

International Journal of Modern Physics E
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Nuclear response to dark matter signals in Ge and Xe odd-mass targets

M. M. Saez

Interdisciplinary Theoretical and Mathematical Sciences Program (iTHEMS), RIKEN, Wako, Saitama 351-0198, Japan and Department of Physics, University of California, Berkeley, CA 94720
manuelasz89@gmail.com

O. Civitarese*

Dept. of Physics, University of La Plata, c.c. 67, (1900) La Plata, Argentina
Institute of Physics. IFLP-CONICET. diag 113 e/63-64.(1900) La Plata. Argentina
osvaldo.civitares@fisica.unlp.edu.ar

T.Tarutina

Dept. of Physics, University of La Plata, c.c. 67, (1900) La Plata, Argentina
Institute of Physics. IFLP-CONICET. diag 113 e/63-64.(1900) La Plata. Argentina
tatiana.tarutina@fisica.unlp.edu.ar

K. Fushimi

Faculty of Astronomy and Geophysics, University of La Plata.
Ps del Bosque s/n (1900) La Plata, Argentina
keiko.fushimi89@gmail.com

Received Day Month Year

Revised Day Month Year

Abstract: The interaction of dark matter particles (WIMPs) with the odd-mass ^{73}Ge and ^{131}Xe target nuclei is analyzed, in the context of the minimal extensions of the SUSY model. The BCS+QRPA technique plus the quasiparticle-phonon coupling scheme is used to describe the nuclear structure part of the calculations. The resulting values for the nuclear spin content of both nuclei are compared to values previously reported in the literature.

Keywords: dark matter, direct detection, spin dependent cross sections

PACS numbers:

1. Introduction

The nature of dark matter (DM) remains one of the most pressing issues in modern physics. Dark matter has not been detected directly yet. Still, there is evidence

*corresponding author

extracted from observations at galactic scales, galaxy clusters and cosmological observables which suggests that much of the Universe is dark [1–5]. A well-established paradigm is that most DM is cold and is made up of weakly interacting massive particles (WIMPs); other promising alternatives are axions [6–8]. The WIMPs as cold dark matter candidates arise naturally in various theories beyond the Standard Model, e.g., the lightest supersymmetric particle is the neutralino χ , with an expected mass between 1 – 1000 GeV [9]. A relevant strategy for searching WIMPs is the direct detection through the elastic scattering of DM particles by nuclei in ultrasensitive low background experiments [10, 11]. WIMPs interact with ordinary matter predominantly through axial-vector (spin-dependent) and scalar (spin-independent) couplings. Given that the cross-section for the spin independent part is proportional to the square of the nuclear mass number, it is convenient to use heavy nuclei as targets, but to be sensitive to the spin-dependent interaction, a target with an odd number of protons or neutrons must be used.

Some leading dark matter experiments use liquid xenon or germanium as a targets [12–15].

In this work, we focus on the spin-dependent (SD) interaction of WIMPs with the odd mass nuclei ^{73}Ge and ^{131}Xe . The SD interaction of DM particles can constrain WIMP properties, giving more substantial limits on the SUSY parameter space than the spin-independent interaction and it is particularly sensitive to the nuclear structure of the involved nuclei [16, 17]. Previous calculations of cross-sections and reaction rates for SD interactions have been performed at zero momentum transfer [18, 19] but recent works have shown that the dependence of the nuclear matrix elements on the momentum transfer cannot always be ignored [16, 17, 20, 21].

The paper is organized as follows. In Section 2 we present a brief description of the dark matter model and the formalism needed to compute direct detection rates, cross-sections and form factors both for zero and finite momentum transfers. In Section 3 we focus on the properties of the target nuclei and their microscopic description using the quasiparticle-phonon coupling model [22]. In Section 4 we present and discuss the results of the calculations and compare our results with those of other works [16, 23, 24]. We shall follow, as closely as possible, the expressions presented in Refs.[23, 24], because of the relevance of both for the purpose of the comparison between the results given by the TDA (Engel et al.[23]), IBM (Pirinen et al. [24]) and QRPA (present work) models, for the nuclear structure part of the calculations.

The conclusions are drawn in Section 5.

2. Dark matter direct detection

2.1. *Dark matter model*

We work within the framework of the MSSM (Minimal Supersymmetric Standard Model), in which the lightest neutralino state can be written as a linear combination

of binos, winos, and higgsinos (\tilde{B} , \tilde{W}_3 , \tilde{H}_1 and \tilde{H}_2)

$$\chi_1^0 = Z_{11}\tilde{B} + Z_{12}\tilde{W}_3 + Z_{13}\tilde{H}_1 + Z_{14}\tilde{H}_2. \quad (1)$$

