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COLLABORATER FOR A CAR-LIKE VEHICLE DRIVEN BY A USER WITH VISUAL INATTENTION

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ABSTRACT

This paper proposes a control system applied to a car-like vehicle driven by a user. The controller is designed to mitigate the negative effects produced by possible visual distractions of this user. In addition, the paper proposes to evaluate the user's visual distraction, defining a vector that has two components: one with respect to the path and the other with respect to the obstacles. These elements can be computed on-line and are associated with two time delays that produce a similar effect of instability on the motion of the vehicle. The proposed scheme considers the distraction in the design through such delays. Finally, experiments using a car simulator are carried out

Key Words: Driver's visual inattention, path following, time delay, driving safety, crash probability.

I. INTRODUCTION

Driving is a complex task that requires the interaction and coordination of various driver skills. Additionally, a high degree of attention and concentration are necessary in order to prevent road accidents. However, sometimes a driver performs secondary tasks, such as changing the radio station, eating, and talking on cell phones, among others. Any activity that distracts the driver or that catches his attention could degrade the driving performance with serious implications for road safety [16]. The visual distraction is the major cause of traffic accidents. For example the National Highway Traffic Safety Administration (NHTSA) estimates that driver inattention contributes to about 25 per cent of vehicle accidents in the USA. For this reason, there is a great interest in developing methods for measuring driver lack of attention while driving, as well as methods to reduce the effects of secondary tasks on driving performance.

The NHTSA classifies distractions into four categories from the view of the driver's functionality: visual distraction, cognitive distraction, auditory distraction, and biomechanical distraction [12]. Most techniques used for the measurement of the distraction are derived from psychological measures based on eye gaze, occlusion techniques, and so on [1], and measures of performance such as distance from the center of the road, speed maintenance, *etc*. [15]. Although several interfaces use binary signals of inattention in order to alert the driver, they do not take into account the environment, or the type of distraction [4]. There are very few studies that incorporate continuous measures of distraction in the control system, such as [11] does.

This paper proposes a control system applied to a car-like vehicle in order to collaborate with a user considering his possible visual distraction. For this purpose, we propose a definition of a vector to evaluate quantitatively the user's visual distraction. This vector has two components: distraction with respect to the path produced by a mismatch between the area of viewing associated with the user's gaze and the current path; and the distraction relative to the obstacles generated by a mismatch between the user's gaze and the position of the called obstacles (pedestrians, other vehicles, *etc*.) that have a significant crash probability. These two signals are continuous and can be evaluated on-line from sensor data. To model the visual distraction within the system, we propose to associate the distraction components with two time delays that produce a similar effect of instability on the vehicle motion following a given path. From this, a stable control scheme is designed considering these delays in order to take advantage of the user's capabilities, and to decrease the negative effects caused by possible visual distractions of the user.

This paper is organized as follows: Section II describes the mathematic preliminaries. In Section III, a measure of visual inattention is proposed and its relationship with a time delay is presented. In Section IV, a control system to mitigate the inattention effects is proposed. In Section V, experiments where a user drives a car simulator are shown. Finally, conclusions are given in Section VII.

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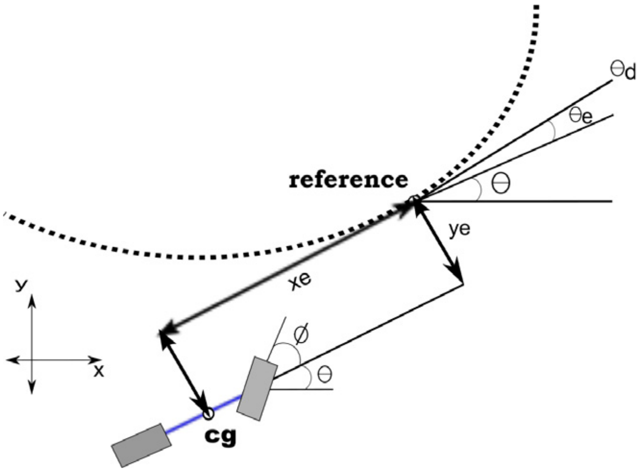

Fig. 1. Pose error.

II. PRELIMINARIES

2.1 Model of a car-like vehicle

There are various models present in the literature to describe a car-like vehicle (Fig. 1). Let us consider a simple dynamic approximation of a car as follows [10]:

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} v_r \cos\theta_e + y_e\dot{\theta} - v \\ v_r \sin\theta_e - x_e\dot{\theta} \\ \dot{\theta}_r - \dot{\theta} \\ \frac{1}{M}u_1 \\ \frac{1}{J}u_2 \end{bmatrix} \tag{1}$$

where $\dot{\theta} = v\dfrac{\tan(\phi)}{L} = w$

θ Orientation vehicle angle.
ϕ Steering angle.
v is the longitudinal velocity of the vehicle.
w is the angular velocity of the vehicle.
v_r is the reference longitudinal velocity of the vehicle.
w_r is the reference angular velocity of the vehicle.
L Vehicle length.
M Mass of the vehicle.
J Inertia moment of the vehicle.
u_1 is the longitudinal force applied to the vehicle.
u_2 is the torque applied to the vehicle.
$x_e = x_r - x$, $y_e = y_r - y$, $\theta_e = \theta_r - \theta$ are the errors between the reference pose given by x_r, y_r, θ_r and the pose of the car-like vehicle.

Now, defining the velocity errors as:

$$\begin{aligned} v_e &= v - v_r \\ w_e &= w - w_r \end{aligned} \tag{2}$$

The model (1) can be represented by:

$$\begin{aligned} \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{v}_e \end{bmatrix} &= \begin{bmatrix} 0 & w_r(t) & -1 \\ -w_r(t) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ v_e \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left(\frac{1}{M}u_1 - \dot{v}_r \right) + \begin{bmatrix} v_r \dfrac{1-\cos(\theta_e)}{\theta_e} & y_e \\ v_r \dfrac{\sin(\theta_e)}{\theta_e} & -x_e \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_e \\ w_e \end{bmatrix} \\ \begin{bmatrix} \dot{\theta}_e \\ \dot{w}_e \end{bmatrix} &= \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_e \\ w_e \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left(\frac{1}{J}u_2 - \dot{w}_r \right). \end{aligned} \tag{3}$$

2.2 Cascade system stability

Let us consider a system $\dot{z} = f(t, z)$, represented by

$$\begin{aligned} \dot{z}_1 &= f_1(t, z_1) + g(t, z_1, z_2)z_2 \\ \dot{z}_2 &= f_2(t, z_2). \end{aligned} \tag{4}$$

where $z_1 \in \mathcal{R}^n$; $z_2 \in \mathcal{R}^n$; $(z_1, z_2) = (0, 0)$ is a point of equilibrium of the system expressed in (4); $f_1(t, z_1)$ is derivable continuous in z_1 and $f_2(t, z_2)$, $g(t, z_1, z_2)$ are continuous on their arguments and locally Lipschitz in z_2 and (z_1, z_2), respectively.

In order to analyze the stability of cascaded systems, a rate growing restriction in the interconnection term is defined by [6] as follows:

$$\|g(t, z_1, z_2)\| \le h_1(\|z_2\|) + h_2(\|z_2\|)\, z_1 \tag{5}$$

where h_1 and h_2 are of class K, both are strictly increasing, and $h_1(0) = h_2(0) = 0$.

Theorem 1. Cascade System Stability [10]. Assuming that the subsystems $\dot{z}_1 = f_1(t, z1)$ and $\dot{z}_2 = f_2(t, z_2)$ are globally exponentially stable and the correlation term $g(t,z_1,z_2)$ fulfills the condition (5), then the cascade system (4) is globally exponentially stable (GES).

2.3 Autonomous controller

The mathematical description of the human operator's behavior performing a task is difficult, often impossible in practice. An alternative to modelling human behavior consists of testing various automatic controllers based on different concepts such as stability, optimization of some functional, *etc*., and choosing the one with a behavior most similar, which is not necessarily the best in terms of autonomous control systems (minimum error, faster convergence rate, *etc.*).

There are many autonomous controllers applied to wheeled robots and vehicles, considering a dynamic model, like [9, 5]. In this paper, the car-like vehicle is described by (1), while the chosen control law approximately fits a driver's normal behavior to follow a given path, and is represented by [10, 6]:

$$\begin{aligned} u_1 &= M(\dot{v}_r + c_1 x_e - c_2 v_e);\, c_1 > 0, c_2 > 0 \\ u_2 &= J(\dot{w}_r + c_3 \theta_e - c_4 w_e);\, c_3 > 0, c4 > 0. \end{aligned} \tag{6}$$

where v_r and w_r are the linear and angular reference velocities, respectively; and c_1, c_2, c_3, c_4 are the controller parameters set by the designer. The closed loop system formed by (1) and (6) is globally exponential stable (GES) [6, 7].

