

Emergence of inflationary perturbations in the CSL model

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The inflationary paradigm is the most successful model that explains the observed spectrum of primordial perturbations. However, the precise emergence of such inhomogeneities and the quantum-to-classical transition of the perturbations has not yet reached a consensus among the community. The Continuous Spontaneous Localization model (CSL), in the cosmological context, might be used to provide a solution to the mentioned issues by considering a dynamical reduction of the wave function. The CSL model has been applied to the inflationary universe before and different conclusions have been obtained. In this letter, we use a different approach to implement the CSL model during inflation. In particular, in addition to accounting for the quantum-to-classical transition, we use the CSL model to generate the primordial perturbations, that is, the dynamical evolution provided by the CSL model is responsible for the transition from a homogeneous and isotropic initial state to a final one lacking such symmetries. Our approach leads to results that can be clearly distinguished from preceding works. Specifically, the scalar and tensor power spectra are not time-dependent, and retains the amplification mechanism of the CSL model. Moreover, our framework depends only on one parameter (the CSL parameter) and its value is consistent with cosmological and laboratory observations.

I. INTRODUCTION

The inflationary paradigm is held among the majority of cosmologists as a successful model for addressing the primordial inhomogeneities that represent the seeds of cosmic structure. In fact, recent observations from the Cosmic Microwave Background (CMB) radiation [1–6] are quite consistent with the standard prediction from the simplest inflationary model, namely, the prediction of a nearly scale-invariant power spectrum. On the other hand, the exact physical mechanism responsible for the generation of the primordial curvature perturbations, associated to a highly Gaussian stochastic classical field is still a matter of debate. In particular, inflation is based on a combination of quantum mechanics and general relativity, two theories that are difficult to merge at both the conceptual and technical level. Therefore, it is expected that when these theories are used in the same footing, some difficulties might arise. Specifically, the quantum-to-classical transition, a subject which has been present since the conception of the quantum theory, is an issue that has not been fully resolved in the early inflationary universe.

A more precise formulation of the problem at hand can be stated as follows: the inflationary universe is characterized by a background spacetime that is completely homogeneous and isotropic. Equivalently, the quantum state of the matter field, namely the vacuum state, is also perfectly homogeneous and isotropic (i.e. it is an eigenstate of the operators associated to the generators

of spatial translations and rotations). On the other hand, the present universe is the result of the evolution of primordial density inhomogeneities.

Henceforth, the problem is: how do the inhomogeneities and anisotropies originate from the initial highly symmetric (the symmetry being the homogeneity and isotropy) state of the universe, described by both the background spacetime and the vacuum state of the matter fields, given that the dynamical evolution, provided by Schrödinger and Einstein equations, does not break the translational and rotational symmetries? In other words, if one considers quantum mechanics as a fundamental theory applicable, in particular, to the universe as a whole, then one must regard any classical description of the state of any system as a sort of imprecise characterization of a complicated quantum-mechanical state. The universe we observe today is clearly well described by an inhomogeneous and anisotropic classical state; therefore, such description must be considered as an imperfect description of an equally inhomogeneous and anisotropic quantum state. Consequently, if we want to consider the inflationary account as providing the physical mechanism for the generation of the seeds of structure, such account must contain an explanation for why the quantum state that describes our actual universe does not possess the same symmetries as the early quantum state of the universe, which happened to be perfectly symmetric. Since there is nothing in the dynamical evolution (as given by the standard inflationary approach) of the quantum state that can break those symmetries, the traditional inflationary paradigm is incomplete in that sense (sometimes, an usual argument is that since the vacuum fluctuations are not zero, somehow the system contains inhomogeneities. See Appendix A for a discussion). Here, we also refer the reader to Refs. [7, 8] where

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the relation between symmetry-breaking vacuum and the reduction of the wave function is discussed.

Earlier works based on decoherence [9–12] and the consideration that the initial vacuum state of the universe evolves into a highly squeezed state [13], led to a partial understanding of the issue, namely, that the predictions from the quantum theory are indistinguishable from those of a theory in which the random fluctuations are the result of a classical stochastic process [14]. Nevertheless, this argument by itself cannot address the fact that a single (classical) outcome emerges from the quantum theory. In other words, decoherence (and the squeezing of quantum states) cannot solve the quantum measurement problem [15, 16], a complication that, within the cosmological context, is amplified due to the impossibility of recurring to the “for all practical purposes” argument in the familiar laboratory situation; i.e. it is not clear how to define in the primordial universe entities such as observers, detectors, etc. Other cosmologists seem to adopt the Everett “many-worlds” interpretation of quantum mechanics plus the decoherence process when confronted with the quantum-to-classical transition in the inflationary universe [17]. In the Everettian formulation, reality is made of a connected weave of ever-splitting worlds, each one realizing one of the alternatives that is opened by what we would call a quantum-mechanical measurement. Regarding this point, we would like to refer the reader to Refs. [18–20] where arguments against the Everett interpretation are presented.

One possible way to address the mentioned issues or, in other words, to avoid the standard “measurement problem” of quantum theory, is to invoke the collapse of the wave function but without relying on any external objects, e.g. [18, 21]. Other approaches to this problem have been based on Bohmian quantum mechanics; see for instance [22, 23].

The idea of invoking a self-induced collapse in order to generate the primordial perturbations has been explored in great detail in previous works, e.g. [24–33]. Moreover, in [34] some generic collapse schemes have been tested using observational data coming from the 7-year release of the WMAP collaboration [35] and the Sloan Digital Sky Survey [36]. Therefore, the attempt to solve the aforementioned issue is not just a matter of philosophical concern but a relevant problem from the theoretical point of view, yielding predictions that can be confronted with observational data.

Furthermore, the proposal of a self-induced collapse of the wave function has been an active line of research since the early ideas of Diósi [37, 38] and Penrose [39], advocating gravity as the main agent triggering the collapse. Also, the Ghirardi-Rimini-Weber (GRW) [40] model was among the first attempts to introduce an objective collapse model. In past decades, the Continuous Spontaneous Localization (CSL) model, which can be viewed as a continuous version of the GRW model, has been regarded as a promising model that can provide a solution to the quantum measurement problem [41, 42]. In

particular, the CSL mechanism is based on a non-linear stochastic modification of the standard Schrödinger equation, in this way, spontaneous and random collapses of the wave function occur all the time, to all particles, regardless they are isolated or interacting. The idea behind the CSL model [43], sometimes referred to as the “amplification mechanism”, is that the collapses must be rare for microscopic systems, in order not to alter their quantum behavior as described by the Schrödinger equation. At the same time, their effect must increase when several particles are held together forming a macroscopic system. Moreover, the testable predictions made by the CSL model are now considered to be feasible within the current available technology [44]. On the other hand, the CSL model is, at this stage, a non-relativistic model and therefore a complete generalization to quantum fields is still under development [45, 46]. Nevertheless, the particular features of the CSL model, e.g. the absence to rely on external agents to solve the measurement problem, would make it a viable candidate to address the issue concerning the emergence of the classical primordial perturbations if it predicts a spectrum consistent with the observational data.

In [47], the CSL model was applied to the inflationary universe for the first time. In this reference, the authors followed an approach in which the collapse affected in the same way all modes of the inflaton field and, as a consequence, the amplification mechanism of the CSL model was lost. Additionally, their predicted scalar power spectrum contained some features that conflicted with the nearly scale-invariant power spectrum, which is consistent with the CMB data. In order to retain the amplification mechanism, other authors [48], proposed a phenomenological manner in which the CSL model would affect each mode of the inflaton field. Additionally, they also assumed that the modification to Schrödinger equation, provided by the CSL mechanism, was by introducing a time-dependent parameter. This last assumption resulted in a prediction for the scalar (and also tensor) power spectrum with the correct shape (for a particular combination of their three free parameters) but that was time-dependent. In order to overcome this shortcoming, the power spectrum was chosen to be evaluated at the end on inflation. However, this choice makes their CSL collapse parameter extremely small ($\sim e^{-120}$) [49].

A shared feature of the works in Refs. [47, 48] is that the authors worked in a joint metric-matter quantization of the perturbations characterized by the Mukhanov-Sasaki variable [50]. On the other hand, in Ref. [51] one followed a different approach to the problem; the authors successfully applied the CSL collapse mechanism to the inflationary universe but within the semiclassical gravity framework. Consequently, as pointed out in [30, 52], the amplitude of the primordial gravitational waves is exactly zero at first order in the perturbations. Therefore, a confirmed detection of primordial gravity waves would make this approach face serious issues.

Another distinction between the approaches in Refs.

[47, 48] and [51] is the specific role played by the self-induced collapse. This difference is subtle but important. The authors of [47, 48] employed the CSL mechanism with the intention of localizing the evolved inflaton vacuum state in an eigenstate of the Mukhanov-Sasaki variable. Meanwhile, the authors of [51] attributed to the self-induced collapse the action of generating the primordial curvature perturbation, irrespectively of which eigenstate the initial state evolves into. Therefore, if no self-induced collapse occurs, the predicted power spectrum in Refs. [47, 48] is exactly the same as the standard one. On the contrary, in Ref. [51] the absence of a collapse results in no perturbations of the spacetime at all, consequently, the predicted scalar power spectrum is exactly zero (see Sect. IID and Appendix B for a more detailed discussion).

In the present work, we adopt the same role for the collapse, provided by the CSL model, as the one presented in [51], but we apply it to the joint metric-matter quantization of the inflaton field, to improve the standard inflationary treatment providing a mechanism for the emergence of the primordial curvature perturbations, and also for driving the quantum-to-classical transition. In this way, we obtained predictions that are different from the ones in [47, 48] and are also quite consistent with the observational data. In particular, our predicted scalar and tensor power spectra are not time-dependent, and the implementation of the CSL mechanism retains the amplification mechanism. Also, our model introduces just one free parameter and in contrast with Ref. [51], it predicts a non-vanishing tensor power spectrum at first order in the perturbations.