The coefficients of the linear combination (Z_{11} , Z_{12} , Z_{13} , Z_{14}) depend on four SUSY parameters, the bino and wino mass parameters (M' and M), the higgsino mass parameter (μ), and the value of $\tan\beta$ related to the ratio of vacuum expectation values of the two Higgs scalars. In the Grand-Unified-Theory (GUT), the parameters M and M' are related by $M' = \frac{5}{3}M \tan^2\theta_W$ [25–27]. The neutralino-quark elastic scattering Lagrangian density in the MSSM is written as [23]

$$\mathcal{L}_{eff} = \frac{g^2}{2M_W^2} \sum_q \left(\bar{\chi}\gamma^\mu\gamma_5\chi\bar{\psi}_q\gamma_\mu A_q\gamma_5\psi_q + \bar{\chi}\chi S_q \frac{m_q}{M_W} \bar{\psi}_q\psi_q \right), \quad (2)$$

where M_W stands for the mass of the W boson, g is the SU(2) coupling constant, and A_q and S_q are defined by [23]

$$\begin{aligned} A_q &= \frac{1}{2}T_{3q}(Z_{13}^2 - Z_{14}^2) \\ &\quad - \frac{M_W^2}{M_{\tilde{q}}^2} \left([T_{3q}Z_{12} - (T_{3q} - e_q)Z_{11} \tan\theta_W]^2 \right. \\ &\quad \left. + e_q^2 Z_{11}^2 \tan^2\theta_W + \frac{2m_q^2 d_q^2}{4M_W^2} \right), \end{aligned} \quad (3)$$

$$\begin{aligned} S_q &= \frac{1}{2}(Z_{12} - Z_{11} \tan\theta_W) \left[\frac{M_W^2}{M_{H_2}^2} g_{H_2} k_q^{(2)} + \frac{M_W^2}{M_{H_1}^2} g_{H_1} k_q^{(1)} \right. \\ &\quad \left. + \frac{M_W^2 \epsilon d_q}{M_{\tilde{q}}^2} \right], \end{aligned} \quad (4)$$

where $M_{\tilde{q}}$ and M_{H_i} are the squark and higgsino masses respectively [28, 29], T_{3q} , e_q , and m_q are the quark weak isospin, charge and mass respectively, and ϵ is the sign of the lightest-neutralino mass eigenvalue [23]. The d_q and $k_q^{(i)}$ parameters for the up and down quarks were taken from [28]. We compute the cross-section using the Lagrangian of Eq. (2). See Ref. [30] and references therein for more details about the calculations.

2.2. Detection rates

The differential recoil rate per unit mass of the detector can be defined as [31]

$$\frac{dR}{dE_{nr}} = \frac{2\rho_\chi}{m_\chi} \frac{\sigma_0}{4\mu^2} F^2(q)\eta, \quad (5)$$

in units of $\text{cpd kg}^{-1} \text{keV}^{-1}$, where cpd stands for counts per day. In the previous equation E_{nr} is the nuclear recoil energy, $q^2 = 2m_A E_{nr}$, m_A is the mass of the

nucleus, m_χ and ρ_χ are the WIMP mass and local density, $\rho_\chi = 0.3 \text{ GeV/cm}^3$ [5, 9], σ_0 is the cross-section at $q = 0$ and $F(q)$ stands for the nuclear form factor. We define the mean inverse-velocity as

$$\eta = \int \frac{f(\vec{v}, t)}{v} d^3v, \quad (6)$$

$f(\vec{v}, t)$ is the WIMP velocity distribution and v is the speed of the WIMP relative to the nucleus.

2.2.1. Velocity distribution

For the velocity distribution of the WIMP, we assume that the model for the DM halo is the Standard Halo Model [32]; therefore, we calculate the velocity distribution from the truncated Maxwell-Boltzmann distribution [31, 33]

$$f(\vec{v}) = \begin{cases} \frac{1}{N(\pi v_0^2)^{\frac{3}{2}}} e^{-|\vec{v}|^2/v_0^2} & |\vec{v}| < v_{\text{esc}} \\ 0 & |\vec{v}| > v_{\text{esc}} \end{cases}, \quad (7)$$

where N is a normalization factor and v_{esc} and v_0 are the escape velocity and the velocity of the Sun, respectively, and their values are $v_{\text{esc}} = 544 \text{ km/s}$ [34] and $v_0 = 220 \text{ km/s}$ [33]. If one considers the laboratory-velocity (\vec{v}_{lab}), the integral of Eq. (6) becomes

$$\eta = \frac{1}{N(\pi v_0^2)^{3/2}} \int \frac{e^{-(\vec{v}+\vec{v}_{\text{lab}})^2/v_0^2}}{v} d^3v. \quad (8)$$

2.2.2. Cross-section: at zero momentum transfer

The total WIMP-nucleus cross-section is a sum over the spin-independent (SI) and spin-dependent (SD) contributions. Since, in this work, we are dealing with the spin dependent channel, the differential cross-section is written [31]:

$$\frac{d\sigma}{dE_{\text{nr}}}(E_{\text{nr}}, v) = \frac{m_A}{2v^2\mu_A^2} \sigma_{SD}(0) F_{SD}^2(q), \quad (9)$$

where μ_A is the reduced mass of the WIMP-nucleus system. For this cross-section we consider all the mediators that contribute to the axial-vector interaction, that is Z and \tilde{q} .

In the zero momentum transfer approximation, we assume that the nuclear form factor F_{SD} is a constant [18]. The cross-section $\sigma_{SD}(0)$ is proportional to the total angular momentum of the nucleus J and it could be written as a function of the neutralino-proton cross-section (σ_{SD}^p) [18]

$$\sigma_{SD}(0) = \sigma_{SD}^p \left[\langle S_p \rangle + \langle S_n \rangle \frac{a_n}{a_p} \right]^2 \left(\frac{\mu_A}{\mu_p} \right)^2 \frac{4(J+1)}{3J}. \quad (10)$$

where μ_p is the reduced mass of the WIMP-proton system and $\langle S_{n(p)} \rangle$ is the spin expectation value of the nucleon over the nuclear wave function, the coefficients $a_{n,(p)}$ are the strengths of the coupling of the WIMPs to the nucleons [23].