Inserting (6) into (3), the model in a closed loop can be written as:

$$\begin{aligned} \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{v}_e \end{bmatrix} &= \begin{bmatrix} 0 & w_r(t) & -1 \\ -w_r(t) & 0 & 0 \\ c_1 & 0 & -c_2 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ v_e \end{bmatrix} \\ &\quad + \begin{bmatrix} v_r \dfrac{1-\cos(\theta_e)}{\theta_e} & y_e \\ v_r \dfrac{\sin(\theta_e)}{\theta_e} & -x_e \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_e \\ w_e \end{bmatrix} \\ \begin{bmatrix} \dot{\theta}_e \\ \dot{w}_e \end{bmatrix} &= \begin{bmatrix} 0 & -1 \\ c_3 & -c_4 \end{bmatrix} \begin{bmatrix} \theta_e \\ w_e \end{bmatrix}. \end{aligned} \tag{7}$$

The previous model can be represented similar to (4) where the subsystem $\dot{z}_1 = f_1(t, z_1)$ is given by:

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{v}_e \end{bmatrix} = \begin{bmatrix} 0 & w_r(t) & -1 \\ -w_r(t) & 0 & 0 \\ c_1 & 0 & -c_2 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ v_e \end{bmatrix}. \tag{8}$$

Considering $c1 > 0$, $c2 > 0$; $w_r(t)$, $\dot{w}_r(t)$ and $v_r(t)$ as bounded signals and w_r persistent exiting (PE), the subsystem (8) is GES [10] and its rate of convergence λ_1 depends on the values of c1 and c2 [7].

On the other hand, the subsystem $\dot{z}_2 = f_2(t, z_2)$ comparing (7) and (4) is as follows:

$$\begin{bmatrix} \dot{\theta}_e \\ \dot{w}_e \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ c_3 & -c_4 \end{bmatrix} \begin{bmatrix} \theta_e \\ w_e \end{bmatrix}. \tag{9}$$

If $c_3 > 0$ and $c_4 > 0$, then (9) is GES [10] and its rate of convergence λ_2 depends on the values of c3 and c4. The interconnection term verifies the condition (5), since:

$$\|g(t, z_1, z_2)\| \le v_r^{max}\sqrt{2} + |z_1|. \tag{10}$$

Since all conditions of Theorem 1 are verified, the system (7) is GES and its rate of convergence λ depends on the $min(\lambda_1,\lambda_2)$ [10].

III. HUMAN VISUAL INATTENTION

Generally, the human inattention is considered to be a binary signal (0–1) and unidimensional (1D) [14]. Instead this work proposes to represent the inattention by a bi-dimensional (2D) continuous signal computed on-line, whose components are the following:

$$i_{visual} = \begin{bmatrix} i_p \\ i_o \end{bmatrix} \tag{11}$$

where i_{visual} is the visual inattention of the user.
i_p is the visual inattention of the user with respect to the path, considering that it has no intersections or forks.
i_o is the visual inattention of the user with respect to the obstacles near to the vehicle.

3.1 Path inattention

The authors define the human's visual attention while he drives a car-like vehicle to follow a given path. This is the ratio of his current sight field that belongs to the path segment situated in the motion direction of the vehicle, with respect to the maximum sight field possible. That is, if a user has high percent of his vision area inside the visible road section, then he has a high attention to the principal activity (driving). Since the user's inattention will be normalized to the range [0,1], it can be defined by the complement to one of the user's attention.

To estimate the sight field of the user, a type-triangle area is set on-line considering the decision sight distance and the human binocular vision field. We take into account the decision sight distance interpolated from Table I [8], where it represents the distance required by a driver to detect unexpected information, recognize danger, select a speed and appropriate path, and complete the maneuver safely and efficiently [2]. The human binocular vision field is assumed in ±0.69 radians, which is compatible with the driver license vision standards considered in Canada (www.eyesite.ca) and USA (www.icoph.org/standards).

This work proposes the definition of the inattention level (i_p) as:

Table I. Decision sight distance.

Velocity(km/h)	50	60	70	80	90	100	110
Distance(m)	75	95	125	155	185	225	265

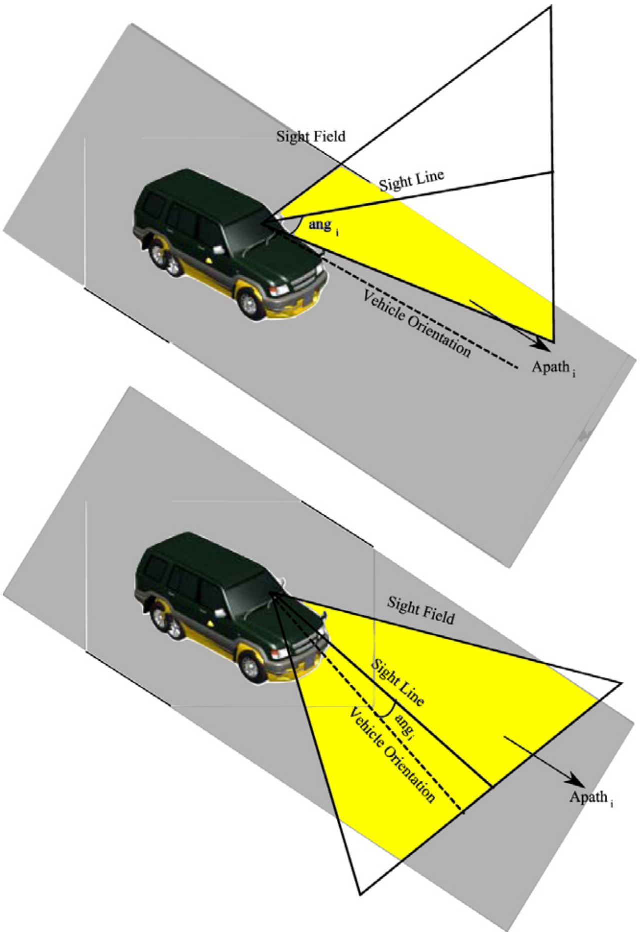

Fig. 2. Area of vision with different view angles *ang*.

$$i_p(t) = 1 - \frac{A_{path}(t)}{A_{path_{max}}(t)} \tag{12}$$

where A_{path} is the area of intersection between the projection of the human visual field on the road plane and the road in the current time instant. Such an area depends on the angle of view, *ang*, of the user as shown in Fig. 2.

$A_{path_{max}}$ is the maximum intersection area that could exist between the user's visible area and the road at the current time instant, and is defined by:

$$A_{path_{max}}(t) = \max_{ang \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]} (A_{path}); \tag{13}$$

3.2 Obstacle inattention

3.2.1 Probability collision

The collision probability is defined as the probability of a crash between a mobile or static obstacle and the vehicle in a finite time. This probability can be scaled to [0,1] and it is measured from the maximal area of intersection between the predicted motion of the vehicle and obstacles:

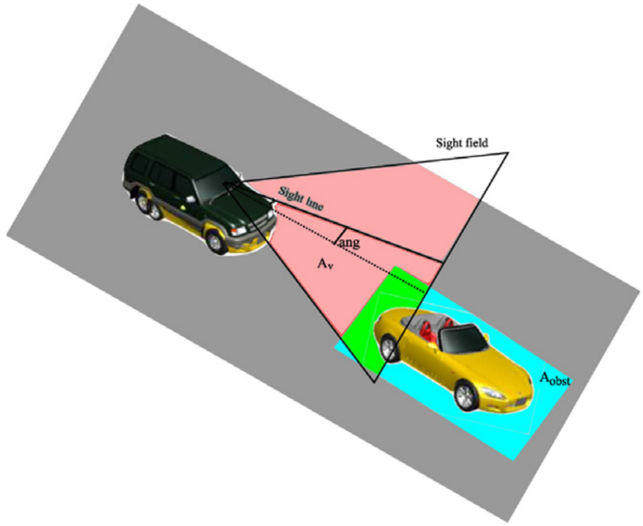

Fig. 3. Area of Vision with obstacles.

$$Pcol = \left(\frac{\max\limits_{t \in [t, t_{max}]} (A_{au} \cap A_{obst})}{A_{au}} \right) \left(1 - \frac{t_{areamax}}{t_{max}} \right) \tag{14}$$

where A_{au} is the area of the vehicle projected on the path surface.

A_{obst} is the obstacle area projected on the path surface.

$max(A_{au} \cap A_{obst})$ is the maximal intersection area between A_{au} and A_{obst} for $t \in [t, t + t_{max}]$.

$t_{areamax}$ is the time elapsed when the maximal intersection between the vehicle area and the obstacle area occurs. By default $t_{areamax}$ is set to t_{max}.

t_{max} is the maximum time during which the prediction of motion is done.

These areas are computed on-line (how is showed in Fig. 3) considering the current values of position and velocity of the vehicle and obstacles, and the intersection of the estimated motion of each one using a linear prediction.

3.2.2 Measure of the user's inattention with obstacles

Considering (14), a measure of the human's visual inattention for one obstacle is proposed as follows:

$$i_o(t) = \left(1 - \frac{(A_v \cap A_{obst})}{\max_{ang \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]} (A_v \cap A_{obst})} \right) Pcol \tag{15}$$

where A_v is the vision area of the user projected in the motion plane.

