Journal Pre-proof

Augmented flow and reduced clogging of particles passing through small apertures by addition of fine grains

Sandip H. Gharat, Luis A. Pugnaloni

PII: S0032-5910(23)00479-5
DOI: https://doi.org/10.1016/j.powtec.2023.118695
Reference: PTEC 118695

To appear in: *Powder Technology*

Received date : 22 February 2023
Revised date : 17 May 2023
Accepted date : 28 May 2023

Please cite this article as: S.H. Gharat and L.A. Pugnaloni, Augmented flow and reduced clogging of particles passing through small apertures by addition of fine grains, *Powder Technology* (2023), doi: https://doi.org/10.1016/j.powtec.2023.118695.

Augmented flow and reduced clogging of particles passing through small apertures by addition of fine grains

Sandip H. Gharat[a], Luis A. Pugnaloni[a]

[a]*Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad Nacional de La Pampa, CONICET, Uruguay 151, 6300 Santa Rosa (La Pampa), Argentina*

Abstract

The e?ect on the flow and clogging of a sample of particles passing through an orifice due to the addition of a second fine-graded species is investigated. The flow rate of the main species is measured for various parameters: the mass ratio of the big species, the particle size ratio and the orifice diameter. We show that when the fine grains are added into the system the flow rate of the larger species can be increased and its clogging significantly reduced. In particular, we were able to flow (without clogging) the big species through an orifice only 1.5 times its particle diameter. This allows for applications such as the alignment of particles in a narrow tube without clogging. A simple state diagram is presented to describe the clogging transition for these binary mixtures. The experimental results are compared with various existing models for the flow rate of binary mixtures.

Keywords: Clogging, Granular discharge, Binary Mixtures

Email address: `luis.pugnaloniX@exactas.unlpam.edu.ar` (Luis A. Pugnaloni)

1. Introduction

The discharge of granular mixtures from silos is observed widely in industries such as chemical, food, mining, ceramic, paint, and pharmaceutical. When a granular material flows through a constriction which is only a few particle diameters across, clogging becomes a mayor issue [1]. In the industry, clogging is generally regarded as a problem to be avoided and different approaches are used to prevent clogging or remediate it by unclogging jammed orifices. Examples of this are the use of vibration [2, 3], placing obstacles before the constriction [4, 5], oscillating the base [6] and introducing air flows [7], among other others.

Recently Madrid et al. [8] studied the flow and clogging of circular grains in a two-dimensional (2D) system under vibration using DEM simulations. They showed that the addition of smaller grains into the original system enhances the flow of the original species. They operated in the clogging regime (small orifices) to enhance the effective flow by making the clogging arches less likely to form, and, when formed, less stable under vibration. One interesting aspect of that study is the fact that the authors focused on the behaviour of the original species (the flow rate of the original particles) rather than the total flow rate of composite binary system. Although the flow rate of the binary system can be higher than the original pure system, the focus was to obtain an improvement in the effective flow rate of the original species. In an earlier study, Benyamine et al. [9] studied the flow rate of binary mixtures and pointed out that the large particles, which would otherwise clog a given orifice, were capable of flowing continuously when they were in a mixture with smaller grains. That study consisted in simple discharges of a three-dimensional (3D) silo with no vibrations applied. However, these authors only reported the flow of the binary system and did not focus on the behaviour of the large grains themselves.

In the past, a few studies have reported on the discharge of binary granular mixtures in silos, but none of these focused on the flow rate of one of the species alone. Apart form the work of Benyamine et al. [9], Arteaga et al. [10] and Humby et al. [11] made significant progress in measuring the flow rate of binary mixtures and their modeling based on a modification of the well established Beverloo equation [12] generally used for roughly monosized systems. In a separate contribution, Madrid et al. [13] studied the discharge of binary mixtures in 3D using both, experiments and simulations, providing results compatible with those form previous references. They also adapted a more recent model for the flow rate, alternative to the Beverloo model, proposed by Janda et al. [14]. Recently, Li et al. [15] studied binary dense granular flows in a two-dimensional hopper using DEM. Also Zhang et al. studied binary mixture discharges via DEM but in a rectangular 3D hopper [16]. The simulation results validate Beverloo's law for the relationship between flow rate and outlet size. Similar studies on the flow and clogging of binary granular mixtures through sieves have been carried out by Chevoir et al. [17] controlling two parameters: the proportion of large grains and the ratio of large grains to sieve hole size. They observed three regimes: steady flows, jamming, and progressive clogging. They provided a quantitative measurement of the flow rate, jamming threshold and clogging time.

When small particles are added to the originally monosized system, these convey an effective reduction of the volume fraction of the original species.

The fine grains tend to flow without clogging while they keep larger grains apart from each other, potentially preventing them from cooperating to form blocking arches at the orifice during flow. With this mechanism in mind, it is natural to connect this work with those from Roussel et al. [18] and Aguirre et al. [19]. On the one hand, Roussel et al. [18] experimentally studied the clogging in filtration processes. They suspended grains in a gel to control the packing fraction of a sample that is then passed through a sieve (a type of filter). On the other hand, Aguirre et al. [19] used a conveyor belt to study flow of circular particles through apertures. They were able to control the area fraction by placing particles apart from each other on the belt before discharge. They reported how the packing fraction is modified by the presence of the outlet. Although there is a point in common with these studies regarding the control of the partial volume fraction of the species of interest, in our experiments there is always the interaction of the fine grains involved. This is different from the particle–gel interaction in the experiments of Roussel et al. [18] or the belt–particle interaction in the work of Aguirre et al. [19]

Inspired by Refs. [8] and [9], we aim at studding the effect of adding a fine-graded sample of grains to a system of monosized particles that flows through a narrow orifice. Like Benynine et al. [9], we carry out experiments in a 3D silo without vibrations and therefore measure the flow only in the continuous flow regime. Like Madrid et al. [8], we consider the fine-graded particles as an additive used to alter the behaviour of the species of interest (the large grains). We perform various experiments and asses the possible enhancement of the flow of the original particles. In particular, we consider how clogging can be avoided by addition of the smaller grains and what is the maximum flow rate of the species of interest that can be obtained for a given orifice. We vary the size of the orifice, the size of the added grains and the mass fraction of the original particles. We present a clogging state diagram for the binary mixture. Based on existing models, we estimate the flow rate of the original particles as a function of their mass fraction and discuss about the limitations of all available models to predict the flow rate for mixtures with large particle size ratios.