2.2.3. Cross-section at finite momentum transfer

The cross-section for this case is also given by Eq. 9 with the form factor

$$F_{SD}^2(q) = \frac{S_{SD}(q)}{S_{SD}(0)}. \quad (11)$$

The axial structure function $S_{SD}(q)$ can be expressed as [23]

$$S_{SD}(q) = a_0^2 S_{00}(q) + a_1^2 S_{11}(q) + a_0 a_1 S_{01}(q). \quad (12)$$

Some results about the structure functions S_{ij} , for the case of ^{73}Ge and ^{131}Xe , are given in Refs. [20, 35]. They are extracted from calculations based on nuclear models, as we shall see next, in the context of the quasiparticle-phonon coupling model. The constants a_0 and a_1 are related to a_n and a_p via [23]

$$\begin{aligned} a_0 &= a_n + a_p, \\ a_1 &= a_p - a_n \end{aligned} \quad (13)$$

where a_n and a_p are expressed in terms of their up and down quark content [36].

3. Nuclear structure aspects of the calculations

To compute the spin expectation values which appear in Eq.(10) we have applied the quasiparticle-phonon coupling model [22]. For the results of calculations performed in the context of other models we shall refer and compare our results with those of Refs.[17],[23] and [24], as well as those obtained using average spin values [37, 38]. Both ^{73}Ge and ^{131}Xe odd-mass targets are nuclei with active neutron excess. To describe their microscopic structure we shall proceed by applying a coupling scheme of neutrons with the low-lying excitations of the even mass ^{72}Ge and ^{130}Xe nuclei. The method is rather well known and it is a matter of textbooks, among them [22] and [39], so we shall briefly outline the main steps of the theoretical framework, which have been presented also in some of our previous works [40]. They are the following:

- Use of the BCS transformations to the quasiparticle basis For neutron states in the single particle states belonging to the $28 \leq N \leq 50$ and $50 \leq N \leq 82$ neutron orbits in the central harmonic oscillator potential with parameters taken from Ref.[22], for Ge and Xe isotopes, respectively, the value of the pairing gap of the even mass ^{72}Ge and ^{130}Xe nuclei have been determined by adjusting the pairing coupling constants to $g = 20/A$ MeV of the monopole pairing interaction for each of them.

- Use of the Quasiparticle Random Phase Approximation (QRPA)

The well known QRPA method [22] is used to diagonalize two- and four-quasiparticle terms of separable monopole and quadrupole interactions of the type

$$\begin{aligned}
H_{(\lambda)} = & \sum_a E_a A^\dagger(a, \lambda\mu) A(a, \bar{\lambda}\bar{\mu}) \\
& + \sum_{a,b} F_{ab:\lambda} A^\dagger(a, \lambda\mu) A(b, \bar{\lambda}\bar{\mu}) \\
& + \sum_{a,b} G_{ab:\lambda} (A^\dagger(a, \lambda\mu) A^\dagger(b, \bar{\lambda}\bar{\mu}) + A(a, \lambda\mu) A(b, \bar{\lambda}\bar{\mu})), \quad (14)
\end{aligned}$$

where $A^\dagger(a, \lambda\mu)$ and $A(a, \bar{\lambda}\bar{\mu})$ denote the creation and annihilation of two-quasiparticle configurations of energy E_a coupled to total angular momentum λ and its projection μ , and where $F_{ab:\lambda}$ and $G_{ab:\lambda}$ are the matrix elements of the separable multipole interaction

$$H_{int} = \sum_{\lambda\mu} \chi_\lambda Q_{\lambda\mu}^\dagger Q_{\lambda\mu}, \quad (15)$$

written in the quasiparticle basis, being $Q_{\lambda\mu}^\dagger$ and $Q_{\lambda\mu}$ the monopole ($\lambda = 0$) and quadrupole ($\lambda = 2$) creation and annihilation operators, respectively. The coupling constant χ_λ , for each channel, is adjusted in order to reproduce the energy $\hbar\omega_\lambda$ of the first excited monopole and quadrupole states of the even mass ^{72}Ge and ^{130}Xe nuclei

$$\frac{1}{\chi_\lambda} = \sum_a \frac{2E_a q_a^2(\lambda)}{E_a^2 - \hbar\omega_\lambda^2}, \quad (16)$$

where the reduced matrix elements of the multipole operator in the quasiparticle basis, $q_a(\lambda)$ for pairs $a = (1, 2)$ of quasiparticles coupled to (λ, μ) are written

$$q_a(\lambda) = (u_1 v_2 - (-1)^c v_1 u_2) \frac{\langle 1 || Q_\lambda || 2 \rangle}{\sqrt{2\lambda + 1}}, \quad (17)$$

u_i and v_i are the quasiparticle occupation factors and $(-1)^c$ is the phase determined by time reversal for multipole operators [22]. The transformation to the monopole and quadrupole one phonon states yields

