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# Computers and Geotechnics



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# Closure to discussion of "Procedure for assessing the liquefaction vulnerability of tailings dams" by Ledesma, O, Sfriso, A, and Manzanal, D

Osvaldo Ledesma<sup>a</sup>, Alejo Sfriso<sup>a,b</sup>, Diego Manzanal<sup>c,\*</sup>

<sup>a</sup> *Universidad de Buenos Aires (UBA), Facultad de Ingeniería, LAME, Buenos Aires, Argentina* 

<sup>b</sup> *SRK Consulting, Argentina* 

<sup>c</sup> ETS de Ingenieros de Caminos, Canales y Puertos, Universidad Politécnica de Madrid, c/Prof. Aranguren 3, Ciudad Universitaria, 28040 Madrid, Spain

### **1. Introduction**

The authors thank the discussers for their comments [\(Reid et al.,](#page-2-0)  [2023\)](#page-2-0), which are mostly related to addressing key uncertainties when completing the analysis proposed by the authors in their work [\(Ledesma](#page-2-0)  [et al., 2022](#page-2-0)), and to the response of the Modified Pastor-Zienkiewicz (MPZ) constitutive model ([Manzanal et al., 2011\)](#page-2-0) employed in the same work.

The authors believe that recognizing and dealing with uncertainties, such as the ones highlighted by the discussers, will provide additional insights for assessing the liquefaction vulnerability of tailings dam systems ([Ledesma et al., 2022](#page-2-0)) using readily available deformation modelling tools.

The authors would also like to highlight that the proposed procedure for assessing liquefaction vulnerability is not a triggering analysis. The proposed actions are not intended to be relevant or realistic for a given dam. However, they should be understood as numerical excursions around a stable configuration to check the stability of such equilibrium. Also, the proposed procedure could be completed using constitutive models other than MPZ. However, there are some key characteristics that the selected constitutive model should have and are described in more detail in [Ledesma et al. \(2022\).](#page-2-0)

#### **2. Uncertainties**

The uncertainties described by the discussers are of two kinds. On the one hand, the discussers identified uncertainties related to how the stress state and key state variables influence the flow liquefaction behaviour of tailings. They focused mainly on:

- The effect that the stress state has on the critical state line (i.e., the value of *M* and the position of the critical state line in the *e* vs *p'* plane as a function of Lode angle).
- The influence that the stress state has on the onset of liquefaction (i. e., the slope of the instability line *ηIL* as a function of Lode angle and of  $K_0$ ).
- The effect that other state variables have on the onset of liquefaction (i.e., the slope of the instability line as a function of the principal stress rotation angle *α* and the state parameter *ψ*.

On the other hand, the discussers highlight uncertainties related to the actual stress state in the field and its evolution over time, deformation, or loading (e.g., the value of  $K_0$  in the field).

These uncertainties are not exclusive to deformation analyses like the ones required to carry out the procedure proposed by the authors. Peak and residual undrained strength used in limit equilibrium analyses have the same embedded uncertainties, although they may not be explicitly recognized in standard design procedures.

Completing deformation analyses using numerical tools do not introduce these uncertainties but require decisions to be made on how to deal with them. One of the critical decisions is the selection of an appropriate constitutive model. This requires identifying which key aspects of the behaviour the constitutive model should reproduce and which models are available for that end. The other important decision is related to which stages of the construction sequence must be modeled, and which initial values of key state variables – such as the void ratio – should be adopted to achieve the expected in-situ conditions (i.e., stress state, void ratio) under which the vulnerability to flow liquefaction of the dam system should be evaluated.

The approach to dealing with these uncertainties should incorporate engineer judgment, parametric analyses, extensive laboratory and field testing, and field monitoring, depending on the phase of the project. In

\* Corresponding author. *E-mail address:* [d.manzanal@upm.es](mailto:d.manzanal@upm.es) (D. Manzanal).

Available online 2 November 2022 0266-352X/© 2022 Elsevier Ltd. All rights reserved. <https://doi.org/10.1016/j.compgeo.2022.105063> Received 28 September 2022; Accepted 29 September 2022



**Fig. 1.** Distribution of the state parameter for  $e_0 = 0.78$ .

particular, the following strategies could be implemented:

- the dependence of *M* and the onset of liquefaction with Lode angle *θ*  can be investigated by a combination of triaxial compression, triaxial extension and direct simple shear tests;
- the effect of the Lode angle on the position of the CSL in the *e* vs *p'*  plane represents a shift of void ratio for plain strain conditions of 0.05 above that of triaxial conditions ([Wanatowski and Chu, 2007](#page-2-0)). Therefore, adopting the results from the triaxial tests would result in a conservative approach in terms of the definition of the state parameter *ψ*;
- $\bullet$  the stress state in the field  $(K_0)$  can be preliminarily addressed by laboratory testing at the design stage and field testing and in-situ monitoring during construction or operation, albeit it is recognized that this is rarely executed in tailings practice. Measuring in-situ stresses is an accomplishable effort which, for instance, is common practice in tunnelling engineering.