The article is organized as follows: in Sect. II we review some basics about the CSL collapse model applied to the inflationary universe, and we obtain the scalar power spectrum within such a framework; in Sect. III we show our results for the power spectrum of tensor modes and the tensor-to-scalar ratio; in Sects. IV and V we make a discussion of our results and compare them with previous works, and finally in Sect. VI we summarize our conclusions.

II. THE CSL MODEL AND THE SCALAR POWER SPECTRUM

In this section, we will summarize some concepts regarding the CSL collapse model, and we will use it to obtain a prediction for the primordial power spectrum of scalar perturbations.

A. Classical description of the perturbations

The inflationary universe is described by Einstein equations $G_{ab} = 8\pi G T_{ab}$ ($c = 1$) and the dynamics of the matter fields dominated by the inflaton. We will assume the simplest inflationary model, which is a sin-

gle scalar field φ in the slow-roll approximation. The background spacetime is accelerating in a quasi-de Sitter type of expansion, characterized by $\mathcal{H} \simeq -1/[\eta(1 - \epsilon)]$, with $\mathcal{H} \equiv a'/a$ the conformal expansion rate, a being the scale factor and the slow-roll parameter is defined as $\epsilon \equiv 1 - \mathcal{H}'/\mathcal{H}^2$ (a prime denotes partial derivative with respect to conformal time η). The energy density of the universe is dominated by the potential of the inflaton field V , and during the slow-roll inflation is satisfied the condition $\epsilon \simeq M_P^2/2(\partial_\phi V/V)^2 \ll 1$, with $M_P^2 \equiv (8\pi G)^{-1}$ the reduced Planck mass. Since we will work in a full quasi-de Sitter expansion, another useful parameter to characterize slow-roll inflation is the second slow-roll parameter, i.e. $\delta \equiv \epsilon - \epsilon'/2\mathcal{H}\epsilon \ll 1$.

The scalar metric perturbations in a FRW background spacetime are generically described by the line element:

$$ds^2 = a^2(\eta) \left\{ -(1 - 2\phi)d\eta^2 + 2(\partial_i B)dx^i d\eta + [(1 - 2\psi)\delta_{ij} + 2\partial_i \partial_j E]dx^i dx^j \right\}. \quad (1)$$

Since we will focus on a joint quantization of the metric and matter perturbations, it is convenient to work with the gauge-invariant quantity known as the Bardeen potential [53] defined as $\Phi \equiv \phi + \frac{1}{a}[a(B - E)']$. The matter sector dominated by the inflaton is separated in an homogeneous part plus small perturbations $\varphi(\eta, \mathbf{x}) = \varphi_0(\eta) + \delta\varphi(\eta, \mathbf{x})$. In a similar way, the perturbations of the inflaton can be modeled by the gauge-invariant fluctuation of the scalar field $\delta\varphi^{(\text{GI})}(\eta, \mathbf{x}) = \delta\varphi + \phi'_0(B - E)$. The two objects Φ and $\delta\varphi^{(\text{GI})}$ can be used to form a new quantity called the Mukhanov-Sasaki variable [50] i.e.,

$$v(\eta, \mathbf{x}) \equiv a \left[\delta\varphi^{(\text{GI})} + \frac{\phi'_0}{\mathcal{H}}\Phi \right]. \quad (2)$$

Written in a gauge-invariant way, and, in the absence of anisotropic stress, the components 00 and 0i of the Einstein perturbed equations at linear order $\delta G_{ab} = 8\pi G \delta T_{ab}$, can be combined to yield

$$\nabla^2 \Phi = -\sqrt{\frac{\epsilon}{2}} \frac{H}{M_P} \left(v' - \frac{z'}{z} v \right) \quad (3)$$

where $z \equiv a\phi'_0/\mathcal{H}$. Equation (3) is expressed in terms of gauge-invariant quantities. Nevertheless, in the longitudinal gauge, Φ represents the curvature perturbation and is related to v exactly in the same way as described in (3). Under the slow-roll approximation, it leads to the following useful expressions: $z'/z \simeq (1 + 2\epsilon - \delta)/(-\eta)$ and $a'/a \simeq (1 + \epsilon)/(-\eta)$, similarly $z''/z \simeq (2 + 6\epsilon - 3\delta)/\eta^2$ and $a''/a \simeq (2 + 3\epsilon)/\eta^2$.

B. Quantization of the perturbations

As mentioned in the Introduction, the CSL model is based on a non-linear modification to the Schrödinger equation. Therefore, it is convenient to begin by presenting the theory of the inflaton in the Schrödinger picture,

where the relevant theory objects are the Hamiltonian and the wave functional. One starts with the action of a scalar field, i.e. the inflaton, minimally coupled to gravity, and then by expanding up to second order in the scalar perturbations one can express the action in terms of the Mukhanov-Sasaki variable $v(\eta, \mathbf{x})$ [54]. The resulting action, expressed in Fourier modes of the field $v(\eta, \mathbf{x})$, reads $\delta^{(2)}S = \frac{1}{2} \int d\eta d^3\mathbf{k} \mathcal{L}$, where

$$\mathcal{L} = v'_\mathbf{k} v_{\mathbf{k}}^* - k^2 v_\mathbf{k} v_{\mathbf{k}}^* - \frac{z'}{z} \left(v_\mathbf{k} v_{\mathbf{k}}^* + v'_\mathbf{k} v_{\mathbf{k}}^* \right) + \left(\frac{z'}{z} \right)^2 v_\mathbf{k} v_{\mathbf{k}}^*. \quad (4)$$

The canonical conjugated momentum associated to $v_\mathbf{k}$ is $p_\mathbf{k} = \partial\mathcal{L}/\partial v_{\mathbf{k}}^*$, i.e.

$$p_\mathbf{k} = v'_\mathbf{k} - (z'/z)v_\mathbf{k}, \quad (5)$$

Since $v(\eta, \mathbf{x})$ is a real field, note that $v_\mathbf{k}^* = v_{-\mathbf{k}}$. Therefore, the Hamiltonian is $H = \frac{1}{2} \int d^3\mathbf{k} (H_\mathbf{k}^R + H_\mathbf{k}^I)$, with

$$H_\mathbf{k}^{R,I} = p_\mathbf{k}^{R,I} p_{\mathbf{k}}^{*R,I} + \frac{z'}{z} \left(v_\mathbf{k}^{R,I} p_{\mathbf{k}}^{*R,I} + v_{\mathbf{k}}^{*R,I} p_\mathbf{k}^{R,I} \right) + k^2 v_\mathbf{k}^{R,I} v_{\mathbf{k}}^{*R,I}, \quad (6)$$

where the indices R, I denote the real and imaginary parts of $v_\mathbf{k}$ and $p_\mathbf{k}$. We now promote $v_\mathbf{k}$ and $p_\mathbf{k}$ to quantum operators, by imposing canonical commutations relations $[\hat{v}_\mathbf{k}^{R,I}, \hat{p}_{\mathbf{k}'}^{R,I}] = i\delta(\mathbf{k} - \mathbf{k}')$.

In the Schrödinger picture, the wave functional $\Psi[v(\eta, \mathbf{x})]$ characterizes the state of the system. Moreover, in Fourier space, the wave functional can be factorized into mode component $\Psi[v(\eta, \vec{x})] = \prod_\mathbf{k} \Psi_\mathbf{k}^R(v_\mathbf{k}^R) \times \Psi_\mathbf{k}^I(v_\mathbf{k}^I)$. From now on, we will deal with each mode separately. Henceforth, each mode of the wave functional, associated to the real and imaginary parts of the canonical variables, satisfies the Schrödinger equation $\hat{H}_\mathbf{k}^{R,I} \Psi_\mathbf{k}^{R,I} = i\partial\Psi_\mathbf{k}^{R,I}/\partial\eta$, with the Hamiltonian provided by (6). The usual assumption is that at an early time τ (i.e. the onset of inflation), the modes are in their adiabatic ground state, which is a Gaussian centered at zero with certain spread. Since the initial quantum state is Gaussian, its form is preserved during the time evolution. For reasons that will become evident in the following, it is convenient to work in the momentum representation; thus, the Gaussian state

$$\Psi^{R,I}(\eta, p_\mathbf{k}^{R,I}) = \exp[-A_k(\eta)(p_\mathbf{k}^{R,I})^2 + B_k(\eta)p_\mathbf{k}^{R,I} + C_k(\eta)] \quad (7)$$

evolves according to Schrödinger equation, with initial conditions given by $A_k(\tau) = 1/2k, B_k(\tau) = C_k(\tau) = 0$ corresponding to the Bunch-Davies vacuum, which is perfectly homogeneous and isotropic in the sense of a vacuum state in quantum field theory (see Appendix A).

In this work, the main reasons for invoking the self-induced collapse of the wave function are precisely to break the homogeneity and isotropy of the initial state, and that the emergence of the seeds of cosmic structures can be achieved. We will accomplish this goal by using

the CSL collapse mechanism, and in the following subsection we will provide a very brief general description of it. For a complete review of the CSL model see for instance [43].

