021) identified and classified the operational invariants present in the aforementioned case, showing that, even though this teacher wanted to teach with an SRP on her own initiative, the operational invariants generating its activity were not compatible with didactic gestures typical in the paradigm of world questioning in which this resource is inscribed, such as formulating new questions and answering them, going in and out of the topic, exploring disciplines and delimiting areas of study. In addition, during an online university course on Didactics of Mathematics, we analyzed how 31 in-service teachers investigated the generative question of the SRP about satellite dishes and how they organized a possible teaching with that question (Otero & Llanos, 2019). The teachers' main difficulties related to their need to control the didactic environment that should be built to develop a codisciplinar SRP, to mathematical and physical modelling, and to the fact that the device is alien to the usual teaching. Teachers had to face two different situations: to study, develop and explore the question that originates the SRP and to make a teaching proposal with it. In the teaching situation, teachers prioritized to the knowledge involved in the program they regularly teach in the secondary school. From our experience with different codisciplinars SRP, we concluded that these devices were too different from the resources that teachers use in regular teaching.

In other papers we proposed teachers to use school problems and we analyzed their use schemes under the same conditions mentioned above. The results showed that the subjects, who were in a training course on ATD, where the usual teaching of school mathematics was questioned, already in the situation of study assumed the habitual role of teacher, without generating any mathematical questioning on the problem, although this was perfectly possible. The fact that, as a first step, each resource is linked to a program theme -considered self-evident and transparent- seems to inhibit any study and questioning activity. That is to say, teachers assimilate the resource with the schemes they have available, typical of the usual teaching. This result is consistent with what Pastré et al. (2006) said about the teaching profession. Among other aspects, they note that the schemes explain both contingent activity and resistance to change. In this case, the schemes of use of teachers for this type of resources, originate in a relatively extensive professional experience and very consolidated in the community of teachers. These schemes persist because they are efficient for the job (Gazzola & Otero, 2022). Due to this, in this research, we selected a problem that allows teaching in the paradigm of questioning the world, diverse mathematical knowledge of the curriculum of the Argentine high school and by means of an SRP.

In this work, we propose teachers to study the so-called pastry box problem (Chappaz & Michon, 2003) that allows to generate an SRP and perform relatively simple modelling activities, can also summon outputs of the subject that lead to the study of various mathematical organizations such as polynomial and rational functions in up to two variables, geometric notions linked to static and dynamic rectangles, proportionality, homotheties, the theorem of Thales and Pythagoras and the geometric progressions and series. The theoretical frameworks are the Theory of Conceptual Fields (TCF) (Vergnaud, 1990, 2013) and the Instrument Approach (Rabardel, 1995), which allow to analyze the instrumental genesis of 55 teachers who are asked to teach with the aforementioned SRP. Mainly, we are interested in describing the instruments generated by teachers and the schemes associated with such instruments.

Situation, activity and schemes

The TCF is a pragmatic theory of the conceptualization of the real that, through the notion of schema, allows to analyze the activity of the subject in situation, the form of the activity, what is preserved and what changes in it, the schemes that the subject puts into play, and the pragmatic and epistemic conditions that produce learning, as well as conceptualization and development in a certain domain. Pragmatic means that the subject acts according to the consequences of his actions (Otero, 2021; Pastré, et al., 2006). To study learning of a specific domain, it is necessary to establish in a precise way, a relationship with that portion of the real, which manifests itself in a situation, in "a task". The situation, says Vergnaud (1998, p. 8), has the character of a task and every complex situation can be analyzed as a combination of tasks, about which it is important to know its nature and its obstacles. A situation, in reality, represents a whole class of situations with epistemological specificities. The subjects adapt to the situations they face, but in reality, it is the schemes they use in the situation, which is modified during the adaptation. Thus, a class of situations summons certain schemes, which develop by virtue of the type of situation. The so-called operative and predicative forms of knowledge are developed in permanent interaction and not in opposition (Vergnaud, 1990, 1998, 2013). The operative form allows the subject to act with a certain event in a situation and the predicative form has the function of identifying the objects of the world, recognizing them, enunciating what we do, generating texts and even books on how certain things are done.

Among the four definitions of the scheme proposed by Vergnaud (1990, 2013), we consider two. A scheme is the invariant organization of activity for a certain type of situations. That is to say that the relationship is between the scheme and the situation, without this implying that neither are unique, on the contrary, the scheme is universal (Vergnaud, 2013). It is remarkable that while the observable behavior that the scheme engenders may vary, the organization is invariant. This means that the schemes are not stereotypes. A scheme is necessarily composed of four classes of components: a goal or several, sub-goals and anticipations, the rules of action, capture and control of information, the operative invariants (concepts in action and theorems in action) and the possible inferences. A concept in action is not a concept, nor a theorem in action is a theorem strictu sensu. In science, concepts and theorems are explicit and their relevance and truth can be discussed. This is not necessarily the case for operative invariants. Explicit concepts and theorems are but the visible part of the iceberg of conceptualization; but without the hidden part formed by the operational invariants, this visible part would be nothing. These operational invariants (concepts in action and theorems in action) are in particular the implicit (or explicit) conceptual basis of the schemes because they allow the selection of relevant information and from it and the goal to be addressed, infer the most appropriate rules of action to manage a situation (Vergnaud, 1990). Regarding Vergnaud (2013), the operational invariants (IO) are the most economical methodological way to access the scheme.