2. Experimental details

2.1. Experimental setup

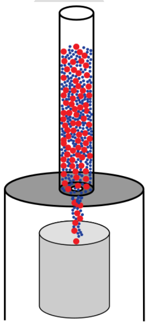


Figure 1: Schematic diagram of the experimental setup. The red circles correspond to the original species (large particles) and the blue circles to the added small particles.

The experimental work was carried out using an acrylic cylindrical silo as sketched in Fig. 1. The height of the silo is 34.5 cm and its internal diameter is 5.0 cm. Acrylic circular disks with orifices of different sizes (3.00 mm to 10.00 mm) are placed at the bottom of the silo to discharge the granular samples.

Three di erent glass bead materials (ρ = 2500 kg/m^3) are used to prepare the binary mixtures. For the big particles we use glass beads of d = 2.0 ± 0.2 mm in diameter. For the fine particles we use either d_f = 0.75 ± 0.2 mm (*fine1*) or d_f = 0.10 ± 0.08 mm (*fine2*) glass beads. A known weight of small (either *fine1* or *fine2*) and big particles were mixed together into a container. We call *system I* the mixtures made of large and *fine1* particles and *system II* the mixtures prepared with large and *fine2* particles. System I corresponds to a particle size ratio d/d_f = 2.6 and system II to d/d_f = 20.0. We define the mass fraction χ of the large particles as $\chi = M_b/(M_b + M_f)$, with M_b and M_f the mass of the large and fine species, respectively. For each system (I or II) we prepared binary mixtures with $0 \leq \chi \leq 1$. Samples were mixed manually in a shallow container till we observed a well mixed system. To reduce segregation during filling, we poured the mixture through a funnel in 10 to 12 small batches, allowing time for each batch to settle. We used ≈ 1.0 kg of granular material in each experiment.

During filling, the bottom orifice remains closed. Once the silo is filled with the binary mixture, the outlet is opened and the material discharges under gravity. A small vessel was kept at the bottom of the silo to collect the discharged material. The time required for the full discharge of the silo is recorded using a stop watch. We calculate the e ective flow rate Q_b of the big particles dividing the total mass of this species that we put into the mixture by the total time required to discharge the mixed material. Each run was repeated three times and we report the average over the three realizations. The range of humidity and temperature during the experiments were 42–66 % and 18–25^{o}C, respectively. Under these conditions the flow is observed to be continuous and cohesion becomes negligible.

2.2. Clogging

Depending on the size of the particles, the mix ratio and the size of the orifice, the system may clog before discharging the full sample in the silo. In such cases we do not measure the flow rate. In some cases clogging occurs in all realizations of the experiment (*full clogging*). Under some circumstances, the flow may clog in some realizations but not in others. We use error bars that extend from the last condition of continuous flow to the first condition of *full clogging* to indicate the range of values of the parameter of interest (particle size, mix ratio, etc.) within which one expects to observe the transition into the clogging regime.

2.3. Bulk density

Binary mixtures may show some degree of variation in the bulk density ρ_b as a function of the mix ratio [10]. We have measured ρ_b for a range of χ for systems I and II. After filling of silo as explained above with a known mass we measure the height of the column and calculate ρ_b. The bulk density as a function of χ for systems I and II is shown in Fig. 2. In agreement with previous studies [9, 10, 13], the bulk density attains a maximum at intermediate mix fractions. The larger the size ratio is, the larger is the maximum.

2.4. Segregation

Segregation during the discharge of silos is common and it affects the efficacy of industrial operations. Segregation occurs due to differences in

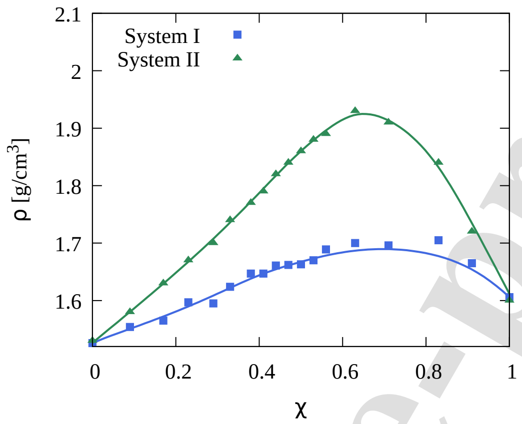


Figure 2: Bulk density of the binary mixture versus mass fraction of the large particles (χ) for systems I and II. The lines are only a guide to the eye.

particle size, shape and density. We have assessed the segregation for system I and II using the 10.0 mm orifice. During discharge, small samples ($\approx$ 50 g) are collected from the flowing material at the start and at the end of the discharge. To avoid the transients at the beginning and at the end of the discharge, the first sample is collected 2-3 seconds after the flow starts and the last sample 2-3 seconds before the silo gets empty. Samples are then segregated using a sieve and the fractions of large and fine particles weighted. Then, we calculated the values of χ for the initial sample (at the beginning of the discharge, χ_i) and the final sample (at the end of the discharge, χ_f). Figure 3 shows the difference $\chi_i - \chi_f$ against χ. For system I, the initial and final samples show the same mass fraction of the large particles, indicating that the flow has not segregated the system. However, for system II, segregation is observed for $\chi \geq 0.7$. By visual inspection we

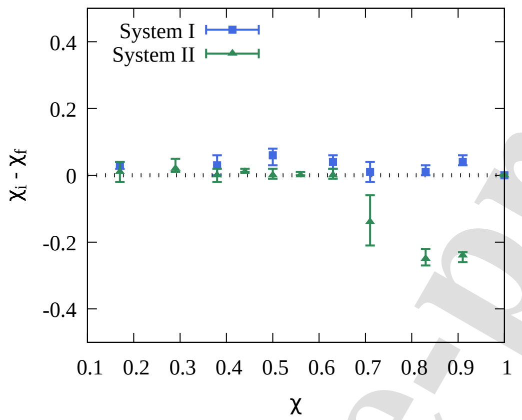


Figure 3: Difference between initial and final mass fraction ($\chi_i - \chi_f$) of the large particles during discharge as a function of the value χ in which the full sample was initially prepared. Experiments correspond to an orifice D = 10.0 mm. The dotted horizontal line is the reference for no segregation. Error bars indicate the maximum and minimum obtained over three realizations.

observed that during flow the small particles percolate between the gaps of the large particles. As a consequence, the flow of system II for $\chi \geq 0.7$ is significantly different from a homogeneous mixture. The initial part of the discharge is mainly composed of fine grains and the final part is composed mostly by big particles. The total flow rate gets affected by this segregation as we will discuss below.