C. CSL collapse mechanism

The CSL collapse mechanism is based on a stochastic non-linear modification of the Schrödinger equation. Ideally, this modification induces a collapse of the wave function toward one of the possible eigenstates of an operator $\hat{\Theta}$, called the collapse operator, with certain rate λ . The self-induced collapse is due to the interaction of the system with a background noise $W(t)$ that can be considered as a continuous-time stochastic process of the Wiener kind. The modified Schrödinger equation drives the time evolution of an initial state as

$$|\Psi, t\rangle = \hat{T} \exp\left\{-\int_{t_0}^t dt' [i\hat{H} + \frac{1}{4\lambda}(W(t') - 2\lambda\hat{\Theta})^2]\right\} |\Psi, t_0\rangle, \quad (8)$$

with \hat{T} the time-ordering operator. The probability associated with a particular realization of $W(t)$ is,

$$P[W(t), t] DW(t) = \langle \Psi, t | \Psi, t \rangle \prod_{t_i=t_0}^t \frac{dW(t_i)}{\sqrt{2\pi\lambda/dt}}. \quad (9)$$

The norm of the state $|\Psi, t\rangle$ evolves dynamically, and Eq. (9) implies that the most probable state will be the one with the largest norm. From (8) and (9), it can be derived the evolution equation of the density matrix operator $\hat{\rho}$, i.e.

$$\frac{d\hat{\rho}}{dt} = -i[\hat{H}, \hat{\rho}] - \frac{\lambda}{2} [\hat{\Theta}, [\hat{\Theta}, \hat{\rho}]]. \quad (10)$$

The density matrix operator can be used to obtain the ensemble average of the expectation value of an operator $\langle \hat{O} \rangle = \text{Tr}[\hat{O}\hat{\rho}]$. Consequently, from (10) it follows that:

$$\frac{d\langle \hat{O} \rangle}{dt} = -i\langle [\hat{O}, \hat{H}] \rangle - \frac{\lambda}{2} \langle [\hat{\Theta}, [\hat{\Theta}, \hat{O}]] \rangle. \quad (11)$$

D. CSL and inflation

In order to apply the CSL to the inflationary regime, we need to establish which are the appropriate observables that emerge from the quantum theory of inflation. A reasonable observable is the curvature scalar perturbation, which is directly related to the temperature anisotropies of the CMB, and in the longitudinal gauge corresponds exactly to the Bardeen potential Φ . A quantization of the Mukhanov-Sasaki variable v yields automatically a quantization of Φ ; therefore, the question that arises here is: what is exactly the relation between the quantum and classical objects? In particular, what is the relation between $\hat{\Phi}$ and Φ ?

To illustrate the point of view we will adopt, let us focus on the temperature anisotropies of the CMB observed today on the celestial two-sphere and its relation to the scalar metric perturbation Φ . Such a relation is approximately given by (i.e. for large angular scales)

$$\frac{\delta T}{T_0}(\theta, \varphi) \simeq \frac{1}{3}\Phi. \quad (12)$$

On the other hand, the observational data are described in terms of the coefficients a_{lm} of the multipolar series expansion $\delta T/T_0(\theta, \varphi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \varphi)$, hence

$$a_{lm} = \int \frac{\delta T}{T_0}(\theta, \varphi) Y_{lm}^*(\theta, \varphi) d\Omega, \quad (13)$$

here θ and φ are the coordinates on the celestial two-sphere, with $Y_{lm}(\theta, \varphi)$ as the spherical harmonics.

Given Eq. (12), the coefficients a_{lm} can be further re-expressed in terms of the Fourier modes associated to $\hat{\Phi}$, i.e.

$$a_{lm} = \frac{4\pi i^l}{3} \int \frac{d^3 k}{(2\pi)^3} j_l(kR_D) Y_{lm}^*(\hat{k}) \Delta(k) \Phi_{\vec{k}}, \quad (14)$$

with $j_l(kR_D)$ the spherical Bessel function of order l of the first kind and R_D is the comoving radius of the last scattering surface. We have explicitly included the modifications associated with late-time physics encoded in the transfer functions $\Delta(k)$. The metric perturbation $\Phi_{\mathbf{k}}$ is the primordial curvature perturbation.

Now, how to relate $\Phi_{\mathbf{k}}$ that appears in Eq. (14) with the quantum operator $\hat{\Phi}_{\mathbf{k}}$ coming from the quantum theory? Evidently, if we compute the expectation value $\langle \hat{\Phi}_{\mathbf{k}} \rangle$ in the vacuum state $|0\rangle$ and identify it exactly with $\Phi_{\mathbf{k}}$, then we obtain exactly zero; while it is clear that for any given l, m , the measured value of the quantity a_{lm} is not zero. As matter of fact, the standard argument is that it is not the quantity a_{lm} that is zero but the average $\overline{a_{lm}}$. However, the notion of average is subtle, since in the CMB one has an average over different directions in the sky, while the average that one normally associates to the quantum expectation value of an operator is related to an average over possible outcomes of repeatedly measurements of an observable associated to an operator in the Hilbert space of the system (clearly the concepts of measurements, observers, etc. are not well defined in the early universe).

Nevertheless, in the standard approach (by invoking decoherence, squeezing of the vacuum, many-world interpretation of quantum mechanics, etc. although we do not subscribe to such postures for the reasons exposed in Ref. [18]) somehow one can make the association $\Phi_{\mathbf{k}} = Ae^{i\alpha_{\mathbf{k}}}$ with $\alpha_{\mathbf{k}}$ a random phase and A being identified with the quantum uncertainty of $\hat{\Phi}_{\mathbf{k}}$, i.e. $A^2 = \langle 0|\hat{\Phi}_{\mathbf{k}}^2|0\rangle$. But the random nature of $\Phi_{\mathbf{k}}$, codified in the random number $e^{i\alpha_{\mathbf{k}}}$, remains unclear. In fact, the relation $\Phi_{\mathbf{k}} = Ae^{i\alpha_{\mathbf{k}}}$ is not valid at all times in the standard approach. It

is only valid after the proper wavelength of the mode is larger than the Hubble radius (or when “the mode has crossed the horizon”). That is, the traditional statement is that during inflation the modes become super-horizon and then occurs the transition $\hat{\Phi} \rightarrow \Phi$. Furthermore, the relation $\Phi_{\mathbf{k}} = \sqrt{\langle 0|\hat{\Phi}_{\mathbf{k}}^2|0\rangle} e^{i\alpha_{\mathbf{k}}}$ could generate some misunderstandings, in the sense that one might think that, since the vacuum fluctuations $\langle 0|\hat{\Phi}_{\mathbf{k}}^2|0\rangle$ are not zero, $\Phi_{\mathbf{k}}$ should be different from zero when the mode is super-horizon, and the spacetime perturbations are “born” (see Appendix A for clarification).

On the other hand, in our approach, the random nature of $\Phi_{\mathbf{k}}$ will come directly from the stochastic aspects of the quantum dynamical reduction, i.e. from the CSL mechanism. Moreover, we will adopt the point of view that the classical characterization of Φ is an adequate description if the quantum state is sharply peaked around some particular value. In consequence, the classical value corresponds to the expectation value of $\hat{\Phi}$ [27]. More precisely, the CSL collapse mechanism will lead to a final state such that the relation

$$\Phi_{\mathbf{k}} = \langle \Psi|\hat{\Phi}_{\mathbf{k}}|\Psi\rangle \quad (15)$$

is valid.

Therefore, in our approach, the coefficients a_{lm} in Eq. (14), will be given by

$$a_{lm} = \frac{4\pi i^l}{3} \int \frac{d^3 k}{(2\pi)^3} j_l(kR_D) Y_{lm}^*(\hat{k}) \Delta(k) \langle \Theta|\hat{\Phi}_{\vec{k}}|\Theta\rangle, \quad (16)$$

where $|\Theta\rangle$ corresponds to the evolved state according to the non-unitary modification of the Schrödinger equation provided by the CSL mechanism. At this point, we encourage the reader to consult Appendix B for a discussion of other possible ways to relate $\hat{\Phi}$ and Φ within the CSL framework.

Additionally, Eqs. (3) and (15) allow us to relate each mode of the curvature perturbation to the quantum field variables. That is,

$$\Phi_{\mathbf{k}} = \langle \hat{\Phi}_{\mathbf{k}} \rangle = \sqrt{\frac{\epsilon}{2}} \frac{H}{k^2 M_P} \langle \hat{p}_{\mathbf{k}} \rangle = \sqrt{\frac{\epsilon}{2}} \frac{H}{k^2 M_P} [\langle \hat{p}_{\mathbf{k}}^R \rangle + i \langle \hat{p}_{\mathbf{k}}^I \rangle], \quad (17)$$

where we used the definition of the momentum provided by (5). Equation (17) relates the classical curvature perturbation with the momentum variable of the quantum field, consequently, this strongly suggest that the collapse operator $\hat{\Theta}$ to be considered in the CSL mechanism is the momentum operator. Thus, if the initial state is the Bunch-Davies vacuum state $\langle \hat{p}_{\mathbf{k}}^{R,I}|0, \tau\rangle \propto \exp[-(\hat{p}_{\mathbf{k}}^{R,I})^2/2k]$, then $\langle 0|\hat{p}_{\mathbf{k}}^{R,I}|0\rangle = 0$, and as a consequence of (17), the curvature perturbation is $\Phi_{\mathbf{k}} = 0$, i.e. the spacetime is perfectly homogeneous and isotropic. It is only after the state has evolved, according to the CSL mechanism, that generically $\langle \hat{p}_{\mathbf{k}}^{R,I} \rangle \neq 0$ and the curvature perturbation is born. This illustrates how the self-induced collapse provided by the CSL model can generate the primordial perturbations.

Furthermore, from the previous discussion, it is evident that we will adopt the CSL mechanism for each mode of the field and the corresponding real and imaginary parts. Therefore, the evolution of the state vector characterizing each mode of the inflaton field, written in conformal time, will be

$$|\Psi_{\mathbf{k}}^{R,I}, \eta\rangle = \hat{T} \exp\left\{-\int_{\tau}^{\eta} d\eta' [i\hat{H}_{\mathbf{k}}^{R,I} + \frac{1}{4\lambda_k}(W(\eta') - 2\lambda_k\hat{p}_{\mathbf{k}}^{R,I})^2]\right\} |\Psi_{\mathbf{k}}^{R,I}, \tau\rangle \quad (18)$$

with $H_{\mathbf{k}}^{R,I}$ given in (6). Moreover, with the previous considerations, the motion equation (11) for the ensemble average of the expectation value of an operator is:

$$\frac{d}{d\eta} \langle \hat{O}_{\mathbf{k}}^{R,I} \rangle = -i \langle [\hat{O}_{\mathbf{k}}^{R,I}, \hat{H}_{\mathbf{k}}^{R,I}] \rangle - \frac{\lambda_k}{2} \langle [\hat{p}_{\mathbf{k}}^{R,I}, [\hat{p}_{\mathbf{k}}^{R,I}, \hat{O}_{\mathbf{k}}^{R,I}]] \rangle. \quad (19)$$

This completes our treatment of the CSL model during inflation.