The goal is the intentional part of the scheme and its role is indispensable in the organization of the activity. The goal is divided into sub-targets, sequentially and hierarchically ordered, which can give rise to numerous anticipations. The goal is only partially conscious and even the consequences of the action are not always foreseeable for the subject. For Vergnaud (2013, p. 138) this intentional character generates important behavioural differences, particularly in education and work.

The concept of schema is relevant for gestures, reasoning, technical and scientific operations, social and linguistic interactions, affectivity and emotions (Vergnaud, 1990, 2013). All registers of activity are present in the situations of work and continuing training, also in the initial training that may occur in the school.

The instrumental approach

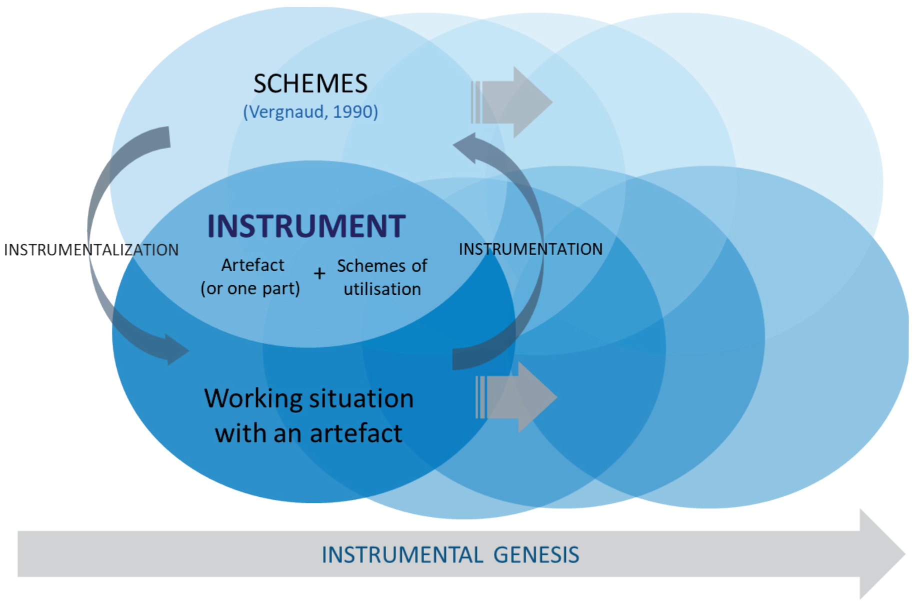
The instrumental approach was proposed by Rabardel (1995) from the Theory of Activity (Vygotsky, 1978) and the TCF (Vergnaud, 1990, 2013). This theory is introduced and developed in the field of cognitive ergonomics and professional didactics. In situations in which people use an artifact, which may or may not be material, an appropriation process takes place, which requires distinguishing between the artifact itself and the instrument that appropriation generates (Figure 1). It is through this process, called by Rabardel (1995) instrumental genesis, that the artifact becomes an instrument for the user. User activity and the situation that promotes it, are decisive. Instruments are generated by the interactions between an artifact and the subject's schemes in a certain situation. The notion of schema refers to the formulation of Vergnaud (1998, 2013), since the schema is not the activity itself, but its invariant organization. Thus, when the activity in a situation is carried out using artifacts, the subject unfolds his repertoire of schematics and organizes an action instrumented by means of a scheme of use of the artifacts in question. An instrument is then a mixed entity, composed of at least one part of the artifact plus a scheme of use of the artifact. Instrumental genesis comprises two interrelated processes (Rabardel, 1995): instrumentation and instrumentalization. Instrumentalization is related to the personalization of the artifact and instrumentation with the appearance of schemes in the subject.

In the instrumentation the limitations and potentialities of an artifact condition the action of the subject who uses it to solve a certain problem. The same artifact can generate different forms of activity organization in different individuals, who will have different assimilation schemes and will construct different operational invariants. Instrumentation is a process directed towards the subject. Instrumentalization, on the other hand, is a process directed towards the artifact, which may be partially included in the instrument, readapted, modified. For example, in the case of the instrumented action of different teachers, it is possible that they act with the same SRP, as shown in the question of satellite dishes. In this case, different instrumentalizations were generated that produced different activities and instruments in the educational situation. It is important to insist on the dialectical character of these processes, which are always unfinished, however much expertise a professional possesses in the use of an instrument, it will always be possible to increase and strengthen it, developing new aspects. Figure 1 shows a diagram about instrumental genesis.

