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Description of Maunder-like events from a stochastic Alpha–Omega model

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Abstract

One of the most impressive solar phenomena is its magnetic activity cycle. The number of sunspots observed in the solar surface varies with a period of approximately 11 years. Superimposed to this cyclic behavior, there are sporadic events such as the Maunder minimum, during which very few sunspots were detected. In the present work, an Alpha–Omega dynamo model is proposed to study these phenomena. We use velocity profiles of the solar interior obtained from helioseismology, which include differential rotation (the Ω-effect), a meridional flow and the turbulent velocity field of the convective region (the α-effect). By simulating the helicoidal and disordered flow of the giant cells through a stochastic component in the α coefficient, we reproduce not only the periodic behavior of the solar cycle, but also sporadic events such as the Maunder minimum. This model suggests the existence of a link between giant cell fluctuations and the irregularities observed in the solar cycle, such as north–south asymmetries and secular variations like the Maunder minimum.

Keywords: Solar cycle; Maunder minimum

1. Introduction

The origin of solar secular events, such as the Maunder minimum (Ribes and Nesme-Ribes, 1993; Beer et al., 1990), has become one of the most actively debated problems in solar physics, with measurable consequences on the Earth's climate (Ribes and Nesme-Ribes, 1993; Svensmark, 1998). The magnetic field observed at the solar surface is believed to be produced by a dynamo process operating in the solar interior. When the field emerges at the surface, it creates bipolar regions in a wide range of sizes and magnetic fluxes. Those regions that are sufficiently large, give rise to sunspots. Sunspot locations are not completely random. At the beginning of the cycle, the first spots appear near a belt at latitudes of 30°. The region of activity widens up and slowly migrates toward the equator as the cycle evolves. The number of spots reaches its maximum in approximately 4 years, and slowly decreases to a minimum.

According to dynamo theory, the number of sunspots and their spatial distribution is in phase with the strong toroidal magnetic field at the base of the convective region. Current dynamo models (Dikpati and Charbonneau, 1999; Nandy and Choudhuri, 2002) are able to explain the mean properties of the cycle, such as the period, the maximum intensity of magnetic fields, and the migration of activity toward the equator as the cycle

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evolves. The angular velocity ω in the Sun has a latitudinal shear at the surface, and a strong radial shear in a thin layer of the solar interior known as the tachocline. Strong toroidal fields are produced in this layer as the end result of the stretching of poloidal field lines caused by differential rotation (Ω-effect). These strong fields eventually rise due to magnetic buoyancy. As they rise, the field is twisted and finally produces new poloidal field. Several models exist to explain the location and source of this conversion Krause and Rädler, 1980; Dikpati and Charbonneau, 1999; Caligari et al., 1995. For historical reasons, we will call α-effect to this process, although we are not considering any particular model to explain its origin. Therefore, the results we present here can apply to any of these theoretical models. A pending question is that although any of these models explain the mean properties of the solar cycle reasonably well, the origin of secular variations such as Maunder minima remains essentially unknown.

We can separate two types of behavior of the solar cycle. In our days, the Sun is active and several sunspots can be seen in its surface (although the cycle shows variations in its period and amplitude). This *normal* activity has been observed for the last three centuries. Fig. 1 shows the distribution of sunspots as a function of solar latitude and time (the so-called *butterfly* diagram), as obtained from the Wolf number data set maintained by the Royal Greenwich Observatory. On the other hand, during certain prolonged periods known as Maunder or Grand minima, the number of spots (Ribes and Nesme-Ribes, 1993) and other indicators of solar magnetic activity (Beer et al., 1990) have decreased dramatically. While normal cycles display a slight asymmetry in the activity between northern and southern hemispheres (Mininni et al., 2002), the asymmetry was much more pronounced during the Maunder minimum (Ribes and Nesme-Ribes, 1993). Observations show that almost all sunspots were located in one hemisphere. Cycles of shorter periods (about 10 years) and lower intensity were observed, while the latitudinal belt of activity was reduced in approximately 10°. All these facts suggested that during minima another type of equatorial symmetry is at work, with the same amount of symmetric and antisymmetric fields (Sokoloff and Nesme-Ribes, 1994). Although several mechanisms have been invoked in the literature, the reasons that force the Sun to change the equatorial symmetry of its global magnetic fields remain unclear (Leighton, 1969; Parker, 1979; Knobloch et al., 1998; Choudhuri, 1992; Ossendrijver et al., 1996).