3. Experimental results

The mass flow rate Q_b of the large species for systems I and II as a function of χ for different orifice diameter is shown in Fig. 4. We note that this is not the total flow rate since there are also small particles flowing

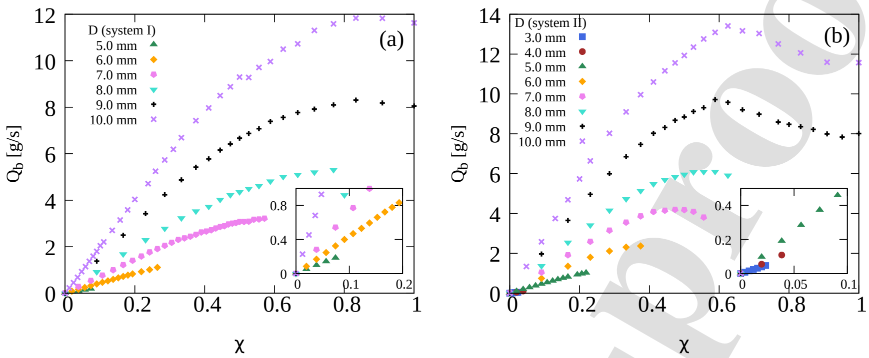


Figure 4: Mass flow rate of the large particles (2.0 mm) versus χ for: (a) system I and (b) system II. Each series corresponds to a different orifice diameter (see legend). The insets show closeups at small χ.

whose mass is not included in the calculation of Q_{b}. As we can see, for small orifices the flow rate cannot be measured when the fraction of big particles is large due to clogging. The smaller the orifice, the lower the maximum value of χ at which a continuous flow can be observed (we will discuss this in detail at the end of this section). Q_{b} generally increases with χ. This is expected since an increase of the proportion of the big particles would imply that the total flow will contain a larger amount of this species. However, for system I using $D = 9.0$ mm and 10.0 mm, the flow is continuous even at $\chi = 1$ and a maximum Q_{b} is observed a $\chi < 1$. This suggests that the addition of a small quantity of a fine species can increase the effective flow rate of the large species. The effect is much more pronounced in system II, where the particle size ratio is 20. The flow rate of the large particles for the large orifices can be increased up to 17% by adding about 40% in mass of the finer grains. For system II, when $\chi > 0.60$, the flow rate Q_{b} decreases rather sharply with the curvature becoming concave up (see Fig. 4(b)). This is due to the segregation observed during flow for the large particle size ratio (see Fig. 3). Note that the case $\chi = 1$ does not present segregation since the system is monodisperse in this limit. If one disregards the segregating mixtures of system II ($0.60 < \chi < 1$) it is still clear that a well mixed continuous discharge with $Q_{\rm b}$ significantly larger than the one observed for the pure system can be obtained.

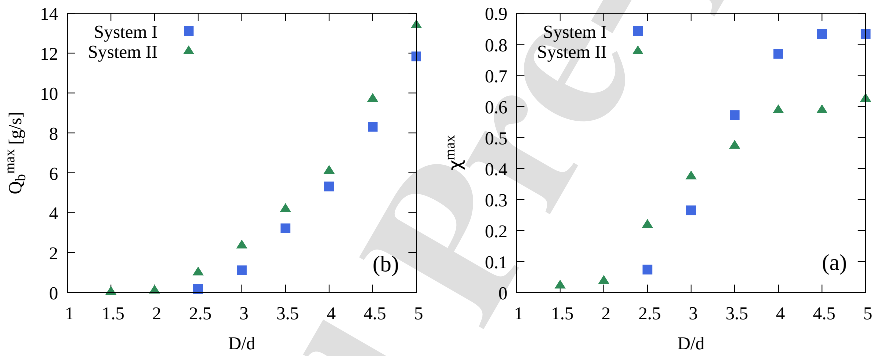


Figure 5: (a) Maximum flow rate $Q_{\rm b}^{\rm max}$ of the big particles as a function of D/d for systems I and II. (b) Mass fraction $\chi^{\rm max}$ at which the maximum flow rate is observed as a function of D/d (d = 2.0 mm).

Figures 5(a) and 5(b) show the maximum flow rate $Q_{\rm b}^{\rm max}$ observed in Fig. 4 and the corresponding $\chi^{\rm max}$ as a function of the dimensionless outlet size D/d, for each system. We note that for small orifices the maximum $Q_{\rm b}^{\rm max}$ is often achieved at the highest value of χ where a continuous flow is possible (see Fig. 4). $Q_{\rm b}^{\rm max}$ increases when D/d increase, as expected. System II yields higher $Q_{\rm b}^{\rm max}$ than system I for all D/d. However, for $D/d \geq 3.5$, $\chi^{\rm max}$ is lower for system II than for system I, which implies that achieving the maximum flow rate of the big particles requires a larger fraction of the fine species if the particle size ratio is large.

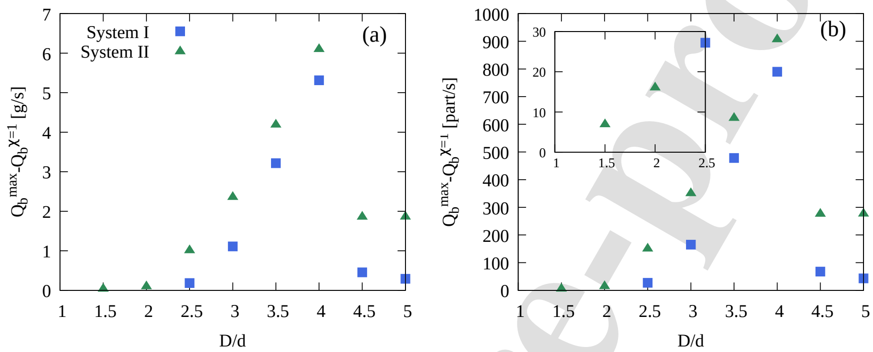


Figure 6: Maximum excess flow ($Q_b^{max} - Q_b^{\chi=1}$) obtained (a) in g/s and (b) in particles per second, versus the dimensionless outlet size for system I and II. Inset: a close-up at small D/d.