If the user is a teacher, his activity with the artifact and the situation that promotes it are decisive. Throughout their professional lives, teachers produce various instrumental genesis and generate knowledge that allows them to decide and act quickly, face changes in a task and ensure productive and viable results. The notion of scheme (Vergnaud, 1990, 2013) is essential to describe the activity because it explains both the genesis of the action and its dynamism as well as its stability and possible resistance to change.

Every instrumental genesis is a process of appropriation and adaptation of a tool by a subject, in order to create one or more instruments to develop a certain activity. Instrumental genesis is driven both by the changes made to the artifact and by the modifications experienced by the subject's schemes. This makes it possible to consider an SRP as an intangible artifact used by a teacher. It is not a single artifact but rather an organized set of them. In this paper, we analyze the interaction of teachers with an SRP and the schemes they construct.

Figure 1



Representation of Instrumental Genesis

Study and research path (SRP)

The ATD defines SRP (Chevallard, 2009) as didactic devices whose main function is to develop the school study in terms of questions in a strong sense. In the questioning paradigm, to study questions interest more than to study answers, being this the norm. The SRP requires developing a process of study and research extended over time. This process begins with a non-trivial question, called generative, from which new and diverse questions are derived. The journey leads to elaborate a possible answer, which is neither immediate nor arbitrary. A teacher engaged in this type of teaching will need to perform and encourage these gestures, as well as use appropriate resources. A decisive issue studying a question, is that the teacher has to study it, explore it and analyze it previously in depth, focusing on mathematical knowledge, in this case, that must be studied, exploiting the potential of the issue and its mathematical and didactic scope. Questioning knowledge calls for carrying out of didactic gestures such as: studying and researching, asking questions and constructing answers, entering into a topic and leaving it when it is necessary, to make the medium of study with the class group, deconstruct and adapt responses existing in the culture , disseminate them and receive responses from others, analyze and synthesize, all of them are actions quite far from the usual teaching.

Methods

This research involved 55 in-service mathematics teachers who, for four months, took a university course in online mathematics didactics. The participants had different training trajectories carried out in non-university tertiary institutions and worked in different regions of Argentina. Most of them work in secondary education and their teaching experience is dissimilar (between 2 and 36 years). The course addresses the fundamentals of ATD and SRP (Chevallard, 2009). In the last month it was proposed to teachers to study and organize a possible teaching with the SRP "La boîte du pâtissier" (Chappaz & Michon, 2003). Teachers have to face two types of situations: the first is to study the problem in depth and the second is to propose a possible organization of teaching. The teaching proposal takes place in several situations in which the teachers first produce a response in groups and then individually, according to the reality of each. During this process there are numerous interactions between the members of each group and with the whole course, including the responsible teachers. Conditions prevent placement in each teacher's classroom from being part of the course.

The teachers answered all the proposed tasks in written form and uploaded them to the Moodle platform, in this paper the 55 individual teaching proposals were analyzed to describe the instruments generated by the teachers. Analysis and meta-analysis techniques (Gürtler & Huber, 2007) are used to identify in each protocol the components of the schemes associated with each instrument: the target, the sub goals and the operative invariants, as well as the "observable" actions in them, which serve as indicators to infer the categories mentioned. In our analysis we differentiate the schemes by goal and also by some sub goals, which generate different actions and reveal the different underlying operating invariants. The results are summarized in Tables 1, 2.

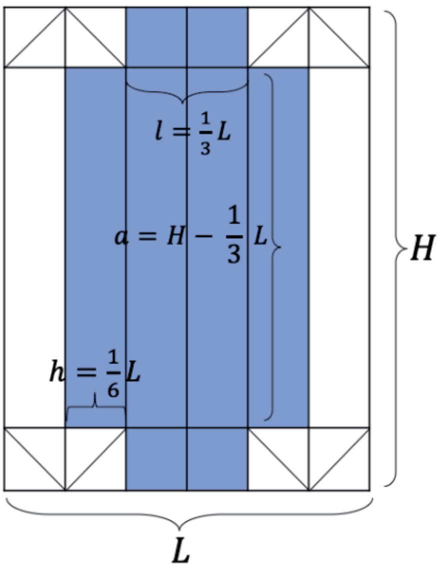
Teachers received the problem as follows:

> Build boxes, following the instructions in the video: https://www.youtube.com/watch?v=gxjpF4bUdDY
> What are height, width and length of the boxes you get if you consider any sheet and how would you calculate its dimensions: V, Sb, total perimeter, etc.?
> How can we make nested boxes with sheets A0, A1, A2, A3, A4, etc.?