2. The role of the giant cells

An understanding of the complex plasma motions and the physical conditions of the turbulent solar convection zone are essential to build a realistic model of the solar dynamo. Recently, long-lived giant cells were detected at the surface of the Sun (Beck et al., 1998) These cells span the entire convective region and their dynamics is therefore expected to have a major impact on the emergence of magnetic flux and solar activity. In particular, it is expected to play a key role in the α-effect, which is directly related to the emergence and twisting of magnetic fields.

Turbulent fluctuations in the convective region are known to affect the twist of magnetic fields only if the fluid motions are helical. Can giant cell motions affect the α-effect? In the Sun, we can observe motions with characteristic length scales ranging from 10^3 km (granules) to 10^5 km (giant cells). Since helical motions are driven by the Coriolis force, we must seek for scales that can be affected by this force. The reciprocal of the Rossby number R_S measures the relevance of the Coriolis force in the dynamics of the plasma in comparison with the convective term, i.e.,

$$R_S^{-1} = \frac{2\Omega}{U/L} \approx \frac{T_{conv}}{T_{rot}}, \qquad (1)$$

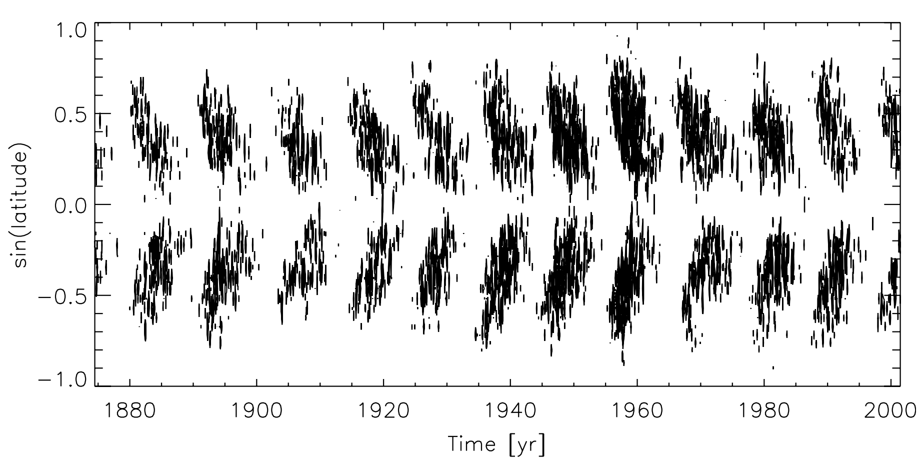

Fig. 1. Butterfly diagram obtained with the Wolf number data from Royal Greenwich Observatory. Each sunspot is indicated by a vertical segment, whose length is proportional to the area covered by the spot.

where U and L are characteristic velocities and lengths for the convective motions, Ω is the rotation rate and $T_{\rm conv}$ and $T_{\rm rot} \approx 27$ days are the convective and rotation periods. Small convective structures like granules, or even supergranules, have turnover times much shorter than $T_{\rm rot}$. As a result, the Coriolis force has a negligible effect on their dynamics, and therefore these convective structures are expected to be essentially nonhelical. On the other hand, giant cells have typical turnover times of about one to four months, which makes their Rossby number of order unity (see Eq. (1)). Thus, we expect giant cells to display helical flows and to have a marked impact on the generation of magnetic fields.

Choudhuri (1992) was the first to assert that these fluctuations could be responsible for the irregularities observed in the Sun. The idea was later explored mostly in linear models by Ossendrijver et al. (1996). Although they were able to obtain Maunder-like events, it was generally believed that the nonlinearities needed in a realistic model could make these events less pronounced (Ossendrijver et al., 1996) or even impossible (Knobloch et al., 1998). This idea led to the construction of simplified models based on low-dimensional chaos which predict long time modulations (Knobloch et al., 1998), although no clear evidence exists supporting this scenario (Carbonell et al., 1994; Mininni and Gómez, 2002).