The main objective of this work is to assess the enhancement the flow of the large species using the addition of fine particles. Hence, in Fig. 6 we plot the maximum excess flow rate $Q_b^{max} - Q_b^{\chi=1}$ of the large particles obtained for both systems (I and II) as a function of D/d. Here, $Q_b^{\chi=1}$ corresponds to the pure big particle system. For values of D/d in which the pure system clogs, we set $Q_b^{\chi=1} = 0$. Parts (a) and (b) in Fig. 6 show the maximum excess flow in g/s and particles per second, respectively. For $D/d > 4$, the pure system does not clog and therefore it has a well defined flow rate. For $D/d \leq 4$, the pure system clogs; hence, all the flow of big particles obtained at χ^{max} is excess flow. This causes the sharp transition at $D/d \approx 4$. Interestingly, for $D/d > 4$, the excess flow obtained is 7-8 times higher for system II than for system I. This implies that a substantial improvement in the flow of non-clogging systems can be obtained by addition of very fine particles. Of course, particles finer than the ones used in system II may cause cohesion to become an issue. We have not explored the effect of cohesion.

In the clogging regime of the pure system (i.e., $D/d \leq 4.0$), we observe the most dramatic effect of adding fine particles to the system. At $D/d = 4.0$, for instance, the pure system clogs whereas the mixed system I and II yield, at the corresponding $\chi^{\max}$ a flow rate of ≈ 800 part/s and ≈ 900 part/s, respectively. This capacity of producing continuous flows by adding smaller grains to a monodisperse system that would otherwise clog was already mentioned by Benyamine et al. [9]. The excess flow decreases with D/d showing always a higher flow for system II than for system I. Noteworthy, for $D/d = 1.5$, where only one big particle at a time can pass through the orifice, we were able to obtain an excess flow of ≈ 7 part/s for system II (see inset in Fig. 6(b)). This would allow to line up large particles without clogging in an industrial process without the need of vibrating the system. The fine particles added can be easily separated afterwards by sieving and then reused.

As we mentioned, clogging occurs for the pure big particle system if $D/d < 4.5$. Adding a fine species allows for a continuous flow only if the fraction χ is below a certain threshold $\chi_{\rm clog}$. Around the flow–clogging transition some systems may clog or flow when carrying out different realizations of the experiment. For a given D/d, we have identified the maximum $\chi^{\max}_{\rm flow}$ at which continuous flow is observed in all realizations and the minimum $\chi^{\min}_{\rm clog}$ at which clogging occurs in all realizations ($\chi^{\max}_{\rm flow} < \chi^{\min}_{\rm clog}$). Figure 7 presents the estimated transition $\chi_{\rm clog} = (\chi^{\min}_{\rm clog} - \chi^{\max}_{\rm flow})/2$ with error bars indicating the two limits $\chi^{\min}_{\rm clog}$ and $\chi^{\max}_{\rm flow}$.

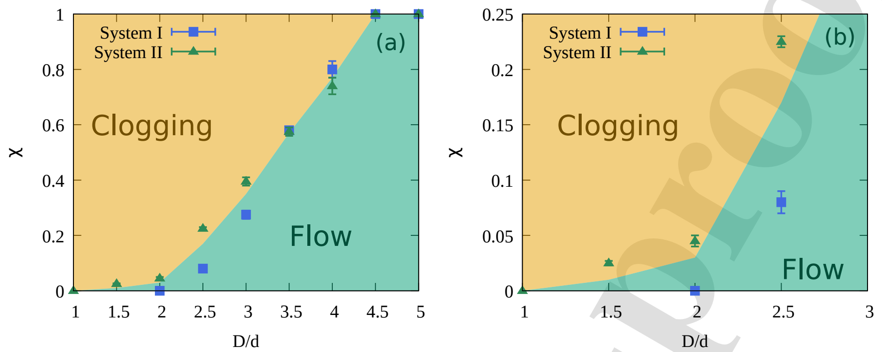


Figure 7: Flow–clogging transition. (a) Mass fraction $\chi_{\rm clog}$ of the large particles beyond which clogging occurs as a function of D/d for system I and II. (b) A close-up at small D/d. Error bars indicate the range in which the transition occurs since the flow in this range may clog in some realizations and not in others. The color areas are only to guide the eye.

The results in Fig. 7 indicate that $\chi_{\rm clog}$ increases with D/d. In particular, for $D/d \geq 4.5$, $\chi_{\rm clog} = 1$ since pure big particles also flow without clogging during discharge. For $D/d \leq 3.0$, system II presents higher values of $\chi_{\rm clog}$ than system I. This means that clogging can be avoided with a smaller fraction of fine particles in system II. Interestingly, for $D/d \leq 2.0$, $\chi_{\rm clog} = 0$ for system I while it remains finite for system II. Since the particle size ratio in system I is $d_{\rm b}/d_{\rm f} \approx 2.6$, the ratio between the orifice diameter and the fine particle diameter is $D/d_{\rm f} < 5.2$, which is close to the clogging diameter even for the pure fine particles in this system. This is not the case for system II, which is able to flow, although with a small fraction of large particles, for $D/d < 2.0$ (see Fig. 7(b)).

Figure 7 shows a crossover at $D/d = 3.5$. For $3.5 < D/d < 4.5$, $\chi_{\rm clog}$ is higher for system I than it is for system II. This occurs due to the segregation process described in section 2 for $\chi \geq 0.7$ (see Fig. 3). For system II, at these values of χ, the relatively few fine particles easily percolate between the gaps of the big species during discharge, most fine grains discharge first and the flow of the big particles occurs at the end, clogging the orifice.

4. Models for the flow rate of the large species

Models for the flow rate of monosized particles are well established in the literature. The two main approaches today are the traditional Haggen-Beverloo scaling [12, 20] and the more recent Mankoc-Janda [14, 21] proposal to avoid the introduction of the *empty annulus* correction.

