

**Dark torsion as the cosmic speed-up**

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It is shown that the recently detected acceleration of the Universe can be understood by considering a modification of the teleparallel equivalent of general relativity, with no need of dark energy. The solution also exhibits phases dominated by matter and radiation as expected in the standard cosmological evolution. We perform a joint analysis with measurements of the most recent type Ia supernovae, baryon acoustic oscillation peak, and estimates of the cosmic microwave background shift parameter data to constrain the only new parameter this theory has.

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**I. INTRODUCTION**

The discovery of an unexpected diminution in the observed energy fluxes coming from type Ia supernovae [1,2] has opened one of the most puzzling and deepest problems in cosmology today. These observations have been interpreted as solid evidence for an accelerating universe dominated by something called dark energy. Although the cosmological constant seems to be the simplest explanation for the phenomenon, several dynamical scenarios have been tried out since 1998 (see e.g., [3–5]). While some authors sustain the idea of the existence of a dark energy, others propose modifications of the Einstein-Hilbert Lagrangian known as  $f(R)$  ([6–14] or [15–17] for recent reviews) as a way to obtain a late accelerating expansion. A great difficulty these theories have, from the point of view of the metric formalism, is that the resulting field equations are 4th order equations, which in many cases makes these hard to analyze. Besides, the simplest cases of the kind  $f(R) = R - \beta/R^n$  have shown difficulties with weak field tests [18,19] and gravitational instabilities [20] and do not present a matter dominated era previous to the acceleration era [21,22]. Alternatively, the Palatini variational approach for such  $f(R)$  theories leads to 2nd order field equations, and some authors have achieved putting observational constraints on these theories [23–25]. However in many cases the equations are still hard to work with, as evidenced by the functional form of the modified Friedmann equation for a generic  $f(R)$ . Recently, models based on modified teleparallel gravity were presented as an alternative to inflationary models [26,27]. In this paper we show a cosmological solution for the acceleration of the Universe by means of a sort of theories of modified gravity, namely  $f(L_T)$ , based on a modification of the teleparallel equivalent of general relativity (TEGR) Lagrangian [28,29], where the torsion will be responsible for the observed

acceleration of the Universe, and the field equations will always be 2nd order equations.

**II. GENERAL CONSIDERATIONS: FIELD EQUATIONS**

Teleparallelism [28,29] uses as the dynamical object a vierbein field  $\mathbf{e}_i(x^\mu)$ ,  $i = 0, 1, 2, 3$ , which is an orthonormal basis for the tangent space at each point  $x^\mu$  of the manifold:  $\mathbf{e}_i \cdot \mathbf{e}_j = \eta_{ij}$ , where  $\eta_{ij} = \text{diag}(1, -1, -1, -1)$ . Each vector  $\mathbf{e}_i$  can be described by its components  $e_i^\mu$ ,  $\mu = 0, 1, 2, 3$  in a coordinate basis; i.e.  $\mathbf{e}_i = e_i^\mu \partial_\mu$ . Notice that Latin indices refer to the tangent space, while Greek indices label coordinates on the manifold. The metric tensor is obtained from the dual vierbein as  $g_{\mu\nu}(x) = \eta_{ij} e_i^\mu(x) e_j^\nu(x)$ . Differing from general relativity, which uses the torsionless Levi-Civita connection, teleparallelism uses the curvatureless Weitzenböck connection [30], whose non-null torsion is

$$T_{\mu\nu}^\lambda = \overset{\mathbf{w}}{\Gamma}_{\nu\mu}^\lambda - \overset{\mathbf{w}}{\Gamma}_{\mu\nu}^\lambda = e_i^\lambda (\partial_\mu e_\nu^i - \partial_\nu e_\mu^i). \quad (1)$$

This tensor encompasses all the information about the gravitational field. TheTEGR Lagrangian is built with the torsion (1), and its dynamical equations for the vierbein imply the Einstein equations for the metric. The teleparallel Lagrangian is [29,31,32]

$$L_T = S_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu}, \quad (2)$$

where

$$S_\rho{}^{\mu\nu} = \frac{1}{2}(K^{\mu\nu}{}_\rho + \delta_\rho^\mu T^{\theta\nu}{}_\theta - \delta_\rho^\nu T^{\theta\mu}{}_\theta) \quad (3)$$

and  $K^{\mu\nu}{}_\rho$  is the contorsion tensor:

$$K^{\mu\nu}{}_\rho = -\frac{1}{2}(T^{\mu\nu}{}_\rho - T^{\nu\mu}{}_\rho - T_\rho{}^{\mu\nu}), \quad (4)$$

which equals the difference between Weitzenböck and Levi-Civita connections.

In this work the gravitational field will be driven by a Lagrangian density that is a function of  $L_T$ . Thus the action

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reads

$$I = \frac{1}{16\pi G} \int d^4x e f(L_T), \quad (5)$$

where  $e = \det(e^i_\mu) = \sqrt{-g}$ . The case  $f(L_T) = L_T$  corresponds to TEGR. If matter couples to the metric in the standard form then the variation of the action with respect to the vierbein leads to the equations

$$e^{-1} \partial_\mu (e S_i^{\mu\nu}) f'(L_T) - e_i^\lambda T^\rho_{\mu\lambda} S_\rho^{\nu\mu} f'(L_T) + S_i^{\mu\nu} \partial_\mu (L_T) f''(L_T) + \frac{1}{4} e_i^\nu f(L_T) = 4\pi G e_i^\rho T_\rho^\nu, \quad (6)$$

where a prime denotes differentiation with respect to  $L_T$ ,  $S_i^{\mu\nu} = e_i^\rho S_\rho^{\mu\nu}$ , and  $T_{\mu\nu}$  is the matter energy-momentum tensor. The fact that equations (6) are 2nd order makes them simpler than the dynamical equations resulting in  $f(R)$  theories.

### III. COSMOLOGICAL SOLUTION AND OBSERVATIONAL CONSTRAINTS

We will assume a flat homogeneous and isotropic Friedmann-Robertson-Walker universe, so

$$e^i_\mu = \text{diag}(1, a(t), a(t), a(t)), \quad (7)$$

where  $a(t)$  is the cosmological scale factor. By replacing in (1), (3), and (4) one obtains

$$L_T = S^{\rho\mu\nu} T_{\rho\mu\nu} = -6 \frac{\dot{a}^2}{a^2} = -6H^2, \quad (8)$$

with  $H$  being the Hubble parameter  $H = \dot{a}a^{-1}$ . As a remarkable feature, the scale factor enters the invariant  $L_T$  just through the Hubble parameter. The substitution of the vierbein (7) in (6) for  $i = 0 = \nu$  yields

$$12H^2 f'(L_T) + f(L_T) = 16\pi G \rho. \quad (9)$$

Besides, the equation  $i = 1 = \nu$  is

$$48H^2 f''(L_T) \dot{H} - f'(L_T) [12H^2 + 4\dot{H}] - f(L_T) = 16\pi G p. \quad (10)$$

In Eqs. (9) and (10),  $\rho(t)$  and  $p(t)$  are the total density and pressure, respectively. It can be easily derived that they accomplish the conservation equation

$$\frac{d}{dt}(a^3 \rho) = -3a^3 H p, \quad (11)$$

whatever  $f(L_T)$  is. Thus, if the state equation is  $p = w\rho$ , then  $\rho$  evolves as  $\rho \propto (1+z)^{3(1+w)}$  ( $z$  is the cosmological redshift).

We are interested in obtaining an accelerated expansion without dark energy but driven by torsion. For this we will try with a kind of  $f(L_T)$  theories:

$$f(L_T) = L_T - \frac{\alpha}{(-L_T)^n}, \quad (12)$$

where  $\alpha$  and  $n$  are real constants to be determined by observational constraints. Although the functional form of (12) is similar to those considered in  $f(R)$  literature, now the guideline towards modified gravity is  $H$  instead of  $R$ . This fact gives to these theories another interesting feature because  $H$  is the most important cosmological variable. For later times the term  $-\alpha/(-L_T)^n$  is dominant, while in early times, when  $H \rightarrow \infty$ , general relativity is recovered. From (9) along with (12), the modified Friedmann equation is

$$H^2 - \frac{(2n+1)\alpha}{6^{n+1}H^{2n}} = \frac{8}{3}\pi G \rho \quad (13)$$

(a functional dependence similar to the results other authors arrived at, through different theoretical motivations such as [33,34]).

Now, replacing  $\rho = \rho_{mo}(1+z)^3 + \rho_{ro}(1+z)^4$ , and calling  $\Omega_i = 8\pi G \rho_{io}/(3H_o^2)$  the contributions of matter and radiation to the total energy density today, Eq. (13) becomes

$$y^n(y-B) = C, \quad (14)$$

where  $y = H^2/H_o^2$ ,  $B = \Omega_m(1+z)^3 + \Omega_r(1+z)^4$ , and  $C = \alpha(2n+1)(6H_o^2)^{-(n+1)}$ . The evaluation of this equation for  $z=0$  allows us to rephrase the constant  $C$  as a function of  $\Omega_i$  and  $n$ :  $C = 1 - \Omega_m - \Omega_r$ . For  $\alpha=0$  (then  $1 = \Omega_m + \Omega_r$ ), the GR spatially flat Friedmann equation  $H^2 = H_o^2 B$  is retrieved. The case  $n=0$  recovers the GR dynamics with cosmological constant  $\Omega_\Lambda = 1 - \Omega_m - \Omega_r$ . Notice the functional simplicity of (13) compared with its analog in  $f(R)$  theories. Compared with GR,  $n$  is the sole new free parameter in (14), since specifying the value of  $n$  and  $\Omega_m$  ( $\Omega_r$ ) the value of  $\alpha$  (in units of  $H^{2(n+1)}$ ) is automatically fixed through the relation (13). In order to obtain  $H(z)$  we solve numerically Eq. (14).

Since the most solid evidence for the acceleration of the Universe comes from measurements of luminosity distances for type Ia supernovae, we will use the most recent compilation of 307 SNe Ia events (the *Union* sample) [2], to put constraints in the  $n - \Omega_m$  plane. The predicted distance modulus for a supernova at redshift  $z$ , for a given set of parameters  $\mathbf{P} = (n, \Omega_m)$ , is

$$\mu(z | \mathbf{P}) = m - M = 5 \log(d_L) + 25, \quad (15)$$

where  $m$  and  $M$  are the apparent and absolute magnitudes, respectively, and  $d_L$  stands for the luminosity distance (in units of megaparsecs),

$$d_L(z; \mathbf{P}) = (1+z) \int_0^z \frac{dz'}{H(z'; \mathbf{P})}, \quad (16)$$

where  $H(z; \mathbf{P})$  is given by the numerical solution of (14). We use a  $\chi^2$  statistic to find the best fit for a set of parameters  $\mathbf{P}$  (marginalizing over  $H_o$ ),

$$\chi_{\text{SNe}}^2 = \sum_{i=1}^{N=307} \frac{[\mu_i(z | \mathbf{P}) - \mu_i^{\text{obs}}(z)]^2}{\sigma_i^2}, \quad (17)$$

where  $\mu_i(z | \mathbf{P})$  is defined by (15), and  $\mu_i^{\text{obs}}$  and  $\sigma_i$  are the distance modulus and its uncertainty for each observed value [2]. As it is known, the measurements of SNe Ia are not enough to constrain  $\Omega_m$  thoroughly. To perform the statistic we also consider, on one hand, the information coming from the baryon acoustic oscillation (BAO) peak detected in the correlation function of luminous red galaxies (LRG) in the Sloan Digital Sky Survey [35]. The observed scale of the peak effectively constrains the quantity (assumed a  $\Lambda$ CDM model),

$$A_{0.35} = D_V(0.35) \frac{\sqrt{\Omega_m H_o^2}}{0.35} = 0.469 \pm 0.017, \quad (18)$$

where  $z = 0.35$  is the typical redshift of the LRG and  $D_V$  is defined as

$$D_V(z) = \left[ \frac{z}{H(z)} \left( \int_0^z \frac{dz'}{H(z')} \right)^2 \right]^{1/3}. \quad (19)$$

On the other hand, we have also included in the statistic the cosmic microwave background (CMB) shift parameter, which relates the angular diameter distance to the last scattering surface with the angular scale of the first acoustic peak in the CMB power spectrum. In order to do this, we have considered a radiation component  $\Omega_r = 5 \times 10^{-5}$ . The CMB shift parameter is given by [36]

$$R_{1089} = \sqrt{\Omega_m H_o^2} \int_0^{1089} \frac{dz}{H(z)} = 1.710 \pm 0.019. \quad (20)$$

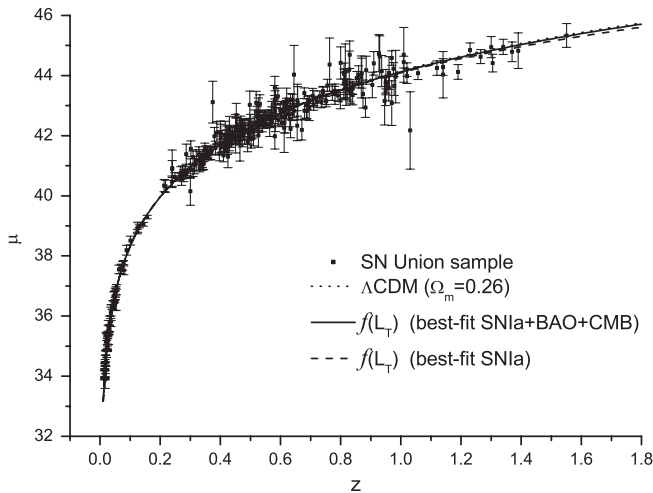


FIG. 1. Hubble diagram for 307 SNe Ia from the *Union* sample [2]. The curves correspond to the concordance model  $\Lambda$ CDM with  $\Omega_\Lambda = 0.74$  and  $\Omega_m = 0.26$  (dotted line); our models with the values corresponding to the best fit  $\Omega_m = 0.42$  and  $n = 1.30$  (dashed line), and also with those coming from the joint analysis of SNIa + BAO + CMB,  $\Omega_m = 0.27$ , and  $n = -0.10$  (solid line).

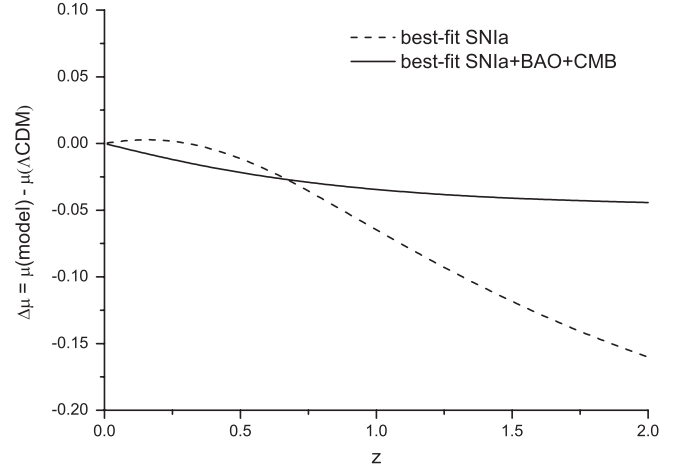


FIG. 2. Distance modulus residual from the  $\Lambda$ CDM model for the same values from Fig. 1.

We can use both parameters since our model presents matter domination at the decoupling time. Figure 1 shows the Hubble diagram for the 307 SNe Ia belonging to the *Union* sample. The curves represent models with values of  $\Omega_m$  and  $n$  obtained from minimizing  $\chi^2$  using only SNIa and SNIa + BAO + CMB as well. For reference, the  $\Lambda$ CDM model with  $\Omega_m = 0.26$  is also shown. The obtained values for the best fit to the SNe Ia data only are  $\Omega_m = 0.42$  and  $n = 1.30$  with the reduced  $\chi_\nu^2 \equiv \chi_{\text{min}}^2/\nu \simeq 1.02$  (or equivalently,  $\Delta\chi_{\text{min}}^2 = -1.1$ ), where  $\nu$  is the number of degrees of freedom.

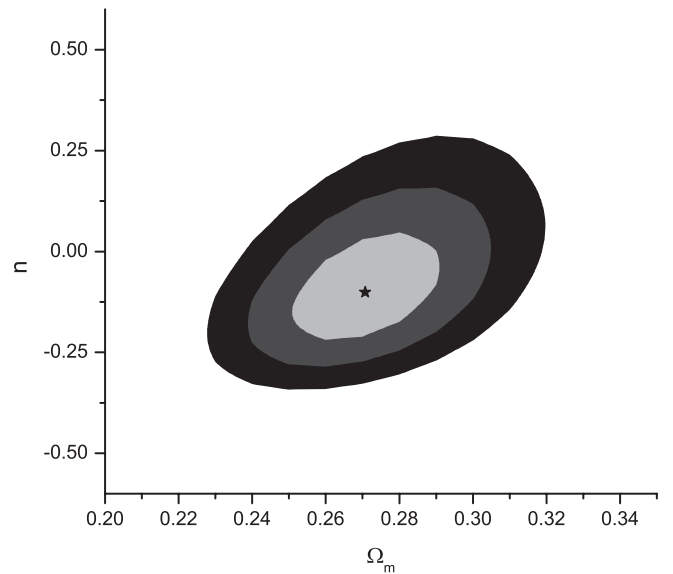


FIG. 3. Confidence intervals at 68.3%, 95.4%, and 99.7% in the  $n - \Omega_m$  plane coming from combining SNe Ia, BAO, and CMB data. The best fit to this joint analysis is reached with the values  $n = -0.10$  and  $\Omega_m = 0.27$ .

In Fig. 2 we plot the distance modulus residual ( $\Delta\mu$ ) from the  $\Lambda$ CDM model to better appreciate the discrepancies between our model and  $\Lambda$ CDM.

Figure 3 shows the confidence intervals at 68.3%, 95.4%, and 99.7% for the joint probability of the parameters  $n$  and  $\Omega_m$ , having combined the SNe Ia data with BAO and CMB parameters. This analysis yields the fact that the best fit to all data is achieved with  $n = -0.10$  and  $\Omega_m = 0.27$  (with a  $\chi^2_{\min}/\nu \approx 1.01$ ,  $\Delta\chi^2_{\min} = -1.2$ ) and also the values of the parameters lie in the ranges (at 68.4% C.L.):  $n \in [-0.23, 0.03]$ ,  $\Omega_m \in [0.25, 0.29]$ .

For our model we have analyzed as well the total and effective equations of state as a function of  $z$ . From (10) and (13) along with (12), one can define a torsion contribution to the density and pressure as

$$\begin{aligned} \rho_T &= \frac{3}{8\pi G} \frac{(2n+1)\alpha}{6^{n+1}H^{2n}}, \\ p_T &= \frac{\alpha}{8\pi G} \left[ (6H^2)^{-(n+1)} \dot{H} [4n(n+1) - 2n] \right. \\ &\quad \left. - 6n(6H^2)^{-(n+1)} H^2 - \frac{(6H^2)^{-n}}{2} \right] \end{aligned} \quad (21)$$

to rewrite the dynamical equations as

$$H^2 = \frac{8\pi G}{3} (\rho + \rho_T), \quad (22)$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{6} [\rho + \rho_T + 3(p + p_T)]. \quad (23)$$

Then, by using (22) and (23) the total and effective equations of state are written as

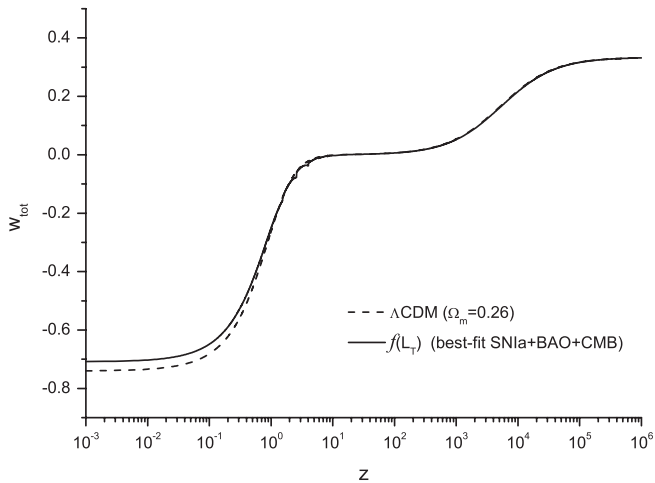


FIG. 4. The curves correspond to the total equation of state as a function of  $z$  expected for the standard concordance model  $\Lambda$ CDM with  $\Omega_\Lambda = 0.74$  and  $\Omega_m = 0.26$  (dashed line), and for our model (solid line) with the values of the best fit coming from SNIa + BAO + CMB,  $\Omega_m = 0.27$ , and  $n = -0.10$ . Three cosmological phases are observed.

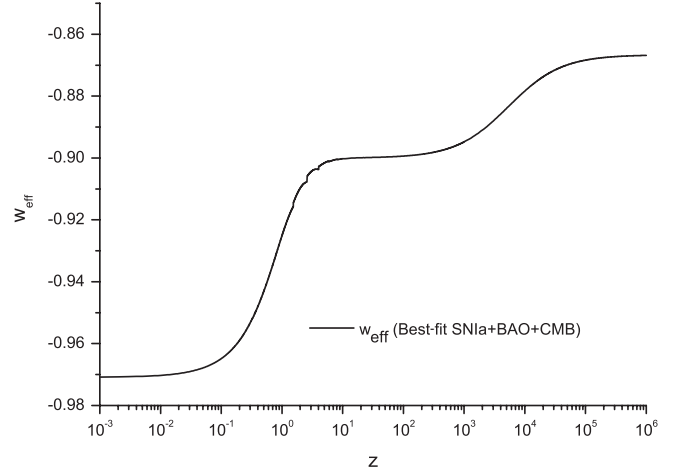


FIG. 5. Effective equation of state as a function of  $z$  for our model with the values of the best fit coming from SNIa + BAO + CMB,  $\Omega_m = 0.27$ , and  $n = -0.10$ .

$$w_{\text{tot}} \equiv \frac{p + p_T}{\rho + \rho_T} = -1 + \frac{2(1+z)}{3H} \frac{dH}{dz}, \quad (24)$$

$$w_{\text{eff}} = \frac{p_T}{\rho_T}. \quad (25)$$

Figure 4 shows the evolution of the total equation of state  $w_{\text{tot}}$  as a function of  $z$  for our model with the values of the best fit. There can be observed the last three phases of the evolution of the universe: radiation dominated ( $w = 1/3$ ), matter dominated ( $w = 0$ ), and late acceleration ( $w \approx -1$ ). Figure 5 shows the effective equation of state coming from the dark torsion contribution. Finally, analyzing (23) we find that our model with  $n = -0.10$  predicts that the transition from deceleration to acceleration occurs at  $z_{\text{acc}} \approx 0.74$  in good agreement with recent works [37].

An interesting point to be highlighted is that Eq. (13) reveals that a value of  $n > 0$ , as the one obtained by considering only SNIa data, implies that the effective dark torsion is of the phantom type [38]. That is, since  $H$  decreases toward the present time, the dark torsion density increases instead of diluting with expansion ( $w_{\text{eff}} < -1$ ). However, when combining the complete data with SNIa + BAO + CMB we can see from Fig. 3 that it is slightly favored ( $1\sigma$  C.L.) as a model with  $n \leq 0$ .

#### IV. CONCLUSIONS

A theory  $f(L_T)$  based on a modification of theTEGR—where torsion is the geometric object describing gravity instead of curvature and its equations are always of 2nd order—is remarkably simpler than  $f(R)$  theories. We have tested the theory  $f(L_T) = L_T - \alpha(-L_T)^{-n}$  with the aim of reproducing the recently detected acceleration of the Universe without resorting to dark energy. We have here performed observational viability tests for this theory by using the most recent SN Ia data, and combined them with

the information coming from the BAO peak and the CMB shift parameter in order to find constraints in the  $n - \Omega_m$  plane. At 68.3% C.L. we found that the values lie in the ranges  $n \in [-0.23, 0.03]$  and  $\Omega_m \in [0.25, 0.29]$ . The values for  $\Omega_m$  are consistent with recent estimations obtained by other authors (see, e.g., [39]). The model with the best-fit values minimizing the  $\chi^2$  that combines SNIa + BAO + CMB data ( $n = -0.10$  and  $\Omega_m = 0.27$ ) exhibits the last three phases of cosmological evolution: radiation

era, matter era, and late acceleration, this last stage having started at  $z_{\text{acc}} \approx 0.74$ .

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