E. The scalar power spectrum

Having established the relation between the objects $\hat{\Phi}$ and Φ , we now focus on the scalar power spectrum. The scalar power spectrum in Fourier space is defined as

$$\overline{\Phi_{\mathbf{k}}\Phi_{\mathbf{k}'}^*} \equiv \frac{2\pi^2}{k^3} \mathcal{P}_s(k) \delta(\mathbf{k} - \mathbf{k}') \quad (20)$$

where $\mathcal{P}_s(k)$ is the dimensionless power spectrum. The bar appearing in (20) denotes an ensemble average over possible realizations of the stochastic field $\Phi_{\mathbf{k}}$. In our approach, the realization of a particular $\Phi_{\mathbf{k}}$ is given by the self-induced collapse that results from the CSL mechanism.

Henceforth, Eq. (17) implies that $\overline{\Phi_{\mathbf{k}}\Phi_{\mathbf{k}'}^*} \propto \overline{\langle \hat{p}_{\mathbf{k}} \rangle \langle \hat{p}_{\mathbf{k}'} \rangle^*}$, where the expectation values are being evaluated at the (evolved) state provided by (18). More explicitly,

$$\begin{aligned} \overline{\langle \hat{p}_{\mathbf{k}} \rangle \langle \hat{p}_{\mathbf{k}'} \rangle^*} &= \overline{\langle \hat{p}_{\mathbf{k}}^R + i\hat{p}_{\mathbf{k}}^I \rangle \langle \hat{p}_{\mathbf{k}'}^R - i\hat{p}_{\mathbf{k}'}^I \rangle} \\ &= \left(\overline{\langle \hat{p}_{\mathbf{k}}^R \rangle^2} + \overline{\langle \hat{p}_{\mathbf{k}}^I \rangle^2} \right) \delta(\mathbf{k} - \mathbf{k}'), \end{aligned} \quad (21)$$

where in the second line we assumed that the CSL model does not induce modes correlations. Thus, from now on we will focus on calculate the quantities $\overline{\langle \hat{p}_{\mathbf{k}}^{R,I} \rangle^2}$. In particular, we will calculate only the real part since the computation for the imaginary part proceeds in the same fashion. In order to simplify the notation, we will omit the index R unless it can create confusion, in which case we will write it explicitly.

By using the Gaussian wave function in the momentum representation (7), and the probability associated to $W(\eta)$ (9), it can be shown that [51],

$$\overline{\langle \hat{p}_{\mathbf{k}} \rangle^2} = \overline{\langle \hat{p}_{\mathbf{k}}^2 \rangle} - \frac{1}{4\text{Re}[A_k(\eta)]}. \quad (22)$$

That is, $(4\text{Re}[A(\eta)])^{-1}$ is the standard deviation of the squared momentum. It is also the width of every packet in momentum space. Thus, to calculate $\overline{\langle \hat{p}_{\mathbf{k}} \rangle^2}$, we only need to find the two terms on the right hand side of (22). The second term will be found from the CSL evolution Eq. (18), and the first one by using Eq. (19).

Let us focus on the second term. Using the general Gaussian state in momentum space (7) and the CSL evolution Eq. (18) for the wave function, the equation of motion for $A_k(\eta)$ results

$$A'_k = \frac{i}{2} + \lambda_k + 2A_k \frac{z'}{z} - 2ik^2 A_k^2. \quad (23)$$

The previous equation can be solved by performing the change of variable $A_k(\eta) \equiv f'(\eta)/[2ik^2 f(\eta)]$, resulting in a Bessel differential equation for f . After solving such an equation, and returning to the original variable A_k , we obtain

$$A_k(\eta) = \frac{q}{2ik^2} \left[\frac{J_{\nu_s+1}(-q\eta) + e^{-i\pi\nu_s} J_{-(\nu_s+1)}(-q\eta)}{J_{\nu_s}(-q\eta) - e^{-i\pi\nu_s} J_{-\nu_s}(-q\eta)} \right], \quad (24)$$

being $q^2 \equiv k^2(1 - 2i\lambda_k)$ and where the initial condition for the Bunch-Davis vacuum $A_k(\tau) = 1/2k$ was used (recall that τ corresponds to the onset of inflation, thus, $\tau \rightarrow -\infty$). J_{ν_s} corresponds to a Bessel function of the first kind of order $\nu_s = 1/2 + 2\epsilon - \delta$. Note that Eq. (24) is exact. However, from the observational point of view, one is interested in modes that are well outside the Hubble radius during inflation, i.e. modes with $k \ll aH$ or equivalently, $-k\eta \rightarrow 0$. Therefore, we can expand $A_k(\eta)$ in the limit $-q\eta \rightarrow 0$ (provided that $\lambda_k \ll 1$):

$$A_k(\eta) \simeq \frac{i\nu_s}{k^2\eta} \left[1 + (-q\eta)^{2\nu_s} \nu_s 2^{-2\nu_s} \frac{\Gamma(-\nu_s)}{\Gamma(\nu_s+1)} e^{i\pi\nu_s} \right]^{-1}, \quad (25)$$

with $\Gamma(\nu)$ the Gamma function. From (25), it is straightforward to obtain

$$\frac{1}{4\text{Re}[A_k(\eta)]} \simeq \frac{k^2 2^{2\nu_s-2} \zeta_k^{-2\nu_s} \sin(\pi\nu_s) \Gamma^2(\nu_s) (-k\eta)^{-2\nu_s+1}}{\pi \sin(2\nu_s\theta_k + \pi\nu_s)} \quad (26)$$

where we have defined $\zeta_k e^{i\theta_k} \equiv \sqrt{1 - 2i\lambda_k}$.

Next, we focus on the first term of (22), i.e. $\overline{\langle \hat{p}_{\mathbf{k}}^2 \rangle}$. It will be useful to define the quantities $Q \equiv \overline{\langle \hat{v}_{\mathbf{k}}^2 \rangle}$, $R \equiv \overline{\langle \hat{p}_{\mathbf{k}}^2 \rangle}$ and $S \equiv \overline{\langle \hat{p}_{\mathbf{k}} \hat{v}_{\mathbf{k}} + \hat{v}_{\mathbf{k}} \hat{p}_{\mathbf{k}} \rangle}$. Thus, the evolution equations for Q , R and S are obtained using (19):

$$Q' = S + 2\frac{z'}{z} + \lambda_k, \quad R' = -k^2 S - 2\frac{z'}{z} R, \quad S' = 2R - 2k^2 Q. \quad (27)$$

Therefore, we have a linear system of coupled differential equations, whose general solution is a particular solution to the system plus a solution to the homogeneous equation (with $\lambda_k = 0$). After a long series of calculations we

find

$$R(\eta) = k^2(-k\eta) \left[C_1 J_{\nu_s}^2(-k\eta) + C_2 J_{-\nu_s}^2(-k\eta) + C_3 J_{\nu_s}(-k\eta) J_{-\nu_s}(-k\eta) \right] + \frac{\lambda_k k^2 \eta}{4\nu_s} \quad (28)$$

where the constants C_1, C_2 and C_3 are found by imposing the initial conditions corresponding to the Bunch-Davies vacuum state: $Q(\tau) = 1/2k, R(\tau) = k/2, S(\tau) = 0$. Equation (28) is exact; expanding it again around $-k\eta \rightarrow 0$ yields

$$R(\eta) \simeq \frac{k}{\pi} 2^{2\nu_s-2} \Gamma^2(\nu_s) \left[1 + \lambda_k \sin \beta_k \cos \beta_k - \frac{\lambda_k k \tau}{2} \times \left(\frac{3}{\nu_s + 1} \sin^2 \beta_k + \frac{\cos^2 \beta_k}{\nu_s} \right) \right] (-k\eta)^{-2\nu_s+1}, \quad (29)$$

with $\beta_k \equiv -k\tau - \nu_s \pi/2 - 3\pi/4$. Also, recall that $R(\eta) \equiv \overline{(\hat{p}_{\mathbf{k}}^2)}$.

At this point, we have the two terms of (22), i.e. $\overline{(\hat{p}_{\mathbf{k}}^2)}$ from Eq. (29), and $1/(4\text{Re}[A_k(\eta)])$ from Eq. (26). Then we can write

$$\overline{(\hat{p}_{\mathbf{k}}^2)} \simeq \frac{k}{\pi} 2^{2\nu-2} \Gamma^2(\nu_s) (-k\eta)^{-2\nu_s+1} F(\lambda_k, \nu_s), \quad (30)$$

where we have defined

$$F(\lambda_k, \nu_s) \equiv \left[1 + \lambda_k \sin \beta_k \cos \beta_k - \frac{\lambda_k k \tau}{2} \left(\frac{3}{\nu_s + 1} \times \sin^2 \beta_k + \frac{\cos^2 \beta_k}{\nu_s} \right) - \frac{\zeta_k^{-2\nu_s} \sin(\nu_s \pi)}{\sin(2\nu_s \theta_k + \nu_s \pi)} \right]. \quad (31)$$

Equation (30) is valid for both the real and the imaginary part of $\hat{p}_{\mathbf{k}}$. Therefore, substituting (30) into (21), and taking into account (17), we obtain

$$\overline{\Phi_{\mathbf{k}} \Phi_{\mathbf{k}'}} = \frac{\epsilon H^2}{k^3 \pi M_P^2} 2^{2\nu-2} \Gamma^2(\nu_s) (-k\eta)^{-2\nu_s+1} F(\lambda_k, \nu_s) \times \delta(\mathbf{k} - \mathbf{k}') \quad (32)$$

Finally, with the definition of the power spectrum (20), and the result in (32), the scalar power spectrum within the CSL model, results¹

$$\mathcal{P}_s(k) = \frac{H^2 2^{2\nu_s-3} \Gamma^2(\nu_s)}{\epsilon M_P^2 \pi^3} (-k\eta)^{-2\nu_s+1} F(\lambda_k, \nu_s) \quad (33)$$

¹ Note that in Eq. (32) the slow-roll parameter ϵ appears in the numerator, while in the expression for the scalar power spectrum (33) appears in the denominator. The reason for this difference is that, in the longitudinal gauge, the scalar curvature perturbation Φ becomes amplified by a factor of $1/\epsilon$ during the transition from inflation to the radiation dominated stage [55, 56], in which the CMB is created (the decoupling era). Therefore in order to obtain a consistent prediction to be compared with the observational data, we must multiply by a factor of $1/\epsilon^2$ the scalar power spectrum obtained during inflation associated to $\overline{\Phi_{\mathbf{k}} \Phi_{\mathbf{k}'}}$.