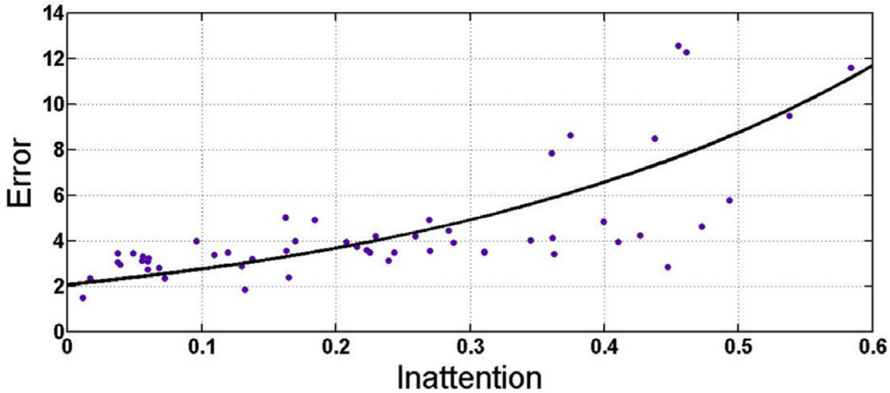

Fig. 4. Error depending on the driver's inattention.

A_{obst} is the area of the obstacle projected in the motion plane. For two or more obstacles, the definition (15) is extended to:

$$i_o = \frac{\sum_{j=1}^{N}(A_v Pcol_j \cap A_{obst_j})^c}{\max_{ang\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]} \sum_{j=1}^{N}(A_v Pcol_j \cap A_{obst_j})^c};\ j = 1, \ldots, N \tag{16}$$

where N is the number of obstacles.

$\sum_{j=1}^{N}(A_v Pcol_j \cap A_{obst_j})^c$ is the sum of intersections of projected areas weighed by the collision probability of the respective obstacles.

$\max_{ang\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]} \sum_{j=1}^{N}(A_v Pcol_j \cap A_{obst_j})^c$ is the maximal sum of intersections weighed by the collision probability for $ang \in [-\pi/2, \pi/2]$.

Based on this definition, the component of the visual inattention respect to obstacles i_o increases if the user is not looking at obstacles that have a high probability of collision.

3.3 Relation between driver's inattention and time delay

The visual inattention with respect to the path i_p, defined in (13), can be linked with a time delay h_p. In order to obtain such relationship, the following process [3] was carried out:

- A test road for the experiments in the simulator is established. In our case, the path is composed for two straight and one curved line.
- Experiments were done in this road, requesting the user to force a visual inattention on the curve. The mean square error (MSE) of the error from the center of the path of each experiment was measured.
- In the same road, replacing the user with the automatic controller proposed as a model of human behavior, experiments for different time delays were carried out, where the mean square error (MSE) of the car position with respect to the center of the path for each experiment was measured.
- From these data, the relationship between the MSE and visual inattention with respect to the path, and the time delay, can be found. We propose the following function:

$$\begin{aligned} h_p &= f^{-1}(g(i_p))\Delta(v, R) \\ &= \frac{1}{b_2} ln\left(\frac{a_1}{a_2} e^{b_1 i_p} + \frac{error(0)}{a_2}\right) \\ &\quad \cdot\left(1 + k_c\left(\frac{v - v_e}{v_{max}} + R_{min}\left(\frac{1}{R} - \frac{1}{R_e}\right)\right)\right) \end{aligned} \tag{17}$$

where a_1, a_2, b_1, b_2 are obtained in this case for the curvature radius $R_e = 200\ m$ and the speed $v_e = 25\ m/s$, v_{max} is the maximum speed of the car and $R_{min} > 0$ is the minimum curvature radius of the road considered significant in practice. The parameter k_c is the gain of $\Delta(v, R)$ established empirically to correct $f^{-1}(g(i_p))$ depending on the car speed and the curvature radius of the road. Such gain must verify that $\Delta(v, R) \geq 0$ for all $0 < v \leq v_{max}$ and $R > R_{min}$.

Let us assume a speed reference set at 90 km/h for a road composed of two straights and one curved line with a curvature radius R = 200m. In addition, we define the mean square error as the norm of the vector difference between the pose of the vehicle (x, y, θ) and the ideal path (the car always travels in the middle of its lane). Then, several experiments and simulations will be performed in order to analyze the relation between user's inattention and time delay. Fig. 4 shows the error obtained for 50 experiments where the user drives the car simulator with different levels of inattention. An exponential function called $g(i_p)$, is proposed to fit the experimental points. The function $g(i_p)$ is set by:

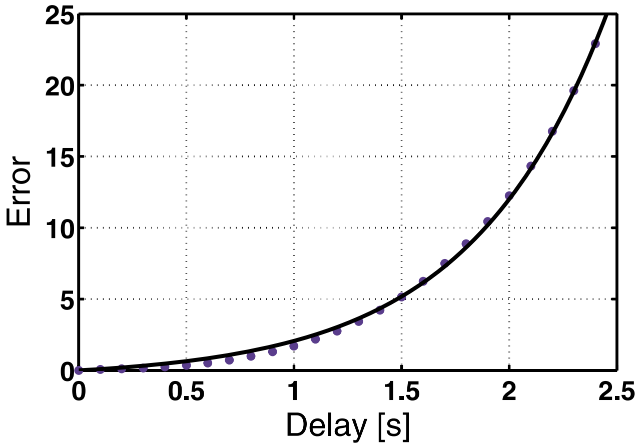

Fig. 5. Error depending on the time delay.

$$error = g(i_p) = a_1 e^{b_1 i_p} \tag{18}$$

where $a_1 = 1.938$ and $b_1 = 3.077$ are the parameter values. Fig. 5 shows the points obtained using the closed loop model (7) for different time delays. The parameters used are $c_1 = 1.0$, $c_2 = 1.5$, $c_3 = 0.1$ and $L = 2.82$. An exponential function called $f(h_p)$, to represent approximately the points set obtained is proposed as follows:

$$error = f(h_p) = a_2 e^{b_2 h_p} \tag{19}$$

where the parameters a_2 and b_2 are set so that (19) as close as possible the points set obtained in the simulation. In our case, $a_2 = 0.00962$ and $b_2 = 4.627$. Equations (18) and (19) support the proposal (17) in which a compensation depending on the speed and the curvature radius is added, too.

Fig. 6 shows the error depending on the time delay and the vehicle speed for three curvature radius R = 200, 100 and 50 m. The error is greater as the vehicle speed or the curvature radius of the road are higher.

Fig. 7 shows the relation between the user's inattention and the time delay h_p for $R_e = 200\ m$ and $v_e = 90\ km/h$. The delay h_p is introduced into the system in order to represent the effects of the human's visual inattention with respect to the path.

In addition, we associate the visual inattention (with respect to the obstacles) with a time delay defined by:

$$h_o = h_{reaction} + k_{obs}(i_o(t)) \tag{20}$$

where $h_{reaction}$ is the reaction time of the user in front of visual stimulus.

k_{obs} is the gain that relates the user inattention and a time delay.

i_o is the visual inattention respect to the obstacles.

That is, a user distracted with respect to the obstacles does not react quickly before a dangerous motion of them.

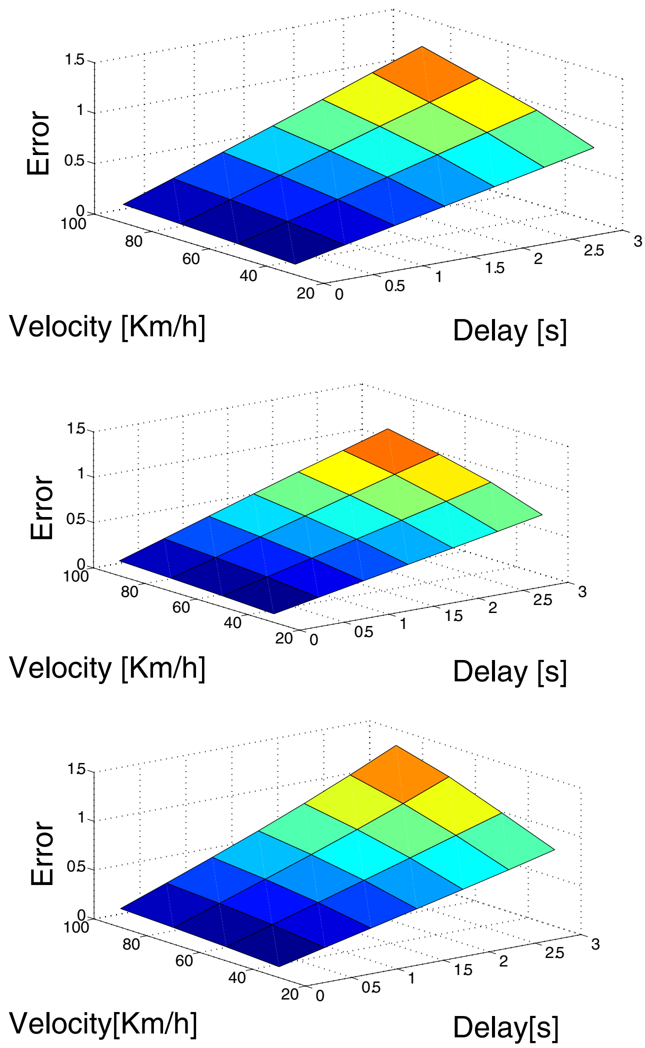

Fig. 6. Error depending on delay and speed: a) R = 50 m, b) R = 100 m, c) R = 200 m.

On the other hand, to fulfill the requirements of the later analysis of stability, h is filtered in order to ensure that $\dot{h} < 1$. This filter rejects abrupt changes in the calculation of h, caused by fast variations in the user's gaze. That is, the filter decreases the effect caused by the controller before short-time distractions which are common in practice, but acts strongly in front of a distraction maintained in time.