A Maunder minimum is an event that lasts about 60 years, and there is evidence suggesting that it was preceded by similar episodes (Beer, 2000). Can therefore the short term fluctuations introduced by the giant cells explain the global change in the symmetry of solar magnetic fields? To answer this question, in the following section we consider a kinematic and axisymmetric dynamo model.

3. The dynamo equations

Kinetic dynamo models describe the evolution of the magnetic field $\mathbf{B}$ for a prescribed velocity field. We assume a large scale axisymmetric velocity field

$$\mathbf{U} = u_r(r,\theta)\hat{r} + u_\theta(r,\theta)\hat{\theta} + r\sin\theta\,\omega(r,\theta)\hat{\phi}, \tag{2}$$

which includes both the differential rotation ω of the Sun and a slow meridional flow. These velocity profiles are derived from recent helioseismic observations (Choudhuri, 1992; Beck, 1999; Cameron and Hopkins, 1998) The magnetic field in terms of its toroidal and poloidal components can be written as

$$\mathbf{B} = B_\phi\hat{\phi} + \nabla\times(A_p\hat{\phi}). \tag{3}$$

Since there is strong observational evidence suggesting that dynamo action takes place at a thin layer at the bottom of the convective region, we reduce the kinematic dynamo equation to a one-dimensional description for A_p and B_ϕ as a function of the solar latitude (Wang et al., 1991, see also Mininni and Gómez, 2002). Considering the observed profiles, and assuming a typical time T_0 and a typical longitude L_0 (the depth of the tachocline), the dimensionless dynamo equation becomes

$$\begin{aligned}
\frac{\partial B_\phi}{\partial t} &= -\left(u_r + \epsilon\frac{\partial u_\theta}{\partial\theta}\right)B_\phi - \epsilon u_\theta\frac{\partial B_\phi}{\partial\theta} \\
&\quad + \left(\Delta\omega\cos\theta - \sin\theta\frac{\partial\omega_{\rm s}}{\partial\theta}\right)A_p + \Delta\omega\sin\theta\frac{\partial A_p}{\partial\theta} \\
&\quad + \left(-\frac{\epsilon^2}{\sin^2\theta} + \epsilon^2\cot\theta\frac{\partial}{\partial\theta} + \epsilon^2\frac{\partial^2}{\partial\theta^2} - 1\right)\frac{B_\phi}{R_M} \\
\frac{\partial A_p}{\partial t} &= -(u_r + \epsilon\cot\theta\, u_\theta)A_p - \epsilon u_\theta\frac{\partial A_p}{\partial\theta} + \alpha B_\phi \\
&\quad + \left(-\frac{\epsilon^2}{\sin^2\theta} + \epsilon^2\cot\theta\frac{\partial}{\partial\theta} + \epsilon^2\frac{\partial^2}{\partial\theta^2} - 1\right)\frac{A_p}{R_M}
\end{aligned} \tag{4}$$

where $R_M = L_0^2/(T_0\eta)$, $\epsilon = L_0/R_\odot \approx 0.1$, $R_\odot$ is the solar radius, η is the magnetic diffusivity (3.39×10^{11} cm^2 s^{-1}, see Dikpati and Charbonneau, 1999), and u_θ and u_r are the radially averaged components of the meridional flow. Note that these equations take into account the observed strong and localized radial shear in the differential rotation, since $\Delta\omega/L_0 = (\omega_{\rm s}-\omega_{\rm c})/L_0 \approx \partial\omega/\partial r$, where $\omega_{\rm c}$ describes the rotation of the core as a rigid body and $\omega_{\rm s}$ is the observed profile at the solar surface.