We will discuss some physical implications from our prediction in Sect. IV.

III. THE CSL MODEL AND THE TENSOR POWER SPECTRUM

Once we have successfully applied the CSL model to the primordial scalar perturbations, we now proceed to focus on the tensor perturbations. As it is well known, these perturbations represent gravitational waves characterized by a traceless, transverse and symmetric tensor field. These properties imply that the gravitational waves are polarized in two ways. As is traditional, we will consider one type of polarization and at the end we will just multiply the final results by a factor of 2.

The action for the tensor perturbations is obtained from the Einstein-Hilbert action by expanding the tensor perturbations $h_{ij}(\mathbf{x}, \eta)$ up to second order [54]. The resulting action for the tensor field $h_{ij}(\mathbf{x}, \eta)$ can be expressed in terms of its Fourier modes $h_{ij}(\mathbf{k}, \eta) = h_{\mathbf{k}}(\eta) e_{ij}(\mathbf{k})$, with $e_{ij}(\mathbf{k})$ representing a time-independent polarization tensor. Performing the change of variable

$$h_{\mathbf{k}}(\eta) \equiv \frac{2}{M_P (e_j^i e_i^j)^{1/2}} \frac{v_{\mathbf{k}}(\eta)}{a(\eta)}, \quad (34)$$

the action can be written as $\delta^{(2)} S_h = \frac{1}{2} \int d\eta d^3\mathbf{k} \mathcal{L}_h$, where

$$\mathcal{L}_h = v_{\mathbf{k}}' v_{\mathbf{k}}'^* - k^2 v_{\mathbf{k}} v_{\mathbf{k}}^* - \frac{a'}{a} \left(v_{\mathbf{k}} v_{\mathbf{k}}'^* + v_{\mathbf{k}}' v_{\mathbf{k}}^* \right) + \left(\frac{a'}{a} \right)^2 v_{\mathbf{k}} v_{\mathbf{k}}^*. \quad (35)$$

By comparing the action in (4) with the one obtained for the tensor perturbations (35), we see that they are equivalent as expected. The only difference is that the action for the scalar perturbations contains a term z'/z while the action for the tensor modes contains the term a'/a . Therefore, the quantization procedure of Sect. II B remains exactly the same by simply replacing $z'/z \rightarrow a'/a$. Now, as was done in Sect. II D, we need to define which are the appropriate observables that emerge from the quantum theory of the tensor modes. By the same argument that allowed us to identify $\Phi_{\mathbf{k}} = \langle \hat{\Phi}_{\mathbf{k}} \rangle$, we will assume that

$$h_{\mathbf{k}} = \langle \Psi | \hat{h}_{\mathbf{k}} | \Psi \rangle \quad (36)$$

is also valid. Hence, it is evident that a quantization of $v_{\mathbf{k}}$ from the action (35) yields a quantization of $\hat{h}_{\mathbf{k}}$. In other words, Eqs. (34) and (36) imply that

$$h_{\mathbf{k}}(\eta) = \frac{2}{M_P (e_j^i e_i^j)^{1/2}} \frac{\langle \hat{v}_{\mathbf{k}}(\eta) \rangle}{a(\eta)}. \quad (37)$$

Thus, (37) relates the quantum field variable $\hat{v}_{\mathbf{k}}$ to the amplitude of the tensor mode $h_{\mathbf{k}}$. Furthermore, this relation is analogous to (17), in which the scalar curvature perturbation is related to the momentum field variable $\hat{p}_{\mathbf{k}}$.

On the other hand, as a matter of physical consistency in the operation of the CSL model, we will keep the assumption that the collapse operator corresponds to the momentum operator, even if the relation between $h_{\mathbf{k}}$ and the quantum matter fields is in terms of $v_{\mathbf{k}}$. In other words, the quantum theory described by actions (35) and (4) are physically equivalent, i.e. they correspond to the action of a scalar field with time dependent mass. Thus, the fields 'do not know' how they will be related to the curvature perturbation. We will deepen this discussion in the next section.

Therefore, since we retained the momentum operator as the collapse operator, the treatment of the CSL model, applied to inflation, remains the same as the one presented in (18) and (19). Moreover, given that the quantization procedure of Sect. IIB stayed unchanged, we will continue working with the wave function shown in (7).

With the relation between $\hat{h}_{\mathbf{k}}$ and $h_{\mathbf{k}}$ already established, we can now focus on the dimensionless tensor power spectrum defined as

$$\overline{h_j^i(\mathbf{k})h_i^{j*}(\mathbf{k}')} \equiv \frac{2\pi^2}{k^3} \mathcal{P}_t(k) \delta(\mathbf{k} - \mathbf{k}'). \quad (38)$$

Furthermore, by using (37) it results in

$$\overline{h_j^i(\mathbf{k})h_i^{j*}(\mathbf{k}')} \propto \overline{\langle \hat{v}_{\mathbf{k}} \rangle \langle \hat{v}_{\mathbf{k}'} \rangle^*} = \left(\overline{\langle \hat{v}_{\mathbf{k}}^{R,I} \rangle^2} + \overline{\langle \hat{v}_{\mathbf{k}}^I \rangle^2} \right) \delta(\mathbf{k} - \mathbf{k}'), \quad (39)$$

Proceeding in a similar manner with which we arrive at (22), we now find that

$$\overline{\langle \hat{v}_{\mathbf{k}}^{R,I} \rangle^2} = \overline{\langle (\hat{v}_{\mathbf{k}}^{R,I})^2 \rangle} - \frac{|A_k(\eta)|^2}{\text{Re}[A_k(\eta)]}. \quad (40)$$

The second term of this expression is obtained from Eq. (25), that is, in the limit $-k\eta \rightarrow 0$ we can write

$$\frac{|A_k(\eta)|^2}{\text{Re}[A_k(\eta)]} \simeq \frac{\sin \pi \nu_t \Gamma^2(\nu_t + 1) 2^{2\nu_t} \zeta_k^{-2\nu_t} (-k\eta)^{-2\nu_t - 1}}{k \sin(2\nu_t \theta_k + \pi \nu_t) \pi}, \quad (41)$$

where $\nu_t = 1/2 + \epsilon$. This is different from $\nu_s = 1/2 + 2\epsilon - \delta$. This distinction arises because we have replaced $z'/z \rightarrow a'/a$ in the whole computation.

Next, in order to obtain the first term in (40), i.e. $\overline{\langle (\hat{v}_{\mathbf{k}}^{R,I})^2 \rangle}$, we note that its evolution equation is already shown in the system of differential equations given by (27), recalling that $Q \equiv \overline{\langle (\hat{v}_{\mathbf{k}}^{R,I})^2 \rangle}$ (and also that $z'/z \rightarrow a'/a$). The solution for Q can be found analytically. However, since we are interested in the modes such that $-k\eta \rightarrow 0$, $Q(\eta)$ becomes

$$Q(\eta) \simeq \frac{2^{2\nu_t}}{k\pi} \Gamma^2(\nu_t + 1) \left[1 + \lambda_k \sin \beta_k \cos \beta_k - \frac{\lambda_k k \tau}{2} \right. \\ \left. \times \left(\frac{3}{\nu_t + 1} \sin^2 \beta_k + \frac{\cos^2 \beta_k}{\nu_t} \right) \right] (-k\eta)^{-2\nu_t - 1}, \quad (42)$$

with β_k being the same as in (29) but replacing $\nu_s \rightarrow \nu_t$. By using (41) and (42) into (40) we write

$$\overline{\langle \hat{v}_{\mathbf{k}}^{R,I} \rangle^2} \simeq \frac{2^{2\nu_t}}{k\pi} \Gamma^2(\nu_t + 1) (-k\eta)^{-2\nu_t - 1} F(\lambda_k, \nu_t). \quad (43)$$

Consequently, Eq. (43) along with (39) imply that

$$\overline{h_j^i(\mathbf{k})h_i^{j*}(\mathbf{k}')} = \frac{8}{M_p^2 a^2} \frac{2^{2\nu_t}}{k\pi} \Gamma^2(\nu_t + 1) (-k\eta)^{-2\nu_t - 1} \\ \times F(\lambda_k, \nu_t) \delta(\mathbf{k} - \mathbf{k}'). \quad (44)$$

Now, by comparing (44) with the definition of the tensor power spectrum (38), we finally obtain

$$\mathcal{P}_t(\eta) = \frac{H^2}{M_p^2 \pi^3} (-\eta)^{-2\nu_t + 1} 2^{2\nu_t + 3} \Gamma^2(1 + \nu_t) \\ \times k^{-2\nu_t + 1} F(\lambda_k, \nu_t), \quad (45)$$

where we have multiplied by a factor of 2 due to the polarization of the gravitational waves, and we used $a(\eta) \simeq -1/(H\eta)$. With the expressions of the scalar and tensor power spectrum at hand, Eqs. (33) and (45) respectively, it is now straightforward to calculate the tensor-to-scalar ratio defined as $r \equiv \mathcal{P}_t(k)/\mathcal{P}_s(k)$. Under the approximation $\nu_s \simeq \nu_t \simeq 1/2$, it leads to $r \simeq 16\epsilon$, which is exactly the same prediction as in the traditional inflationary scenario. This result is also consistent with the one obtained in [57] in which the tensor-to-scalar ratio remained unchanged when considering a generic collapse scheme.