IV. DRIVER ASSISTANCE

This work proposes a control scheme to mitigate the effects of possible visual distractions of a user driving a car-like vehicle. Fig. 8 shows the proposed control system, where the control action applied to the vehicle is computed by:

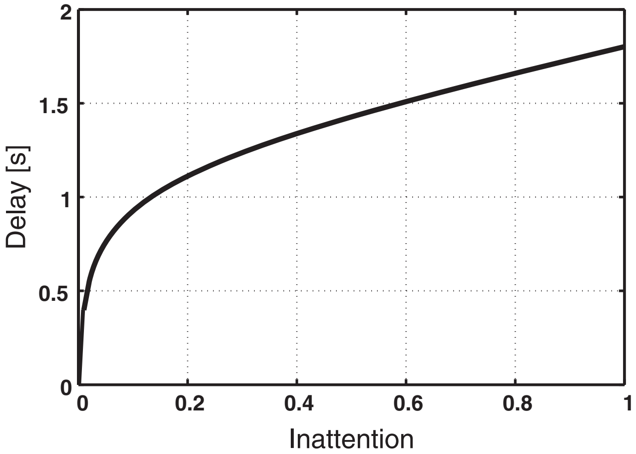

Fig. 7. Inattention vs Delay.

$$\begin{aligned} u &= u_{human} + \Delta Control \\ u &= u_{human} + u_{exp} - k(t) \end{aligned} \tag{21}$$

where $\Delta Control$ is the contribution of the controller in order to decrease the negative effect produced by the user inattention.

$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}^T$ is the command applied to the vehicle.

$u_{human} = \begin{bmatrix} u_{1_{human}} \\ u_{2_{human}} \end{bmatrix}^T$ is the command generated by the user.

$u_{exp} = \begin{bmatrix} u_{1_{exp}} \\ u_{2_{exp}} \end{bmatrix}^T$ is the pattern command obtained from an autonomous controller, called an expert, which points out a desired behavior.

$u_h = \begin{bmatrix} u_{1_h} \\ u_{2_h} \end{bmatrix}^T$ is the control command produced by a delayed model that represents a distracted user.

$k(t)$ is a vector gain bounded for all t defined by:

$$k(t) = \begin{bmatrix} k_{vel} \\ k_{vol} \end{bmatrix}^T. \tag{22}$$

Here, the expert command (u_{exp}) is represented by a tracking controller like (6) that guarantees the global exponential stability in closed loop, where the reference of the expert controller is set as follows:

$$v_{r_{exp}} = v_r - k_1 y_e - k_2 curv_{prox} \tag{23}$$

$$w_{r_{exp}} = w_r + k_1 y_e \tag{24}$$

where $k_1 > 0$ and $k_2 > 0$ are gains, $curv_{prox}$ is the curvature of the next segment of the path, v_r is the reference of linear velocity calculated by averaging the vehicle speed in a time window, w_r is the reference of angular velocity computed from the linear velocity reference and the curvature of the road. This reference is described considering a careful user who tends to decrease the velocity when the next segment is a curve, or the vehicle is so far from the middle of its lane. The command u_{exp} can be expressed like (6) as follows:

$$\begin{aligned} u_{1_{exp}} &= \dot{v}_{r_{exp}} + c_1 x_e - c_2 v_{e_{exp}} \\ u_{2_{exp}} &= \dot{w}_{r_{exp}} + c_3 \theta_e - c_4 w_{e_{exp}} \end{aligned} \tag{25}$$

where c_1, c_2, c_3, c_4 are bounded and establish the desired behavior, and $\dot{v}_{r_{exp}}$ and $\dot{w}_{r_{exp}}$ are the derivative of the linear and angular velocity reference of the expert. The commands (25) also can be expressed by:

$$\begin{aligned} u_{1_{exp}} &= \dot{v}_r + c_1 x_e - c_2 v_e + \Delta u_{1_{exp}} \\ u_{2_{exp}} &= \dot{w}_r + c_3 \theta_e - c_4 w_e + \Delta u_{2_{exp}} \end{aligned} \tag{26}$$

where $\Delta u_{1_{exp}}$ and $\Delta u_{1_{exp}}$ are bounded and represent the difference between the expert reference and the user reference, and $\dot{v}_r$ and $\dot{w}_r$ are the derivatives of the user's linear and angular velocity reference, respectively.

The control commands of a model of a distracted user are represented by (6), including the time delay h_p, as follows:

$$\begin{aligned} u_{1_h} &= \dot{v}_{r_h} + c_1' x_{e_h} - c_2' v_{e_h} \\ u_{2_h} &= \dot{w}_{r_h} + c_3' \theta_{e_h} - c_4' w_{e_h} \end{aligned} \tag{27}$$

where $c_1'(t)$, $c_2'(t)$, $c_3'(t)$, and $c_4'(t)$ are bounded, $h = h_p$ is the time delay produced by the visual inattention respect to the path, $\dot{v}_{r_h} = \dot{v}_r(t - h_p)$ is the derivative of the delayed velocity reference, $\dot{w}_{r_h} = \dot{w}_r(t - h_p)$ is the derivative of the delayed angular velocity reference of the user and $x_{e_h} = x_e(t - h_p)$, $\theta_{e_h} = \theta_e(t - h_p)$, $v_{e_h} = v_e(t - h_p)$, $w_{e_h} = w_e(t - h_p)$ are the delayed errors between the reference pose and the vehicle pose. Finally, the control law can be written like (21) considering (26) and (27) as follows:

$$\begin{aligned} u_1 &= \dot{v}_r + c_1 x_e - c_2 v_e + \Delta u_{1_{exp}} + u_{1_{human}} \\ &\quad - k_{vel}(\dot{v}_{r_h} + c_1' x_{h_e} - c_2' v_{e_h}) \\ u_2 &= \dot{w}_r + c_3 \theta_e - c_4 w_e + u_{2_{human}} + \Delta u_{2_{exp}} \\ &\quad - k_{vol}(\dot{w}_{r_h} + c_3' \theta_{e_h} - c_4' w_{e_h}). \end{aligned} \tag{28}$$

4.1 Stability of the system

In this part, the stability of the closed loop system will be analyzed using the stability theory available for delayed systems.

Assumption 1. Let us assume that the control commands of a user can be represented like (6), as follows:

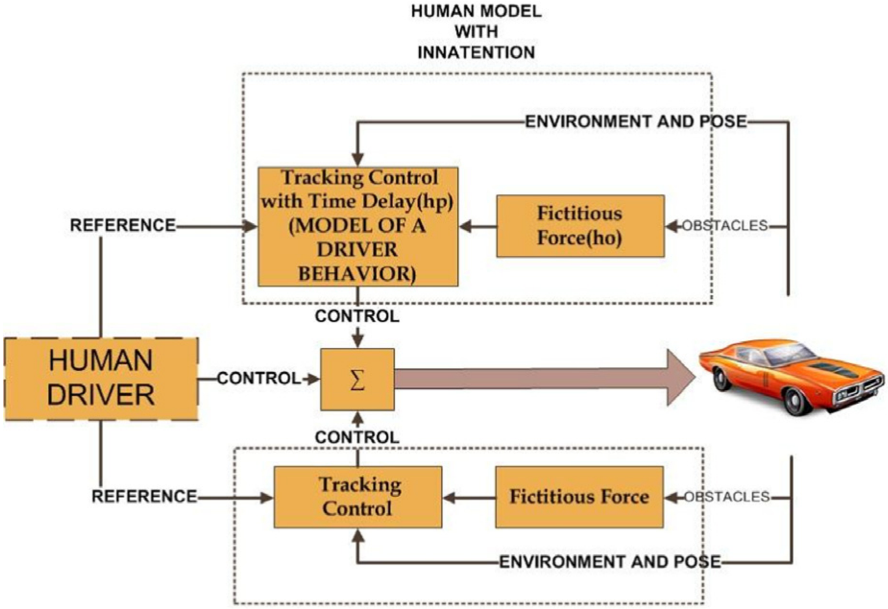

Fig. 8. Scheme of controller.

$$\begin{aligned} u_{1_{human}} &= \dot{v}_{r_h} + c_1''x_{e_h} - c_2''v_{e_h} \\ u_{2_{human}} &= \dot{w}_{r_h} + c_3''\theta_{e_h} - c_4''w_{e_h} \end{aligned} \tag{29}$$

where $c_1''(t)$, $c_2''(t)$, $c_3''(t)$, and $c_4''(t)$ are bounded, unknown and different for each user.