Beverloo equation for the mass flow rate Q is

$$Q = C\rho_{\mathrm{b}}\sqrt{g}(D - kd)^{5/2}. \tag{1}$$

with C and k empirical non-dimensional constants. According to Nedderman [23], the values obtained by fitting experimental data are $0.55 < C < 0.63$ (typical value $C = 0.58$) and $1.4 < k < 3.0$ (typical value $k = 1.5$). Recently, Darias et al. [22] found in a first principle derivation that $C = \pi\sqrt{2}/8 \approx 0.56$.

The approach taken by the Pamplona group [14, 21] avoids the use of the $-kd$ correction to the orifice diameter by introducing an exponential function that corrects the actual density (due to the dilation caused by the flow) at the orifice. The flow rate becomes

$$Q = \frac{\pi\sqrt{2}}{8}\rho\sqrt{g}\varphi_{\infty}[1 - e^{-(D/d)/\alpha}]D^{5/2}, \tag{2}$$

where ρ is the material density of the particles (not the bulk density), φ_∞ represents the volume fraction at the center of the orifice for infinitely large orifices and α characterizes the effect of dilation caused by the proximity of the orifice edges. According to Madrid et al. [13] $\varphi_\infty = 0.638$ and $\alpha = 8.524$. Note that the prefactor in Eq. (2) coincides with the value of C in Eq. (1) obtained by Darias et al. [22]

In Fig. 8(a) we show the flow rate for the pure systems (the large particles and the two different fine grains used) as a function of D. Figure 8(b) displays the rescaled flow rate $[Q/(\rho\pi\sqrt{g}d^{5/2})]^{2/5}$, which is independent of d when plotted against D/d, in view of both the Beverloo and Janda models. Note that these systems are not perfectly mono-sized; however, a good fit to the Beverloo and Janda models is obtained by setting $d = 2.0$ mm for the large particles, $d_\mathrm{f} = 0.75$ mm for the *fine1* particles and $d_\mathrm{f} = 0.1$ mm for the *fine2* particles. As we can see, the plot of the rescaled flow rate shows a negligible difference between the predictions of each model. However, in Fig. 8(a) small differences can be appreciated. In particular, the Beverloo equation predicts a zero flow rate for $D < kd$ whereas the Janda model predicts a positive Q even for orifices smaller than the particle size. In any case, for the values of D at which continuous flow is observed both approaches provide equally accurate flow rates.

For binary mixture, predicting the flow rate of the large particles at low concentration ($\chi \ll 1$) can be done by simply assuming that the total flow rate is controlled by the fine grains. The flow rate of the fine grains can be calculated by using the available Beverloo [12] and Janda [14] expressions. In the case of Beverloo we have

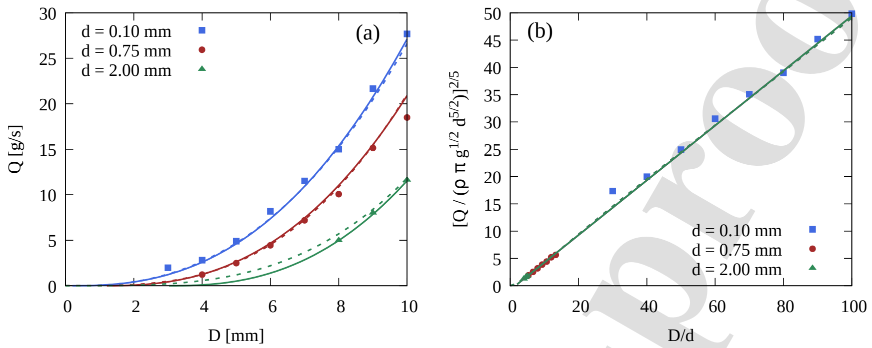


Figure 8: (a) Flow rate Q as a function of orifice diameter D for pure systems (not mixed) with different particle sizes d. (b) Scaled flow rate $[Q/(\rho\pi\sqrt{g}d^{5/2})]^{2/5}$ as a function of the scaled orifice diameter D/d. The solid lines correspond to the Beverloo model [Eq. (1) with $C = \pi\sqrt{2}/8$ and $k = 1.5$]. The dashed lines correspond to the Janda model [Eq. (2) with the parameters provided in Ref. [13] $\varphi_\infty = 0.638$ and $\alpha = 8.524$].

$$Q_{\rm f}^0 = \frac{\pi\sqrt{2}}{8}\rho_{\rm b}\sqrt{g}(D - kd_{\rm f})^{5/2} \qquad (\text{for } \chi = 0). \tag{3}$$

Here we take the typical value of k for spherical grains used by others (i.e., $k = 1.5$) [23]. The mass flow rate of the large species for χ 1 can be approximated as

$$Q_{\rm b} \approx \chi Q_{\rm f}^0 \qquad (\text{for } \chi \quad 1). \tag{4}$$

Figure 9 shows the predicted flow rate of the large particles using Eqs. 3 and 4. As we can see, this linear approximation for small χ works well up to $\chi \approx 0.3$ for all orifice sizes. This model is sufficient when considering small orifices ($D \leq 6.0$ mm) since a continuous flow is only observed for $\chi < 0.3$. However, for $D > 6.0$ mm ($D/d > 3.0$) continuous flow is achieved beyond $\chi = 0.3$ and the prediction of the linear model becomes poor. Hence, a prediction for the flow rate of the mixture for $\chi > 0.3$ is desirable.