IV. DISCUSSION

With the power spectra within the CSL framework calculated, we will make a few remarks. Let us rewrite the scalar and tensor power spectra in the following suggestive forms:

$$\mathcal{P}_s(k) = A_s k^{n_s - 1} F(\lambda_k, \nu_s), \quad \mathcal{P}_t(k) = A_t k^{n_t} F(\lambda_k, \nu_t) \quad (46)$$

where we define

$$A_s \equiv \frac{H^2 (-\eta)^{-2\nu_s + 1} 2^{2\nu_s - 3} \Gamma^2(\nu_s)}{\epsilon M_p^2 \pi^3}, \quad n_s - 1 \equiv -2\nu_s + 1, \quad (47a)$$

$$A_t \equiv \frac{H^2 (-\eta)^{-2\nu_t + 1} 2^{2\nu_t + 3} \Gamma^2(\nu_t + 1)}{M_p^2 \pi^3}, \quad n_t \equiv -2\nu_t + 1. \quad (47b)$$

The quantities, A_s and A_t correspond to the amplitude of the scalar and tensor power spectrum respectively. From the definition of A_s , it can be shown [58] that this amplitude is practically a time-independent quantity [i.e. $d/d\eta\{A_s\} = \mathcal{O}(\epsilon^2, \delta^2)$]. The same argument applies for A_t . Actually, the amplitudes A_s and A_t coincide exactly with the ones from standard predictions. Moreover, since the amplitudes are constants, it is customary to evaluate

the power spectra at some conformal time η_* , which is usually taken to be the conformal time at the horizon crossing $-k\eta_* = 1$. However, from our point of view, the self-induced collapse of the wave function generates the primordial perturbations. Therefore, we cannot evaluate the power spectra at some conformal time, e.g. at the horizon crossing, if the collapse has not taken place yet (or more precisely the CSL mechanism has not concluded), but this evaluation is purely conventional, our prediction for the power spectra is time independent.

The shapes of the power spectra (that is, its dependence on k) are of the form $k^{n_s-1}F(\lambda_k, \nu_s)$ and $k^{n_t}F(\lambda_k, \nu_t)$ for the scalar and tensor power spectrum, respectively. Let us focus on the function $F(\lambda_k, \nu)$ shown in (31) (where $\nu = \nu_s, \nu_t$), and make the approximation $\nu_s \simeq \nu_t \simeq 1/2$, i.e.

$$F(\lambda_k, 1/2) = 1 + \lambda_k[\cos(-k\tau)\sin(-k\tau) - k\tau] - \sqrt{\frac{2}{1 + \sqrt{1 + 4\lambda_k^2}}}, \quad (48)$$

where we used the definitions of β_k , ζ_k and θ_k . Furthermore, if the following assumptions are valid: i) $-k\tau \gg 1$ and ii) $\lambda_k \ll 1$, then

$$F(\lambda_k, 1/2) \simeq -\lambda_k k\tau. \quad (49)$$

Henceforth, if $\lambda_k = \lambda_0/k$, then F becomes essentially scale-invariant and the only dependence on k in the power spectra is of the form k^{n_s-1} and k^{n_t} . In fact, from (47) and the definitions of ν_s and ν_t , we have $n_s - 1 = -4\epsilon + 2\delta$ and $n_t = -2\epsilon$. That is, the spectral indices are exactly the same as in the standard approach. Thus, if $\lambda_k = \lambda_0/k$ we recover the standard prediction, what was also noted in [51] but within the semiclassical gravity and just for the scalar case. In Sect. V A we will say more about the validity of i) and ii).

Finally, it is interesting to observe that if $\lambda_k = 0$, i.e. the vacuum state is evolving according to the unmodified Schrödinger equation and no self-induced collapse occurs, then $F(0, \nu) = 0$ (with $\nu = \nu_s, \nu_t$). Consequently, $\mathcal{P}_s = \mathcal{P}_t = 0$. This implies that there are no primordial perturbations and the spacetime is perfectly homogeneous and isotropic. On the other hand, the primordial perturbations are generated by “switching on” the self-induced collapse $\lambda_k \neq 0$. In this case, $\Phi_{\mathbf{k}}$ is created randomly, and its power spectrum is obtained from $\langle \hat{\Phi}_{\mathbf{k}} \rangle \langle \hat{\Phi}_{\mathbf{k}}^* \rangle$. Note that one could argue that maybe the cancellation in the case $\lambda_k = 0$ occurs only in the limit $-k\eta \rightarrow 0$, i.e. only affects the super-horizon modes. We have checked that even if one considers the exact analytic expressions of $\langle \hat{p}_{\mathbf{k}}^2 \rangle$ and $1/(4\text{Re}[A_k(\eta)])$, as well as $\langle (\hat{v}_{\mathbf{k}}^{R,I})^2 \rangle$ and $|A_k(\eta)|^2/(\text{Re}[A_k(\eta)])$ in the tensor case, one arrives at the same result, that is, the spacetime does not contain perturbations of any scale.

V. CSL, SQUEEZING, GAUGE INVARIANCE AND PREVIOUS WORKS

A. CSL parameter

We begin this section by giving a rough estimate for the value of the CSL parameter λ_k . As mentioned in the previous section, if assumptions i) and ii) are valid, then by taking $\lambda_k = \lambda_0/k$ the standard prediction for the power spectra is recovered. Assumption i) reads $-k\tau \gg 1$. An estimate for τ can be obtained by assuming that the energy scale at the onset of inflation is 10^{16} GeV, and that inflation lasts ~ 70 e-folds. Hence, $\tau \simeq -10^7$ Mpc. Since the observational range for k is $10^{-6} \text{ Mpc}^{-1} \leq k \leq 10^{-1} \text{ Mpc}^{-1}$, assumption i) is verified. On the other hand, assumption ii) is $\lambda_k = \lambda_0/k \ll 1$. Thus, by using again the range for k , the CSL parameter must satisfy $\lambda_0 \ll 10^{-6} \text{ Mpc}^{-1}$. We can check that this is indeed the case from (46) along with Friedmann equation. This yields $P_s(k) \propto \frac{V}{M_P^2 \epsilon}(-\lambda_0\tau)$ and as a consequence, if $\lambda_0 \simeq -1/\tau$, our model prediction is consistent with the standard accepted result. Moreover, since $\tau \simeq -10^7$ Mpc, $\lambda_0 \simeq 10^{-7} \text{ Mpc}^{-1} \simeq 10^{-21} \text{ s}^{-1}$, in which case, assumption ii) is valid.

Thus, if the CSL parameter is of the form $\lambda_k = \lambda_0/k$, with $\lambda_0 \simeq 10^{-21} \text{ s}^{-1}$, the model is compatible with the CMB observational data. This result is also consistent with findings of the CSL collapse mechanism applied to situations different from the cosmological context. In particular, the fact that λ_k depends on the wave number k captures the spirit of the ‘amplification mechanism’ of the CSL model, which states that micro- and macro-objects do not behave the same way. For example, micro-objects tend to exhibit quantum features (e.g. superposition in their wave functions) while macro-objects do not. In our model, the particular dependence on k of the CSL parameter (which generically sets the strength of the self-induced collapse) implies that the self-induced collapse affects the large-scale modes more than the low-scale modes; in some sense, the low-scale modes behave quantum mechanically while large-scale modes can be treated classically. It is also interesting to compare our value $\lambda_0 \simeq 10^{-21} \text{ s}^{-1}$ with the one suggested by the GRW model ($\lambda_{\text{GRW}} = 10^{-16} \text{ s}^{-1}$), adopted later in the CSL model with the mass-density operator as the collapse operator [41, 42]. Our estimated value for λ_0 makes the strength of the self-induced collapse five orders of magnitude weaker than the one in the GRW model. However, our estimate was based on very robust assumptions regarding the inflationary era, particularly, the energy scale of inflation and the number of e-foldings. Note that the value of the parameter ϵ can also change the value of λ_0 by a couple of orders of magnitude. Thus, a more careful analysis including the full angular power spectrum of the CMB data is required in order to constrain the value of the CSL parameter. This subject is left for a future work.

B. Squeezing of the modes

It is usually argued that during inflation, the Bunch-Davies vacuum evolves toward a squeezed state; that is, a state in which its uncertainty is increased in one variable and decreased in another one, such that the product of the uncertainties satisfy the minimum value allowed by the Heisenberg uncertainty principle. In the Schrödinger picture, the uncertainties of the real and imaginary parts of $\hat{p}_{\mathbf{k}}$ and $\hat{v}_{\mathbf{k}}$ (associated to scalar perturbations) are given by

$$\Delta^2 \hat{p}_{\mathbf{k}}^{R,I}(\eta) = \frac{1}{4\text{Re}[A_{\mathbf{k}}(\eta)]}, \Delta^2 \hat{v}_{\mathbf{k}}^{R,I}(\eta) = \frac{|A_{\mathbf{k}}(\eta)|^2}{\text{Re}[A_{\mathbf{k}}(\eta)]}. \quad (50)$$

