Persisting black holes in a bouncing universe

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Resumen / El objetivo de este trabajo es investigar si un agujero negro puede estar presente en un universo que atraviesa una fase de contracción, rebote y expansión. Para ello, modelamos una inhomogeneidad embebida en universo de Friedmann-Lemaître-Robertson-Walker mediante la métrica generalizada comovil de McVittie, la cual tiene en cuenta la interacción de la masa central con el fluido cosmológico. Calculamos los horizontes atrapados, regiones espacio-temporales y determinamos la estructura de los conos de luz. Del análisis de la estructura causal concluimos que el agujero negro sobrevive al rebote y continúa su existencia en la fase de expansión. Esto implica que los modelos cosmológicos de rebote permiten la existencia de agujeros negros en todas las fases del universo, y a su vez que los agujeros negros provenientes de la época de contracción podrían jugar algún rol en la etapa de expansión.

Abstract / We analyze whether a black hole could exist and persist in a universe that goes through a phase of contraction, bounce and subsequent expansion. To this end, we investigate the comoving generalized McVittie metric that represents an inhomogeneity embedded in Friedmann-Lemaître-Robertson-Walker universe and allows interaction with the cosmic fluid. We compute the trapping horizons, spacetime regions and determine the light cone structure. The analysis of the causal structure leads us to conclude that a dynamical black hole survives the cosmological bounce and continues to exist in the expanding phase of the universe. This result implies that bouncing cosmologies admit black holes at all epochs and that these black holes might play some role in the expanding phase of the universe.

Keywords / black hole physics — gravitation — cosmology: theory

1. Introduction

Though the ΛCDM model is the most successful cosmological model up to date, being able to explain most of the available data Planck Collaboration et al. (2020); Ade et al. (2016), it is deficient in several aspects. One of its major problems is the initial cosmological singularity. Bouncing cosmologies provide an alternative to overcome this problem. In these models the universe contracts from a very diluted phase and then smoothly evolves into a bounce that leads to the current expansion epoch as described by the ΛCDM model. As the universe contracts, the temperature and density increase erasing all structure in the process. Black holes, however, might survive the bounce and play some role in the subsequent expanding phase of the universe.

The survival of black holes to a cosmological bounce, however, is unclear. Some authors have explored the problem using different approaches (Carr & Coley, 2011; Clifton et al., 2017; Gorkavyi & Tyul'bashev, 2021). Since black holes are essentially spacetime regions with a particular curvature, the global evolution of the universe should affect their horizons, especially close to a bounce. The whole process is dynamical, and hence cannot be investigated using the standard static solutions.

In a previous series of works Pérez et al. (2021a,b) we considered the evolution of the McVittie metric before, during, and after a cosmic bounce and showed that although the metric describes a black hole in the past of the bounce, the trapping horizon disappears close to it, when it merges with the cosmic horizon. In the McVittie metric, however, the central mass does not interact with the cosmic fluid (its mass remains constant), a situation that does not seem realistic. In this work, we deal with this problem and we investigate a black hole described by a generalized McVittie metric. These solutions are able to represent the interaction of the central object with the cosmic fluid and evolve with the universe.

2. Scale factor of the bouncing cosmological model

There are many mechanisms that could generate a cosmological bounce, either by classical or quantum effects. Novello & Bergliaffa (2008). We choose a scale factor that was derived by Celani and collaborators (Celani et al., 2017) considering quantum corrections to the classical evolution of the scale factor Pinto-Neto & Fabris (2013). It has the form

$$a(T) = a_b \left[1 + \left(\frac{T}{T_b}\right)^2\right]^{1/3}. \tag{1}$$

Here, T is the cosmic time and T_b fixes the bounce time scale, where 10^{-41} s $< T_b < 10^{-4}$ s Frion et al. (2020). We adopt a value close to the upper limit ($T_b = 10^{-4}$ s), so we consider a classical bounce for simplicity.

3. Comoving Generalized McVittie spacetime

The McVittie metric (McVittie, 1933) is a solution of Einstein field equations that describes an inhomogeneity embedded in a Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological background. This solution was later generalized by Faraoni & Jacques (2007). The corresponding line element in isotropic coordinates (T, r, θ, ϕ) reads

$$ds^2 = -\frac{\left[1 - \frac{Gm(T)}{2c^2 r}\right]^2}{\left[1 + \frac{Gm(T)}{2c^2 r}\right]^2} c^2 dT^2 \qquad (2)$$
$$+ a(T)^2 \left[1 + \frac{Gm(T)}{2c^2 r}\right]^4 \left[dr^2 + r^2 d\Omega^2\right].$$

Here, $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, $a(T)$ is the scale factor of the cosmological background model and $m(T)$ is a function that depends on the cosmic time T.