At the surface, the differential rotation profile is given by observations (Beck, 1999)

$$\omega_{\rm s}(\theta) = a + b\cos^2(\theta) + c\cos^4(\theta), \tag{5}$$

where $a = 2.913\times10^{-6}$ rad s^{-1}, $b = -0.405\times10^{-6}$ rad s^{-1}, $c = -0.422\times10^{-6}$ rad s^{-1}. The rotation of the core is $w_{\rm c} = 2.243\times10^{-6}$ rad s^{-1}, a value slightly larger than the one inferred from helioseismology. The meridional velocity field is assumed to be incompressible (although in a stratified medium) with the mass flux given by the following stream function

$$\psi(\theta,r) = -\psi_0\sin^p(\theta)\cos^q(\theta)[r^n(r-r_{\min})(r_{\max}-r)], \tag{6}$$

where $p = 2.5$, $q = 1$ to fit the observed meridional velocity field at the surface (Cameron and Hopkins, 1998). We adopt a density profile in the convection zone given by $\rho(r) = \rho_0 r^{-n}$, and assume $n = .5$, $r_{\min} = 0.6\ R_\odot$, $r_{\max} = R_\odot$, and ψ_0 and ρ_0 to obtain a meridional flow at the surface of about 20 m s^{-1}.

The process responsible for the conversion of toroidal field into poloidal field is nonlinearly quenched by the Lorentz force when the magnetic field is large (Dikpati and Charbonneau, 1999), and therefore we assume

$$\alpha(\mathbf{B}) = \frac{\alpha_0}{1 + B_\phi^2/B_0^2}\sin(\theta)\cos(\theta), \tag{7}$$

where B_0 is related to the saturation value of the magnetic field, which (from simulations) is known to take place when $B_\phi \approx 10^4$–10^5 G. The value chosen for the amplitude of α_0 is 14 cm s^{-1}, to yield results compatible with observations (Ossendrijver et al., 1996; Choudhuri, 1992). Note that all the velocity profiles and parameter values were chosen to fit as closely as possible the observations of the Sun. These equations were numerically integrated using a finite difference scheme with 500 points in latitude, and a predictor–corrector method to evolve in time with a time step of $T_0/500$. The integration is made in both solar hemispheres, and the boundary conditions at the poles are $B_\phi = \partial A_p/\partial\theta = 0$. The reduction of the dynamo equations to a 1D model allow us to perform long-time simulations at sufficiently high latitudinal resolution. As a result, we can study the effect of giant-cell fluctuations on the temporal evolution of the cycle with very good statistics.

4. Maunder-like events

To model the fluctuations in α introduced by giant cell motions we consider each cell as a turbulent eddy. The life time of these eddies is of the order of their turnover time (from one to a few months), and their typical size is about $5 - 6 \times 10^5$ km in longitude and 10^5 km in latitude (Beck et al., 1998). Therefore, we add to α_0 a fluctuating component $\delta\alpha$ modeled as normally distributed noise with these spatial and temporal correlations, which are derived from observations. The dispersion of these fluctuations satisfies $\delta\alpha/\alpha \approx 1$, which is a manifestation of the fact that fluctuations are expected to be strong in turbulent flows such as the solar convective region. A heuristic argument in this direction was given by Choudhuri (1992). Assuming that α_0 is the average in longitude for a given latitude, then the fluctuations about this average would be of the order of $\delta\alpha/\alpha \approx 1/\sqrt{N}$, where N is the number of giant cells as we go around longitudes. For the typical giant cell sizes reported by Beck et al. (1998), is $N \approx 8$ and therefore $\delta\alpha/\alpha \approx 0.35$.

Fig. 2 shows the toroidal magnetic field obtained after integrating the system for a large number of solar cycles. The solutions show long periods of normal magnetic activity. Compare for instance the last cycles in Fig. 2 with those in Fig. 1, displaying intense and well defined cycles. As the cycle evolves, magnetic activity below this latitude migrates toward the equator. A normal magnetic cycle lasts for about 22 years, and every 11 years the global magnetic field reverses. Fluctuations can be observed both in the amplitude and period of the cycles, as well as in slight asymmetries between hemispheres. These epochs of normal activity can last very long, ranging from 100 to 700 years.

However, from time to time these fluctuations excite strong asymmetries between hemispheres, as can be observed at the center of Fig. 2. When these events take place, the amplitude of the cycle decreases substantially, the period becomes shorter, the latitudinal span of magnetic activity shrinks, and the magnetic activity virtually shuts off in a way strongly reminiscent of solar Grand minima. Usually, these events show a strong asymmetry between hemispheres, as was reported for the Maunder minimum (Ribes and Nesme-Ribes, 1993). In Fig. 2, the northern hemisphere is more active than the southern hemisphere during this particular minimum. These events typically last for 60–100 years, and can be statistically observed every several centuries. Note that the events are not periodic, and that the average separation time can be reduced by increasing the amplitude of the fluctuations. In many cases, these periods are preceded by an unusually strong hemispheric asymmetry during the normal cycle. If confirmed, this asymmetry could be used as an indication that the Sun is approaching a minimum of activity.