The video shows how to make the rectangular box with a sheet of dimensions y, being the dimension where the folds are made. From the unfolded box and making certain geometric considerations, its dimensions are obtained (Figure 2).

The magnitudes depend on the dimensions of the sheet, this leads to study polynomial functions in two variables, which are geometrically surfaces in R^3. The variables are reduced if one or both sides of the sheet are parameterized, either the surface, volume or perimeter of the box (Figure 3), and also if both sides of the sheet are parameterized, for example by constructing the boxes with DIN sheets[1], or more generally, with sheets forming successive similar rectangles. It is important to note that, if *L* is a parameter, all functions will be linear, and this is too restricted, then, the parameter should be *H*.

FIGURE 2



Displayed box

Adopting volume or surface as a parameter, rational equations are obtained in two variables and it is possible to express *H* as a function of *L*, generating families of hyperbolic functions that represent isosurface and isovolume curves:

$$S = \frac{1}{3}L.H - \frac{1}{9}L^2 \text{ then } H(L) = \frac{L}{3} + \frac{3s}{L}, L > 0$$

$$V = \frac{1}{18}L^2H - \frac{1}{54}L^3 \text{ then } H(L) = \frac{L}{3} + \frac{18\,v}{L^2}, L > 0$$

It has been identified that in service teachers have difficulty teaching the characteristics and differences between constants, variables and parameters, and the relative character of the latter two, either when they are carrying out mathematical

1 https://en.wikipedia.org/wiki/ISO_216

modelling activities and they have to interpret the solutions of a differential equation or when they are generalizing a problem (Otero et al., 2017). This difficulty is the product of a widespread and consolidated trans-positive process, which is observed even in university courses in mathematics, because in general the parameters are treated as unique and fixed.

The boxes will be nested if they are constructed with series of sheets represented, represented by similar rectangles, that is to say that their homologous sides must be proportional and the dimensions of the sheets form geometric progressions. If in addition the sides of the sheets retain the ratio $\frac{H}{L} = \lambda \frac{H}{L} = \lambda$, being τ an irrational number, the rectangles are called dynamic. A particular case is that of DIN series sheets where $\frac{H}{L} = \sqrt{2}$. This is a usefulness ratio because it solves the problem of the division or the doubling of similar rectangles, since, by bending the paper by the mediatrix of the major side, two identical sheets of the following format are obtained, preserving the proportion of the sides of their predecessor. Could exist other notable proportions between the sides of the sheets, such as the golden number (aureus), whose discovery is related to the study of the renowned Fibonacci succession. If the constant of proportionality between the sides of the rectangles is a rational number, these are called static. In this case, successive semblable rectangles also generate nested boxes. The way of constructing the DIN sheets also assumes that the ratio between the sides of each sheet is the same as that between the homologous sides of two successive sheets. This involves interesting techniques and geometric properties of rectangles linked to the division of segments into synthetic geometric. To reflect this, in the model proposed in Figure 3, two proportionality constants are used: λ refers to the module of the rectangles, and k is the ratio of similarity between them.

Figure 3 synthesizes possible study paths and mathematical knowledge that could be taught. It is remarkable the importance of carrying out an in-depth study, to identify a wide set of mathematical works belonging to the field of geometry, arithmetic, algebra and analysis, closely related to each other and to the problem posed, to analyze its didactic-mathematical potential.

