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Toward a dynamo model for the solar tachocline

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Abstract

The generation of magnetic field on the Sun is caused by a dynamo mechanism, and is related with the differential rotation of the solar interior. Helioseismologic observations show that strong velocity gradients concentrate in a thin layer located at the base of the convective region, known as the tachocline. This remarkable observational finding lends support to a theoretical model for the solar dynamo, based on a magnetohydrodynamic extension of the so called shallow water approximation. We developed a numerical code to integrate the dynamic equations. Our analysis shows an initial stage during which the magnetic energy grows exponentially fast. In a subsequent stage we find that magnetic energy keeps growing until it reaches equipartition with the mechanical energy of the flow.

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1. Introduction

One of the traditional dynamo models for the Sun, the so-called $\alpha - \Omega$ dynamo, assumes the competition of two processes: the generation of a toroidal field (i.e., along the direction of rotation) by shearing a pre-existing poloidal field (i.e., with only radial and azimuthal components) by differential rotation (Ω-effect), and a source to re-generate the poloidal field out of the toroidal component. For instance, the re-generation of the poloidal field can be caused by helical turbulence driven by the Coriolis force (α-effect, see [1]). According to mean field theory, the α-effect corresponds to a net electromotive force exerted by microscopic helical motions and inducing a large-scale magnetic field.

Recently, helioseismologic observations of the Sun have established the profiles of the flows in the solar interior. It was found that the differential rotation of the Sun has a strong shear in a thin layer at the base of the convective region, called *tachocline*. This layer comprises less than 3% of the solar radius [2]. In this region, the magnetic flux tubes must be intense enough as to travel through the convection zone without being destroyed by the turbulent motions [3]. As the strongest velocity shear (i.e., the Ω-effect) is concentrated in the solar tachocline, it is now believed that the solar dynamo must also operate in this thin region. Considering that the tachocline is thin, an extension of the "shallow water" model including magnetic fields (hereafter "shallow MHD" or SMHD) has been recently presented [4], although within the ideal approximation (i.e., neglecting dissipative effects). Assuming dynamo action in the tachocline, leads to consider whether some kind of α-effect (traditionally related to helical turbulence in the convective zone) can operate in this thin region of the solar interior.

Evidence of the existence of turbulence in the solar tachocline has recently been presented [2]. The aim of this work is to study the feasibility and the efficiency of turbulent magnetic field amplification in SMHD, with the aid of direct simulations in this particular geometry. The SMHD approximation is of course much simpler than the more general three-dimensional MHD, and therefore the existence of a mechanism for the amplification of magnetic fields in this geometry might bring a new insight to the present status of the theory. The SMHD approximation has also been used to describe problems involving a free surface flow of conducting fluids in laboratory and industrial environments [5].

In Section 2 we briefly present the shallow water approximation. The characteristics of the magnetic field generation observed in our simulations are given in Section 3, and the main results are summarized in Section 4.

2. The SMHD equations

The shallow water approximation has been extensively used in the context of hydrodynamics to model the dynamics of free surface fluids in channels [6], ocean and atmospheric dynamics [7], the atmospheres of jovian planets [8], and mass accretion in binary stars [9].

A generalization of this approximation to the MHD case has been developed [4], which can be used to model the dynamics of thin layers in the interior of stars (such as the solar tachocline) and dynamo mechanisms in thin accretion disks. It has also been used for the theoretical description of free surface flows of conducting fluids in laboratory and industrial environments [5,6]. The equations of Navier–Stokes for the velocity field, the induction equation for the magnetic field and mass continuity equation in this context reduces to

$$\frac{\partial \boldsymbol{u}}{\partial t} = -\boldsymbol{u} \cdot \nabla \boldsymbol{u} + \boldsymbol{B} \cdot \nabla \boldsymbol{B} + g \nabla h + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{F} \,, \tag{1}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\boldsymbol{u} \cdot \nabla \boldsymbol{B} + \boldsymbol{B} \cdot \nabla \boldsymbol{u} + \eta \nabla^2 \boldsymbol{B} \,, \tag{2}$$

