


# Symmetry in Nonlinear Dynamics and Chaos

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Nonlinear dynamics and chaos have collaborated to increase our understanding of several phenomena. In this Special Issue, some recent advances in nonlinear dynamical systems and chaotic behaviors are reviewed.

This Special Issue consists of eight papers recently published in MDPI's journal *Symmetry* under the general thematic title "Symmetry in Nonlinear Dynamics and Chaos" (see [1–8]). The content of these eight published papers covers the following subjects:

- Thermoablation in the treatment of tumorous bones [1].
- Analyses of a hyperchaotic nonlinear dynamical system [2].
- Shunting inhibitory cellular neural networks [3,5].
- Predictive regularity of symmetry groups to generate an algebraic theory of patterns [4].
- The nonlinear dynamics of the lithosphere–atmosphere–ionosphere coupling and its relation with earthquakes [6].
- The nonlinear dynamics of the impact of the non-constant velocity shockwave over straight surfaces [7].
- An analysis method for two-symbol symbolic time series [8].

In Ref. [1], Pakhmurin and coauthors analyze the influence of thermoablation treatment on the mechanical properties of bone tissue. They study the impact of such a thermal treatment on the structural properties of pig femurs in a range of 60–100 °C. The authors use an approach involving a periosteal arrangement of heating elements, generating gradual bone-dry heating from its periphery to its center. Inside the 60–90 °C range of heating, they did not find reduced structural characteristics in the samples with the thermoablation treatment in comparison to samples of healthy bones not subjected to this treatment. However, for 100 °C, they obtained a reduction in the elasticity modulus and an increment in the elastic strain (strain to failure). The authors show that the treatment of bone tissue in a thermal range of 60–90 °C can be utilized in studies of thermoablation efficiency against bone tumors.

In Ref. [2], Nestor and coauthors develop the nonlinear dynamic analyses of a new 4D hyperchaotic dynamical system with two positive Lyapunov exponents. They analyze the system theoretically and numerically to search for the hyperchaotic behavior, which is used in image cryptosystem design. The hyperchaotic system possesses a rich dynamical behavior, which includes a single unstable equilibrium point, self-excited attractors, hysteresis phenomenon, hidden attractors, and hyperchaotic functioning adequate for securing information. The authors also propose an image-encryption scheme using the hyperchaotic behavior of the 4D map. They use the original image SHA-256 hash value to generate the secret key of the cryptosystem. Finally, the authors, using performance studies, discover that the cryptosystem has elevated quality in terms of the performance/complexity trade-off compared to other algorithms.

Two papers were dedicated to shunting inhibitory cellular neural networks [3,5]. Akhmet and coauthors analyze shunting inhibitory cellular neural networks with continuous time-varying rates and inputs [3]. They introduce a new model with compartmental passive decay rates consisting of periodic and Poisson stable components and implement



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synchronized inputs and rates to obtain Poisson stable outputs. Further, they present numerical simulations of Poisson stable outputs and inputs that confirm the viability of theoretical results. In a second paper, Akhmet and coauthors study shunting inhibitory cellular neural networks with a generalized piecewise constant argument [5]. The system's impulsive part displays symmetry concerning the differential part. The authors examine the existence and asymptotic properties of the unique Poisson stable motion and develop simulation examples. They also compare the impulsive shunting inhibitory cellular neural networks with former neural network models. The results could be implemented in parallel image and signal processing, pattern recognition, and control and synchronization in deterministic and stochastic processes.

Rupe and Crutchfield generalize the predictive regularity of symmetry groups to provide an algebraic theory of patterns [4]. This theory is constructed from a core principle of future equivalence. The authors demonstrate how a minimal semiautomaton and its semigroup algebra generalize translation symmetry to partial and hidden symmetries. Further, they exhibit how future equivalence local versions can be used to obtain patterns in space-time. For higher dimensions, the authors specify two local representations that capture several features of space-time patterns and show that a previously developed local space-time variant of future equivalence is not a faithful generator of these patterns. The introduced theory defines and quantifies patterns in a wide range of classical field theories.

In Ref. [6], Politis and coauthors investigate the dynamics of stratospheric potential energy modifications before significant earthquakes ( $M > 6.7$ ). They analyze the atmospheric gravity waves as the principal mechanism for the energy transmission from the lower atmosphere to the stratosphere and mesosphere associated with atmospheric disturbances observed before strong earthquakes. The atmospheric gravity waves are generated by temperature disturbances, which are studied through stratospheric potential energy. The authors examine 11 cases of significant earthquakes that have occurred in the last 10 years in different geographic zones. They analyze the temperature profile at the location of each of the examined earthquakes. Via the temperature profile, the authors found the potential energy and determined the altitude range for which prominent pre-seismic disturbances were observed. The potential energy time series at specific altitudes within the determined "disturbed" range is studied using the criticality analysis method, called the "natural time" (NT) method. The purpose of this is to find evidence of an approach to a critical state prior to the earthquake. They find criticality indications in the fluctuation in potential energy between 1 and 15 days preceding the earthquakes, except in one case.

Monaldi and coauthors carry out numerical studies about the complex nonlinear dynamics generated by the impact of shockwaves traveling with non-constant velocity over straight surfaces [7]. They investigate two types of shockwaves generated by sudden energy released: cylindrical and spherical. The authors develop several numerical tests considering different distances between the origin of the shockwave and the reflecting surface. They use the Kurganov, Noelle, and Petrova (KNP) scheme implemented in the rhoCentralFoam solver of the OpenFOAM<sup>TM</sup> free software. The results of the shockwave Mach number, the reflected angle, and the position of the triple point are compared with pseudo-steady theory, numerical simulations, and experimental studies. The authors find accuracy for the reflected angle and minor differences in the Mach number. However, they detect that the triple-point position is more difficult to predict with high accuracy.

In Ref. [8], Contoyiannis and coauthors introduce a method to analyze two-symbol symbolic time series based on the time-to-space mapping achieved through a device of current carrying circular rings. They propose an algorithm that quantizes the stratified magnetic field, which uses the theory of prime numbers to estimate the magnetic field generated by the device. The authors use a two-symbol symbolic time series to find the flow directions of the rings currents, which allows them to obtain a time-to-space mapping of the system dynamics. They determine the dynamics of the original system by analyzing the space allocation of resulting quantized magnetic field values from randomness (lack of dynamics) to the criticality (dynamics at all scales). The authors find that the proposed

symbolic time-series analysis scheme can consider information related to both symbols, which allows them to analyze the two-symbol time series of short length, where the probabilities of occurrence of the two symbols are different. Further, the introduced method generates a unique way to determine the dynamics of the underlying complex system. However, this behavior does not occur with the study of waiting times in the time domain, which produces an ambiguous quantitative result.

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