FIGURE 3

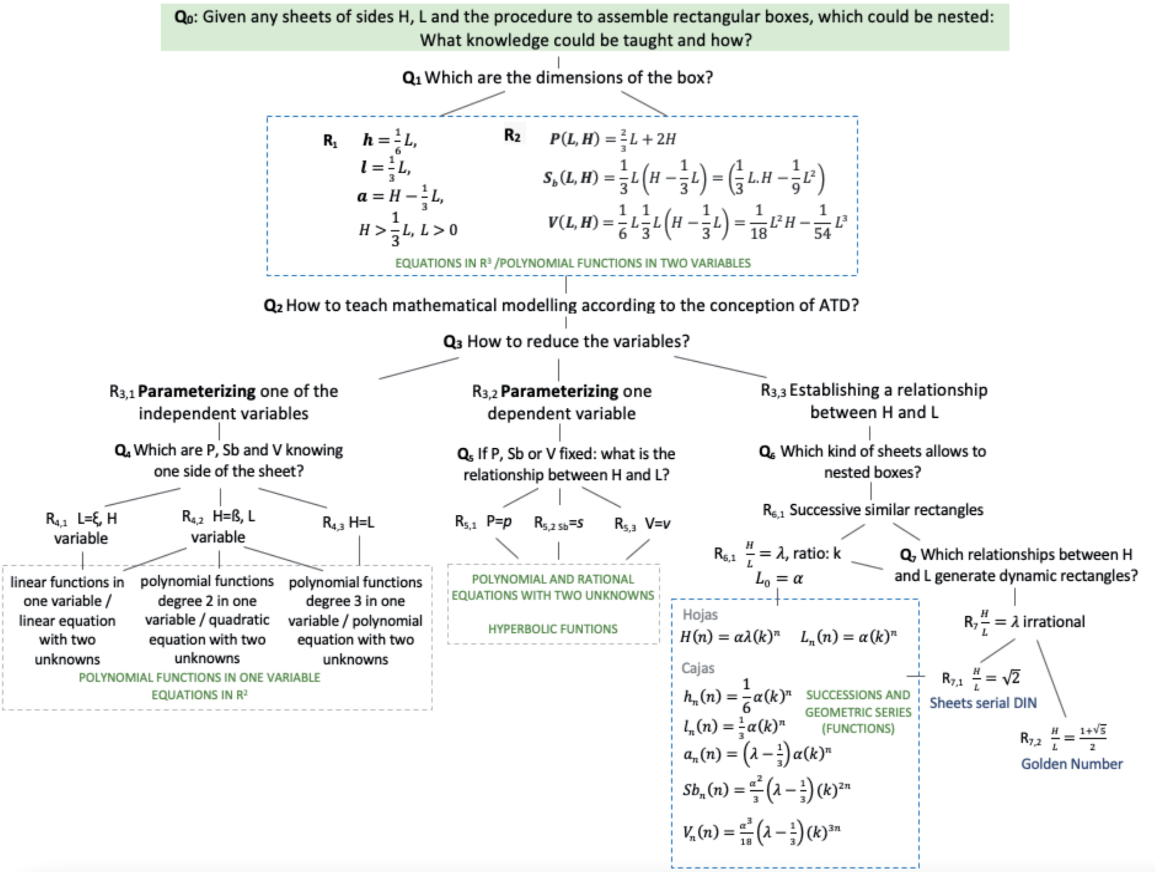


Diagram of possible study paths

DATA ANALYSIS

When teachers have to propose a possible organization of teaching, two goals are identified, from which they are separated into two groups. A part of the teachers (25/55) carries out their teaching proposal on polynomial functions from grade one to three and the rest (30/55) on geometric sequences. This drives modifications to the artifact and different instruments are generated (Tables 1 & 2).

Regarding the goal of teaching polynomial functions from grade one to three (Table 1), several sub-goals are identified, according to how teachers propose that students obtain the formulas from the sides of the box and certain dimensions associated with it. In the scheme (E_{11}) the sub-goal is that students will get the formulas by generalization. To achieve this, teachers assign specific measures to the sides of the sheets, with which the dimensions of the boxes will be calculated and students are expected to write the formulas by direct observation of those calculates dimensions. Alternatively, in the diagrams E_{12} and E_{13} the formulas will be obtained from the geometric relations that are evidenced in the unfolded sheet, when building and disassembling the box. The difference between these two schemes, resides in which of the sides of the sheet the teachers will parameterize. Thus, some of them will fix the side of the sheet that has the six folds (E_{12}) and then only teach affine function, and others, will fix the side of the sheet that does not have them (E_{13}) and teach polynomial functions from degree one to three. The schemes identified are part of three different instruments.

The first instrument is linked to the E_{11} numerical scheme. Teachers instruct the students to look at the video and make several boxes with sheets whose measurements have been set in advance. Students must assemble and fill Tables with sides, they have obtained by numerical calculations. Then the teachers request to obtain from the numbers, generalized two variable formulas. Since the common goal is to teach functions of a variable, teachers make the decision to parameterize one side of the sheet or some dimension, without analyzing the consequences of this action (Figure 3). The characteristic OIs of this scheme are: "*Formulas are obtained by generalisation, knowing both sides of the sheets*"; "*Variables are reduced by setting any dimension (ad hoc)*".

The second instrument is linked to the E_{12} scheme, in which the formulas come from assembling the box, disassembling it and analyzing the folds, taking as a data the side of the sheet having six folds. In this case, any formula obtained will be linear and the affine function will be taught. The distinctive OI of this scheme is "*It is necessary to fix the side of the sheet having six folds*". In this case, the didactic mathematical consequences are not questioning nor considered.

The third instrument is linked to the scheme E_{13}, similar to the previous one excepting that the parameter is the side of the sheet without six folds, so now according to the calculated magnitude, linear, quadratic and cubic polynomial functions could be taught. The distinctive OI of this scheme is "*It is necessary to fix the side of the sheet without six fold*". This case is the one that has a greater mathematic scope, although it cannot be said that these results arise from questioning the mathematical didactic consequences of having fixed the chosen side. This fact is related to the pragmatic nature of the action proposed by the TCF, since the subject, is not always aware of all the sub goals nor the expected consequences of the action.