$$\frac{\partial h}{\partial t} = -\nabla \cdot (h \boldsymbol{u}) \,. \tag{3}$$

The unknown $\boldsymbol{u} = \boldsymbol{u}(x, y)$ and $\boldsymbol{B} = \boldsymbol{B}(x, y)$ are the two-dimensional (i.e., such that $\hat{z} \cdot \boldsymbol{u} = 0 = \hat{z} \cdot \boldsymbol{B}$) velocity and magnetic fields, and $H = H_0[1 + h(x, y, t)]$ is the local height of the layer, where H_0 is the equilibrium height. The last term in Eq. (1) is an external force, and ν and η are respectively the kinematic viscosity and magnetic diffusivity. The effect of magnetic diffusivity is essential, since some degree of magnetic reconnection is required for the enhancement of the magnetic field.

The total energy in SMHD involves kinetic, magnetic and potential energy, i.e.,

$$E = \int \left[\frac{(1+h)}{2} \left(|\boldsymbol{u}|^2 + |\boldsymbol{B}|^2 \right) + \frac{g H_0}{2} (1+h)^2 \right] \mathrm{d}x \, \mathrm{d}y \,. \tag{4}$$

A relevant quantity in dynamo theory is the kinetic helicity which in this geometry takes the following expression,

$$\Gamma = \frac{1}{2} \int H_0^2 (1+h)^2 [\nabla (\nabla \cdot \boldsymbol{u}) \times \boldsymbol{u} - (\nabla \cdot \boldsymbol{u})(\boldsymbol{\nabla} \times \boldsymbol{u})] \cdot \hat{z} \, \mathrm{d}x \, \mathrm{d}y \,. \tag{5}$$

Note that the kinetic helicity in Eq. (5) partially corresponds to vertical swirling motions, i.e., to a correlation between vortices an up-down flows (since $u_z \propto \nabla \cdot \boldsymbol{u}$). Mean field theory establishes that the α-effect is proportional to the kinetic helicity in the flow. Also, direct numerical simulations confirm that helical turbulence is a key ingredient for the generation of large-scale magnetic fields [10].

3. Generation of magnetic fields in SMHD

We integrated the system of Eqs. (1)–(3) in a square box with periodic boundary conditions. Spatial derivatives were computed using a pseudo-spectral scheme [11], with the $\frac{2}{3}$ rule to control aliasing truncation errors. The equations were evolved in time using a second order Runge–Kutta method. We performed several runs with 256^2 grid points. We are interested on general properties of hydrodynamic and magnetohydrodynamic turbulence under this approximation, and therefore we will not introduce differential rotation in our simulations. In other words, we are only exploring the α-effect, i.e., the ability of microscale helical motions to generate large-scale magnetic fields.

We first integrate Eq. (1) applying a helical forcing $\boldsymbol{F}$ centered at $k_F = 2$ to develop a purely hydrodynamic turbulent regime ($\boldsymbol{B} = 0$ during this stage). As showed in our Eq. (5), the α-effect in SMHD takes place in regions where the divergence ($\nabla \cdot \boldsymbol{u}$) and the vorticity ($\nabla \times \boldsymbol{u}$) are strongly correlated. Therefore, the external force $\boldsymbol{F}$ has been chosen with a highly correlated divergence and curl to induce this regime. In Fig. 1 we show the behavior of the kinetic (i.e., $\int (1+h)/2|\boldsymbol{u}|^2 \,\mathrm{d}x\,\mathrm{d}y$) and potential energy (i.e., $\int gH_0/2(1+h)^2 \,\mathrm{d}x\,\mathrm{d}y$, see Eq. (4) as a function of time. We can clearly see that in spite of the intermittent behavior of both series, the system has reached a stationary regime.

In a second stage of the simulations, we add a non-helical and very small magnetic seed. This initial magnetic field is generated by a δ-correlated vector potential at a spatial scale smaller than the forcing scale ($k_{seed} = 8$). The run was continued with the same helical force $\boldsymbol{F}$ in the Navier–Stokes equation, to study the growth of magnetic energy. Fig. 2 shows the mechanical (i.e., kinetic plus potential) and magnetic energy as a function of time.