Now, setting (28) into (1), the closed loop system can be written as:

$$\begin{aligned} \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{v}_e \end{bmatrix} &= \begin{bmatrix} 0 & w_r(t) & -1 \\ -w_r(t) & 0 & 0 \\ c_1 & 0 & -c_2 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ v_e \end{bmatrix} \\ &+ \begin{bmatrix} v_r \dfrac{1-\cos(\theta_e)}{\theta_e} & y_e \\ v_r \dfrac{\sin(\theta_e)}{\theta_e} & -x_e \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_e \\ w_e \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} (\Delta u_{1_{exp}} + u_{1_{human}} - k_{vel}(\dot{v}_{r_h} + c_1'x_{h_e} - c_2'v_{e_h})) \\ \begin{bmatrix} \dot{\theta}_e \\ \dot{w}_e \end{bmatrix} &= \begin{bmatrix} 0 & -1 \\ c_3 & -c_4 \end{bmatrix} \begin{bmatrix} \theta_e \\ w_e \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ 1 \end{bmatrix} (\Delta u_{1_{exp}} + u_{2_{human}} - k_{vol}(\dot{w}_{r_h} + c_3'\theta_{e_h} - c_4'w_{e_h}). \end{aligned} \tag{30}$$

Putting (27) and (29) into (30), the system represented in state space can be written as:

$$\begin{aligned} \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{v}_e \\ \dot{\theta}_e \\ \dot{w}_e \end{bmatrix} &= \begin{bmatrix} 0 & w_r(t) & -1 & v_r \dfrac{1-\cos(\theta_e)}{\theta_e} & y_e \\ -w_r(t) & 0 & 0 & v_r \dfrac{\sin(\theta_e)}{\theta_e} & -x_e \\ c_1 & 0 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & c_3 & -c_4 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ v_e \\ \theta_e \\ w_e \end{bmatrix} \\ &+ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ c_1'' - k_{vel}c_1' & 0 & -c_2'' + k_{vel}c_2' & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_3'' - k_{vol}c_3' & -c_4'' + k_{vol}c_4' \end{bmatrix} \begin{bmatrix} x_{e_h} \\ y_{e_h} \\ v_{e_h} \\ \theta_{e_h} \\ w_{e_h} \end{bmatrix} \\ &+ \begin{bmatrix} 0 \\ 0 \\ \Delta u_{1_{exp}} + \dot{v}_{r_h} - k_{vel}\dot{v}_{r_h} \\ 0 \\ \Delta u_{2_{exp}} + \dot{w}_{r_h} - k_{vol}\dot{w}_{r_h} \end{bmatrix}. \end{aligned} \tag{31}$$

Now, the model (31), can be written as:

$$\dot{x} = f(x) + g(x, x_h) + P; \tag{32}$$

where P is a perturbation. Considering that the subsystem $\dot{x} = f(x)$ is the same as that of (7), then it is GES with a convergence rate λ. Ensuring that k_{vol} and k_{vel} are bounded and continue through their design, then $|g|$ is bounded, too. Now, using Theorem 2 (Appendix A), it is possible to assume the stability of the system (32), regardless of the perturbation P, if the parameters c_1, c_2, c_3 and c_4 (autonomous control) are chosen such that the convergence rate λ is sufficiently strong in order to assure the stability of the system. Finally, due to the fact that P in (30) is bounded but non vanishing, the solutions of the whole system (31) are ultimately bounded [7].

4.2 Effects of the obstacles

This work proposes a fictitious force that modifies the longitudinal velocity reference like the behavior of a user when he sees obstacles. Such force and velocity reference are computed from:

$$ff(t) = k_{fic} i_o(t) \tag{33}$$

where ff is the fictitious force.
k_{fic} is a gain parameter, and
i_c is the inattention with respect to the obstacles. This fictitious force is used to modify the reference of longitudinal velocities, as follows:

$$\begin{aligned} v_{r_{expobst}} &= v_{r_{exp}} + ff(t) \\ v_{r_{obst}} &= v_r + ff(t-h_o) \end{aligned} \tag{34}$$

where $v_{r_{expobst}}$ is the new expert reference of longitudinal velocity,
$v_{r_{obst}}$ is the new user reference of longitudinal velocity and
h_o is the time delay produced by the inattention respect to the obstacles defined in (20).

This variation is bounded since i_o is also bounded and it can be absorbed by the term P in the system (32), so the stability proof is valid too.

4.3 Design of $k(t)$

The design of $k(t)$ of (21), is not intended to optimize the stability condition (the system will tend to an automatic controller) but rather to take advantage of the best information of the user control actions and collaborate in a way such that he does not reject any external help.

According to this policy, we propose involving the following two restrictions:

$$u_{human} \le u \le u_{exp}. \tag{35}$$

If $\Delta Control$ (21) is non null,

$$\Delta Control \propto |u_{\exp} - u_{human}||u_{expert} - u_h|\, sign(u_{exp}) \tag{36}$$

where (35) implies that the command applied must be bounded between the human command and a pattern command (u_{exp}), considering that best one is generated by a controller like an expert driver. Additionally, (36) states that $\Delta Control$ is proportional to the difference $|u_{exp} - u_h|$ between the model of the expert and the model of the distracted user and it always has the same sign of u_{exp}. In our case, the function $sign(x)$ is defined by a vector with components $sign(x_i)$ equal to -1 when $x_i < 0$ or 1 if $x_i \ge 0$. In order to comply with the conditions of design (21), (35) and (36), we propose defining $k(t)$ (22) as follows:

$$k_{vel} = \left(u_{1_{exp}} - k_{rel} \tanh\left(\left|u_{1_{exp}} - u_{1_h}\right|\right)\left(u_{1_{exp}} - u_{1_{human}}\right)\right)\Big) \tag{37}$$

where u_{1_h} is the accelerator command of the model with time delay.
$u_{1_{exp}}$ is the accelerator command of the expert.
$u_{1_{human}}$ is the accelerator command of the user.
$k_{rel} = 0.5 + \lim_{ks\to\infty}(1/\pi) arctan(ks(u_{1_{exp}} - u_{1_{human}}))$ with $ks \gg 1$.

$$k_{vol} = \left(u_{2_{exp}} - \left(\tanh\left(\left|u_{2_{exp}} - u_{2_h}\right|\right)\left(u_{2_{exp}} - u_{2_{human}}\right)\right)\right) \tag{38}$$

where u_{2_h} is the steer command of the model with time delay.
$u_{2_{exp}}$ is the steer command of the expert.
$u_{2_{human}}$ is the steer command of the user.
ε is an infinitesimal value in order to avoid a singularity. Putting (37) and (38) in (21), and considering that $(u_{exp} - u_{human}) = sign(u_{exp} - u_{human})|u_{exp} - u_{human}|$, we obtain:

$$\begin{aligned} u &= u_{human} + u_{exp} - \left(u_{exp} - \begin{bmatrix} k_{rel} & 0 \\ 0 & 1 \end{bmatrix} sign(u_{exp} - u_{human}) \right. \\ &\quad \left. tanh\left(\left|u_{exp} - u_h\right|\right)\left|u_{exp} - u_{human}\right| \right) \\ u &= u_{human} + \overbrace{\begin{bmatrix} k_{rel} & 0 \\ 0 & 1 \end{bmatrix} sign(u_{exp} - u_{human}) tanh\left(\left|u_{exp} - u_h\right|\right)}^{k_{aux}} \\ &\quad \left|u_{exp} - u_{human}\right| \\ u &= u_{human} + k_{aux}\left|u_{exp} - u_{human}\right| \end{aligned} \tag{39}$$

where $|k_{aux}| \le 1$ with $k_{aux} > 0$ if $u_{exp} > u_{human}$. Therefore, the condition (35) is verified. From (21) and (39), we obtain:

$$\begin{aligned} \Delta Control &= u_{human} + u_{exp} - \left(u_{exp} - \begin{bmatrix} k_{rel} & 0 \\ 0 & 1 \end{bmatrix}\right. \\ &\quad sign(u_{exp} - u_{human}) tanh\left(\left|u_{exp} - u_h\right|\right) \\ &\quad \left. \left|u_{exp} - u_{human}\right| \right) \\ \Delta Control &= \begin{bmatrix} k_{rel} & 0 \\ 0 & 1 \end{bmatrix} sign(u_{exp} - u_{human}) tanh\left(\left|u_{exp} - u_h\right|\right) \\ &\quad \left|u_{exp} - u_{human}\right| \end{aligned} \tag{40}$$

From (40), $\Delta Control \propto |u_{exp} - u_{human}|$ and $\Delta Control \propto |u_{exp} - u_h|$. From this, (36) is fulfilled, and therefore, the definition of $k(t)$ verifies the restrictions (35) and (36).

V. EXPERIMENTS

Next, several experiments have been made to show the performance of the designed control system, where the user provides motion control signals through a steering wheel and gas pedal, to drive a car simulator over a given road. Simultaneously, the direction of view of the user is estimated from image processing software applied to the frames captured on-line by a webcam, pointing to the user's head. To perform the experiments, a laptop HP dv4-400 (Core 2 Duo, 2.4 Ghz, 4 Gb RAM) with onboard webcam (1.3 Mp) and a Logitech steering wheel (Driving Force Pro) were used. The car simulator used is SPEED-DREAMS (http://www.speed-dreams.org/), which was chosen because it is an open source simulator and provides much information about the car and its environment. The model of the car implemented in the simulator is more complete and complex that the model presented in this paper, because it takes many variables into account, like force of the wind, friction with the road, and others. The Watson 5.0 program, developed by the Artificial Intelligence Laboratory at MIT (http://groups.csail.mit.edu/vision/vip/index.htm) is used to provide the pose (position angle, translation, and variance) of the head relative to the first image acquired. In addition, Watson 5.0 can be used with a stereo web cam. This program sends the computed data through the TCP/IP protocol. Fig. 9 shows the experimental setup used. In addition, software was developed to process the data and save it for later analysis and visualization using MATLAB (www.mathworks.com). Fig. 10 shows how the hardware and software components are linked. The program Interface2 (Fig. 10) estimates the visual inattention of the user (with respect to path and obstacles), using the information provided by the Watson and SPEED DREAMS programs. The longitudinal velocity reference v_r is set as the average of the car speed computed in the last 2 s, and the angular velocity reference w_r is computed based on the longitudinal velocity reference and the radius of the path provided by the car simulator. Fig. 11 shows the variables used to compute the compensated commands applied to the car simulator (accelerator command, steering command and brake command).

5.1 Path inattention compensation

In Fig. 12, an enlargement of the path is presented where the user has had a significant visual inattention with respect to the path and he doesn't make an optimal maneuver when the car goes onto the curve (t = 53.24 s). That behavior produces an increase of the error with respect to the center of path. In Figs 13 and 14, the behavior of the controller in this situation is shown.

In Fig. 13, it can be observed that when the inattention produces a time delay, the steering command applied is away

Fig. 9. Experimenter setup.

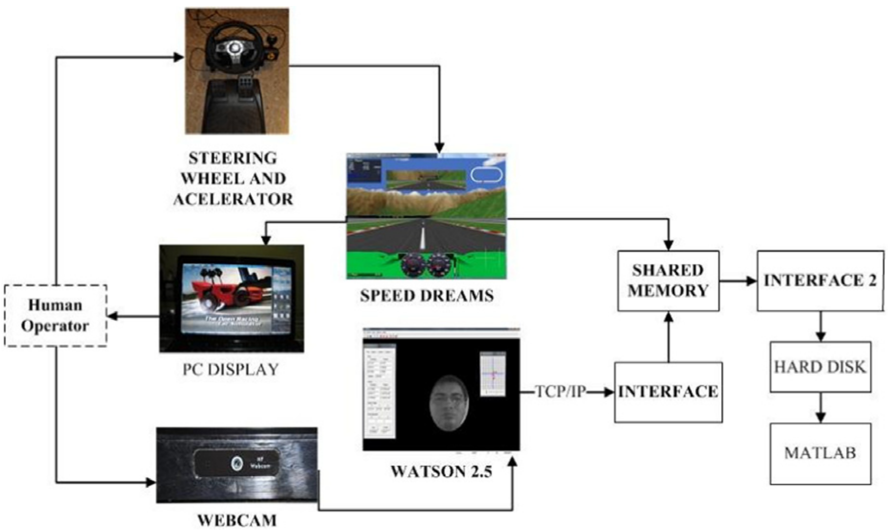

Fig. 10. Hardware and software components.

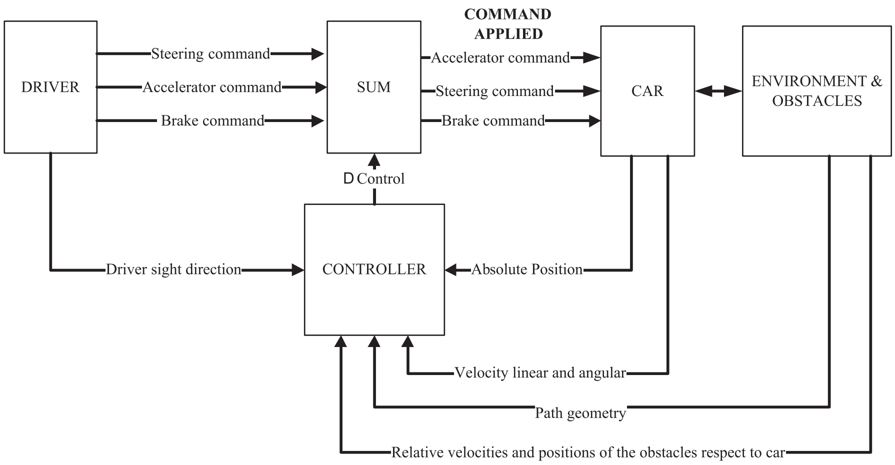

Fig. 11. Variables used for the proposed controller.

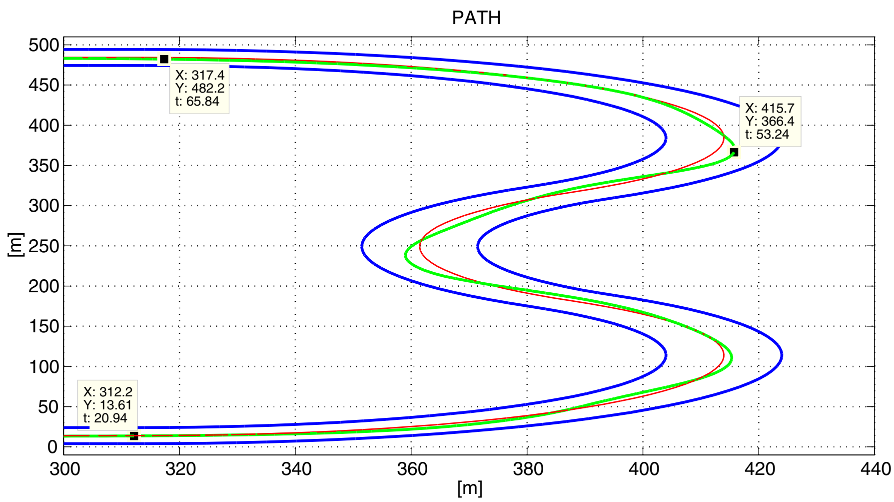

Fig. 12. Path followed by the car.

from the human command tending to the command expert. In addition, the applied command is always set between the human and expert command values.

On the other hand, the effects of the controller in the accelerator command when there is visual inattention can be viewed in Fig. 14. For this case, the proposed control causes a decrement of the accelerator command given by the user.

The designed controller never rises the accelerator command since an increase of the velocity not produced by the human could be dangerous and confusing for him. Generally, the expert accelerator command tends to decelerate the system because the reference speed of it is more conservative. The accelerator command is a representation of the virtual gas pedal.

Figs 15 and 16 show that the components of k are bounded in practice.

Finally, Table II shows the results (average of mean square error MSE and number of crashes) of 10 experiments, with and without controller, into the system in a car simulator for various users, using the path shown in Fig. 17. It is observed that the proposed collaborator decreases the mean square error (MSE) from the center of road and reduces the total number of crashes.

5.2 Obstacle inattention compensation

Fig. 18 shows a situation where a distracted user tries to overtake another car. The controller acts through the brake command when a user does not see an obstacle that has a significant crash probability (t = 48.9 s). When the self user recovers his attention, he can overtake the obstacle without interference of the controller (t = 57.5 s).

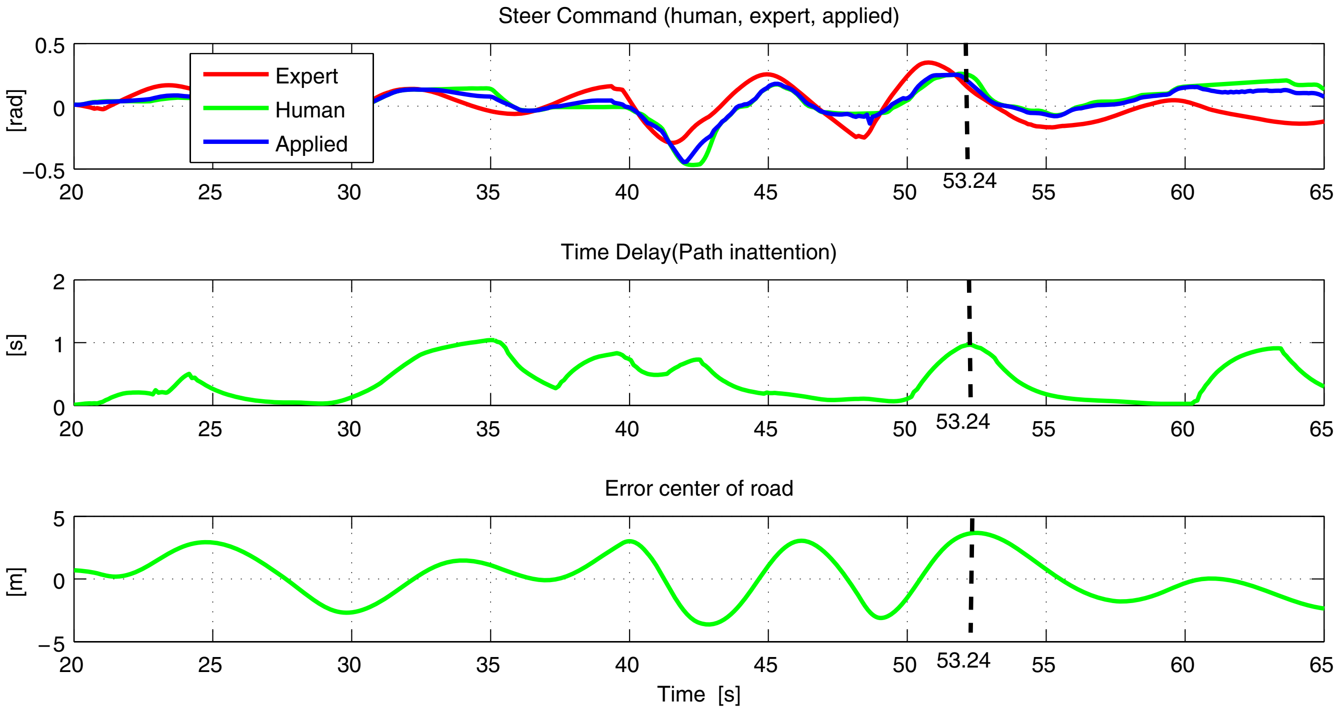

Fig. 13. Steer command, time delay, error to center of road of a user with a significant visual inattention respect to path.

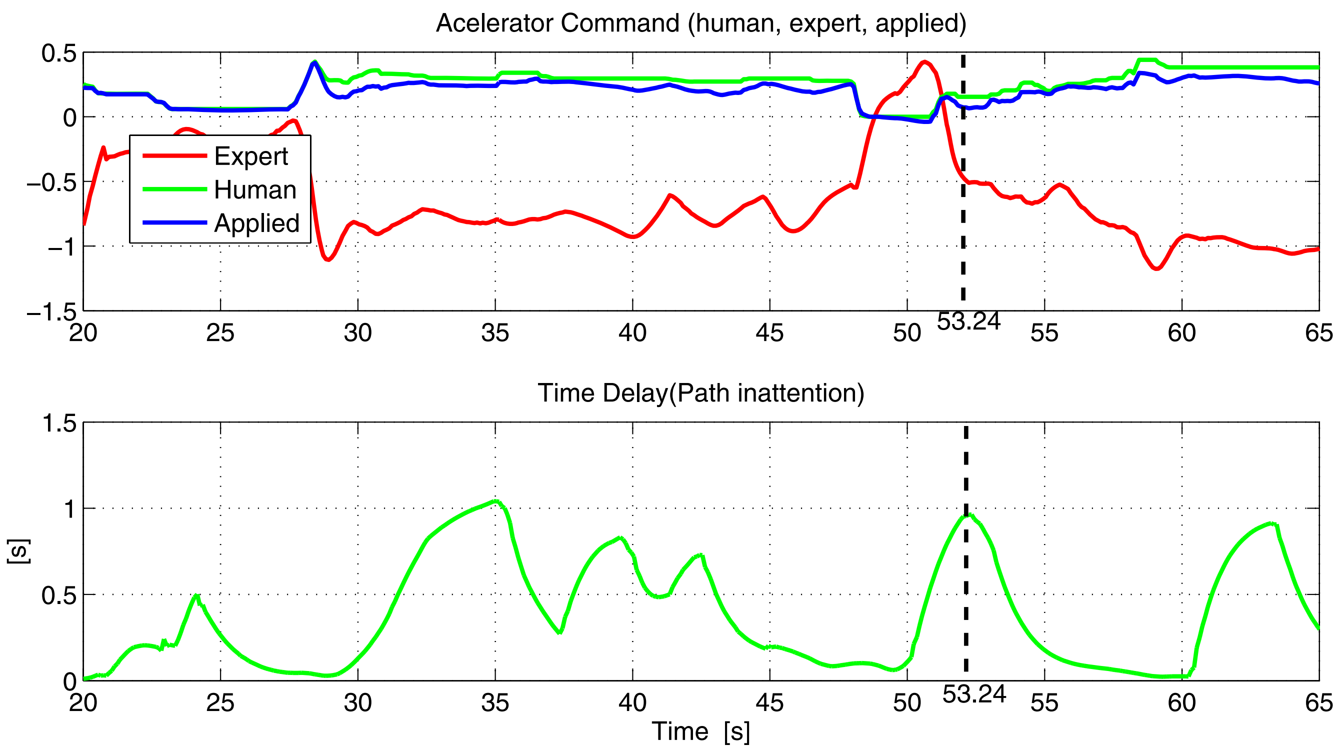

Fig. 14. Acelerator command, time delay of a user with a significant visual inattention respect to path.

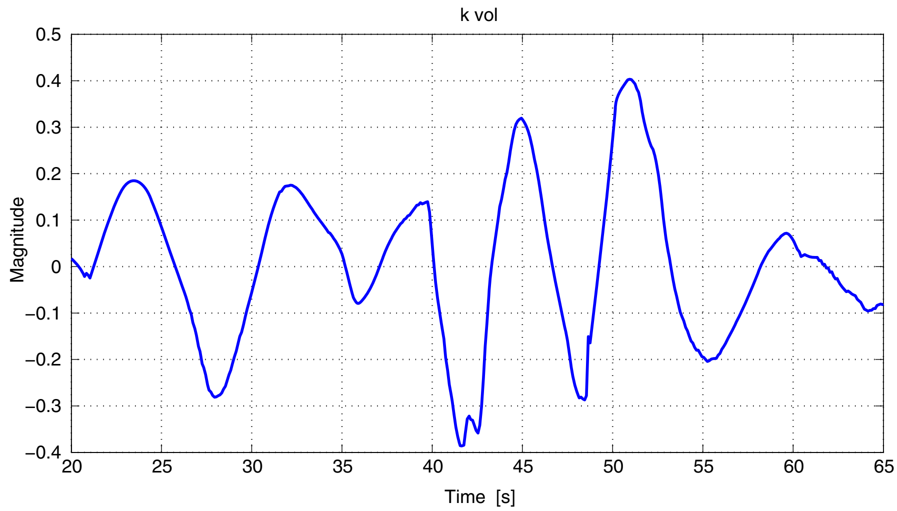

Fig. 15. Magnitude of K_{vol}.

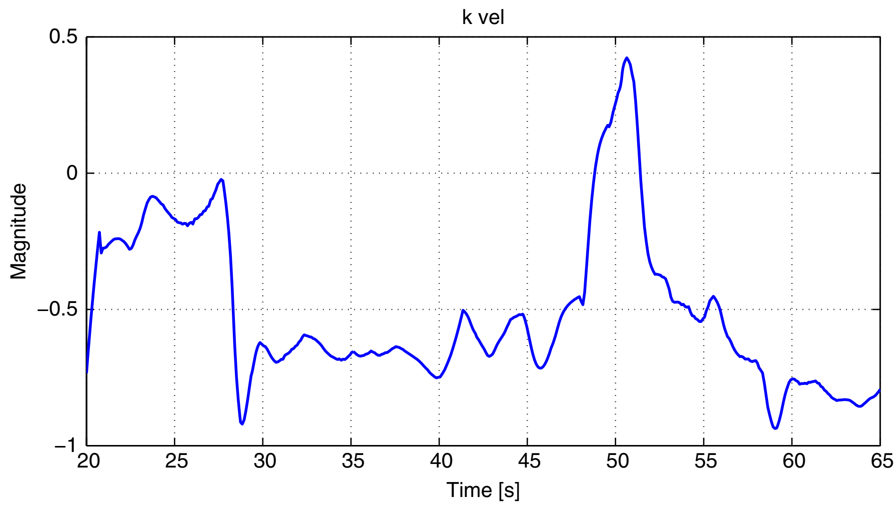

Fig. 16. Magnitude of K_{vel}.

Table II. Comparison of system with controller and without controller.

Activated Controller	MSE from center (valid experiments)	crashes/N^o experiments	% Path inattention
Yes	2.8646[m]	1/10	6.88
No	5.1277[m]	4/10	1.1

VI. CONCLUSIONS

In this paper, a new definition of the driver's visual inattention has been proposed. Such a definition can be computed on-line from the information measured about the state of the vehicle and its environment, as well as the state of the user. In addition, the user inattention can be represented by a time delay. Thus, the control theory of delayed systems can be used to design a controller to mitigate the effects of the visual inattention on the safety of a user driving a car-like vehicle. The parameters of the proposed control structure can be set so that $\Delta Control$ (21) can be sufficiently bounded so as not to disturb the common driving. Finally, experiments using a car simulator have shown that the designed system provides friendly help to the user when faced with visual distractions.

VII. APPENDIX

7.1 TIME DELAY SYSTEMS STABILITY [13]

Theorem 2. Let us consider a delayed system represented by:

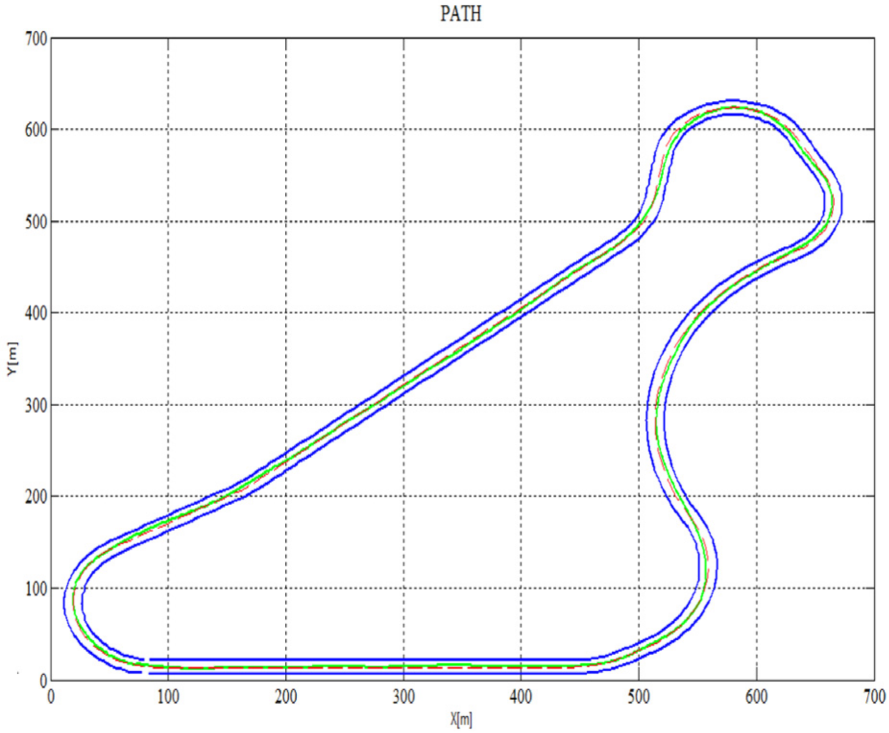

Fig. 17. Experimental path.