These last two schemes share actions that lead to OIs: "*To write the formulas, the geometric relations arising from the folds of the sheet, must be analyzed*", "*To study functions must be used GeoGebra*" and "*GeoGebra motives students*". The OIs linked to GeoGebra were also identified in previous work (Parra & Otero, 2021). It is remarkable that using GeoGebra has become widespread in teaching, particularly it has replaced the hands-on graphic representations of the formulas of functions. Also, the simultaneous use of sliders to variate parameters, mostly programmed by the teacher, without consider the control of variables, joined to the analysis of the notable points in an ostensive way, have been added. As a result, this software becomes an instrument that does not fully exploit the enormous potential of this device.

TABLE 1

Schemes related to teaching polynomial functions

Goal 1: Teach polynomial functions up to grade 3 in one variable			
Sub-goals	**Actions**	**OI**	**Fr**
E_{11} Students will get the formulas by arming the boxes with sheets whose sides have been fixed by the teacher. Students will get the formulas if the teacher has set any dimension.	Instruct students to watch the video and assemble the box with sheets of known sides. Instruct students to complete a table with the measurements of the sheet and box. Instruct students to find by means of the numbers of the table, the relationships between the dimensions of the sheets and the box, writing them. (If they couldn't, the teacher will do it). Indicate to the students to formulate the perimeter, surface of the base, volume of the box or some of them. Fix any dimension of sheets or box. Define the function to be taught.	Students have to assemble the boxes with their hands. Formulas are obtained from generalizing, knowing both sides of the sheets. To teach about functions I need your formulas. The formulas should be reduced to one variable. Variables are reduced by setting any dimension (ad hoc). Teacher defines the function.	$\frac{8}{25}$
E_{12} Students will get the formulas from the displayed box. Students will get the formulas if the teacher has set the measure on the side of the sheet having six folds.	Fix the side of the sheet having six folds. Instruct students to watch the video and assemble the box. Ask students to disassemble the box and analyze the folds. Instruct students to form: height, width, length, perimeter, base surface, box volume. (affine). Define the Affine Function. Indicate using GeoGebra to represent the formulas and to study the function from the graph (notable points, growth, decrease, sets of positivity and negativity).	The formulas should be reduced to one variable. It is necessary to fix the side of the sheet having six folds. Students have to assemble the boxes with their hands. To write the formulas, the geometric relations arising from the folds of the sheet, must be analyzed. To teach the functions, their formulas are needed. Teacher defines the function. To study functions must be used GeoGebra GeoGebra motives students.	$\frac{4}{25}$

Table 1

Sub-goals	Actions	OI	Fr
E_{13} Students will get the formulas from the displayed box. Students will get formulas if the teacher sets the measurement on the side of the sheet without six folds.	Fix the side of the sheet without six folds. Instruct students to watch the video and assemble the box. Ask students to disassemble the box and analyze the folds. Ask students to formulate the height, width and length of the box and also: Perimeter of the box (Affine function) and/or Surface of the base of the box (Quadratic function) and/or Volume of the box (Cubic function). Define the function to be taught. Indicate using GeoGebra to represent the formulas and to study the function from the graph (notable points, growth, decrease, sets of positivity and negativity).	The formulas should be reduced to one variable. It is necessary to fix the side of the sheet without six folds. Students have to assemble the boxes with their hands. To write the formulas, the geometric relations arising from the folds of the sheet, must be analyzed. To teach functions, their formulas are needed. Teacher defines the function. To study functions must be used GeoGebra GeoGebra motives students.	$\frac{13}{25}$

The three schemes have in common the OIs: "*Students have to assemble the boxes with their hands*", "*To teach functions, their formulas are needed*", "*The formulas should be reduced to one variable*" and "*Teacher defines the function*". In all three cases and according to the goal, once the formula is reached, teachers define the function from it and focus on analyzing parameters and notable points, as is usual in traditional teaching.

When the objective of the teachers is to teach the geometric sequences (Table 2) they use the DIN A sheets and propose to obtain the formulas of the displayed box. Three sub-goals are identified sharing the intention that students should obtain the formulas of the umpteenth term of successions. The difference between them refers to how teachers propose to find the proportionality ratio between the sides of the sheets.

The instrument linked to the scheme E_{21} is numerical and characterizes a third of the teachers. The proportionality ratio is obtained from the specific measures of the sheets. The characteristic OIs are: "*For students the simplest way to get the relationships between the sides of the sheets is numerically*" and "*The formulas of the sequences are obtained from the numbers*".

The scheme E_{22} is the most frequent. The scheme E_{22} is the most frequent. The teacher informs the students the constant of proportionality between the sides of the DIN sheets or indicates them to search for it on the internet. The distinctive OI is: "*Students must be informed the value of the proportionality constant between the sides of the sheets*".