$$H_{phonon} = \sum_{\lambda\mu} \hbar\omega_\lambda \Gamma_{(\lambda\mu)}^\dagger \Gamma_{\lambda\mu} + \text{constant}, \quad (18)$$

where $\Gamma_{(\lambda\mu)}^\dagger$ and $\Gamma_{(\lambda\mu)}$ are one phonon creation and annihilation operators which are written

$$\begin{aligned}
\Gamma_{(\lambda\mu)}^\dagger &= \sum_a (X_a A^\dagger(a, \lambda\mu) - Y_a A(a, \bar{\lambda}\bar{\mu})) \\
\Gamma_{(\lambda\mu)} &= \sum_a (X_a A(a, \lambda\mu) - Y_a A^\dagger(a, \bar{\lambda}\bar{\mu})), \quad (19)
\end{aligned}$$

with forward and backward going amplitudes X_a and Y_a , respectively.

- Diagonalization of the one-quasiparticle plus one phonon couplings
The last step towards the determination of the wave functions and energies of low-lying states belonging to the odd mass Ge and Xe isotopes consists of transforming the remaining terms of the Hamiltonian, the so-called H_{31} terms of it [41], to the one quasiparticle plus one phonon basis, that is by diagonalizing the interaction

$$H_{qp-ph} = \sum_{\lambda} \Lambda_{\lambda} (Q_{\lambda}(\Gamma_{\lambda}^{\dagger} + \Gamma_{\lambda}))_0, \quad (20)$$

where the quantities Λ_{λ} are the strength of the couplings whose expression in terms of the QRPA amplitudes and matrix elements of the multipole operators is given by

$$\Lambda_{\lambda} = -\chi_{\lambda} \sum_a (X_a + Y_a) q_a(\lambda). \quad (21)$$

- Wave functions of the odd-mass nuclei
The diagonalization of the term H_{qp-ph} of Eq.(20) in the basis of one quasiparticle coupled to n ($n=0,1$) phonon states of multipolarity λ gives the wave functions

$$|IM\rangle = \sum_{j,n=0,1} \alpha(j, n; I) |j \otimes \lambda; IM\rangle, \quad (22)$$

being $\alpha(j, n; I)$ the corresponding amplitudes.

The nuclear response to the interaction with dark matter particles, as described by the cross section of Eq.(10), is given by the expectation value of the spin channel taken on the wave function of the states belonging to the spectrum of the odd-mass nuclei, which is written

$$\langle IM | S_{1\mu} | IM \rangle = \frac{1}{\sqrt{2I+1}} \langle IM 1\mu | IM \rangle \langle I || S_1 || I \rangle, \quad (23)$$

where

$$\begin{aligned} \langle I || S_1 || I \rangle &= (2I+1) \sum_{j, n_{\lambda}, j', n'_{\lambda}} \alpha(j, n_{\lambda}; I) \alpha(j', n'_{\lambda}; I) (-1)^{l+1+1/2+I} \\ &\quad \sqrt{(2j+1)(2j'+1)} \begin{Bmatrix} j & \lambda & I \\ I & 1 & j' \end{Bmatrix} \begin{Bmatrix} l & 1/2 & j \\ 1 & j' & 1/2 \end{Bmatrix} \\ &\quad \sqrt{6} (-u_j u'_j + v_j v'_j) \delta(n_{\lambda} : n'_{\lambda}) \delta(\lambda, \lambda'). \end{aligned} \quad (24)$$

In this expression n and n' are the number of phonons of each the initial and final configurations, respectively, and the radial and orbital quantum numbers of the involved quasiparticle states j and j' should be equal.

4. Results

We have calculated the expected signal for a xenon detector and for a germanium detector, considering neutralino masses between 10 and 100 GeV, by applying the formalism of Section 2.2. For the calculation of the rates, we have taken into account the limits imposed on the cross-section mass plane [42] given by the Xenon1T exclusion limit [43].

To start with, and following the steps discussed in the previous sections we have calculated the spectrum of the odd-mass ^{73}Ge and ^{131}Xe nuclei. This is done in two steps, namely:

4.1. Nuclear structure of the even-mass mother nuclei:

The calculation of the wave function of the low-lying quadrupole excitations in the even-even mother nuclei, is needed in order to determine the strength functions and coupling coefficients to be used in the calculation of the odd mass isotopes. To verify the accuracy of the description, which is based in the use of the Quasiparticle Random Phase Approximation, we have calculated E2 matrix elements and B(E2) transition probabilities and compared the results with the available data and other theoretical calculations.