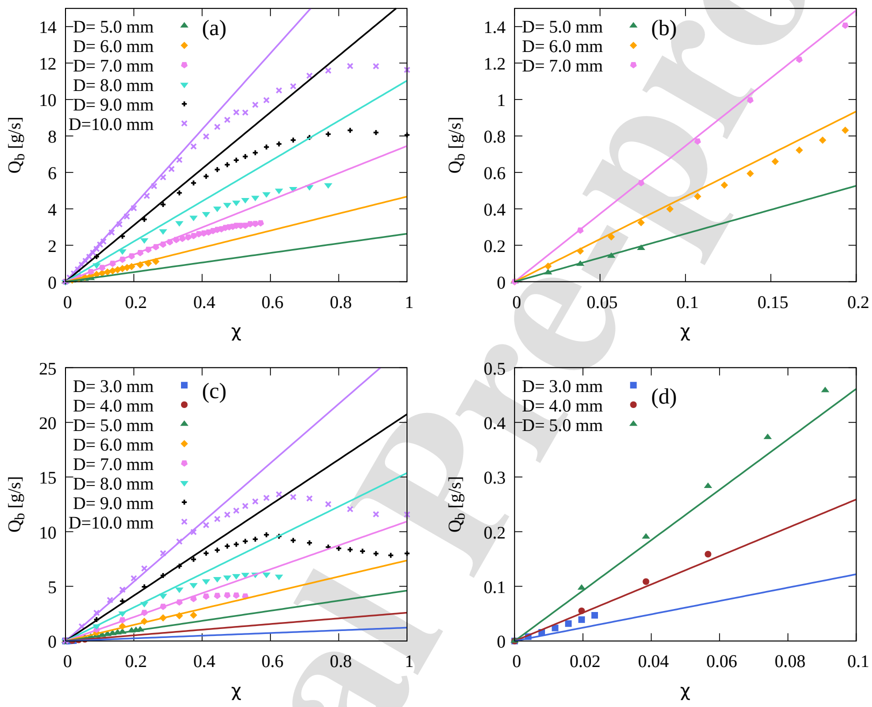


Figure 9: Same data as in Fig. 4. The solid lines correspond to Eqs. (3) and (4). (a) For system I. (b) A zoomed-in plot for the small orifices in part (a). (c) For system II. (d) A zoomed-in plot for the small orifices in part (c).

Previous studies on the flow of binary mixtures have put forward different expressions for the total flow rate of the mixture. Arteaga et al. [10] proposed a simple correction to the Beverloo formula where the particle diameter d is replaced by an effective diameter of the binary mixture defined as $d_{\mathrm{eff}} = \chi d_{\mathrm{b}} + (1-\chi)d_{\mathrm{f}}$. Moreover, Arteaga et al. [10] put special effort in measuring the bulk density ρ_{b} during discharge since this is different from the static value. They introduced in Eq. (1) the dynamic bulk density obtained for each χ, which looks similar to the ones presented in section 2 (see Fig. 2). However, thanks to recent works from Madrid et al. [13] and Zhou et al. [24] we know that the relevant density at the outlet is not affected by the mix ratio.

Humby et al. [11] suggested that, after trying different expressions, using an effective diameter based on the volume to area ratio might provide a better prediction to the flow rate of the mixture. For this they define $d_{\mathrm{eff}} = (N_{\mathrm{b}} d_{\mathrm{b}}^3 + N_{\mathrm{f}} d_{\mathrm{f}}^3)/(N_{\mathrm{b}} d_{\mathrm{b}}^2 + N_{\mathrm{f}} d_{\mathrm{f}}^2)$. Here, N_{b} and N_{f} are the number of particles of the large and fine species, respectively. The number of particles can be obtained from χ using the expression $N_{\mathrm{b}}/N_{\mathrm{f}} = \chi(d_{\mathrm{f}}/d)^3/(1-\chi)$. As Arteaga et al., these authors use the Beverloo equation with this new definition for d_{eff}.

More recently, Benyamine et al. [9] and Madrid et al. [13] have adapted the Janda model to binary mixtures. However, they have provided different solutions to this problem. Madrid et al. [13] used Eq. (2) and replaced d by d_{eff} as defined by Arteaga [i.e., $d_{\mathrm{eff}} = \chi d_{\mathrm{b}} + (1-\chi) d_{\mathrm{f}}$]. In contrast, Benyamine et al. [9] propose that the *dilatancy factor* $1 - e^{-(D/d)/\alpha}$ of the large and fine species have to be weighted by χ, which leads to

$$Q = \frac{\pi\sqrt{2}}{8}\rho\sqrt{g}\varphi_\infty\{\chi[1 - e^{-(D/d_{\mathrm{b}})/\alpha}] + (1-\chi)[1 - e^{-(D/d_{\mathrm{f}})/\alpha}]\}D^{5/2}. \quad (5)$$

Based on the suggestion by Humby, one can also propose a correction to the Madrid approach by using $d_{\mathrm{eff}} = (N_{\mathrm{b}} d_{\mathrm{b}}^3 + N_{\mathrm{f}} d_{\mathrm{f}}^3)/(N_{\mathrm{b}} d_{\mathrm{b}}^2 + N_{\mathrm{f}} d_{\mathrm{f}}^2)$. We will call this the Madrid–Humby model in the following.

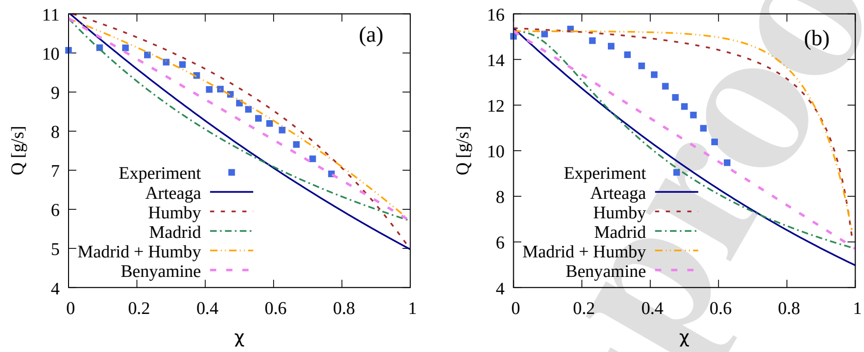


Figure 10: Total flow rate Q of the binary mixture for D = 8.0 mm, for system I (a) and II (b). Lines correspond to the various models: Arteaga [10], Humby [11], Madrid [13], Madrid–Humby (see main text) and Benyamine [9].

Figure 10 displays the total flow rate (not to be confused with Q_b) of mixed systems I and II as a function χ for an orifice of 8.0 mm along with the five different available models. It is clear that none of the models is fully satisfactory. In most previous studies, the flow rate is plotted as a function of D and this may obscure the subtle differences between the models. Here, when plotted for a given D as a function of χ, differences are appreciable. For system I (where $d_b/d_f = 2.6$), the Madrid–Humby model seems to be the best, providing the experimentally observed concave down curvature. However, for system II (for which $d_b/d_f = 20.0$) none of the models provides a good fit to the data. The Humby and Madrid–Humby models seem to be reasonable at least for $\chi < 0.3$.