TABLE 2

Schemes related to teaching geometric sequences

Goal 2: teaching geometric sequences			
Sub - goals	**Actions**	**OI**	**Fr**
E_{21} Students shall obtain numerically the ratio of the sides of the DIN A sheets and formulate the nth term of the successions.	Instruct students to watch the video, assemble the box, disassemble it and analyze the folds. Ask students to formulate: height, width and length of the box, base surface and volume or any of them. Give students the measurements of sheets A (or some successive) and ask them to find the relationship between the sides numerically. Ask students to calculate numerically the magnitudes of the boxes that are built with DIN A sheets and write the sequence of numbers. Indicate students analyze if the values found are successively lower (boxes can be nested). Generalize from the measurements and formulate the successions of the box: height, width, length, base surface, volume. Define geometric succession by the nth term	Students have to assemble the boxes with their hands. To write the formulas of the box, the geometric relations arising from the folds of the sheet, must be analyzed. For students the simplest way to get the relationships between the sides of the sheets is numerically. Boxes nest if they can be placed inside each other. The nth term of each succession must be formulated. The formulas of the sequences are obtained from the numbers. Teacher has to define successions.	$\frac{10}{30}$
E_{22} Students will formulate the nth term of successions, being the constant of proportionality an information.	Instruct students to watch the video, assemble the box, disassemble it and analyze the folds. Ask students to formulate: height, width and length of the box, base surface and volume or any of them. Provide students with information about the proportionality of the sides of the sheets. Ask students to look online for the proportionality constant between the sides of the sheets. Ask students to formulate the nth term for successions on the sides of boxes and/or other dimensions using that reason. Define geometric succession by the nth term. Ask students to calculate some values of the sequences and/or plot the above formulas analyzing whether the values are successively lower (and thus can be nested).	Students have to assemble the boxes with their hands. To write the formulas of the box, the geometric relations arising from the folds of the sheet, must be analyzed. Students must be informed the value of the proportionality constant between the sides of the sheets. The nth term of each succession must be formulated. Successions must be formulated algebraically. Teacher has to define successions. Boxes nest if they can be placed inside each other.	$\frac{12}{30}$

Table 2

Sub - goals	Actions	OI	Fr
E23 Students will obtain geometrically (Thales) the proportion of DIN A sheets and formulate the nth term of successions.	Instruct students to watch the video, assemble the box, disassemble it and analyze the folds. Ask students to formulate: height, width and length of the box, base surface and volume or any of them. Ask students to find (geometrically) the ratio of proportionality between the sides of the DIN A sheets. Ask students to formulate the nth term of some or all possible successions related to the DIN A sheets and the box. Define geometric succession by the nth term. Ask students to find the ratio between all sides and dimensions of the successive boxes.	Students have to assemble the boxes with their hands. To write the formulas of the box, the geometric relations arising from the folds of the sheet, must be analyzed. The ratio of proportionality between the sides of the sheets DIN A should be obtained from the geometric relations. The nth term of each succession must be formulated. Successions must be formulated algebraically. Teacher has to define successions. The boxes are nested if all their sides and dimensions are proportional.	$\frac{8}{30}$

The scheme E_{23} is the lowest frequency and corresponds to the most mathematically evolved instrument. The teacher must propose students obtain and justify geometrically the ratio between the sides of the sheets by means of Thales' theorem. The distinctive OIs are: "*The ratio of proportionality between the sides of the sheets DIN A should be obtained from the geometric relations*" and "*The boxes are nested if all their sides and dimensions are proportional*". This invariant refers appropriately to the nesting condition of the boxes, allowing to increase the scope of the contents to be taught.

The last two schemes share the OI: "*Successions must be formulated algebraically*" and all schemes include the OIs: "*The nth term of each succession must be formulated*" and "*Teacher has to define successions*". Once the sequence has been defined by the formula of the nth term, teaching is proposed to continue in a relatively traditional way, without exploiting the didactic-mathematical potentialities of the problem.

Discussion

When teachers have to use the problem to teach, they generate various instruments. Those who have chosen to teach polynomial functions generate three instruments with their respective schemes, and the others who, will teach geometric sequences, also produce three different instruments. This separation would be related to teachers are making links with the curriculum, driven by an operational invariant, that has been already identified in other works, consisting in associate each problem or task with only one theme of the taught program (Gazzola & Otero, 2022). This is the first modification that teachers have made in the resource: to segment and break down the mathematical knowledge they are going to teach. Although this is an efficacious decision for the teacher, it is not well oriented with respect to the paradigm of questioning the world, but in the opposite direction, because the teachers cut out the knowledge to be studied. For example, those who decided to teach polynomial functions and correctly fixed one of the independent variables ($R_{4,2}$ from Figure 3), did not justify that action nor did they consider studying rational functions and equations ($R_{3,2}$). On the other hand, those who decided to teach sequences ($R_{3,3}$) focused on DIN sheets ($R_{7,1}$) and did not consider static and dynamic rectangles nor their implications. These claims are based on the absence of operational invariants related to knowledge mentioned above.