At the beginning, the magnetic energy grows exponentially from the initial energy value of the small-scale seed. After this stage, the magnetic energy saturates and reaches a regime of approximate equipartition with the mechanical energy. Indeed, our direct simulations verify that the statistical correlation between vortices and up–down flow motions produces the expected rise in the magnetic energy. However, this value of magnetic energy cannot be maintained for long times if the magnetic energy remains at small scales, since resistive dissipation might be non-negligible. Therefore, for the mechanism to be efficient, magnetic energy must grow also at large

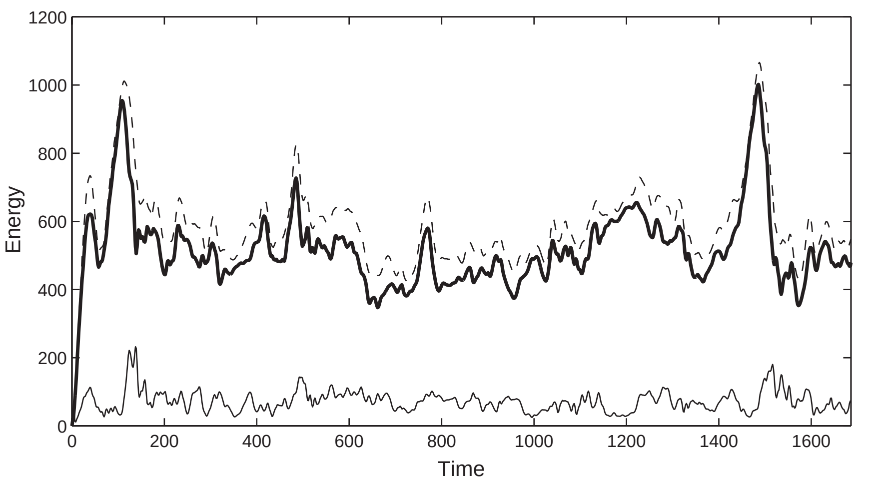

Fig. 1. Kinetic (thick trace), potential (thin trace) and total mechanical energy (dotted line) vs. time.

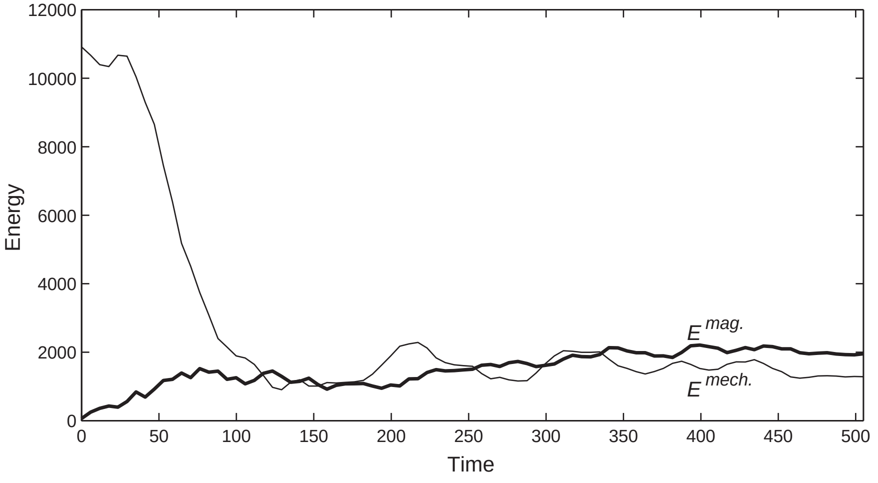

Fig. 2. Mechanical (thin trace) and magnetic (thick trace) energy vs. time.