Henceforth, by using (26) and (41), we see that

$$\begin{aligned} \Delta^2 \hat{v}_{\mathbf{k}}^{R,I}(\eta) &\sim (-k\eta)^{-2\nu_s-1} \\ \Delta^2 \hat{p}_{\mathbf{k}}^{R,I}(\eta) &\sim (-k\eta)^{-2\nu_s+1} \end{aligned} \quad (51)$$

as $-k\eta \rightarrow 0$. Thus, the uncertainty in $\hat{v}_{\mathbf{k}}^{R,I}$ increases while the uncertainty in $\hat{p}_{\mathbf{k}}^{R,I}$ decreases as inflation takes place. This is an expected result; in fact, it is normally used to show that inflation induces squeezing in the momentum variable [9, 13, 47, 48]. Furthermore, recall that in our approach, the self-induced collapse generates the primordial curvature perturbation. In the longitudinal gauge, our point of view was reflected in Eq. (17); therefore, the uncertainty in the momentum and in the curvature perturbation operator Φ are directly related, and since during inflation $\Delta^2 \hat{p}_{\mathbf{k}}^{R,I}(\eta)$ decreases, $\Delta^2 \hat{\Phi}_{\mathbf{k}}^{R,I}(\eta)$ also decreases. On the other hand, if we had chosen the comoving gauge, the curvature perturbation would be given by $\mathcal{R} = v/z$, where $z \simeq a$. Consequently, in our approach, there would be a relation between the uncertainties $\Delta^2 \hat{\mathcal{R}}_{\mathbf{k}}^{R,I}(\eta) = \Delta^2 \hat{v}_{\mathbf{k}}^{R,I}(\eta)/a^2$, and since during inflation $a(\eta) \simeq -1/\eta$ then $\Delta^2 \hat{\mathcal{R}}_{\mathbf{k}}^{R,I}(\eta) \sim (-k\eta)^{-2\nu+1}$. Thus, it decreases in the same fashion as $\Delta^2 \hat{p}_{\mathbf{k}}^{R,I}(\eta)$. The tensor modes follow this behavior, i.e. since $\hat{h}_{\mathbf{k}} \simeq \hat{v}_{\mathbf{k}}/a$, the uncertainty in $\hat{h}_{\mathbf{k}}^{R,I}$ decreases even if the uncertainty in $\hat{v}_{\mathbf{k}}^{R,I}$ increases. One could argue that this behavior occurs irrespectively of whether there is a quantum collapse or not. However, it should be borne in mind that, in our approach, if there is no quantum collapse then the classical curvature perturbation is exactly zero in spite of its quantum uncertainty $\Delta^2 \hat{\Phi}_{\mathbf{k}}^{R,I}(\eta)$ (in the longitudinal gauge) or $\Delta^2 \hat{\mathcal{R}}_{\mathbf{k}}^{R,I}(\eta)$ (in the comoving gauge) decreases as inflation takes place. We should also mention that even if the Bardeen potential Φ , the Mukhanov-Sasaki variable v and \mathcal{R} are gauge-invariant quantities, the curvature perturbation, characterized by the spatial Ricci scalar, is not [58]. Hence, when a physical interpretation of these mathematical entities is needed, one must choose a specific gauge: the Bardeen potential represents the curvature perturbation in the longitudinal gauge, the Mukhanov-Sasaki variable represents the inflaton perturbation in a spatially flat gauge, and the quantity \mathcal{R} represents the curvature perturbation in the comoving gauge.

In summary, our conceptual point of view regarding the collapse states the following: if there is no quantum collapse ($\lambda_k = 0$), then $\Phi_{\mathbf{k}} = \Phi_{\mathbf{k}}^R + i\Phi_{\mathbf{k}}^I \propto \langle 0|\hat{p}_{\mathbf{k}}^R|0\rangle + i\langle 0|\hat{p}_{\mathbf{k}}^I|0\rangle = 0$, with the uncertainty in $\hat{\Phi}_{\mathbf{k}}^{R,I}$ decreasing as $(-k\eta)^{-2\nu_s+1}$, and therefore the spacetime is perfectly homogeneous and isotropic. It is only by introducing a self-induced collapse, provided in this case by the CSL mechanism, that $\lambda_k \neq 0$, then $\Phi = \Phi_{\mathbf{k}}^R + i\Phi_{\mathbf{k}}^I \propto \langle \hat{p}_{\mathbf{k}}^R \rangle + i\langle \hat{p}_{\mathbf{k}}^I \rangle \neq 0$ and the uncertainty of the curvature perturbation operator decreases as $\Delta^2 \hat{\Phi}_{\mathbf{k}}^{R,I}(\eta) \simeq (-k\eta)^{-2\nu_s+1}/[\zeta_k \cos(\pi\theta_k)]$. Also, note that since we are treating the self-induced collapse as a sort of 'measurement process', it must act on the hermitian operators, say $\hat{\Phi}_{\mathbf{k}}^R$ and $\hat{\Phi}_{\mathbf{k}}^I$. Thus, the uncertainties that are important from the 'measurement like process' are those corresponding to $\Delta^2 \hat{\Phi}_{\mathbf{k}}^R$ and $\Delta^2 \hat{\Phi}_{\mathbf{k}}^I$.

C. Comparison to previous works

As we have mentioned in the Introduction, the CSL model has been applied to the inflationary universe in previous works by several authors, and they have reached different conclusions with each other. Here we will present a brief summary of their results and a comparison to our findings.

The first work we will address is the one presented in [47]. There, the authors applied the CSL model to inflation using the Mukhanov-Sasaki variable as the collapse operator, and assumed that the CSL parameter λ to be the same for all modes, i.e. independent of k . Those assumptions led to a scalar power spectrum with two branches, one that is scale-invariant, and another one with a spectral index $n_s = 4$ in an evident conflict with the CMB data. In order to suppress the conflicting branch, the CSL parameter had to be severely constrained. Furthermore, the fact that λ is independent of k fails to incorporate the amplification mechanism that is crucial in the CSL model. In some sense, if λ is the same for all modes, then small and large-scale modes exhibit a classical behavior. Another issue mentioned by the authors is that the uncertainty in the Mukhanov-Sasaki variable increases even if such field variable corresponds to the CSL collapse operator.

In order to address some of the issues of [47], other authors proposed that the CSL parameter is of the form $\lambda(k, \eta) = \lambda_0(k/k_0)^\beta/(-k\eta)^\alpha$ (with k_0 a pivot scale), i.e. a function of both, the conformal time and k , while retaining the Mukhanov-Sasaki variable as the collapse operator [48, 49]. Hence, since the CSL parameter depends on k , the amplification mechanism of the CSL model is recovered. On the other hand, their model introduced three free parameters: λ_0, α and β . However, by taking into account the tensor modes [49], the modification to the scalar and tensor spectral indices could be captured in just one effective free parameter $\delta = 3 + \alpha - \beta$ and λ_0 . In fact, if $\delta = 0$, then their model predicts a scale-invariant scalar power spectrum. Furthermore, if $1 < \alpha < 2$, then, as inflation goes on, the squeezing oc-

curs in the Mukhanov-Sasaki variable rather than its conjugated momentum. This is because, in their approach, the CSL parameter depends on the conformal time. Nevertheless, this dependence on η is inherited in their final prediction for the scalar and tensor power spectra; thus, they choose to evaluate their power spectra at the end of inflation. This choice, in turn, translates into an extremely small value for the CSL parameter $\lambda_0 \simeq e^{-120}$, however, a higher value can be achieved by bringing down the energy scale of inflation.

References [48, 49] can be considered as an improved version of [47]; nevertheless, they share some features. First, in both approaches, if $\lambda = 0$, which means no self-induced collapse, their prediction for the (scalar) power spectrum is exactly the same as in the standard approach.² This contrasts with the result presented in this letter, since in our approach, if $\lambda_k = 0$ there are no curvature perturbations at all. As a consequence, $\mathcal{P}_s = 0 = \mathcal{P}_t$. This difference ultimately boils down to the quantum objects that are going to be identified with the power spectrum. In the framework of [47, 48], the power spectrum corresponds exactly to the Fourier transform of the two-point quantum correlation function, that is, $\mathcal{P}_s(k) \simeq \langle \Psi | \hat{v}_{\mathbf{k}} \hat{v}_{\mathbf{k}'} | \Psi \rangle$, where the state $|\Psi\rangle$ corresponds to the post-collapse state, or to the Bunch-Davies vacuum if $\lambda = 0$. In our approach, we have assumed that, once the collapse took place, it can be assumed that $\Phi_{\mathbf{k}} = \langle \Psi | \hat{\Phi}_{\mathbf{k}} | \Psi \rangle$. Therefore, the quantum collapse is the mechanism by which the curvature perturbation emerges. After the curvature perturbation was generated, as a result of a stochastic process, one can calculate the power spectrum associated to a classical stochastic field, i.e. $\mathcal{P}_s(k) = \overline{\Phi_{\mathbf{k}} \Phi_{\mathbf{k}'}} = \langle \Psi | \hat{\Phi}_{\mathbf{k}} | \Psi \rangle \langle \Psi | \hat{\Phi}_{\mathbf{k}'}^* | \Psi \rangle$. Another aspect that the authors in [47, 48] share is that they treat the field variable v as their collapse operator and then argue why it would be a desirable feature to obtain a squeezing in that variable. From our point of view, within the cosmological context, the choice of the collapse operator as the field variable is equally valid as the choice of the momentum operator as we have done in the present paper. In fact, we motivated this choice by Eq. (17), but we could have chosen to work in the comoving gauge, in which $\mathcal{R} = v/z$ represents the curvature perturbation, and then have argued to set v as the collapse operator. We think that in the absence of a complete relativistic version of the CSL mechanism, it is not entirely clear which operator plays the role of the collapse operator. On the other hand, we also think that the squeezing of a variable does not necessarily denote that the system has become classical. For instance, squeezed states are regarded as highly non-classical states in Quantum Optics [59, 60].

Therefore, in our approach neither the squeezing of the momentum nor the squeezing of the field variable are crucial for the classicalization of the perturbations. Moreover, as we have argued, if the squeezing occurs in the momentum variable, which is also our choice for the collapse operator, then by (17), the “squeezing” also occurs in the curvature perturbation operator (because the longitudinal gauge was selected). Additionally, (17) allows us to establish the relation $\mathcal{P}_s(k) \propto \langle \Psi | \hat{p}_{\mathbf{k}} | \Psi \rangle \langle \Psi | \hat{p}_{\mathbf{k}'}^* | \Psi \rangle$. Likewise, here the tensor modes are directly related to the field variable through (37), and in this case the collapse operator is still the momentum operator. Thus, even if the uncertainty in $\hat{v}_{\mathbf{k}}^{R,I}$ increases, the uncertainty in $\hat{h}_{\mathbf{k}}^{R,I}$ (which is associated to the tensor curvature perturbation), does not become larger. This is because of the relation $\Delta^2 \hat{h}_{\mathbf{k}}^{R,I} \simeq \Delta^2 \hat{v}_{\mathbf{k}}^{R,I} / a^2$. In both the scalar and the tensor cases, we have focused on the uncertainties of the quantum operators that are directly related to the observables of the CMB; namely, the operators associated to the curvature perturbations.