Fig. 3 shows the magnetic energy at mid-latitudes contained in the toroidal magnetic field (which is expected to be in phase with the sunspot number). Several Maunder-like events can be observed in this time series, separated by a long period of normal behavior. From abundances of ^{14}C and ^{10}Be we know that the solar Maunder minimum was probably preceded by similar

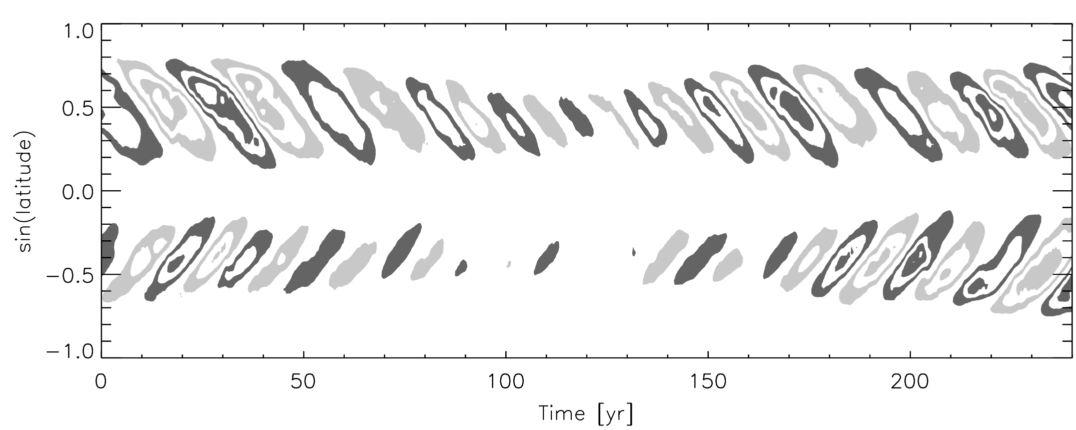

Fig. 2. Contour levels of toroidal magnetic fields, showing the occurrence of a Maunder-like event. Contours filled with dark (light) gray correspond to positive (negative) levels.

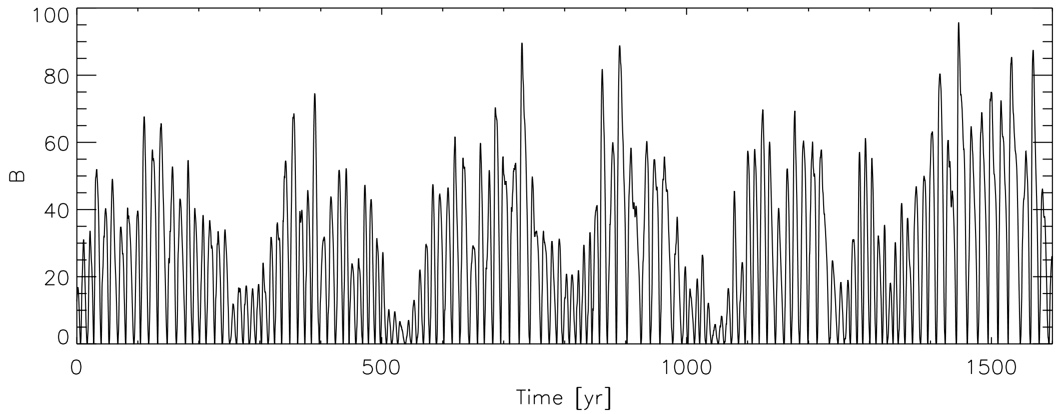

Fig. 3. Magnetic energy at mid-latitudes for a numeric integration equivalent to 1600 years.

episodes, the previous one taking place about 200 years before (Beer, 2000). Also, the increase of cycle-related solar magnetic fields can be observed in Fig. 2, reminiscent of the one observed in the first half of the last century (Solanki et al., 2000). Charbonneau et al. (2004) recently performed a numerical study of a two-dimensional Babcock-Leighton dynamo model, which also shows Maunder-like minima as a result of a stochastic forcing. In their study, the stochastic driver is not intended to specifically simulate giant cell convection, and therefore its spatial and temporal correlation properties are different from those used in the present paper.

5. Conclusions

We apply a simple Alpha–Omega axisymmetric dynamo model and perform time extended numerical integrations to study the generation of Maunder-like events. The alpha term contains a stochastic part to imitate the helical flows associated to giant convective cells. These Maunder-like events arise as a result of the change in the symmetry of magnetic fields, as suggested by observations (Ribes and Nesme-Ribes, 1993) and correctly described by our model. This origin for the Maunder minimum was already suggested in the literature (Sokoloff and Nesme-Ribes, 1994; Knobloch et al., 1998; Ossendrijver et al., 1996), although the catalyst of this symmetry change was not clear. Our model shows that the statistical properties of giant cells can generate this change. Moreover, we show that a nonlinear and realistic model of the solar dynamo can yield solutions displaying periods of normal activity intertwined with Maunder events. The presently available records of ^{10}Be and ^{14}C show cyclic variability of 11 years as well as irregularly distributed intervals of very low solar activity, with typical durations of 100 years. Note that our model explain both the secular variations and the regular cycle without the need of external modulations or changes in solar parameters.

Secular variations of the Sun and low sunspot activity are known to have profound effects on the solar envelope and its output (Ribes and Nesme-Ribes, 1993). On the other hand, a causal connection between solar Grand minima and the Earth's climate has been reported (Ribes and Nesme-Ribes, 1993; Svensmark, 1998). However, a correct assessment of the impact of these changes can only be made once all the relevant physical mechanisms are known. In summary, we believe that our model provides a perhaps unexpected link between giant cell fluctuations and global properties of the solar cycle.

Acknowledgments

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References

Beck, J.G., Duvall Jr., T.L., Scherrer, P.H. Nature 394, 653, 1998.
Beck, J.G. Sol. Phys. 191, 47, 1999.
Beer, J., Blinov, A., Bonani, G., Hofmann, H.J., Finkel, R.C. Nature 347, 164, 1990.
Beer, J. Space Sci. Rev. 94, 53, 2000.
Caligari, P., Moreno-Insertis, F., Schüssler, M. ApJ 441, 886, 1995.
Cameron, R., Hopkins, A. Sol. Phys. 183, 263, 1998.
Carbonell, M., Oliver, R., Ballester, J.L. A&A 290, 983, 1994.
Charbonneau, P., Blais-Laurier, G., St-Jean, C. ApJ 616, L183, 2004.
Choudhuri, A.R. A&A 253, 277, 1992.
Dikpati, M., Charbonneau, P. ApJ 518, 508, 1999.
Knobloch, E., Tobias, S.M., Weiss, N.O. MNRAS 297, 1123, 1998.
Krause, F., Rädler, K.-H. Mean-Field Magnetohydrodynamics and Dynamo Theory. Pergamon, Oxford, 1980.
Leighton, R.B. ApJ 156, 1, 1969.
Mininni, P.D., Gómez, D.O. ApJ 573, 454, 2002.
Mininni, P.D., Gómez, D.O., Mindlin, G.B. Phys. Rev. Lett. 89, 061101, 2002.
Nandy, D., Choudhuri, A.R. Science 296, 1671, 2002.

Ossendrijver, A.J.H., Hoyng, P., Schmitt, D. A&A 313, 938, 1996.
Parker, E.N. Cosmical Magnetic Fields: Their Origin and Their Activity. Oxford University Press, New York, 1979.
Ribes, J.C, Nesme-Ribes, E. A&A 276, 549, 1993.
Sokoloff, D., Nesme-Ribes, E. A&A 288, 293, 1994.
Solanki, S.K., Schüssler, M., Fligge, M. Nature 408, 445, 2000.
Svensmark, H. Phys. Rev. Lett. 81, 5027, 1998.
Wang, Y.-M., Sheeley, N.R., Nash, A.G. ApJ 383, 431, 1991.