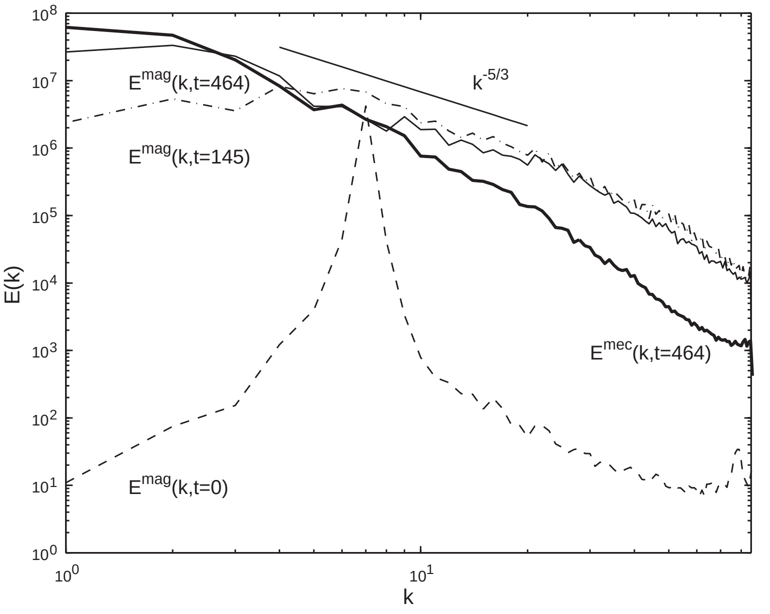

Fig. 3. Energy power spectra. The thick line correspond to the mechanical energy. Magnetic energy spectra taken at different times are plotted with different traces, as indicated. The Kolmogorov slope is displayed for reference.

scales. In Fig. 3 we show the spectral evolution of the magnetic energy, starting from the seed spectrum and progressively shifting toward large scales until it saturates at equipartition. The thick line corresponds to the spectrum of mechanical energy.