In this work, we focus on a particular class of generalized McVittie models that corresponds to the choice $m(T) = m_0$, where m_0 is a constant. Under this prescription, the line element (2) is

$$ds^2 = -\frac{\left[1 - \frac{Gm_0}{2c^2 r}\right]^2}{\left[1 + \frac{Gm_0}{2c^2 r}\right]^2} c^2 dT^2 \qquad (3)$$
$$+ a(T)^2 \left[1 + \frac{Gm_0}{2c^2 r}\right]^4 \left[dr^2 + r^2 d\Omega^2\right].$$

This metric is usually referred as Comoving Generalized McVittie (CGMcV) spacetime. In the limit $a(T) \to 1$, the Schwarzschild metric in isotropic coordinates is recovered, and if $m_0 \to 0$, we obtain the Friedmann-Lemaître-Robertson–Walker (FLRW) cosmological spacetime.

In order to determine whether the metric (3) represents a black hole that exists and survives in a universe that goes through a cosmological bounce, a full analysis of the causal structure of the spacetime is necessary. This includes studying the existence of trapping horizons, the determination of regular trapped and anti-trapped regions, and the computation of the trajectories of ingoing and outgoing radial null geodesics.

Trapping horizons are defined as the surfaces where null geodesics change their focusing properties Hayward (1994). Mathematically, these horizons are determined by the condition $\theta_{\rm in}\theta_{\rm out} = 0$ where $\theta_{\rm in}$ ($\theta_{\rm out}$) stands for the expansion of ingoing (outgoing) radial null geodesics with tangent field n^a (l^a). Spacetime regions can be classified as:

- *Regular* if $\theta_{\rm in}\theta_{\rm out} < 0$.
- *Anti-trapped* if $\theta_{\rm in}\theta_{\rm out} > 0$, where $\theta_{\rm in} > 0$ and $\theta_{\rm out} > 0$.
- *Trapped* if $\theta_{\rm in}\theta_{\rm out} > 0$, where $\theta_{\rm in} < 0$ and $\theta_{\rm out} < 0$.

Trapped regions are a key feature that allow to identify the presence of a black hole Hayward (1994): in the trapped region of a black hole ingoing and outgoing null rays are converging and remain confined and enclosed by a horizon.

The expansion of the null vector l^a (n^a) when the geodesic to which it is tangent is not necessarily affinelly-parametrized can be computed using the expression Faraoni (2015)

$$\theta_{\rm out} = \left[g^{ab} + \frac{l^a n^b + n^a l^b}{(-n^c l^d g_{cd})}\right] \nabla_a l_b, \qquad (4)$$

$$\theta_{\rm in} = \left[g^{ab} + \frac{l^a n^b + n^a l^b}{(-n^c l^d g_{cd})}\right] \nabla_a n_b. \qquad (5)$$

The analysis of the causal structure is much simpler if performed in Painlevé-Gullstrand (PG) coordinates $(\tilde{t}, \tilde{r}, \theta, \phi)$. Under this coordinate transformation, the line element (3) now takes the form*

$$ds^2 = a^2(\tilde{t}, \tilde{r}) d\tilde{s}^2, \qquad (6)$$
$$d\tilde{s}^2 = -c^2\left(1 - \frac{2Gm_0}{c^2\tilde{r}}\right) d\tilde{t}^2 + 2\, c\sqrt{\frac{2Gm_0}{c^2\tilde{r}}} d\tilde{t} d\tilde{r}$$
$$+ d\tilde{r}^2 + \tilde{r}^2 d\Omega^2. \qquad (7)$$

3.1. Trapping horizons

The outgoing (ingoing) radial null geodesic congruence of (7) have tangent fields (Nielsen & Visser, 2006):

$$l^\mu = \left(\frac{1}{c}, 1 - \sqrt{\frac{2Gm_0}{c^2\tilde{r}}}, 0, 0\right), \qquad (8)$$

$$n^\mu = \left(\frac{1}{c}, -1 - \sqrt{\frac{2Gm_0}{c^2\tilde{r}}}, 0, 0\right). \qquad (9)$$

We use these two vectors to calculate $\theta_{\rm out}$ and $\theta_{\rm in}$. In terms of the scale factor given by (1), the dimensionless cosmic time $\bar{T} \equiv T/T_b$ and $x \equiv \tilde{r}/r_g$ ($r_g \equiv Gm_0/c^2$), the expansions take the form

$$\theta_l = \frac{2}{r_g\, x}\left[f_1(x) + g(\bar{T}, x)\left(1 + \frac{\sqrt{\frac{2}{x}}}{f_2(x)}\right)\right],$$

$$\theta_n = \frac{-2}{r_g\, x}\left[f_2(x) + g(\bar{T}, x)\left(-1 + \frac{\sqrt{\frac{2}{x}}}{f_1(x)}\right)\right].$$

$$g(\bar{T}, x) = \frac{2\, r_g\, x}{3\, c}\frac{\bar{T}}{T_b}\frac{1}{\left(1 + \bar{T}^2\right)^{4/3}},$$

$$f_{1(2)}(x) = 1 \mp \sqrt{\frac{2}{x}}.$$

We show in Fig. 1 a plot of the location of the trapping horizons and the different spacetime regions.

3.2. Radial null geodesics and light cones

We obtain the equation that determines the trajectories of outgoing and ingoing radial null geodesics by setting $\widetilde{ds}^2 = 0$ and $d\theta = d\phi = 0$

$$\left.\frac{dx}{d\bar{T}}\right|_{\pm} = \frac{c\, T_b}{a(\bar{T}) r_g}\left(\pm 1 - \sqrt{\frac{2}{x}}\right), \qquad (10)$$

*Notice that there are two intermediate coordinate transformations: $(T, r, \theta, \phi) \to (T, \tilde{r}, \theta, \phi)$, isotropic to radius coordinate; $(T, \tilde{r}, \theta, \phi) \to (\tau, \tilde{r}, \theta, \phi)$, cosmic time to conformal time, and then to PG coordinates.

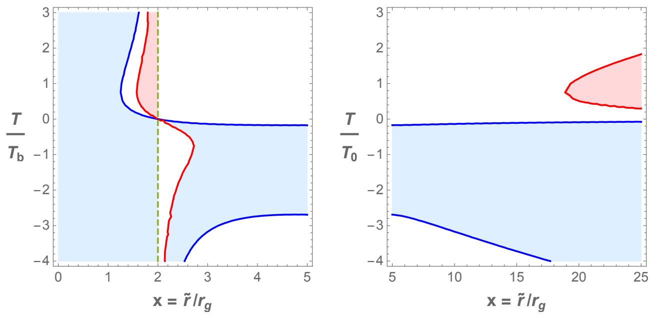

Figure 1: The blue and red lines indicates the conditions $\theta_{\rm out} = 0$ and $\theta_{\rm in} = 0$, respectively. The dashed green line denotes the surface $\tilde{r} = 2r_g$. The trapped regions are painted in light blue, the anti-trapped are coloured in light pink and the regular zones are in white. Here, $T_b = 10^{-4}$ s and $m_0 = 10\ M_\odot$.

where the "+" ("−") corresponds to the outgoing (ingoing) case. We integrate this equation and show the result in Fig. 2. The dotted curves represent the null ingoing geodesics while the dashed curves the null outgoing ones. The grey shadow regions show some light cones and the black arrow indicates the local future direction.

The trajectories of ingoing radial null geodesics have a negative slope for all values $\bar{T}$ and x. Those ingoing null rays that go through the surface $x = 2$, end up at the singular surface $x = 0$. These geodesics can cross the surface $x = 2$ in only one way: from $x > 2$ to $x < 2$ since the region enclosed by $x = 2$ is trapped.

Outgoing null geodesics are expanding in the region $x > 2$ for all values of the cosmic time. As they get closer to $x = 2$, the slope of the trajectories becomes smaller and in the limit $x \to 2$, the slope goes to zero. In the trapped region ($x < 2$), the slope of outgoing null rays is negative and these geodesics are interrupted at the singularity.

The light cone structure makes evident that the surface $x = 2$ acts as a one way membrane behaving like an event horizon that is present at all cosmological epochs of the universe (contraction, bounce and expansion). Thus, we conclude that the comoving generalized McVittie spacetime in a bouncing cosmological model includes a dynamical black hole at all times

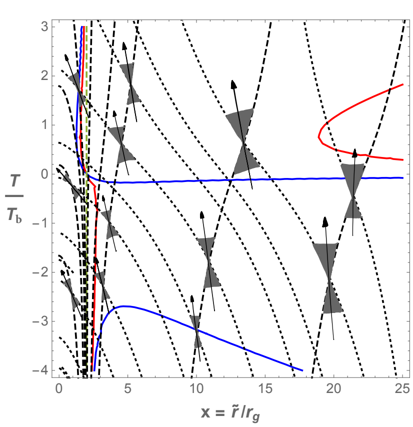

Figure 2: The dotted (dashed) curves represent the null ingoing (outgoing) radial geodesics. The grey shadow regions show some light cones and the black arrow indicates the future direction. Here, $T_b = 10^{-4}$ s and $m_0 = 10\ M_\odot$.

4. Conclusions

We have analyzed the causal structure of the comoving generalized McVittie spacetime in a bouncing cosmological model. We have computed the trapping horizons, spacetime regions, and derived the trajectories of radial null rays. We have probed that a dynamical black hole is present at all stages of the cosmic evolution, before, during and after the bounce.

If black holes survive through a cosmological bounce, they might play an important role in the subsequent expanding phase of the universe. For instance, these surviving black holes might contribute to a fraction of the total dark matter component or provide the seeds for the formation of galaxies Carr & Silk (2018); Carr & Kühnel (2020). Some of these issues will be explored in a future work.

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