Based on the comparison of the different models available, we decided to use the Madrid–Humby model for the total Q and estimate $Q_b = \chi Q$. In Fig. 11, we present Q_b as a function of χ for systems I and II and for

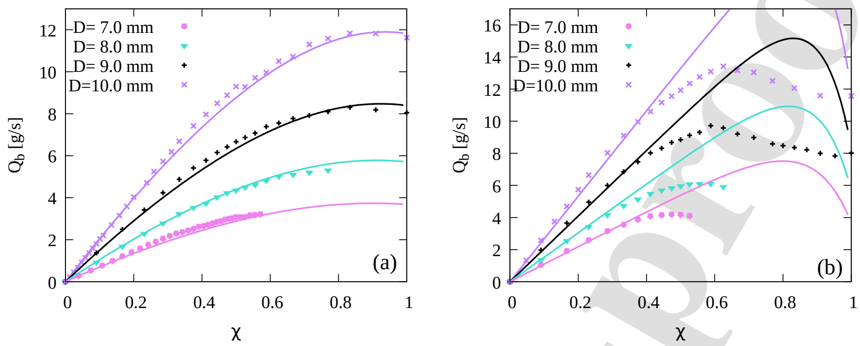


Figure 11: Same as Fig. 4 for the flow rate of the big particles. The solid lines correspond to $Q_{\mathrm{b}} = \chi Q$ where Q is taken from the Madrid–Humby model using Eq. (2) and $d_{\mathrm{eff}} = (N_{\mathrm{b}} d_{\mathrm{b}}^3 + N_{\mathrm{f}} d_{\mathrm{f}}^3)/(N_{\mathrm{b}} d_{\mathrm{b}}^2 + N_{\mathrm{f}} d_{\mathrm{f}}^2)$. (a) For system I, (b) for system II. We only include results for $D \geq 7.0$ mm, for which the linear model is not able to fit data beyond $\chi = 0.3$.

orifices $D \geq 7.0$ (for smaller D the linear model discussed above is already satisfactory). It is clear that the Madrid–Humby model provides a very good estimate of the flow rate of the big particles in the entire range of χ for for system I, while it fails for $\chi > 0.3$ for system II. For these very fine grains the Madrid–Humby model, although it outperforms all other models discussed, is not a significant improvement with respect to the linear model.

5. Conclusions

We have experimentally studied the discharge in a three dimensional silo using binary mixtures of big and fine spherical glass beads for two different system: $d_{\mathrm{b}}/d_{\mathrm{f}} \approx 2.6$ and $d_{\mathrm{b}}/d_{\mathrm{f}} \approx 20.0$. Instead of studying the flow of the entire mixture we have focused only on the flow of the large species. We have shown that the flow of the big species can be enhanced by the addition of small grains. In particular, the clogging is reduced by adding the fine grains without the need of vibrating the system. Interestingly, we were able to flow the big particles (at ≈ 7 part/s) through orifices only 1.5 times the particle diameter by adding grains 20 times smaller than the original grains. This provides a very effective and inexpensive mechanism to enhance the flow of big species to the limit of making them flow continuously one at a time though an orifice. Since the added particles are much smaller than the original ones, they can be later removed using standard sieving techniques.

We have used existing models for the flow rate of binary mixtures to predict the flow of the main large species. We found that a hybrid model (Madrid–Humby) provides a very good estimate of the flow rate of the big particles in the entire range of χ for particle size ratio 2.6, but fails for $d_\mathrm{b}/d_\mathrm{f} = 20.0$. This suggests that modeling the flow of mixtures with large particle size ratios is still a challenge.

Acknowlegments

This work was funded by CONICET (Argentina) under grant PIP-717.

References

[1] I. Zuriguel, Invited review: Clogging of granular materials in bottlenecks, Pap. Phys. 6 (2014) 060014.

[2] C. Mankoc, A. Garcimartín, I. Zuriguel, D. Maza, L. A. Pugnaloni, Role of vibrations in the jamming and unjamming of grains discharging from a silo, Phys. Rev. E 80 (2009) 011309.

[3] C. Lozano, G. Lumay, I. Zuriguel, R. Hidalgo, A. Garcimartín, Breaking

arches with vibrations: the role of defects, Phys. Rev. Lett. 109 (6) (2012) 068001.

[4] I. Zuriguel, A. Janda, A. Garcimartín, C. Lozano, R. Arévalo, D. Maza, Silo clogging reduction by the presence of an obstacle, Phys. Rev. Lett. 107 (27) (2011) 278001.

[5] K. Endo, K. A. Reddy, H. Katsuragi, Obstacle-shape e□ect in a two-dimensional granular silo flow field, Phys. Rev. Fluids 2 (9) (2017) 094302.

[6] K. To and H.-T. Tai, Flow and clog in a silo with oscillating exit, Phys. Rev. E 96 (2017) 032906.

[7] Y. Zhou, P. Y. Lagrée, S. Popinet, P. Ruyer, P. Aussillous, Gas-assisted discharge flow of granular media from silos, Phys. Rev. Fluids 4 (12) (2019) 124305.

[8] M. A. Madrid, C. M. Carlevaro, L. A. Pugnaloni, M. Kuperman, S. Bouzat, Enhancement of the flow of vibrated grains through narrow apertures by addition of small particles, Phys. Rev. E 103 (2021) L030901.

[9] M. Benyamine, M. Djermane, B Dalloz-Dubrujeaud, P. Aussillous, Discharge flow of a bidisperse granular media from a silo, Phys. Rev. E 90 (2014) 032201.

[10] P. Arteaga, U. Tüzün, Flow of binary mixtures of equal-density granules in hoppers - size segregation, flowing density and discharge rates, Chem. Eng. Sci. 45 (1) (1990) 205-223.

[11] S. Humby, U. Tüzün, A. B. Yu, Prediction of hopper discharge rates of binary granular mixtures, Chem. Eng. Sci. 53 (3) (1998) 483-494.

[12] W. A. Beverloo, H. A. Leniger, J. Van de Velde, The flow of granular solids through orifices, Chem. Eng. Sci. 15 (3) (1961) 260–269.

[13] M. Madrid, K. Asencio, D. Maza, Silo discharge of binary granular mixtures, Phys. Rev. E 96 (2017) 022904.