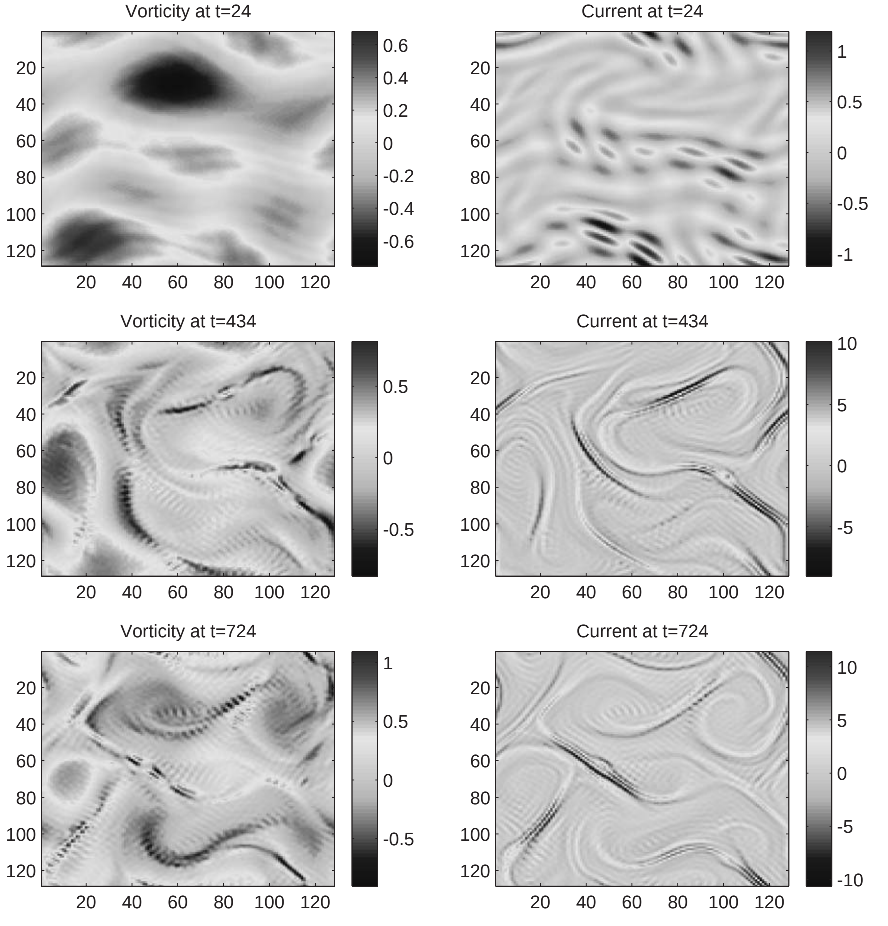

Fig. 4. Vorticity patterns $\boldsymbol{z} \cdot \nabla \times \boldsymbol{u}$ at different times are shown on the left panels, while patterns of electric current density $\boldsymbol{z} \cdot \nabla \times \boldsymbol{B}$ are displayed on the right panels.

In Fig. 4 we show the spatial distribution of vorticity and electric current density at different times. We see the formation of fine structures in both fields, which are the regions where the energy dissipation preferentially takes place.

In Fig. 5 we show the evolution of magnetic energy for three runs containing different levels of kinetic helicity. The connection between the content of kinetic helicity in the turbulent flow and the efficiency in generating magnetic fields becomes clearly apparent.

4. Conclusions

Several models for the re-generation of the solar poloidal field have been proposed, which differ from one another on the physical mechanisms and the

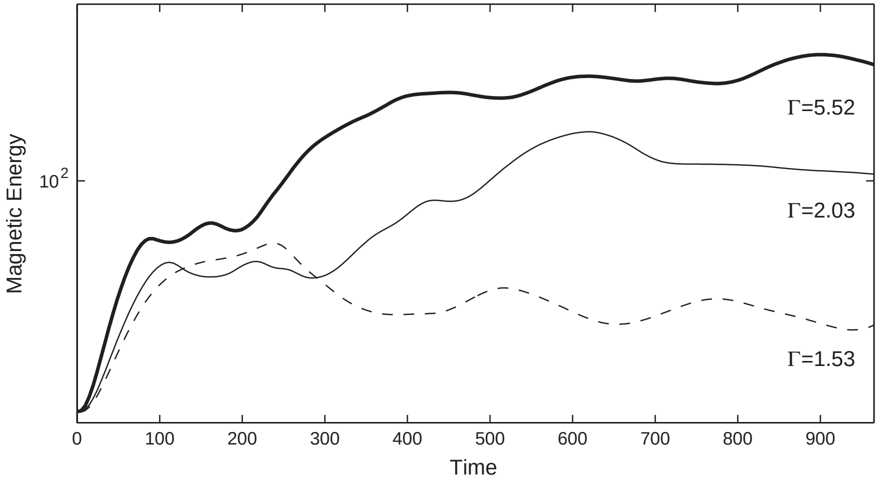

Fig. 5. Magnetic energy vs. time for three simulations with different levels of kinetic helicity (indicated).

location of this source. The main goal of this work is to show that this process can also take place in a thin layer under the SMHD approximation. With the aid of numerical simulations of the SMHD equations, we show that a considerable amplification of the magnetic field takes place in regions where the vorticity and the divergence (i.e., up–down flows perpendicular to the tachocline) of the velocity field are correlated.

Although the magnetic field is indeed amplified, in order to have a realistic dynamo model several ingredients must be added. While the effect of differential rotation (Ω-effect) is relatively well understood, the conversion of toroidal into poloidal field has many details that should be taken into account. The solar tachocline is not an isolated layer. Therefore, after a transient amplification of magnetic field in this layer, the magnetic flux is expected to rise and interact with the upper layers of the solar atmosphere. Later amplification or conversion could take place in the convective region or at the solar surface. In spite of the simplicity of our model, the fact that the magnetic energy can efficiently grow until it reaches equipartition with the mechanical energy, lead us to believe that the SMHD approximation can play a significant role in understanding some key features of magnetohydrodynamic dynamos, with potential applications to solar and stellar interiors and accretion disks.

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