In the teaching situation, among the 25/55 teachers who will teach polynomial functions, a third of them propose that the mathematical model arise from a kind of generalization of numbers. We attribute this to the fact that teachers anticipated difficulties in using this resource in the classroom and to that mathematical modelling activities are scarce in the usual teaching. In both study and teaching situations, teachers obtained expressions in two variables or proposed that students do it, but quickly leave them aside, because they do not consider these kinds of expressions o be teachable in secondary school. Therefore, when they have to think about teaching, everyone decides that it is necessary to reduce the entire algebraic expressions to a variable. To achieve this, a few fixed the side of the sheet that has the six folds, without noticing the didactic difficulty that this decision generates, because first-degree expressions are obtained for the perimeter, surface and volume. The remaining teachers setted the other side, enabling the treatment of polynomial functions up to grade three. These decisions were made in action and would not have arisen from a previous analysis. Then, the question is why teachers did not avoid in time, the encounter of students with the functions in two variables, rather than putting these functions aside without giving reasons. This could be due to the insufficient analysis prior to the conception of the teaching, since the control of the variables and the existing possibilities (Figure 3) were not considered.

In all the instruments generated in the teaching situation, the problem is used only at the beginning, and then it is continued in a traditional way. The notion of function is omnipresent in the knowledge taught in secondary school, where classically only polynomial functions of first and second degree in one variable are studied. First an encounter with the "definition" in polynomial form and with the associated parameters is provoked. Graphical representations are made without consider function families, while parameters are interpreted in terms of visual characteristics of the Cartesian graph. In the school, from an algebraic point of view, equations in R2 are reduced to equations of a variable, equalling to zero the dependent variable of the associated function. Leaving aside the equations of first and second degree in two variables, the potentiality of algebraic calculus and the equivalence relations that justify the equational techniques are hided. Techniques for solving equations are presented as a set of unmotivated and unjustified "per se" rules (for example, it will be talk about "rules of the passage of terms", even in the first university courses).

In this case, it is not strange the teachers' weak questioning about the reduction of variation, nor about what knowledge could be taught if any of them is fixed. None of the 55 professors analyzed that, by parameterizing the perimeter, volume or surface of the base, it would have been possible to study even the hyperbolic functions of the third degree. In other words, regarding to the analysis summarized in Figure 3, the teachers followed the traditional path, without questioning knowledge to teach.

It is important to note here that we do not attribute this action to a limitation of the mathematical knowledge of teachers, but that, in the teaching situation, the operational invariants linked to the productive aspects of the activity prevail over the constructive ones (Pastré et al., 2006) that assure a relative success in the management of the teaching. Thus, even within the framework of voluntary vocational training where it is proposed to conceive teaching from a strong question, being this a novelty for teachers, the relationship between their schemes and the situation, leads them to link the question directly with a certain mathematical knowledge of the taught school program. This is also because, in regular professional practice, it is neither necessary, nor frequent, nor effective, to question knowledge profoundly. The this generates uncertainty and makes teachers act as if mathematical knowledge were transparent and unquestionable. This is contrary to the type of activity required to develop an SRP, where it is required to enable gestures proper to questioning, which expand the scope of knowledge.

Regarding to nested boxes problem, teachers only achieved to relate the problem to geometric sequences, after various interactions with their colleagues and with the course teachers. This is why only in the latest version of the teaching proposal (30/55) teachers refer to geometric progressions, that is to say that initially, the successions were not on their "radar". This is an auspicious fact, that happens in the teaching situation: more than half of them are interested in teaching geometric sequences. It is important to note here that this knowledge belongs to the official curriculum of secondary education, but it is not usually taught. However, it must also be said that only eight of the thirty professors who dealt with the nesting of boxes, took into account that it was necessary to leave the theme geometric sequences, to enter into the study of the proportionality characterizing the sides of the DIN A sheets looking for mathematical reasons, and then come back to the former question. In addition, they only considered the DIN A sheets and not the broader mathematical organization of the dynamic rectangles. This is also attributed to an insufficient study and research prior to the conception of teaching.

Conclusion

In this research we analyze the use schemes of 55 mathematics teachers in service during instrumental genesis with a study and research tour. After several reformulated proposals based on various interactions, teachers generate instruments with their respective use schemes, whose description allows them to understand the difficulties of teaching in the questioning paradigm. The device summons knowledge of the program that is usually taught and others, which are not usually taught, the latter being those that enable some gestures of questioning, as happened with the geometric sequences. The a-priori didactic - mathematical analysis carried out by teachers is weak and insufficient to develop teaching from questions, where the questioning of knowledge is indispensable. It should also be noted that this type of teaching is very complex and that the study of a question is divergent while the time available is limited. The time and interactions that allow teachers to evolve the instruments to develop teaching based on questioning, during the course and outside it, should be increased.

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