# The role of Absorbing Markov Chains in childhood cancer

David H. Margarit<sup>a,b</sup>, Marcela V. Reale<sup>a,c</sup>, Ariel F. Scagliotti<sup>b,d</sup>, Lilia M. Romanelli<sup>a,b</sup>

<sup>a</sup>Instituto de Ciencias (ICI), Universidad Nacional de General Sarmiento (UNGS), J. M. Gutiérrez 1150, Los Polvorines, Malvinas Argentinas, B1613, Buenos Aires, Argentina <sup>b</sup>Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Godoy Cruz 2290, Ciudad Autónoma de Buenos Aires, C1425, Argentina

<sup>c</sup>Departamento de Ingeniería e Investigaciones Tecnológicas, Universidad Nacional de La Matanza (UNLaM), Florencio Varela 1903, San Justo, La Matanza, B1754, Buenos Aires, Argentina

<sup>d</sup>Facultad Regional Mendoza (FRM), Universidad Tecnológica Nacional (UTN), Coronel Rodríguez 273, Ciudad de Mendoza, M5500, Mendoza, Argentina

### Abstract

Absorbing Markov Chains are an important mathematical tool used for different applications in science. On the other hand, cancer and its metastases in children have a significant impact on health due to their degree of lethality. Therefore, the aim of this work is to model the metastatic pathways of the main childhood cancers worldwide. The probabilities of generating metastases, from a primary site to secondary and tertiary sites, were characterized by constructing a directed graph and the associated transition matrix. In addition, the time of absorption and the probabilities of absorption by each absorbing state were calculated.

Keywords: Absorbing Markov Chains, Metastasis, Mathematical Biology.

#### 1. Introduction

Cancer is the deadliest disease in the world according to the World Health Organization (WHO), one in six deaths is due to this disease that afflicts people at any age (children, adults and the elderly)[1]. The determining factors are very varied[2]. In general, cancer is the uncontrolled replication of cells of a tissue, altering its intrinsic properties and damaging it, even causing death[3]. Cancer can not only establish itself in the organ where it

Preprint submitted to Physica A

December 12, 2022

originates, but it can spread and colonize other tissues or organs, generating a new one there. This phenomenon is known as metastasis[3]. Although it is estimated from which organ to which other metastasis can be generated, it is not known what these routes will be like when there is subsequent cancer (metastasis of metastases) or when it will end up in a tissue where it will not spread further[3]. In children, childhood cancer seriously affects their health, causing premature deaths and serious side effects in those who manage to recover[4]. It cannot be ignored, and it is part of the motivation of this work, that in paediatric cancer the most common cancers are not the same as in adults[5, 6], as well as the sites of metastasis or the frequency with which they occur.

From a mathematical point of view, Markov Chains promote a tool that is used in different fields due to its potential for predictability based on probabilities that are determined by linear algebra. Markov Chains are used not only in mathematics but also in other science [7, 8, 9]. In this sense, it is possible to extrapolate and use them in the biological systems, in particular, in cancer and metastasis

In previous work, we used Markov Chains for statistics of Argentina[10]. Now, having into account the importance of modelling mathematically routes the metastasis in childhood cancer, we analyse metastatic pathways in the main haematological and solid cancers for children aged 0-14 years. The data used here was obtained from the International Agency for Research on Cancer (IARC)[11], belonging to WHO.

In this way, by using Markov Chains, we will analyse the different steps between the metastases from one organ (primary site) to others, whether they are secondary or tertiary sites. Those organs with a low probability of generating metastases are called absorbing states, therefore, to contemplate this phenomena this work is developed through Absorbing Markov Chains. Thus, we will probabilistically analyse how organs are related to childhood metastases and the role of absorbing states through current information.

#### 2. Methodology

There are three general ways to spread cancer cells and generate new cancer in another organ: through the blood circulation, lymph nodes and transcoelomic tissues[12]. Blood vessels are the primary route to spread to distant organs. Meanwhile, lymphatic vessels provide a route to local lymph

nodes, and frequently after metastasis, they spread through the blood. Although the spreading through the blood seems to be quite common, the lymphatic spread appears to depend on the location of the primary tumour. For example, cancer of bone and soft tissue (sarcomas) firstly spread through the blood, while kidney cancer spreads through the lymphatic system. Transcoelomic spread is rare and appears to be restricted to mesotheliomas and ovarian carcinomas[13].

As the current WHO-IARC statistics, those cancers that commonly affect children are related to the circulatory and lymphatic systems (leukaemia, Hodgkin's lymphoma, non-Hodgkin's lymphoma)[11]. Subsequently, solid tumours are the ones that continuous the list (brain, kidney, liver, ovary, testicular, thyroid, nasopharynx, lip and oral cavity, and bone)[11]. However, bone, liver, and brain are organs that, when they are primary sites, do not generate metastases in other organs (or at least there is a very low probability of this happening)[11, 12], these are called absorbing states or sites. It is worth clarifying that we will not distinguish between cancer subtypes with the intention of being clearer and more global in the analysis. Fig. 1 shows the directed graph and how cancer can migrate from a certain primary cancer to another organ generating a secondary one.

Then, from the most reported cancers in children by WHO-IARC statistics[11], we found that these primary sites can generate metastasis in the following sites:

- Leukaemia: ovaries, testicles and brain. [14, 15, 16]
- Hodgkin lymphoma: testicles and brain. [15, 17, 18]
- Non-Hodgkin lymphoma: testicles and brain. [15, 17, 18]
- Kidney: brain, liver and bones. [5, 19, 20]
- Ovaries: liver and bones. [21, 22]
- Testicles: brain, liver and bones. [5, 23, 24]
- Thyroids: brain, liver and bones. [25, 26, 27]
- Nasopharynx: lip/oral cavity and liver.[28, 29]
- Lip/oral cavity: nasopharynx and bones.[30, 31]

- Brain: usually does not metastasize to another organ.
- Liver: usually does not metastasize to another organ.
- Bones: usually does not metastasize to another organ.

In general, brain, liver and bone cancer do not spread to other organs. Therefore, if they generate metastasis is only on themself.[32, 33]

For simplicity, and considering the name of the organs, cancers will be referred to by a symbol as depicted in Table 1.

Organ	Name	Type
Leukaemia	Le	Haematological
Hodgkin lymphoma	Н	Lymphatic
Non-Hodgkin lymphoma	NH	Lymphatic
Kidney	K	Solid
Ovary	Ο	Solid
Testicular	Te	Solid
Nasopharynx	N	Solid
Thyroid	Th	Solid
Lip, oral cavity	LO	Solid
Brain	Br	Solid
Liver	Li	Solid
Bone	Bo	Solid

Table 1: Symbols for organs depending on their type

In the cancers of Table 1, it does not appear the lung. Even though the lung is known to be the organ where the majority of organs can develop metastasis, this is not the case for paediatric cancer[34, 35, 36].

#### 3. Construction of the transition matrix and absorbing states

Let  $X_0$  (primary site) be the organ where cancer originated, and  $X_1$  is the state of the process where a new cancer is formed coming from a primary site. We consider it as a step of the Markov matrix when a new tumour has already been found in another organ. Likewise, the same concept applies to the second step matrix (this will be discussed later). In terms of the Markov Chain formalism[37], the above implies that the transition matrix has been



Figure 1: Graph of the routes of metastasis of the most common childhood cancers. Nodes in green: from these organs, metastasis is generated mainly in 2 organs; in red: these organs generate metastasis mainly in 3 organs; finally, in black, the absorbing states, these generally do not metastasise.

built under the assumption of the existence of the tumour in  $X_0$  that will develop metastasis in  $X_1$ . Then, the probability of an organ (i) develops metastasis into another (j) is:

$$p_{ij} = P\left[X_1 = j | X_0 = i\right]$$
(1)

where i, j = 1, 2, 3, ..., m refer to the m = 12 organs taken into account in Table 1.

In other words,  $X_1$  (metastasis from the primary site or a secondary site) will have a probability of being metastasis from some primary site called  $X_0$ . If the tertiary site (metastasis from the secondary site) exists, it will be labelled as  $X_2$ .

Generalizing these probabilities  $p_{ij}$  for all the *m* cancers, we construct the called transition matrix called *P*.

$$P = [p_{ij}], \ P \in \mathbb{R}^{m \times m} \text{ with } p_{ij} \ge 0$$
(2)

Since there is no general agreement on the probability of metastasis in children, because this differs in the reported cases, we will consider that the sites of metastasis are equiprobable. That is, if an organ *i* has  $\rho$  possible metastases, the probability  $p_{ij}$  will be  $\frac{1}{\rho}$ .

In terms of the current statistics and data, we built the matrix transition P as follows:

A way to extract information from P: in Fig. 2a and 2b there, are two examples that describe the probabilities of generating metastasis in determinate organs from specific primary sites (thyroids and lip and oral cavity).



Figure 2: Probabilities of thyroids and lip and oral cavity of generating metastasis given by P matrix.

We can observe in the P matrix the existence of Absorbing States in the system. These are states where once they are reached it is impossible to leave[37]. In terms of probability:

$$p_{ii} = 1 \text{ and } p_{ij} = 0 \tag{3}$$

In our system, we can see that in the P matrix there are three absorbing states (the organs brain, liver and bone). In these particular organs, when cancer is originated there, the probability of metastases in another organ is virtually null. However, if the cancer forms elsewhere (first or second), it can metastasise to one of these sites, but no new metastases will form from there. Therefore, it can be considered an absorbing matrix.

#### 4. Transition matrix for tertiary sites

A quite important application of Markov Chains is to analyse the probability when a process goes from state i, through an intermediate state k, to state j in two steps[37]. That is:

$$p_{ij}^{(2)} = P\left[X_2 = j | X_0 = i\right] = \sum_{k=1}^m p_{ik} \cdot p_{kj}$$
(4)

In the context of our models, we can estimate quantitatively the probability of metastasis being generated in a tertiary site, that is, a metastasis of

metastases. Generalizing this for all organs, based on Eq. 4, the matrix of probabilities of metastases from metastases (cancer to the tertiary site from the primary site) is given by:

$$P \cdot P = P^{(2)} = \left[ p_{ij}^{(2)} \right]$$
 (5)

		Le	H	NH	K	0	Te	Th	N	LO	Br	Li	Bo	
$P^{(2)} =$	Le	0	0	0	0	0	0	0	0	0	$\frac{4}{9}$	$\frac{5}{18}$	$\frac{5}{18}$	
	H	0	0	0	0	0	0	0	0	0	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	
	NH	0	0	0	0	0	0	0	0	0	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{1}{6}$	
	K	0	0	0	0	0	0	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	
	0	0	0	0	0	0	0	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	
	Te	0	0	0	0	0	0	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	
	Th	0	0	0	0	0	0	0	0	0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	
	N	0	0	0	0	0	0	0	$\frac{1}{4}$	0	0	$\frac{1}{2}$	$\frac{1}{4}$	
	LO	0	0	0	0	0	0	0	0	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	
	Br	0	0	0	0	0	0	0	0	0	1	0	0	
	Li	0	0	0	0	0	0	0	0	0	0	1	0	
	Bo	0	0	0	0	0	0	0	0	0	0	0	1	

The transition matrix of the second step  $(P^{(2)})$  shows that, in general, from a primary site there is a higher probability, compared to P, of having reached an absorbing state (liver, liver or brain). That is, for metastases of metastases, an absorbing state always has a greater than zero probability of being reached. To quantify this information, such as the number of steps to complete an absorption state, both in general and for a particular organ, we will develop the following section.

## 5. Absorptive capacity by non-absorbing states and number of expected metastases

In Absorbing Markov Chains, the number of steps before the system is absorbed and the probability of absorption by any absorbing state can be found. In order to calculate this information, each transition matrix must be represented in its canonical form[37], called J. The canonical J matrix is composed by 4 sub-matrices  $(N, A, O \text{ and } I_a)$ . These smaller matrices contain elements of probability that originate a absorbing states and n nonabsorbing states. Thus, there are a + n = m states of the system. For our model, the matrix J is represented by:



- Sub Matrix  $N: N \in \mathbb{R}^{n \times n}$  has the probabilities of going from a nonabsorbing state to another non-absorbing state.
- Sub Matrix A:  $A \in \mathbb{R}^{n \times a}$  has the probabilities of going from a nonabsorbing state to another absorbing state.
- Sub Matrix  $O: O \in \mathbb{R}^{a \times n}$  represents the probabilities of going from an absorbing state to another non-absorbing state (zeros matrix).
- Sub Matrix  $I_a$ :  $I_a \in \mathbb{R}^{a \times a}$  this represents the probabilities of remaining inside of an absorbing state (an identity matrix).

Let be fundamental matrix[37]  $F = (I - N)^{-1}$ , where I is an identity matrix with the same dimensions as N. Starting in a transient state, the expected number of steps (in our case, a step in the metastasis propagation from one organ to another) before being absorbed by an absorbing state is given by:

$$T = F \cdot c \tag{6}$$

where c is a column vector whose entries are ones.

$$T = \begin{bmatrix} Le & 5\\ H & \frac{3}{2}\\ NH & \frac{3}{2}\\ K & 1\\ T & 1\\ Te & 1\\ Th & 1\\ N & 2\\ LO & 2 \end{bmatrix}$$

As can be seen, the expected number of steps T for metastasis generated in a primary site and ending in an absorbing organ is at most (or close to) 2. Haematological or lymphatic cancers have a number of steps between 1 and 2, which implies that more than one metastasis can occur from the secondary site. Although these cancers do not metastasise in adults (except for very isolated events), in children they can develop a branching metastatic pathway. In the cases of the nasopharynx and lip/oral cavity, they are generally absorbed in the second step of metastasis until they reach one of the three absorbing organs. Finally, the ovaries, the testicles and the thyroid will generate a metastasis and will remain in an absorbing state, something that improves the prevention prognosis.

The vector T can be interpreted as a degree of tolerance of the human body. Generally, if the primary sites are not treated, they are known to trigger eventual metastases (in one or more organs), and from there to other sites. Therefore, this information acquired by T is an important tool for early diagnosis and potential predictions about cancer sites and pathways depending on their origin.

On the other hand, the probability of absorption of any non-absorbing state by any absorbing states is depicted by the matrix Z[37]:

$$Z = ((I - N)^{-1}) .A$$
(7)

The Z matrix has a similar meaning as the vector T, except that we observe specifically for each primary site the probability of being absorbed by a certain absorbing organ (states). These results also provide sensitive information for the treatment of childhood cancer. As can be seen from the Z matrix, leukaemia and both lymphomas are more likely to end (if they metastasise) in the brain. In the case of the nasopharynx and lip/oral cavity, they finish their route in the liver or in the bones. Finally, the rest of the organs end up being absorbed by any of the three absorbing states. It is important to highlight the difference between the probabilities of absorption and the routes where cancers generated in solid and non-solid tumours can end up.

#### 6. Conclusions

Through the use of Absorbing Markov Chains, we were able to quantitatively characterize the metastatic pathways of the main childhood cancers.

It is important to highlight that it was found that in no more than two steps the absorption states (liver, brain, and bones) are reached. In addition, the probabilities of knowing specifically in which absorbing organ the pathway end, depending on the primary site, were characterized.

In the previous section, the matrices T and Z provide relevant information for any preventive treatment in children who are going through this disease.

As a future work, different age groups will be considered in order to compare them, taking into account different age ranges, sex or pathologies related to some types of cancer in particular. Additionally, we will work with a model that implements machine learning to automatically optimize the model when available statistics are updated.

#### Acknowledgements

This work is partially financed by PIP - CONICET N° 11220200100439CO. The authors acknowledge funding from European Union's Horizon 2020 MSCA-RISE-2016 under grant agreement N°734439 (INFERNET Project: New algorithms for inference and optimization from large-scale biological data).

#### References

- [1] World Health Organization Cancer. Available at: https://www.who.int/news-room/fact-sheets/detail/cancer. Accessed 17/09/2022.
- [2] Ansari, K. K., & Jha, A. (2022). Causes of cancer in the world: comparative risk assessment of nine behavioural and environmental risk factors. Cureus, 14(9).
- [3] Hiatt, H. H., Watson, J. D., & Winsten, J. A. (1977). Origins of human cancer. Book A. Incidence of cancer in humans; Book B. Mechanisms of carcinogenesis; Book C. Human risk assessment. Cold Spring Harbor Laboratory, Box 100, Cold Spring Harbor, NY 11724, USA.
- [4] Kattner, P., Strobel, H., Khoshnevis, N., Grunert, M., Bartholomae, S., Pruss, M., ... & Debatin, K. M. (2019). Compare and contrast: paediatric cancer versus adult malignancies. Cancer and Metastasis Reviews, 38(4), 673-682.
- [5] Bouffet E, Doumi N, Thiesse P, Mottolese C, Jouvet A, Lacroze M, Carrie C, Frappaz D, Brunat-Mentigny M (1997). Brain metastases in children with solid tumors. *Cancer*, **79(2)**, pp. 403-410.
- [6] Paulson V, Rudzinski E & Hawkins D. (2019). Thyroid Cancer in the Pediatric Population. Genes, 10, 723.
- [7] Zhang, B. (2021). Probability, Markov Chain, and their applications. In Journal of Physics: Conference Series, 1848(1), 012145.
- [8] Ghosh, S., Chatterjee, N. D., & Dinda, S. (2021). Urban ecological security assessment and forecasting using integrated DEMATEL-ANP and CA-Markov models: A case study on Kolkata Metropolitan Area, India. Sustainable Cities and Society, 68, 102773.

- [9] Phelan, T., & Eslami, K. (2022). Applications of markov chain approximation methods to optimal control problems in economics. *Journal of Economic Dynamics and Control*, 104437.
- [10] Margarit, D. H., & Romanelli, L. (2016). A mathematical model of absorbing Markov chains to understand the routes of metastasis. Biomath, 5(1), ID-1607281.
- [11] World Health Organization Agency for Research on Cancer. Cancer Today. Available at: https://gco.iarc.fr/today/online-analysis-table. Accessed 17/09/2022.
- [12] Welch, D. R., & Hurst, D. R. (2019). Defining the hallmarks of metastasis. Cancer research, 79(12), 3011-3027.
- [13] Brabletz T, (2012). To differentiate or not routes towards metastasis, Nature Reviews Cancer, 12, pp. 425-436.
- [14] Whiteley A, Price T, Cantelli G & Sipkins D (2021). Leukaemia: a model metastatic disease. *Cancer*, 21, pp. 461-475.
- [15] Dörücü, D., Şekerci, Ç. A., & Yücel, S. (2021). Testicular and Paratesticular Tumors in Children. Bull Urooncol, 20(4), pp. 200-205.
- [16] Oktem, O., Kim, S. S., Selek, U., Schatmann, G., & Urman, B. (2018). Ovarian and uterine functions in female survivors of childhood cancers. *The Oncologist*, 23(2), pp. 214-224.
- [17] Sidnet Kimmel Cancer Center Centers & Clinics - Pediatric Oncology. Available at: https://www.hopkinsmedicine.org/kimmel\_cancer\_center/cancers \_we\_treat/pediatric\_oncology/ becoming\_our\_patient/cancer\_types/lymphoma.html. Accessed 09/09/2022.
- [18] Allen, C. E., Kelly, K. M., & Bollard, C. M. (2015). Pediatric lymphomas and histiocytic disorders of childhood. *Pediatric Clinics*, 62(1), pp. 139-165.
- [19] PDQ Pediatric Treatment Editorial Board. PDQ Wilms Tumor and Other Childhood Kidney Tumors Treatment. Bethesda, MD: National Cancer Institute. Updated 10/18/2022. Available

at: https://www.cancer.gov/types/kidney/patient/wilms-treatment-pdq. Accessed 09/09/2022. [PMID: 26389390]

- [20] Children's National. Pediatric Wilms Tumor and Kidney Tumors. Available at: https://childrensnational.org/visit/conditionsand-treatments/cancer/wilms-tumor-and-kidney-tumors. Accessed 09/09/2022.
- [21] Sakai, S., Nomura, K., Abe, T., Hayashi, K., Tsutsuno, T., Mizushima, H., ... & Ikawa, Y. (2021). Neuroblastoma with ovarian and pancreatic metastasis. *Journal of Pediatric Surgery Case Reports*, **73**, 101996.
- [22] Min Zhang & Jimei Sun (2013). Bone metastasis from ovarian cancer: Clinical analysis of 26 cases. Saudi Medical Journal, 34 (12), pp. 1270-1273.
- [23] Jarvisa H, Cost N & Saltzmana A (2021). Testicular tumors in the pediatric patient. Seminars in Pediatric Surgery, 30 (4), 151079.
- [24] Nagasawa, M., Johnin, K., Hanada, E., Yoshida, T., Okamoto, K., Okada, Y., ... & Kawauchi, A. (2015). Advanced childhood testicular yolk sac tumor with bone metastasis: a case report. Urology, 85(3), pp. 671-673.
- [25] Francis G, Waguespack S, Bauer A, Angelos P, Benvenga S, Cerutti J, Dinauer C, Hamilton J, Hay I, Luster M, Parisi M, Rachmiel M, Thompson G & Shunichi Yamashita S (2015).Management Guidelines for Children with Thyroid Nodules and Differentiated Thyroid Cancer. *Thyroid* 25 (7), pp. 716-759.
- [26] Pereira, J. O., Egido, L. P., García-Casillas, M. A., Del Cañizo, A., Fanjul, M., de la Torre, M., ... & de Agustín, J. C. (2021). Thyroid surgery in pediatric patients: causes and results. *Cir Pediatr*, 34, pp. 9-14.
- [27] Prpić, M., Franceschi, M., Jukić, T., Kust, D., Dabelić, N., Varjačić, T., ... & Kusić, Z. (2019). Differentiated thyroid cancer in pediatric population (≤ 18 years): postoperative treatment with radioactive iodine (I-131). Acta Clinica Croatica, 58(1), pp. 119–127.

- [28] Vijayasekharan, K., Velu, U., Godkhindi, V. M., & Munisamy, M. (2021). Pediatric Nasopharyngeal Carcinoma. *Cancer Therapy*.
- [29] González-Motta, A., González, G., Bermudéz, Y., Maldonado, M. C., Castañeda, J. M., Lopéz, D., & Cotes-Mestre, M. (2016). Pediatric nasopharyngeal cancer: case report and review of the literature. *Cureus*, 8(2).
- [30] Irani, S. (2016). Distant metastasis from oral cancer: A review and molecular biologic aspects. *Journal of International Society of Preven*tive & Community Dentistry, 6(4), pp. 265-271.
- [31] Becker, M., Stefanelli, S., Rougemont, A. L., Poletti, P. A., & Merlini, L. (2017). Non-odontogenic tumors of the facial bones in children and adolescents: role of multiparametric imaging. Neuroradiology, 59(4), 327-342.
- [32] Stanford Childrens. Brain Tumors in Children. Available at: https://www.stanfordchildrens.org/es/topic/default?id=brain-tumorsin-children-90-P02745. Accessed 09/09/2022.
- [33] Jhon Hopkins Medicine. Brain Tumors in Children. Available at: https://www.hopkinsmedicine.org/health/conditions-anddiseases/brain-tumor/pediatric-brain-tumors. Accessed 09/09/2022.
- [34] Weldon, C. B., & Shamberger, R. C. (2008). Pediatric pulmonary tumors: primary and metastatic. In *Seminars in pediatric surgery*, **17(1)**, pp. 17-29.
- [35] Kayton, M. L., He, M., Zakowski, M. F., Moreira, A. L., Lau, C., Chou, A. J., ... & Ladanyi, M. (2010). Primary lung adenocarcinomas in children and adolescents treated for pediatric malignancies. *Journal of Thoracic Oncology*, 5(11), pp. 1764-1771.
- [36] Ritwik, P., & Chrisentery-Singleton, T. E. (2020). Oral and dental considerations in paediatric cancers. *Cancer and Metastasis Reviews*, **39(1)**, pp. 43-53.
- [37] Modica G., Poggiolini (2013). A First Course in Probability and Markov Chains (United Kingdom: John Wiley & Sons, Ltd) pp. 187-195.