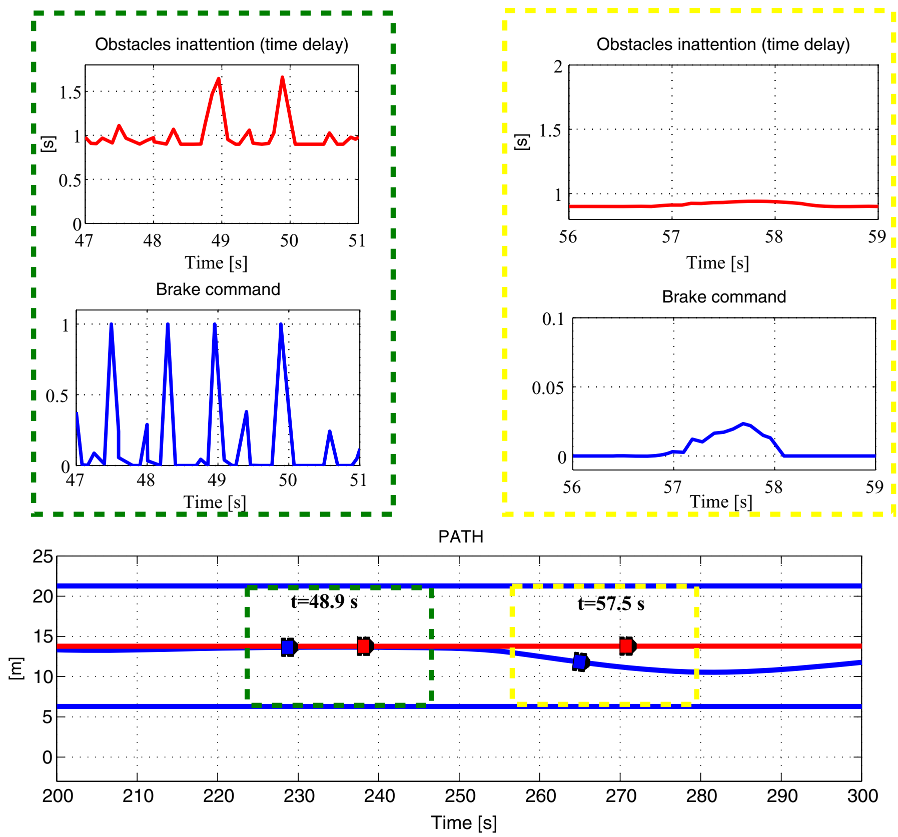

Fig. 18. Effect of the obstacles inattention.

$$\dot{x} = f(x) + g(x, x_h); \tag{41}$$

where $x_h = x(t-h)$, $0 \le h(t) \le h_m$, $\dot{h}(t) < \tau < 1$ and $\dot{x} = f(x)$ is globally exponentially stable with a convergence rate λ. Proposing a Lyapunov function as:

$$V = \frac{1}{2}x^T x + \frac{\frac{1}{2}|g|}{1-\tau}\int_{t-h(t)}^{t} x^T(z)x(z)dz > 0; \tag{42}$$

$$\dot{V} = x^T\dot{x} + \frac{\frac{1}{2}|g|}{1-\tau}x^T x - \frac{\frac{1}{2}|g|(1-\dot{h})}{1-\tau}x(t-h)^T x(t-h). \tag{43}$$

Inserting (41) into (43), $\dot{V}$ along the system trajectories can be obtained as follows

$$\begin{aligned}\dot{V} &= x^T f(x) + x^T g(x, x(t-h)) + \frac{\frac{1}{2}|g|}{1-\tau}x^T x\\ &- \frac{\frac{1}{2}|g|(1-\dot{h})}{1-\tau}x(t-h)^T x(t-h).\end{aligned} \tag{44}$$

Now, using norm properties

$$\begin{aligned}x^T g(x, x(t-h)) &\le |x||g|[|x| + |x(t-h)|]\\ &\le |g||x|^2 + |g||x||x(t-h)|\\ &\le |g||x|^2 + \frac{|g|}{2}|x|^2 + \frac{|g|}{2}|x(t-h)|^2\\ &\le \frac{3}{2}|g||x|^2 + \frac{1}{2}|g||x(t-h)|^2\end{aligned} \tag{45}$$

Introducing (45) into (44), yields,

$$\begin{aligned}\dot{V} &\le x^T f(x) + \frac{|g|}{2}\left[3 + \frac{1}{1+\tau}\right]x^T x\\ &+ \frac{|g|}{2}\left(1 - \left[\frac{1-\dot{h}}{1-\tau}\right]\right)x^T(t-h)x(t-h)\end{aligned} \tag{46}$$

The third term of the right handside in (46) is negative definite because $\dot{h} < \tau < 1$. Now considering $x^T f(x) \le -\lambda x^T x$ (Lemma 1, [13]) then (44) can be written as:

$$\dot{V} \le -\lambda x^T x + |g|\left[\frac{2 - \frac{3}{2}\tau}{1-\tau}\right]x^T x. \tag{47}$$

From (47), the system 41 is globally exponentially stable if $\lambda > |g|\left[\frac{2 - \frac{3}{2}\tau}{1-\tau}\right]$.

REFERENCES

1. Ahlstrm, C. and K. Kircher, Review of real-time visual driver distraction detection algorithms, *Proc. 7th Int. Conf. Meth. Tech. Behav. Res. (MB '10)*, New York, pp. 2:1–2:4 (2010).
2. Berardo, M. and V. Irureta, *Influencia de la Correcta Evaluacion del tiempo de Percepcion y Reaccion, (In Spanish)*, II Congreso Iberoamericano de Seguridad Vial (CISEV), Instituto Vial Iberoamericano (2009).
3. Chavez, D., E. Slawinski, and V. Mut, *Modeling the Inattention of a Human driving a car*, Symposium on Analysis, Design, and Evaluation of Human-Machine Systems, IFAC (2010).
4. Dong, Y., Z. Hu, K. Uchimura, and N. Murayama, *Driver Inattention Monitoring System for Intelligent Vehicles: A Review*, Intelligent Transportation Systems IEEE Transactions, pp. 875–880 (2010).
5. Hsieh, C.-S. and D.-C. Liaw, (2010), Continuous- and discrete-time fixed-gain controller designs for the control of vehicle lateral dynamics. *Asian J. Control.* Vol. 14, No. 2, pp. 359–372 (2010).
6. Jakubiak, J., A. Lefeber, K. Tchon, and H. Nijmeijer, *Two observer-based tracking algorithms for a unicycle mobile robot*, *Int. J. Appl. Math. Comput. Sci.*, Vol. 12, pp. 513–522 (2002).
7. Khalil H., *Nonlinear Systems Second Ed.*, Prentice Hall, Upper Saddle River, New Jersey (1996).
8. Leclair R., *Manual Centroamericano: Normas para el Diseno Geometrico de las Carreteras Regionales (In Spanish)*, Secretara de Integracin Econmica Centroamericana, SIECA (2001).
9. Malagari, S. and B. J. Driessen, (2010), Globally exponential controller/observer for tracking in robots without velocity measurement. *Asian J. Control.* Vol. 14, No. 2, pp. 309–319 (2010).
10. Panteley, E., E. Lefeber, A. Lora, and H. Nijmeijer, *Exponential Tracking Control of a Mobile Car Using a Cascaded Approach*, Proc. IFAC Workshop Motion Control, Grenoble, France (1998).
11. Pohl, J., W. Birk, and L. Westervall, *A driver-distraction-based lane-keeping assistance system*, Proc. IMechE Vol. 221 Part I: J. Systems and Control Engineering, No. 4, pp. 541–552 (2007).
12. Ranney, T., *Driver Distraction: A Review of the Current State-of-Knowledge*, National Highway Traffic Safety Administrations (2008).
13. Slawinski, E., V. Mut, and J. Postigo, Stability of systems with time-varying delay, *Latin Am. Appl. Res.*, Vol. 36, No. 1, pp. 41–48 (2006).
14. Smith, P., M. Shah, and N. Da Vitoria Lobo, *Determining driver visual attention with one camera*, Intelligent

Transportation Systems, IEEE Transactions on, Vol. 4, No. 4, pp. 205–218 (2003).
15. Wu, Q., *An overview of driving distraction measure methods*, Computer-Aided Industrial Design Conceptual Design on, pp. 2391–2394 (2009).
16. Young, K., M. Regan, and M. Hammer, *Driver Distraction: A Review of the Literature*, Monash University Accident Research Centre, Report No. 206 (2003).

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