The one-phonon energies, of the even mass nuclei ^{72}Ge and ^{130}Xe , were fixed at the experimentally determined values of 0.691 MeV (0_1^+) and 0.834 MeV (2_1^+) in ^{72}Ge and 0.536 MeV (2_1^+) in ^{130}Xe , respectively. As an example, we are quoting the values listed in Table III of Ref.[44] and compare them with our results for the case ^{130}Xe (see Table 1)

Transition($I_i \rightarrow I_f$)	B(E2)(W.u.)	Source
$2^+(\text{first exc.}) \rightarrow 0^+(\text{g.s.})$	32(3)	(experimental value)(a)
	21 ($e_n=0.5e, e_p=1.5e$)	(b)
	20 ($e_n=0.5e, e_p=1.5e$)	(c)
	18.10 ($e_n=0.5e, e_p=1.5e$)	(present value) (d)
	22.84 ($e_n=0.84e, e_p=1.68e$)	(present value) (e)

Table 1. Experimental and calculated value for the B(E2) transition from the low-lying quadrupole excitation to the ground state of ^{130}Xe . The values have been taken from the work of Ref.[44], and they are the experimental value (a), and the theoretical results (b) and (c) obtained by using shell model interactions. The values indicated by (d) and (e) are the results of the the present QRPA calculations

4.2. Nuclear structure of the odd-mass nuclei:

As mentioned before, the choice of the odd mass Ge and Xe nuclei is basically related to the importance of the mother nuclei in experiments related to double beta decay processes, as it is well documented in the literature. Here, we have assumed that

both of the even mass mother nuclei are spherical, although this assumption for the case of Ge may not be taken with care due to deformations [45, 46]. In both odd-mass nuclei the coupling to octupole states of the mother even mass nuclei is not taken into account since these states are lying at higher energies [45, 47].

The calculated amplitudes of the wave function of the ground state of each of the odd-mass nuclei ^{73}Ge ($I^\pi=9/2^+$) and ^{131}Xe ($I^\pi=3/2^+$) are written

$$\begin{aligned} |^{73}\text{Ge}(I^\pi = 9/2^+)_{g.s.}\rangle &= 0.9816|4g_{9/2}\rangle - 0.0946|4g_{7/2} \otimes 2^+\rangle \\ &\quad - 0.1291|4g_{9/2} \otimes 0^+\rangle - 0.1046|4g_{9/2} \otimes 2^+\rangle \\ |^{131}\text{Xe}(I^\pi = 3/2^+)_{g.s.}\rangle &= 0.9950|4d_{3/2}\rangle + 0.0897|4d_{3/2} \otimes 2^+\rangle \\ &\quad + 0.0619|4d_{5/2} \otimes 2^+\rangle. \end{aligned} \quad (25)$$

As done for the case of the even mass nucleus ^{130}Xe , and in order to get an idea about the validity of the quasiparticle-phonon coupling scheme for the odd-mass nucleus ^{131}Xe , we show in Table 2 the comparison between the experimental and the calculated values of the energies and spin assignments for the low-lying states. As it is seen from the results the agreement is quite good, except for the negative parity state with $j^\pi=9/2^-$ tentatively identify at about 350 keV, which in the quasiparticle phonon coupling scheme is predicted with a shift of the order of 500 keV respect to the $j^\pi=11/2^-$ state, both members of the $h_{11/2}$ multiplet. Notice that the shift is just of the order of the one phonon energy.

Energy (MeV)		
$j(\pi)$	Exp	Theory
3/2(+)	g.s	0.0
1/2(+)	0.090	0.098
11/2(-)	0.170	0.130
9/2(-)	0.350	0.620
5/2(+)	0.570	0.534
3/2(+)	0.570	0.560

Table 2. Energy levels of low-lying states in ^{131}Xe . Experimental values are compared to the results obtained by applying the quasiparticle-phonon coupling model

Then, from the analysis of the results provided by the QRPA method for the even mass nuclei and for the quasiparticle -phonon coupling scheme, one may conclude by saying that the results of the nuclear structure part of the calculations are rather acceptable when compared to data, for ^{130}Xe and ^{131}Xe . The case of ^{73}Ge is a bit difficult, because of deformation effects in the even mass core ^{72}Ge . Nevertheless the small splitting between the ground state (9/2)+ and the first excited (1/2)- state (which is a pure $p(1/2)$ state) is well reproduced.

4.3. Expectation value of the spin operator:

Next, we proceed with the calculation of the expectation value of the spin operator, which is the main purpose of the present work. We have calculated the spin expectation values for the ^{73}Ge and ^{131}Xe target nuclei.

In Table 3 we summarize our results for the expectation value of the spin operator in the neutron channel and compare them to the results of previous calculations quoted in Refs. [23, 24, 38]. The results for the proton channel are negligible because the cancellation induced by their occupation numbers in the BCS approach.

nucleus	$J\pi$	Ref.	$\langle S_n \rangle$	$\langle S_p \rangle$
^{131}Xe	3/2+	This work	-0.1349	0.0
		QTDA Engel et al. [23]	-0.236	-0.041
		IFBM-2 Pirinen et al. [24]	-0.188	-0.0222
^{73}Ge	9/2+	This work	0.2871	0.0
		Hybrid Dimitrov et al. [38]	0.378	0.03
		CEFT Klos et al. [38]	0.439	0.031

Table 3. Calculated spin expectation values for neutrons $\langle S_n \rangle$ and protons $\langle S_p \rangle$ of ^{131}Xe and ^{73}Ge in different models. QTDA: Quasi Tamm-Dancoff Approximation [23], IFBM-2: microscopic interacting boson-fermion model [24], CEFT: chiral effective field theory.

As it is seen from these results the inclusion of backward going amplitudes, that is by the way of the QRPA formalism, reduces the value of $\langle S_n \rangle$ making it closer to the value calculated by Pirinen et al. [24], for ^{131}Xe , but definitively smaller, for the case of ^{73}Ge , than the value given by Dimitrov et al. [38] in their Hybrid model, and that of Klos et al. [48] obtained using the chiral effective field theory (CEFT) model.

4.4. WIMPs-nucleus cross section

Next, we have calculated the cross section given by Eq.(10) for different values of the mass of the WIMPs. The results are given in Table 4.

The results of Table 4, for the detection of a fermionic cold-dark-matter particle such as the neutralino, have been obtained by fixing the SUSY parameters entering the equations introduced in Section 2.1. The s-quark-mass and the Higgs pseudo-scalar mass were fixed at the values 1500 GeV and 500 GeV, respectively [29]. We fixed also the parameter $\tan\beta = 10$ [26, 27, 25], and varied the parameter μ . The value of the parameter M was determined as a function of μ and m_χ .

Results for the target nucleus ^{73}Ge are shown in Figs.1-4.4, respectively, while those for the ^{131}Xe detector are shown in Figs.4-6. In the upper panel of the aforementioned figures we show the spin-dependent differential direct detection rate (see Eq. 5) as a function of the nuclear recoil energy for different WIMP's masses and for June 2, where the laboratory speed is maximum. Solid-lines represent the results

nucleus	J	m_χ [GeV]	σ_{SD} [cm ²]
¹³¹ Xe	3/2+	10	1.60×10^{-45}
		50	1.21×10^{-46}
		100	1.79×10^{-46}
⁷³ Ge	9/2+	10	4.75×10^{-45}
		50	2.66×10^{-46}
		100	5.04×10^{-46}

Table 4. Calculated cross-sections following Eq. 10 for different WIMP's masses using our nuclear model.

obtained with our spin expectation value, dashed-lines are the ones corresponding to Ref. [23], and dotted-lines are those taken from Ref. [24].

In Table 5 we are listing the ratio (Γ) between the recoil rates obtained with different spin expectation values. From the behaviour of Γ we note that, for lower WIMP masses, there is a higher sensitivity upon the spin expectation value making relevant the choice of the nuclear model to be used. Different values of the spin content can amplify the rates by up to 2.5 times. The results are independent of the approximation used for the transfer of momentum.

nucleus	m_χ [GeV]	ratio(a)	ratio(b)
⁷³ Ge	10	0.512	0.707
	50	0.573	0.803
	100	0.570	0.799
¹³¹ Xe	10	0.525	0.702
	50	0.737	0.862
	100	0.726	0.854

Table 5. Ratios of the counting rates for the calculated spin expectation values, as a function of the WIMPs mass m_χ . The columns denoted as ratio (a) and ratio (b) correspond to: (a) $\frac{dR/dE_{nr}|_{\langle S_n \rangle=0.43}}{dR/dE_{nr}|_{\langle S_n \rangle=0.28}}$, and (b) $\frac{dR/dE_{nr}|_{\langle S_n \rangle=0.37}}{dR/dE_{nr}|_{\langle S_n \rangle=0.28}}$, for the case of ⁷³Ge, respectively, and (a) $\frac{dR/dE_{nr}|_{\langle S_n \rangle=-0.23}}{dR/dE_{nr}|_{\langle S_n \rangle=-0.13}}$, and (b) $\frac{dR/dE_{nr}|_{\langle S_n \rangle=-0.18}}{dR/dE_{nr}|_{\langle S_n \rangle=-0.13}}$ for ¹³¹Xe.

5. Conclusions

In this work, we have studied the nuclear response in dark matter direct detection experiments. Particularly, we have focus the attention on odd-mass germanium and xenon isotopes as detectors. We have described dark-matter particles within the Minimal Supersymmetric Standard Model and studied the effects due to different masses and SUSY parameters within the limits established by the latest results of the Xenon1T collaboration. We have computed the cross-sections and differential

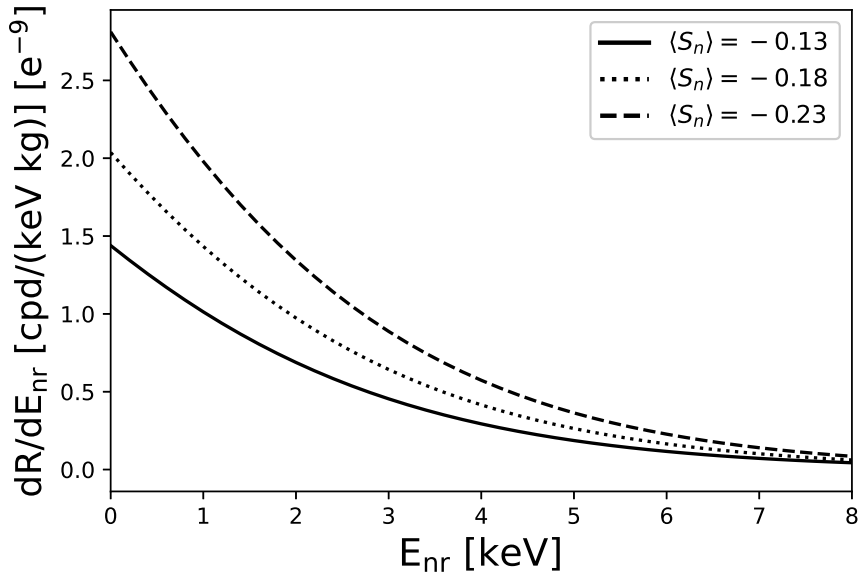


Fig. 1. Results for $m_\chi = 10$ GeV for ^{73}Ge . Upper panel: differential recoil rate dR/dE_{nr} as a function of the nuclear recoil energy E_{nr} . Solid line: spin expectation value (this work); dashed line: spin expectation value from Ref. [48], dotted line spin expectation value from Ref. [38].

rates adopting the quasiparticle-vibration coupling model for the description of the nuclear structure of the odd-mass nuclei ^{73}Ge and ^{131}Xe . We have obtained spin expectation values which are smaller than those reported in the literature until now. The differential rates obtained using our results are smaller than those obtained with other spin contents. Germanium was more sensitive to the changes in the spin content than Xenon. We also found that there is a strong sensitivity, with respect to the nuclear spin expectation value, for light WIMPs masses. In contrast, when working with heavy mass neutralinos, the dependence with the transfer of momentum becomes relevant. However, in spite of these details, the general trend of the results of the QRPA and quasiparticle-phonon coupling model agree quite satisfactory with those obtained with the QTDA of Engel et al. [23] and more closely to the results provided by the use of the IFBM (Interacting Fermion Boson Model) of Pirinen et al. [24]. The changes in the recoil rate due to the zero momentum transfer approximation generates differences of about 30 percent with respect to the case of finite momentum case, especially at high energies.

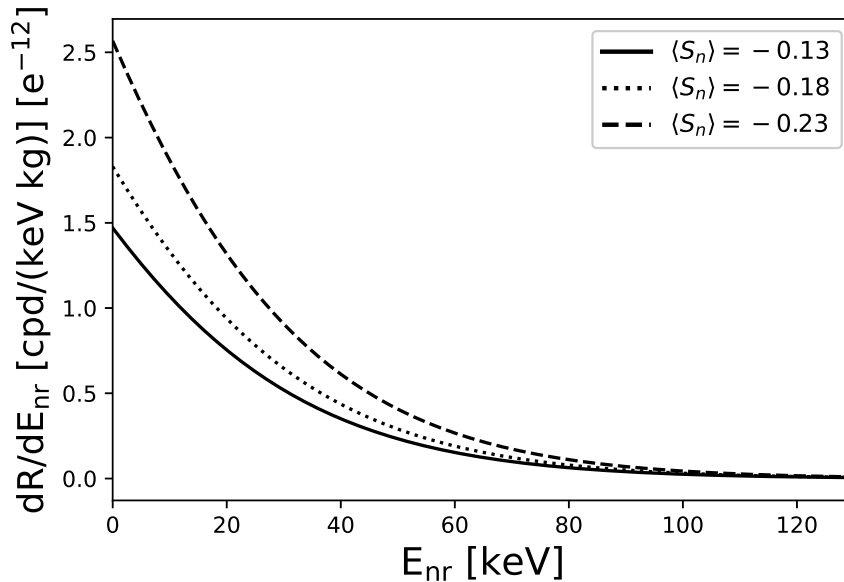


Fig. 2. Same as Figure 1 but for $m_\chi = 50$ GeV.

Acknowledgments

This work has been partially supported by the National Research Council of Argentina (CONICET) by the grant PIP 11220200102081CO, and by the Agencia Nacional de Promoción Científica y Tecnológica (ANPCYT) PICT 140492. O.C and T.T are members of the Scientific Research Career of the CONICET. Discussions with Dr M. E.Mosquera are gratefully acknowledged.

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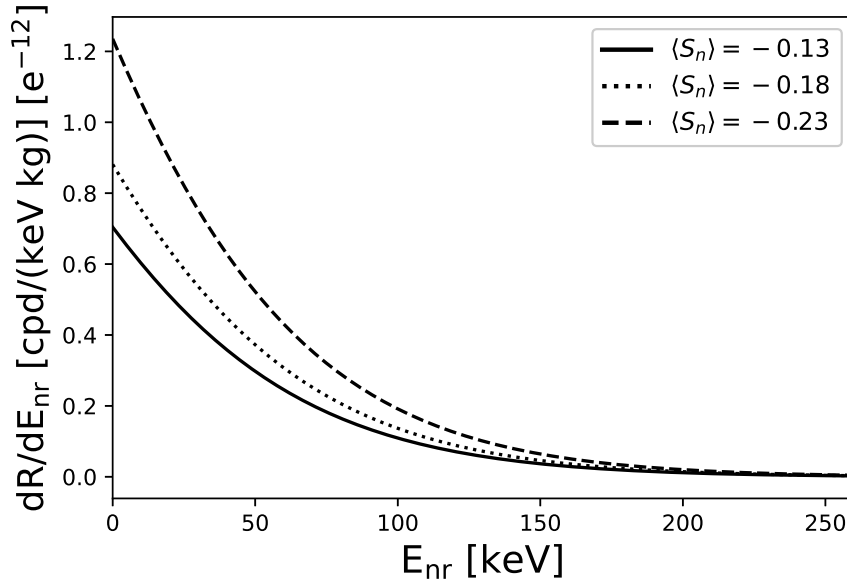


Fig. 3. Same as Figure 1 but for $m_\chi = 100$ GeV.

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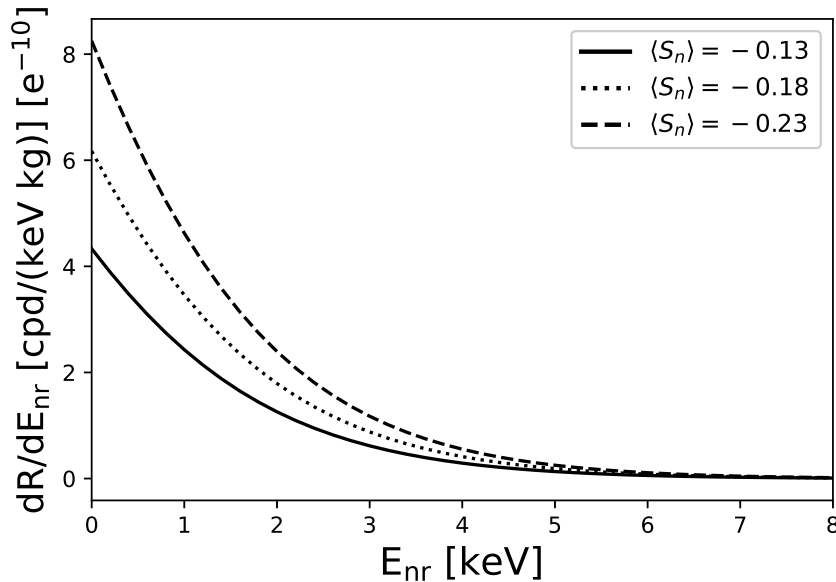
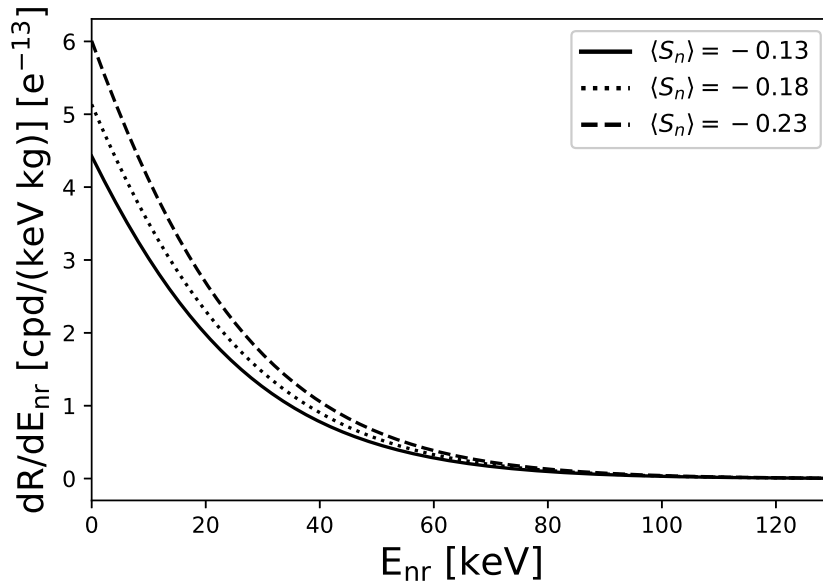


Fig. 4. Results for $m_\chi = 10$ GeV for ^{131}Xe . Upper panel: differential recoil rate dR/dE_{nr} as a function of the nuclear recoil energy E_{nr} . Solid line: spin expectation value (this work); dashed line: spin expectation value from Ref. [23], dotted line spin expectation value from Ref. [24].

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Fig. 5. Same as Figure 4 but for $m_\chi = 50$ GeV.

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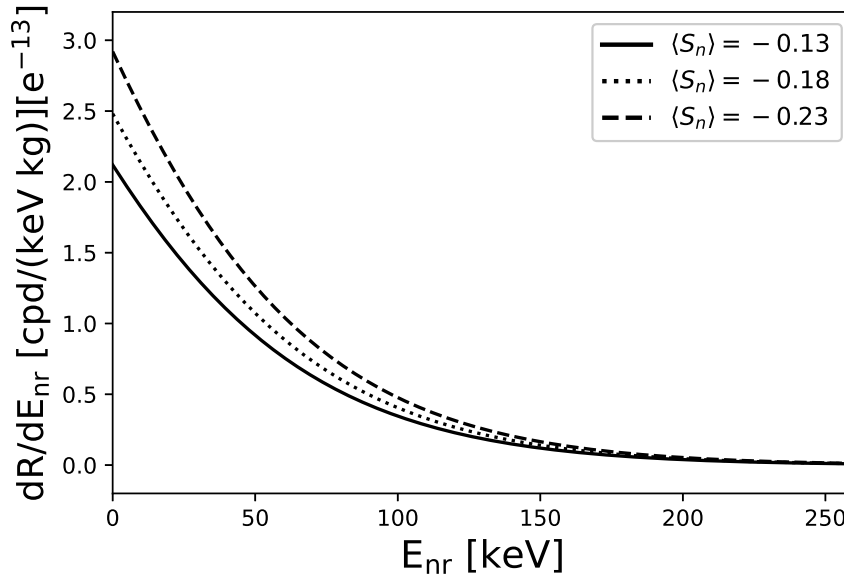


Fig. 6. Same as Figure 4 but for $m_\chi = 100$ GeV.

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