[14] A. Janda, I. Zuriguel, D. Maza, Flow rate of particles through apertures obtained from self-similar density and velocity profiles, Phys. Rev. Lett. 108 (2012) 248001.

[15] C. Li, X. Li, T. Jiao, F. Hu, M. Sun, D. Huang, Influence of grain bidispersity on dense granular flow in a two-dimensional hopper, Powder Technol. 401 (2022) 117271.

[16] Z. Zhang, Y. Liu, B. Zheng, R. Li, P. Sun, Discharge characteristics of binary particles in a rectangular hopper with inclined bottom, Comput. Part. Mech. 8 (2021) 315.

[17] F. Chevoir, F. Gaulard, N. Roussel, Flow and jamming of granular mixtures through obstacles, Europhys Lett. 79 (2007) 14001.

[18] N. Roussel, T. L. H. Nguyen, P. Coussot, General Probabilistic Approach to the Filtration Process, Phys. Rev. Lett. 98 (2007) 114502.

[19] M. A. Aguirre, R. De Schant, J.-C. Géminard, Granular flow through an aperture: Influence of the packing fraction, Phys. Rev. E 90 (2014) 012203.

[20] G. H. L. Hagen, Ber. Verhandl. K. Preuss. Akad. Wiss. Berlin 17 (1852) 35-42.

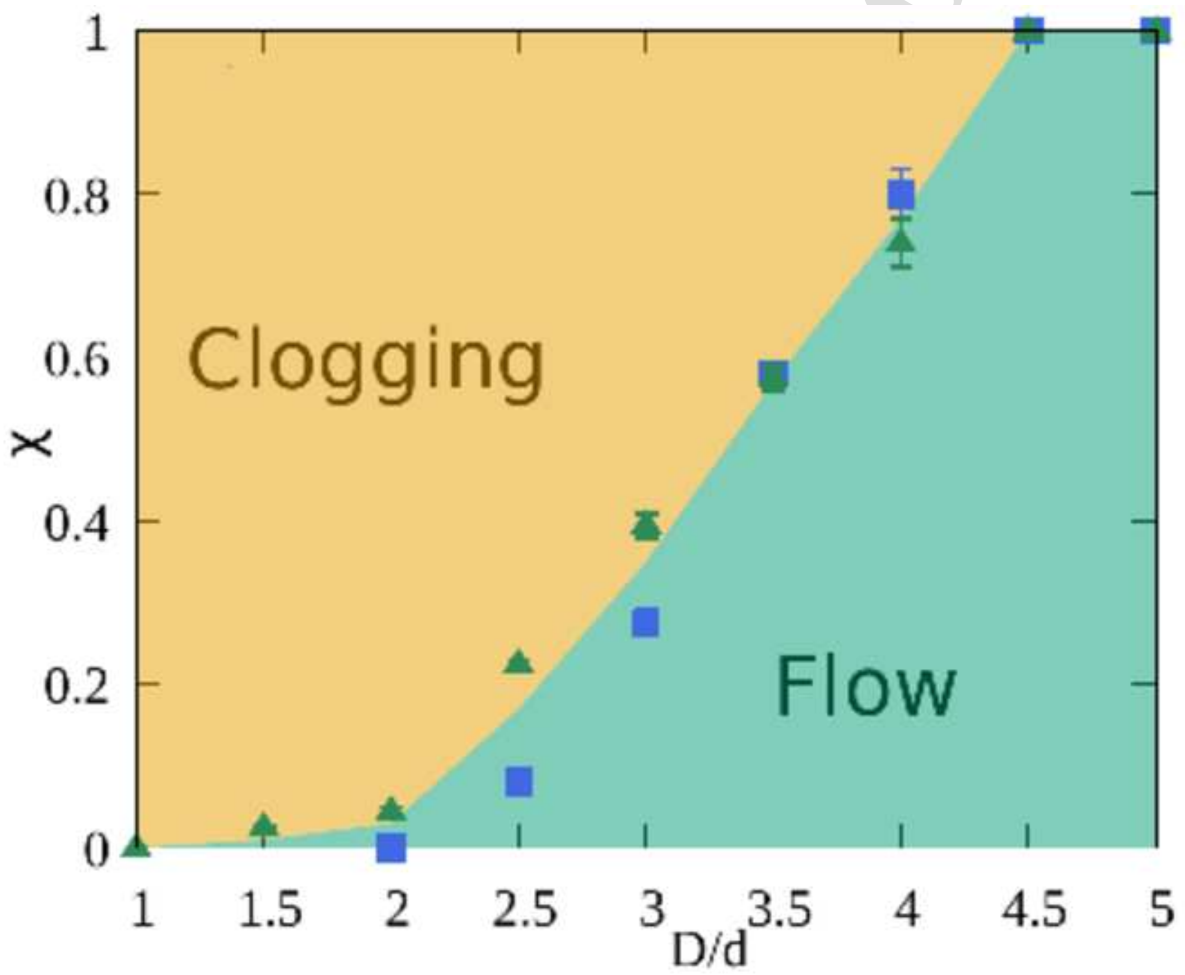
[21] C. Mankoc, A. Janda, R. Arévalo, J. Pastor, I. Zuriguel, A. Garcimartín, D. Maza, The flow rate of granular materials through an orifice, Granul. Matter 9 (2007) 407-414.

[22] J. R. Darias, M. A. Madrid, L. A. Pugnaloni, Differential equation for the flow rate of discharging silos based on energy balance, Phys. Rev. E 101 (2020) 052905.

[23] R. M. Nedderman, Statics and Kinematics of Granular Materials, Cambridge University Press, Cambridge (2005).

[24] Y. Zhou, P. Ruyer, P. Aussillous, Discharge flow of a bidisperse granular media from a silo: Discrete particle simulations, Phys. Rev. E 92 (2015) 062204.

Graphical Abstract (for review)



The discharge of grains can be increased by the addition of a second finer species

Clogging can be significantly reduced by the addition of a second finer species

Particles can be made to flow without clogging through very narrow orifices

Current flow models for mixtures are still unsatisfactory for large size ratios

Sandip H. Gharat: Conceptualization, Methodology, Formal analysis, Investigation, Writing - Original Draft
Luis A. Pugnaloni: Conceptualization, Validation, Formal analysis, Writing - Review & Editing, Project administration

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: