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Mathematical models for rescheduling Ecuador's 2020 professional football league season disrupted by COVID-19

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ABSTRACT

The year 2020 saw the world turned upside down by the coronavirus pandemic. Countless human activities were suspended or cancelled as the virus spread across the globe. In this paper, we show how the regular season matches of Ecuador's professional football league were rescheduled due to the disruption caused by the pandemic. As with many others, this league had to reschedule its remaining games to fit within in a much shorter period of time than originally planned. To address this problem, we developed two mathematical models that designed new match calendars. The first one, a round assignment model, rescheduled the various rounds in the season still to be played while the second one, a day assignment model, took the solutions of the first model as input to assign the matches within each round to specific days. The implementation of our models secured a well-balanced number of days off before each match across all of the teams. Also, it enabled the league to conclude a full season without cancelling any matches or changing the schedule format, unlike what occurred in many other leagues, and won the approval of all stakeholders including league officials, players, team coaches, the TV broadcaster and fans.

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sports analytics, sports and pandemic, mathematical programming, football

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1. INTRODUCTION

The COVID-19 pandemic had an enormous impact on every aspect of life around the world in 2020 and 2021. Among the many consequences of the coronavirus as it spread across the globe were more than 250 million confirmed cases of the infection and over 5 million deaths (Our World in Data, 2021), coupled with a severe economic contraction and the paralysis or suspension of countless human activities. Latin America and its sporting activities were no exception, as both were hit hard by the effects of the pandemic. Many annual sports competitions were already underway when severe health measures were adopted by the region's governments, resulting in the cancellation of matches, stadium attendance reductions or prohibitions, and curbs on team travel. Sports league organizers found themselves facing the question of how to reschedule the remainder of their seasons. In this scenario, the deployment of scientific tools for rescheduling matches to fit a reduced time period and meet various other previously unimagined restrictions took on heightened importance.

A number of publications have studied various aspects of the sports league disruptions caused by COVID-19. Most have focussed on the problem of ranking teams, which was already a subject of research before the pandemic (see e.g. Veghes 2014), but gained special importance in cases of league seasons that were cut short due to the COVID-19 disruptions. For example, Csató (2021a) identified a set of desired axioms for ranking unfinished seasons and provided scoring rules that satisfied all of them. Radojicic et al. (2021) proposed a methodology based on hierarchical cluster analysis that can be applied when an unforeseen circumstance requires that a season be terminated before its scheduled completion. Van Eetvelde et al. (2021) introduced a tool called Probabilistic Final Standing Calculator to calculate the final standings of seasons that have remained unfinished.

The present paper reports on the case of the Ecuadorian professional football league, whose annual season calendars have been drawn up since 2019 through a collaborative effort between league officials and our research team using mathematical models from the field of sports scheduling. Although the literature in this area is extensive (see, for example, the surveys in Durán 2021; Kendall et al. 2010; Rasmussen – Trick 2008; Ribeiro 2012; and Van Bulck et al. 2020), to our knowledge no previous publications have presented applications dealing with the specific problem of redefining sports competition schedules in the wake of the pandemic.

In this study, we describe how the First Division season of the Ecuadorian league was mathematically redesigned to take account of the disruptions caused by COVID-19 and the many limitations the pandemic imposed. Our approach was built around two models solved in consecutive stages. The first one, a round assignment model, rescheduled the various rounds in the season still to be played while the second one, a day assignment model, took the solutions of the first model as input to assign the matches within each round to specific days. Although the problem of assigning matches to rounds has been thoroughly explored, assigning matches to specific days within rounds has been much less studied, and it is the solution of this latter problem that the present analysis aims to address with a novel approach. But both problems were aggravated by the effects of the pandemic, and both required the incorporation of previously unaddressed considerations that the two models proved capable of capturing efficiently.

Briefly, then, the main contribution of this study is the documentation of a real-world implementation that demonstrates the practical benefits of applying analytical tools to improve the organization and management of a sports competition. This has also been the message of previous studies (e.g., Csató 2021b; Wright 2009) but in the present case there is an added dimension in that the proposed approach was implemented in the midst of a pandemic on a world scale.

The remainder of this article is organized into four sections. Section 2 provides the necessary background on the Ecuadorian league's First Division and sets out the problem to be solved and the factors that had to be considered. Section 3 develops the solution approach adopted for the rescheduling of the season's remaining matches and details the formulation of the day assignment model. Section 4 reports the results obtained for the Ecuadorian case. Section 5 presents our conclusions and some remarks regarding future extensions.

2. DESCRIPTION OF THE PROBLEM

The First Division of Ecuador's professional football system was recently named by the International Federation of Football History & Statistics, in its annual *World's Strongest National Leagues* (IFFHS 2021) ranking for 2020, as one of the ten best leagues in the world. There are various metrics for rating leagues, but the IFFHS's positioning of Ecuador together with its national side and the recent performance of its top league teams places the country's football system in a solid position within South America, generally considered one of the two strongest regions in world football. Proof of the foregoing are the national side's qualification for the 2022 World Cup, the team Independiente del Valle's capture of the 2019 Copa Sudamericana, and Barcelona's run to the semi-finals in the 2021 Copa Libertadores, among the most notable events.

The 16 teams currently making up the Ecuadorian league's First Division play a mirrored double round robin over the course of a season divided into two phases. Each phase is thus a single round robin, meaning each team faces every other team once. In the second phase the matchups between each pair of teams follow the same order as the first except that each team's home-away status is reversed. There are 15 rounds of matches in each phase and thus a total of 30 in the whole season, distributed over the calendar year from February to December. In each round there are eight matches, mainly played over an extended weekend from Friday through Monday although in a few cases mid-week matches are scheduled between Tuesday and Thursday. Following the second phase, the season champion is decided in a two-legged playoff between the winners of each phase, and the combined final standings of the two phases determine which teams qualify for the following year's international cup competitions and which will be relegated to the Second Division.

The actual day of each match within each weekend or mid-week round is decided by league officials, subject to a long list of conditions submitted by the teams, the television broadcaster carrying the games, and the league itself. In simple applications, these decisions could all be made using manual methods either based on random assignment or following a template. For most practical cases such as professional leagues, however, there is a wide range of desirable conditions that must be met in order to arrive at a satisfactory schedule, which can usually be achieved by mathematical modelling techniques.

The list of conditions includes multiple criteria relating to scheduling fairness, organizational logistics, and match attractiveness to the public and for broadcast on television. In general terms, the organizers' objectives are to make the schedules as fair as possible for all of the teams while also boosting the league's profile and acceptance among fans and the media. These criteria are all well-established in the sports scheduling literature, as can be observed in a recent classification by Van Bulck et al. (2020) of the objective functions and constraints most commonly used in the field.

The pandemic hit Latin America with full force in March of 2020, triggering high COVID-19 case rates that led to the interruption of much of the countries' normal activities. Sporting events in particular were cancelled at both professional and recreational levels. When Ecuador's First Division season was suspended, only the first four rounds and some of the fifth round matches had already been played.

Then, in April, officials of *LigaPro de Ecuador*, the entity in charge of organizing the First Division calendar asked our research team to prepare a number of different scenario analyses and proposals for rescheduling the rounds to be played so that the season could be completed within the year, on the assumption that it would resume at some point between June and August. Our initial proposals consisted of several alternatives for short formats of less than the usual 30 rounds. The main idea of these alternatives was to split the teams into high-altitude and lowland regions, so that teams within the same region would play each other without the need of long travels. We also designed another proposal for the full complement of the 30 rounds involving all teams, which would end before the end of the year but with shorter periods off between each one. *LigaPro* opted to retain this original 30-round format, with resumption of the season slated for mid-August. This implied that in little more than four months, the teams would have to complete the 26 rounds that were left to play when the season was suspended, in addition to the finals. The first condition imposed by league officials for the redesigned schedule was that the initial rounds feature matches between teams from the same city or nearby cities to keep journeys to a minimum because of the pandemic travel restrictions.

Thus, the original season assignments for the first phase were of little use and the new ones, as well as retaining the original constraints, had to incorporate this additional limitation on team trip distances for the first four rounds following resumption. The combined effect of this restriction and the fact that various matches of the season had already been played led inevitably to a respecification of the overall scheduling problem for the first phase.

To fit the 26 rounds into the four remaining months of the year, two rounds had to be scheduled for almost every week instead of just the usual weekend round. This was accomplished by adding a large number of mid-week rounds (Tuesdays through Thursdays). But that in turn posed the further challenge of formulating a different mathematical assignment model that would incorporate a series of new conditions. These included ensuring that each team had at least three days off between any two consecutive matches, whether national or international, and that the number of days off each team in any matchup had since its last game was similar.

This latter restriction reflected the scheduling fairness principle, which meant avoiding such situations as a Thursday match where one of the teams had just played the previous Sunday while its opponent had not played since the Friday before that, thus giving the latter team the advantage of two more days off.

Such considerations have emerged as an important criterion in the recent sports scheduling literature (Atan – Çavdaroğlu 2018; Goossens et al. 2020; Lambers et al. 2021; Van Bulck – Goossens 2020), prompted not only by the inequities of matchups in which the teams are not equally rested (Scoppa 2015) but also by the player fatigue and risk of injuries caused by excessive match loads, that is, too many games in too short a period (Bengtsson et al. 2013; Dupont et al. 2010). In addition, the model had to consider the television broadcaster's requirements such as certain minimum and maximum numbers of matches per day.

3. SOLUTION APPROACH

The solution approach consists of two mathematical models solved in consecutive stages to produce new match calendar designs. The first one is a round assignment model that re-schedules the various rounds of the season still to be played while the second one is a day assignment model that takes the solutions of the first model as input and assigns the matches within each round to specific days.

3.1. Round assignment model

Mathematical programming techniques lie at the heart of both the original scheduling and the rescheduling of the 2021 *LigaPro* season. In either instance, the decision variables in the scheduling problem are binary variables x_{ijk} that take a value of 1 if team i plays at home against team j in round k and zero otherwise. Other decision variables capture sequences of two consecutive matches with the same home-way status, denoted home breaks or away breaks as the case may be. The many considerations conditioning the problems are expressed as constraints of equality or inequality between the decision variables.

One of the constraints aims at striking a balance in each phase and for each team between away matches in Ecuador's high-altitude regions and those in the lowlands. Another constraint attempts to avoid assigning away matches to a team on dates very close to those it is scheduled to play in international tournaments. Yet others seek to assign teams to away matches around dates when their home stadiums are not available and to accommodate requests from the television broadcasters regarding the rounds in which the season's most important games are played. The rescheduling model retains most of the original model's constraints, relaxing some of them by incorporating them in the objective function with penalties in order to minimize violations. It also contains additional constraints to take into account the matches played before the season suspension and the restriction on the initial rounds after resumption to matches between teams from nearby home ground locations.

Both schedules were designed using integer linear programming models whose formulations are broadly similar to those found in the literature. The reader is therefore referred to a number of other applications relating to those discussed here. A case involving Ecuadorian football is developed in Recalde et al. (2013) while more recent football examples pertain to Argentina (Durán et al. 2021a; 2021b), Belgium (Van Bulck et al. 2019), Chile (Alarcón et al. 2017), Iceland (Gunnarsdóttir 2019) and the South American qualifying rounds for the FIFA World Cup (Durán et al. 2017).

The solution method for the round assignment model is a two-step decomposition technique commonly applied in sports scheduling problems. In the first step, home-away patterns are created for the teams that determine, for each round, which teams play at home and which away. Then, in the second step, the model is solved considering all of the constraints but with the teams' home-away statuses for each round, or at least some of them, already set by the aforementioned patterns. By so decomposing the problem in order to define the home-away patterns in a step previous to solving it, the overall solution process is drastically accelerated.

A seminal work in the development of the decomposition approach is Nemhauser and Trick (1998), which applied it to an American college basketball league. Since then, multiple variants of the methodology have appeared in the literature and are surveyed in Rasmussen and Trick (2008). A recent variation applied to Argentina's professional football league created home-away patterns using clusters of teams based on geographical criteria (Durán et al., 2021), a technique that considerably reduced execution times for the Ecuadorian problem.

Fast generation of solutions is important for the practical implementation of model results. In our case the decomposition approach allowed us to readily produce some 30 different rescheduling proposals for the *LigaPro*, which were analyzed together with league officials before settling on the final version. The procedure involved generating multiple alternatives and submitting them to the officials in batches of two or three, which were then returned to us with observations that were addressed in the next iterations. This was repeated as many times as was necessary until the definitive solution was arrived at. It was thus crucial to have an optimization tool that was sufficiently fast and flexible to find candidate options in a matter of minutes, or at most, a few hours.

3.2. Day assignment model

Once a solution of the round assignment model that assigns matches to rounds has been selected, the assignment of matches to specific days within each round is normally carried out manually by league officials as each round approaches. However, with the shortened time window for completing the season due to the pandemic, the criteria relating to days off between actual match days within a round were much more difficult to satisfy. It was this new and unprecedented challenge that prompted the formulation of a day assignment model. Incorporating the requirements of the broadcasters televising the matches added further complexities to the model design. In what follows, we list the notation for the various elements of the day assignment model and then set out its development.

- Sets and parameters

I: set of teams.

K: set of rounds.

S: set of tuples (i,j,k) predetermined by the schedule such that team i must play at home against team j in round k.

D: set of days of the week.

$D_k \subset D$: set of days of the week on which round k can be played.

Fixed: set of tuples (i,j,k,d) such that team i must play at home against team j on day d of round k.

$IPop \subset I$: set of the most popular teams.

IPair: set of team pairs *(i,j)* such that *i* cannot play a match on the same day as *j* (unless they play against each other).

$DRepeat \subset D$: set of days such that a popular team cannot play on the same day of the week two weeks in a row.

DRest: set of pairs of days $(d, \widehat{d})$ such that day d is very close to day $\widehat{d}$ (less than three days apart), for which a team cannot play on day d in a given round and day $\widehat{d}$ in the following round.

Inter: set of tuples *(i,k,d)* such that team *i* cannot play on day *d* in round *k* due to the days-off requirement for its participation in international tournaments.

$u_{i,j}$: maximum difference between days off of teams i and j before their match.

n_d: maximum number of matches that can be played on day d.

p_d: maximum number of matches that can be played on day d featuring a popular team.

$t_{k,d,\widehat{d}}$: number of days between day d in round k-1 and day $\widehat{d}$ in round k.

- Decision variables

$y_{i,j,k,d}$: equal to 1 if team i plays at home against team j in round k on day d, and 0 otherwise.

$w_{i,k,d,\widehat{d}}$: equal to 1 if team i plays on day d in round k-1 and on day $\widehat{d}$ in round k, and 0 otherwise.

$\Delta_{i,j}$: difference between team i and team j in their respective number of days off before their matchup.

- Objective function

$$min \sum_{i \in I} \sum_{\substack{j \in I: \\ j \neq i}} \Delta_{i,j} \tag{1}$$

The objective function (1) minimizes the sum of the differences, over all pairs, between the two teams in each pair in their respective number of days off. The model formulation also considers the constraints below.

- Resting conditions

$$\Delta_{i,j} \leq u_{i,j} \quad \forall i \in I,\ j \in I : j \neq i \tag{2}$$

$$\sum_{\substack{j \in I: \\ j \neq i}} \left(y_{i,j,k,d} + y_{j,i,k,d} + y_{i,j,k+1,\widehat{d}} + y_{j,i,k+1,\widehat{d}} \right) \leq 1 \quad \forall i \in I, (d,\ \widehat{d}) \in DRest,\ k \in K : k < |K| \tag{3}$$

Constraint (2) ensures that the difference between two teams in their respective number of days off before their matchup does not exceed the corresponding upper bound $u_{i,j}$. In practice, this upper bound was equal to 2 if the match between teams i and j was on a weekend round preceded by a weekend round, and equal to 1 if their match was on a weekend (midweek) round preceded by a midweek (weekend) round. Constraint (3) prohibits a team from playing two matches on days very close together in consecutive rounds (e.g., a Thursday and the following Saturday, or a Sunday and the following Tuesday) in order to satisfy the requirement that any team has a minimum of three days off between consecutive matches, as recommended by FIFA.

- Logical relationships among variables

$$\sum_{\substack{j \in I: \\ j \neq i}} \left(y_{i,j,k-1,d} + y_{j,i,k-1,d} + y_{i,j,k,\hat{d}} + y_{j,i,k,\hat{d}} \right) \leq 1 + w_{i,k,d,\hat{d}} \quad \forall i \in I,\ k \in K : k>1,\ d \in D_{k-1},\ \hat{d} \in D_k \tag{4}$$

$$2w_{i,k,d,\hat{d}} \leq \sum_{\substack{j \in I: \\ j \neq i}} \left(y_{i,j,k-1,d} + y_{j,i,k-1,d} + y_{i,j,k,\hat{d}} + y_{j,i,k,\hat{d}} \right) \quad \forall i \in I,\ k \in K : k>1,\ d \in D_{k-1},\ \hat{d} \in D_k \tag{5}$$

$$\Delta_{i,j} \geq t_{k,d_1,\hat{d}_1} w_{i,k,d_1,\hat{d}_1} - t_{k,d_2,\hat{d}_2} w_{j,k,d_2,\hat{d}_2} \quad \forall d_1 \in D_{k-1}, \hat{d}_1 \in D_k,\ d_2 \in D_{k-1},\ \hat{d}_2 \in D_k, (i,\ j,\ k) \in S{:}k>1 \tag{6}$$

$$\Delta_{i,j} \geq t_{k,d_2,\hat{d}_2} w_{j,k,d_2,\hat{d}_2} - t_{k,d_1,\hat{d}_1} w_{i,k,d_1,\hat{d}_1} \quad \forall d_1 \in D_{k-1},\ \hat{d}_1 \in D_k,\ d_2 \in D_{k-1},\ \hat{d}_2 \in D_k,\ (i,\ j,\ k) \in S{:}k>1 \tag{7}$$

Constraints (4) and (5) are used in calculating the difference between two teams in their respective number of days off before their matchup. Constraints (6) and (7) express the logical relations for calculating the w variables.

- TV requirements and popular teams

$$\sum_{i \in I} \sum_{\substack{j \in I: \\ j \neq i}} y_{i,j,k,d} \leq n_d \quad \forall k \in K,\ d \in D \tag{8}$$

$$\sum_{i \in IPop} \sum_{\substack{j \in I: \\ j \neq i}} \left(y_{i,j,k,d} + y_{j,i,k,d} \right) \leq p_d \quad \forall k \in K,\ d \in D \tag{9}$$

$$\sum_{\substack{j \in I: \\ j \neq i}} \left(y_{i,j,k,d} + y_{j,i,k,d} + y_{i,j,k+1,d} + y_{j,i,k+1,d} \right) \leq 1 \quad \forall i \in IPop,\ d \in DRepeat,\ k \in K : k<|K| \tag{10}$$

$$y_{i,j,k,d} = 1 \quad \forall (i,\ j,\ k,\ d) \in Fixed \tag{11}$$

Constraint (8) imposes a maximum number of matches played on a given day depending on the television broadcaster's requests. Similarly, constraint (9) imposes a maximum number of matches played on a given day featuring a popular team (so that the most attractive matches of a given round are not played all on the same day). Constraint (10) prohibits a popular team from playing on the same day of the week two weeks in a row. This is motivated by the television broadcaster's desire to maintain diversity in its programming (for example, a given popular team cannot play two consecutive Fridays). Constraint (11) sets an exact day for certain matches where so decided by league officials, based typically on special requests from the television broadcaster.

- Safety requirements

$$\sum_{\substack{h\in I:\\ h\neq i,h\neq j}} \left(y_{i,h,k,d} + y_{h,i,k,d} + y_{j,h,k,d} + y_{h,j,k,d}\right) \leq 1 \quad \forall(i,\ j)\in IPair,\ d\in D,\ k\in K \tag{12}$$

Constraint (12) makes sure that traditional rivals whose home grounds are in the same city do not play on the same day (except in matches where they play against each other) to avoid the increased risk of street disturbances by team supporters.

- International tournaments

$$\sum_{\substack{j\in I:\\ j\neq i}} \left(y_{i,j,k,d} + y_{j,i,k,d}\right) = 0 \quad \forall(i,\ k,\ d)\in Inter \tag{13}$$

Constraint (13) guarantees that teams participating in international tournaments have at least three days off before their tournament matches.

- Consistency with round assignment

$$\sum_{d\in D_k} y_{i,j,k,d} = 1 \quad \forall(i,\ j,\ k)\in S \tag{14}$$

Constraint (14) ensures that matches to be played in a given round as previously determined by the round assignment model are assigned to a day within that round.

- Domain constraints

$$y_{i,j,k,d}\in\{0,\ 1\} \quad \forall i\in I,\ j\in I: j\neq i,\ k\in K,\ d\in D_k \tag{15}$$

$$w_{i,k,d,\widehat{d}}\in\{0,\ 1\} \quad \forall i\in I,\ k\in K: k>1,\ d\in D_{k-1},\ \widehat{d}\in D_k \tag{16}$$

$$\Delta_{i,j}\geq 0 \quad \forall i\in I,\ j\in I: j\neq i \tag{17}$$

Constraints (15) to (17) define the variables' domains.

The elements set forth above thus form a mixed integer linear programming model that can be solved in a matter of seconds by solvers such as Gurobi or CPLEX. The fact that the assignments of matches to rounds are inputted to the day assignment model makes it considerably easier to solve than the round assignment model, so in the former case there is no need for any decomposition algorithm or other sophisticated procedure. Note also that constraints (11), (13) and (14) eliminate a large number of variables, which therefore could have been omitted altogether with certain constraints rewritten accordingly. Nevertheless, we have chosen to maintain them in order to list all of the constraints explicitly and provide thereby a fuller presentation of the model's formulation.

4. RESULTS

This section presents the results of the implementation of our model designs for the rescheduling of the Ecuadorian league in the wake of the pandemic. The first two subsections discuss

Table 1. Pre- and post-pandemic distribution of each team's home and away games

Team abbreviation	Full name	Pre-pandemic schedule				Post-pandemic schedule			
		Home breaks	Away breaks	@ High-altitude	@ Lowland	Home breaks	Away breaks	@ High-altitude	@ Lowland
AUC	Aucas	1	1	4	3	2	1	3	2
BAR	Barcelona	0	0	5	2	1	1	4	1
CUE	Cuenca	1	1	4	3	1	1	3	2
DEL	Delfín	1	0	5	2	1	0	4	1
EME	Emelec	0	0	5	3	1	1	4	2
GYQ	Guayaquil City	0	1	5	3	0	1	4	1
IND	Independiente del Valle	1	1	4	3	1	1	3	3
LIP	Liga Portoviejo	1	1	5	3	2	1	3	2
LIQ	Liga de Quito	1	1	5	3	0	1	3	3
MAC	Macará	0	1	5	3	1	1	3	2
MSH	Mashuc Runa	1	1	5	3	2	1	2	3
NAC	C.D. El Nacional	1	1	4	3	0	1	3	3
OLM	Olmedo	0	1	5	3	0	1	3	2
ORE	Orense	1	0	5	2	2	1	3	2
TEC	Técnico Universitario	1	0	4	3	1	1	4	1
UCQ	Universidad Católica	1	1	5	3	0	0	2	3

Source: authors.

Team\Round	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
AUC	DEL	@TEC	MSH	@EME	GYQ	@OLM	LIQ	@IND	CUE	@ORE	MAC	NAC	@BAR	@UCQ	LIP
BAR	TEC	@DEL	LIP	@LIQ	OLM	@IND	NAC	@CUE	ORE	@MAC	EME	@GYQ	AUC	@MSH	UCQ
CUE	LIQ	@LIP	UCQ	@NAC	IND	ORE	@DEL	BAR	@AUC	TEC	@MSH	@MAC	OLM	@EME	GYQ
DEL	@AUC	BAR	TEC	@LIP	NAC	@MAC	CUE	@GYQ	EME	@IND	ORE	@OLM	UCQ	@LIQ	MSH
EME	@ORE	GYQ	@MAC	AUC	@UCQ	LIP	@TEC	LIQ	@DEL	MSH	@BAR	IND	@NAC	CUE	@OLM
GYQ	OLM	@EME	@ORE	MAC	@AUC	UCQ	@MSH	DEL	@LIQ	NAC	@LIP	BAR	@IND	TEC	@CUE
IND	MSH	@LIQ	OLM	UCQ	@CUE	BAR	@LIP	AUC	@TEC	DEL	@NAC	@EME	GYQ	@ORE	MAC
LIP	@NAC	CUE	@BAR	DEL	@TEC	@EME	IND	@ORE	MAC	@UCQ	GYQ	LIQ	@MSH	OLM	@AUC
LIQ	@CUE	IND	@NAC	BAR	@ORE	MSH	@AUC	@EME	GYQ	@OLM	UCQ	@LIP	MAC	DEL	@TEC
MAC	UCQ	@OLM	EME	@GYQ	@MSH	DEL	@ORE	TEC	@LIP	BAR	@AUC	CUE	@LIQ	NAC	@IND
MSH	@IND	NAC	@AUC	@TEC	MAC	@LIQ	GYQ	@OLM	UCQ	@EME	CUE	@ORE	LIP	BAR	@DEL
NAC	LIP	@MSH	LIQ	CUE	@DEL	TEC	@BAR	@UCQ	OLM	@GYQ	IND	@AUC	EME	@MAC	ORE
OLM	@GYQ	MAC	@IND	ORE	@BAR	AUC	@UCQ	MSH	@NAC	LIQ	@TEC	DEL	@CUE	@LIP	EME
ORE	EME	@UCQ	GYQ	@OLM	LIQ	@CUE	MAC	LIP	@BAR	AUC	@DEL	MSH	@TEC	IND	@NAC
TEC	@BAR	AUC	@DEL	MSH	LIP	@NAC	EME	@MAC	IND	@CUE	OLM	@UCQ	ORE	@GYQ	LIQ
UCQ	@MAC	ORE	@CUE	@IND	EME	@GYQ	OLM	NAC	@MSH	LIP	@LIQ	TEC	@DEL	AUC	@BAR

Away game in high-altitude region | Away game in lowland region | Popular team

Red: Away break games | Blue: Home break games

Fig. 1. Pre-pandemic scheduling of first phase matches
Source: authors

the results obtained, while the third compares them to the schedules adopted by other leagues following the outbreak of COVID-19.

4.1. Results of the round assignment model

Data on the distribution of home and away matches in each Ecuadorian team's schedule before and after the pandemic suspension (hereafter "pre-pandemic" and "post-pandemic") are set out in Table 1. A more detailed view of the two sets of match assignments for the 15 first phase rounds are shown in Figs 1 and 2, respectively. Note that each match appears twice, once for each participating team. The "@" symbol indicates away games.

Notable among the characteristics of the schedules is the good balance of away matches between the high-altitude and lowland regions of the country. More specifically, the pre-pandemic schedules assigned every team either four of five away matches in high-altitude regions, and either two or three in lowland regions. Post-pandemic, the corresponding numbers

Team\Round	5	6	7	8	9	10	11	12	13	14	15
AUC	@UCQ	NAC	LIQ	@IND	MAC	@BAR	GYQ	CUE	@ORE	@OLM	LIP
BAR	ORE	@CUE	EME	@GYQ	OLM	AUC	@MSH	UCQ	@MAC	@IND	NAC
CUE	GYQ	BAR	@MAC	ORE	@MSH	TEC	@DEL	@AUC	IND	@EME	OLM
DEL	EME	@GYQ	ORE	@LIQ	NAC	@OLM	CUE	@IND	MSH	UCQ	@MAC
EME	@DEL	MSH	@BAR	LIP	@TEC	@NAC	IND	@OLM	LIQ	CUE	@UCQ
GYQ	@CUE	DEL	@LIP	BAR	@IND	UCQ	@AUC	@LIQ	NAC	@MSH	TEC
IND	@NAC	MAC	@TEC	AUC	GYQ	@LIP	@EME	DEL	@CUE	BAR	@ORE
LIP		@ORE	GYQ	@EME	@UCQ	IND	LIQ	@MSH	OLM	MAC	@AUC
LIQ	@OLM	UCQ	@AUC	DEL	@ORE	MAC	@LIP	GYQ	@EME	@TEC	MSH
MAC		@IND	CUE	TEC	@AUC	@LIQ	NAC	@ORE	BAR	@LIP	DEL
MSH		@EME	@OLM	UCQ	CUE	@ORE	BAR	LIP	@DEL	GYQ	@LIQ
NAC	IND	@AUC	@UCQ	OLM	@DEL	EME	@MAC	TEC	@GYQ	ORE	@BAR
OLM	LIQ	@TEC	MSH	@NAC	@BAR	DEL	@UCQ	EME	@LIP	AUC	@CUE
ORE	@BAR	LIP	@DEL	@CUE	LIQ	MSH	@TEC	MAC	AUC	@NAC	IND
TEC		OLM	IND	@MAC	EME	@CUE	ORE	@NAC	@UCQ	LIQ	@GYQ
UCQ	AUC	@LIQ	NAC	@MSH	LIP	@GYQ	OLM	@BAR	TEC	@DEL	EME

Away game in high-altitude region | Away game in lowland region | Popular team

Red: Away break games | Blue: Home break games

Fig. 2. Post-pandemic rescheduling of first phase assignments
Source: authors

were two to four and one to three, respectively, but note that these results refer only to a part of the schedule since the rest had already been played before the suspension. Also, in 25 out of the 30 matches played in the first four rounds following resumption, the home team and the away team were from the same region, while in the other 5 matches the away team travelled to the closest city outside its region.

Another noteworthy characteristic is the well-balanced distribution of home and away matches over the length of the phase. The pre-pandemic schedule assigned each team at least seven and at most eight home games and no more than one break at home and another away. The post-pandemic assignments relaxed the upper bound on home breaks to two.

The schedules also achieved a reasonable spacing over the phase of matchups involving the most popular teams in the league (Barcelona, Emelec and Liga de Quito), in every case satisfying a lower bound of two intervening games against other sides.

Starting at Round 5 after the pandemic suspension, the rescheduling of the first phase assigned new dates for 86 matches, as can be seen in Fig. 2. Of those 86, only the six shown within rectangles were assigned to the same round they had been assigned to in the original scheduling. This underlines the importance of rescheduling with optimization models, for it is

Table 2. Number of days off for each team as of its match day in each round, first phase of the tournament

Team\Round	6	7	8	9	10	11	12	13	14	15	Min	Max	Avg
AUC	3	3	3	3	4	5	7	5	8	6	3	8	4.7
BAR	3	5	3	3	3	3	9	5	8	7	3	9	4.9
CUE	3	3	4	4	3	5	7	6	8	7	3	8	5.0
DEL	3	3	4	5	3	4	7	7	7	7	3	7	5.0
EME	4	4	3	3	4	4	5	8	8	5	3	8	4.8
GYQ	3	4	4	4	3	4	5	8	7	7	3	8	4.9
IND	3	4	3	5	3	4	7	6	7	7	3	7	4.9
LIP		4	4	4	3	3	7	6	9	5	3	9	5.0
LIQ	3	3	3	3	4	4	6	8	6	9	3	9	4.9
MAC		3	4	3	4	4	7	6	9	7	3	9	5.2
MSH		4	3	3	3	3	8	8	6	8	3	8	5.1
NAC	4	4	3	4	3	3	7	7	6	8	3	8	4.9
OLM	3	4	3	3	4	3	6	7	8	8	3	8	4.9
ORE	3	3	4	3	4	3	8	6	7	8	3	8	4.9
TEC		3	3	4	3	3	8	8	5	8	3	8	5.0
UCQ	3	4	3	4	3	3	8	7	7	5	3	8	4.7

Source: authors.

unlikely that a manual approach based on the original assignments would have been able to reallocate so many matches to different rounds while still satisfying all of the applicable conditions. As for the second phase, the match-to-round assignments were the same as those scheduled before the pandemic so in this case the rescheduling did not require the running of the first-stage model.

4.2. Results of the day assignment model

To illustrate the effectiveness of the day assignment model for the second phase, some important results are summarized in Tables 2–5.

As may be appreciated in Tables 2 and 3, the number of days off enjoyed by each team before each match was never less than three. Also, this number was well-balanced across all of the teams. Moreover, the average number was similar for all of them, varying only slightly from 4.7 to 5.2 days in the first phase and 4.6–4.8 days in the second phase. As for days-off comparisons of the two teams in each match, Tables 4 and 5 indicate that the differences between them were relatively minor. Among the 188 matches for which the teams involved both had games in the previous round, the maximum difference was 2 days (but even this was the case on

Table 3. Number of days off for each team as of its match day in each round, second phase of the tournament

Team\Round	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Min	Max	Avg
AUC	3	6	9	7	3	4	3	4	7	6	4	4	3	4	3	9	4.8
BAR	3	8	6	7	3	3	6	4	6	7	3	4	3	3	3	8	4.7
CUE	5	6	7	5	4	3	6	3	5	7	4	3	4	3	3	7	4.6
DEL	3	7	8	6	3	3	5	4	6	6	4	5	3	4	3	8	4.8
EME	3	5	9	7	3	3	4	4	6	8	3	3	3	3	3	9	4.6
GYQ	3	6	7	8	3	3	4	3	6	9	3	3	3	3	3	9	4.6
IND	3	8	6	6	4	5	3	3	7	8	3	3	3	4	3	8	4.7
LIP	4	6	7	6	4	4	4	4	6	7	3	3	3	5	3	7	4.7
LIQ	3	7	7	6	5	4	3	3	7	7	4	4	3	4	3	7	4.8
MAC	3	5	8	7	3	5	3	5	6	6	3	5	3	3	3	8	4.6
MSH	4	5	7	8	4	3	5	4	5	6	4	4	4	4	3	8	4.8
NAC	3	6	8	6	3	3	6	3	5	9	3	3	4	3	3	9	4.6
OLM	3	7	5	8	4	3	5	3	6	7	3	3	4	3	3	8	4.6
ORE	4	5	6	7	4	5	4	4	6	5	4	3	4	4	3	7	4.6
TEC	3	7	6	8	3	4	4	3	6	8	3	3	4	5	3	8	4.8
UCQ	4	6	6	8	3	3	5	4	6	6	3	5	3	3	3	8	4.6

Source: authors.

Table 4. Difference between teams in their respective number of days off as of the day of the match between them, first phase of the tournament

Team\Round	6	7	8	9	10	11	12	13	14	15	Min	Max	Avg
AUC	−1	0	0	0	1	1	0	−1	0	1	−1	1	0.1
BAR	0	1	−1	0	−1	0	1	−1	1	−1	−1	1	−0.1
CUE	0	0	0	1	0	1	0	0	0	−1	−1	1	0.1
DEL	0	0	1	1	−1	−1	0	−1	0	0	−1	1	−0.1
EME		−1	−1	−1	1	0	−1	0	0	0	−1	1	−0.3
GYQ	0	0	1	−1	0	−1	−1	1	1	−1	−1	1	−0.1
IND		1	0	1	0	0	0	0	−1	−1	−1	1	0.0
LIP		0	1	0	0	−1	−1	−1	0	−1	−1	1	−0.3
LIQ	0	0	−1	0	0	1	1	0	1	1	−1	1	0.3
MAC		0	1	0	0	1	−1	1	0	0	−1	1	0.2
MSH		0	0	−1	−1	0	1	1	−1	−1	−1	1	−0.2
NAC	1	0	0	−1	−1	−1	−1	−1	−1	1	−1	1	−0.4
OLM		0	0	0	1	0	1	1	0	1	0	1	0.4
ORE		0	0	0	1	0	1	1	1	1	0	1	0.6
TEC		−1	−1	1	0	0	1	1	−1	1	−1	1	0.1
UCQ	0	0	0	0	0	0	−1	−1	0	0	−1	0	−0.2

Source: authors.

just three occasions, which involved only weekend rounds, so the games were not too close to each other). For all the others, the difference was just 1 day or none at all. Another interesting virtue of the model revealed in Tables 2 and 3 is that the days-off difference across the various matches of a given round could potentially be as many as four days, but such inequities were effectively prevented by the model.

More generally, we note that the schedules defined by the optimization models enabled the entire set of rounds for the two phases of the 2020 season including the finals to be completed on 29 December of that year in an exciting finish, when Barcelona de Guayaquil defeated Liga Deportiva Universitaria de Quito on penalties after both matches of the two-legged playoff had ended in a draw.

4.3. Comparisons with other leagues

As has just been demonstrated, the day assignment model allowed the Ecuadorian football league season to be completed despite the reduced time period for fitting in all of the matches, and every requested conditions and restrictions were also satisfied. By contrast, most of the other South American leagues were unable to complete their 2020 seasons within the year. In Argentina, the

Table 5. Difference between teams in their respective number of days off as of the day of the match between them, second phase of the tournament

Team\Round	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Min	Max	Avg
AUC	0	1	0	−1	−1	0	0	1	1	0	1	0	0	−1	−1	1	0.1
BAR	0	2	−1	−1	−1	0	0	0	0	−1	0	0	−1	0	−1	2	−0.2
CUE	1	0	−1	−1	0	0	0	−1	−1	1	1	0	1	0	−1	1	0.0
DEL	0	0	1	0	0	0	1	0	−1	1	1	0	0	0	−1	1	0.2
EME	0	0	0	−1	−1	−1	1	0	1	1	0	0	−1	0	−1	1	−0.1
GYQ	0	1	−1	1	0	0	−1	0	1	2	0	0	−1	0	−1	2	0.1
IND	0	1	0	1	1	1	0	0	1	−1	0	0	−1	1	−1	1	0.3
LIP	−1	−2	−1	−2	1	−1	0	−1	0	−2	−1	−1	−1	1	−2	1	−0.8
LIQ	0	1	1	−1	1	0	−1	0	1	1	1	−1	0	−1	−1	1	0.1
MAC	0	0	1	−1	0	0	−1	1	0	0	−1	1	−1	−1	−1	1	−0.1
MSH	1	−1	1	1	−1	0	0	0	−1	−1	0	1	1	0	−1	1	0.1
NAC	−1	−1	1	0	0	0	1	0	−1	1	−1	0	1	−1	−1	1	−0.1
OLM	0	−1	−1	1	1	0	0	0	−1	−1	−1	0	1	0	−1	1	−0.1
ORE	0	−1	1	1	0	0	0	0	−1	−1	0	0	1	1	−1	1	0.1
TEC	0	0	−1	2	0	1	1	0	1	1	0	0	1	1	−1	2	0.5
UCQ	0	0	0	1	0	0	−1	0	0	−1	0	0	0	0	−1	1	−0.1

Source: authors.

finals were held in January 2021 after a shortened season. The Brazilian and Chilean leagues played a full season, but that meant pushing the playoffs back to February 2021, and in Uruguay the season ended later still with the championship-deciding match played in March.

A comparison with the Spanish and Italian leagues in 2020, two of the most important in the world, is particularly worthy of note. In both cases, the season was also rescheduled by reducing the spacing between rounds using methods we presume were strictly manual, but unlike Ecuador there were significant discrepancies between teams in days off before matches, creating a major problem of sporting fairness. Thus, over the last 10 rounds of the Spanish season there were 14 matches in which the day-off difference between the two teams was two days while in three other matches the difference was three days. In Italy, meanwhile, there were seven matches over the last 11 rounds where the difference was two days.

5. CONCLUSIONS

This paper has developed an optimization approach to reschedule a season match calendar, motivated by the real-world case of the Ecuadorian football league which like so many others was disrupted by the COVID-19 pandemic. The main complications in the rescheduling process were the large number of matches still to play and the shortened time period in which to organise them. This in turn meant that schedule fairness in terms of maintaining a balance across teams in their numbers of days off between matches became a major issue. Traditional manual or random scheduling methods were ill-suited for the challenge posed by this problem. By contrast, the optimization approach reported here was able to find solutions that addressed this and other difficulties arising from the pandemic, and thus proved to be an effective tool in supporting Ecuadorian league officials' rescheduling decisions. The implementation of our approach enabled the league to conclude a full season without cancelling any matches or changing the schedule format, unlike what occurred in many other leagues.

The methodology was built around two models, the first one assigning matches to rounds and inputting the results to a second one that assigned matches to days within rounds. Although designed specifically for the Ecuadorian case, the day assignment model could be applied more generally to other cases given that the basic problem it was designed to solve is an important one faced by all sports competition organizers, yet has not been extensively explored in the literature. The proposed formulation, agreed to by league officials, is based on minimizing the differences, over all team pairs, between the two teams in each pair in their respective number of days off. However, this global minimization does not guarantee that no inequalities between teams will be generated. Other approaches could therefore be considered. One such alternative would be to minimize the difference between the two teams with the most and least accumulated rest days, respectively. Another would be a lexicographic minimization method that first minimizes the days off for the team with the largest difference, then sets the result as a constraint for the team(s) where the maximum is reached, and repeats these steps for the second-largest case, iterating the procedure in decreasing order of size of difference for the rest of the teams. In addition, since the twin problems of round and day assignment solved here by separate models are interrelated, another interesting challenge for future research would be to tackle the development of a single formulation that integrates the two. Finally, an approach that merits exploration at a more qualitative level would be to conduct interviews in order to gauge the level of satisfaction with the schedules among the various stakeholders in the process.

To sum up, despite the crisis faced by the Ecuadorian football league due to the COVID-19 pandemic, the 2020 season was completed within the usual time period under conditions that met with the approval of all the key actors including league officials, players, coaches, the television broadcaster and football fans. Commenting on the results of our approach, Mr. David Constante, competitions director at LigaPro de Ecuador, had this to say:

Due to the pandemic, we suspended the season in March 2020 after only four rounds, and began the process of redesigning the rest of the schedule in collaborations with this scientific team. Various alternatives were proposed, and we opted to maintain the original setup but with all games rescheduled starting from the fifth round of the first phase of the season. The primary objective was to keep travel in the early rounds to a minimum. Since the teams' days off between matches would have to be reduced, we had to implement a system of assigning match days within each round that would ensure the teams in each game were sufficiently and equally rested. This was made possible thanks to the sophisticated mathematical methods applied by our team of expert collaborators. As a result, the season was completed on time and remained competitive right to the end.

We conclude simply by noting that LigaPro has continued using the methodologies described in this paper for the scheduling of the 2021 and 2022 seasons.

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