The authors also encourage using different constitutive models when performing these vulnerability analyses together with a proper parametric analysis varying key inputs to evaluate the change in the response.

#### **3. MPZ capabilities**

The MPZ model was developed assuming that the material was isotropic and, therefore, it was formulated in terms of the three invariants of the effective stress tensor *p*, *q* and *θ*. The implementation of the MPZ model used [\(Ledesma et al., 2021](#page-2-0)) captures some but not all of the behavioural aspects highlighted by the discusser and described in

the previous sections.

The location and slope of the critical state line in the *e* vs *p'* plane is unique in the model and does not change depending on the Lode angle. Also, the onset of liquefaction is independent of the rotation of principal stresses. The influence of principal stress rotation angle *α* on the stress–strain response and on the stress path, including the onset the liquefaction has recently been implemented in the MPZ model [\(Garcia-](#page-2-0)[Garcia et al., 2023](#page-2-0)).

The response of the MPZ model for oedometric compression tests is presented in Fig. 1. The same constitutive parameter as those used to represent the behaviour of the Fundão tailings were used (Ledesma [et al., 2022](#page-2-0)). Fig. 1a shows the evolution of the void ratio with the vertical stress, while Fig. 1b presents the evolution of the ratio between the horizontal and vertical stress,  $K_0$ . The test was set to start in an isotropic condition with  $K_0 = 1.0$ , which then evolves, reaching a minimum value of  $K_0 = 0.60$  and a maximum value of  $K_0 = 0.65$  for a vertical stress  $\sigma'$ <sup>*V*</sup> = 1500 kPa. These values fall within the expected range, as pointed out by the discussers.

The functional form of  $M(\theta)$  that was employed in the implementation of the MPZ model used by the authors is described in detail in [Ledesma et al. \(2021,2022](#page-2-0)). [Willam and Warnke \(1975\)](#page-2-0) yield function was adopted as it can be set to match the shape of the well-known failure surfaces criteria by [Matsuoka and Nakai \(1974\)](#page-2-0) and by [Lade and Duncan](#page-2-0)  [\(1975\)](#page-2-0) with a neglectable error by varying one parameter  $\rho$ . This functional form of  $M(\theta)$  results in a surface on the principal stress space as shown in Fig. 2 (adapted from [Ledesma 2022\)](#page-2-0).

The results of element tests simulations of drained triaxial compression and simple shear tests are presented in [Fig. 3](#page-2-0) to further illustrate the influence that the functional form of  $M(\theta)$  has on the response of the model. These simulations were conducted for effective



**Fig. 2.** Representation of Matsuoka-Nakai (MN), Lade-Duncan (LD), William-Warnke (WW) yield criteria.

<span id="page-2-0"></span>

**Fig. 3.** Simulations of triaxial compression and direct simple shear tests for a confining stress of 500 kPa and an initial void ratio *e* = 0.75.



**Fig. 4.** Simulations of undrained triaxial compression and direct simple shear tests for a confining stress of 500 kPa and a void ratio *e* = 0.92.

confining stress of 500 kPa and an initial void ratio of 0.75. The constitutive parameters are included in Manzanal et al. (2011) and Ledesma (2022) and were initially used to reproduce experimental results on Toyoura Sand (Verdugo and Ishihara 1996). The effect of the Lode angle on the response of the model is illustrated by comparing the results of the triaxial tests (TX) against those of direct simple shear tests (DSS) in Fig. 3. On the other hand, the effect of the functional form of *M*  (*θ*) can be seen when comparing the results of DSS simulation with the different values of  $\rho$  where  $\rho = 1$  is equivalent to the Drucker-Prager criterion,  $\rho = 0.87$  is equivalent to Lade-Duncan, and  $\rho = 0.71$  is equivalent to Matsuoka-Nakai.

The influence that  $M(\theta)$  has on the onset of liquefaction was also investigated by performing similar simulations under undrained conditions. The results of these simulations are presented in Fig. 4, for confining stress of 500 kPa and a void ratio  $e = 0.92$  with the same constitutive parameters as the ones used for the drained simulations. The effect on the onset of liquefaction is equivalent to a reduction in the slope of the instability line – which is not a parameter of the model – when the stress states differ from an axisymmetric condition.

The authors thank the discussers for their interest in the behaviour of the model under constant shear drained stress paths (CSD). It is an interesting research topic that the authors would like to explore in the future.

## **Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

#### **Data availability**

No data was used for the research described in the article.

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