Finally, it is also worthwhile to mention that the conceptual point of view adopted in this paper, regarding the role of the collapse during inflation, was the same as the one followed in [51]. However, in such a work, the authors applied the CSL model to inflation within the semi-classical gravity framework, and one important difference with our work is that, within their approach, the amplitude of the tensor modes is exactly zero at first order in the perturbations. Additionally, the authors in [51] did not consider a full quasi-de Sitter background spacetime and therefore their final expression for the scalar power spectrum does not contain a specific prediction for the scalar spectral index.

VI. CONCLUSIONS

In this work, we have adopted an important role for the self-induced collapse of the wave function characterized by the CSL model, i.e. the collapse acts as the main agent for the emergence of the primordial curvature perturbations. We are aware that other people in the scientific community might prefer to adopt the Everett “many-world” interpretation of quantum theory plus decoherence in the case of the inflationary universe. However, we have presented that a dynamical reduction of the wave function, provided by the CSL model, can be a viable alternative explanation in the sense that is consistent with observations and also addresses the standard “measurement problem” of quantum theory.

Specifically, by focusing on the curvature perturbation operators, we have presented predictions for the scalar and tensor power spectra (46) that are quite consistent with the standard prediction. In particular, the scalar and tensor spectral indices are exactly the same, as well as the scalar A_s and tensor A_t amplitudes. The tensor-to-scalar ratio is also of the same form as in the tra-

² Actually, in the result presented in [48, 49] if $\lambda_0 = 0$ one gets a divergent quantity for the power spectra; nevertheless, we think this is because of the truncated terms in their series expansions and not because an actual problem with their model.

ditional approach. On the other hand, the addition of the CSL mechanism is reflected in the appearance of the function $F(\lambda_k, \nu)$ [defined in (31)] multiplying the standard prediction for the scalar and tensor power spectra. However, if $\nu \simeq 1/2$ and the CSL parameter is of the form $\lambda_k = \lambda_0/k$, then $F(\lambda_k, \nu)$ becomes practically independent of k , and we recover the standard prediction for the power spectra. This allows us to give a rough estimate for the CSL parameter $\lambda_0 \simeq 10^{-21} \text{ s}^{-1}$ (similar to the one suggested by the GRW model, $\lambda_{\text{GRW}} = 10^{-16} \text{ s}^{-1}$), which can be refined by using the full CMB data set, what is left for a future work. On the other hand, small variations of the recipe $\lambda_k = \lambda_0/k$ will result in different predictions from the standard case and that could be confronted with the observational data.

Our prediction for the power spectra differs from the ones obtained in previous works [47–49] in which the CSL mechanism was also applied to the inflationary universe. In particular, in contrast to Ref. [47], we have captured the amplification mechanism by assuming a k dependence of the CSL parameter. Additionally, our final result for the power spectra is not time dependent and we have only one free parameter λ_0 to be adjusted using the observational data. This is different from the findings of Ref. [48], in which their final predictions for the power spectra depend on the conformal time and their model introduces more than one free parameter. The reasons for these differences are traced back to the conceptual point of view regarding the collapse of the wave function during inflation. Specifically, if $\lambda_k = 0$, within the framework of Refs. [47, 48], then their model recovers the standard prediction. On the other hand, in our approach, if $\lambda_k = 0$, there is no quantum collapse. Then the tensor and scalar power spectra are exactly zero, which means that the spacetime does not contain any perturbations at all. Another important (conceptual) difference from previous research is that the squeezing of the field and momentum variables seems to play a relevant role in the approach of Refs. [47–49]. In our approach, we have shown that the uncertainty of the curvature perturbation operator (either in the comoving or in the longitudinal gauge) decreases as inflation goes on; however, the self-induced collapse is still needed for generating the primordial perturbation.

Our result, unlike the case of semiclassical gravity, predicts non-null tensor modes at first order in the perturbations, and it is also consistent with the findings of [57], where the tensor spectrum and the tensor-to-scalar ratio were calculated within a generic collapse scheme.

We conclude that, from the phenomenological point of view, the CSL mechanism can successfully be used to improve the standard inflationary paradigm by explaining the origin of the primordial perturbations and driving the quantum-to-classical transition. However, there are still some aspects concerning the CSL model that need to be addressed further. For example, it is well known that the CSL model violates energy conservation, which can lead to divergences in the energy-momentum tensor. Also,

the CSL model is nonrelativistic, hence, the choice of the collapse operator and the k dependence in the λ_k parameter is at this point purely phenomenological. Therefore, we think that the early universe provides a natural laboratory to test new ideas that would allow us to reach a deep understanding of Nature.

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Appendix A: Homogeneity and isotropy of the vacuum state

The vacuum state is defined by $\hat{a}_{\mathbf{k}}|0\rangle = 0$, which is the state of the quantum field after a few e -folds of inflation; usually such a vacuum state is chosen to be the Bunch-Davies vacuum. Moreover, the operator generating spatial translations is $\hat{P}_j = \sum_{\mathbf{k}} k_j \hat{a}_{\mathbf{k}}^\dagger \hat{a}_{\mathbf{k}}$ with $j = 1, 2, 3$. Therefore, applying a spatial translation to the vacuum, given by the displacement vector \mathbf{S} , yields $\exp[i\mathbf{S}_j \hat{P}_j]|0\rangle = |0\rangle$, thus, the vacuum state is homogeneous. One can similarly check the isotropy of the vacuum state by considering the behavior of the state under rotations. Consequently, the state of the field is homogeneous and isotropic.

During the inflationary regime, the quantum field is usually represented by the Mukhanov-Sasaki field variable \hat{v} [see Eq. (2)]. One can easily show that, for a mode $\hat{v}_{\mathbf{k}}$ of the quantum field, one has $\langle 0|\hat{v}_{\mathbf{k}}|0\rangle = 0$ and $\langle 0|\hat{v}_{\mathbf{k}}^2|0\rangle \neq 0$, hence, the quantum uncertainty $\Delta^2 \hat{v}_{\mathbf{k}} \equiv \langle 0|\hat{v}_{\mathbf{k}}^2|0\rangle - \langle 0|\hat{v}_{\mathbf{k}}|0\rangle^2$ is clearly not zero.

Thereupon, the fact that the quantum uncertainty is not zero (or that the system contains vacuum fluctuations) does not mean that the system contains inhomogeneities of any definite size. As we have shown, the quantum state characterizing each mode of the field $\hat{v}_{\mathbf{k}}$ is homogeneous and isotropic and the symmetries of the quantum state encode the symmetries of the physical system.

Appendix B: Possible ways to relate $\hat{\Phi}$ and Φ

In this appendix we will broaden the discussion to other possible ways to relate the quantum operator associated to the curvature perturbation $\hat{\Phi}$ and its classical stochastic value Φ within the CSL framework.

As we mentioned in Sect. IID, for a Fourier mode, the usual way to relate $\hat{\Phi}_{\mathbf{k}}$ and $\Phi_{\mathbf{k}}$ is through $\Phi_{\mathbf{k}} =$

$\sqrt{\langle 0|\hat{\Phi}_{\mathbf{k}}^2|0\rangle}e^{i\alpha_{\mathbf{k}}}$ with $\alpha_{\mathbf{k}}$ a random phase (the relation is valid only when the mode has become super-horizon). Note that since $\langle 0|\hat{\Phi}_{\mathbf{k}}|0\rangle = 0$, the quantum uncertainty $\Delta^2\hat{\Phi}_{\mathbf{k}}$ is exactly equal to $\langle 0|\hat{\Phi}_{\mathbf{k}}^2|0\rangle$.

Although the works in Refs. [47–49] differ in some ways of implementing the CSL model to the inflationary universe, they do share the same approach in relating $\hat{\Phi}_{\mathbf{k}}$ and $\Phi_{\mathbf{k}}$. That is, they both use the relation

$$\Phi_{\mathbf{k}} = \left(\Delta^2\hat{\Phi}_{\mathbf{k}}\right)_{\text{CSL}}^{1/2} e^{i\alpha_{\mathbf{k}}} \quad (\text{B1})$$

where $(\Delta^2\hat{\Phi}_{\mathbf{k}})_{\text{CSL}}^{1/2}$ represents the square root of the quantum uncertainty associated to the operator $\hat{\Phi}_{\mathbf{k}}$ evaluated in the state that results from the non-unitary evolution given by the CSL mechanism, explicitly

$$(\Delta^2\hat{\Phi}_{\mathbf{k}})_{\text{CSL}} = \langle \Theta|\hat{\Phi}_{\mathbf{k}}^2|\Theta\rangle - \langle \Theta|\hat{\Phi}_{\mathbf{k}}|\Theta\rangle^2. \quad (\text{B2})$$

It is thus clear that if there is no CSL evolution, i.e. if $|\Theta\rangle = |0\rangle$ then the standard way of relating $\hat{\Phi}_{\mathbf{k}}$ and $\Phi_{\mathbf{k}}$ coincides with the one provided by Eq. (B1). That can explain the fact that if one considers the CSL parameter $\lambda_k = 0$ in the expressions for the scalar power spectrum in Refs. [47–49], i.e. the CSL evolution is “switched off,” then the prediction for the scalar power spectrum is exactly the same as the standard one. Moreover, that could also explain why in both works the CSL parameter λ_k has to be extremely small in order to fit the predictions for the scalar power spectrum with the observational data. That is, if $\lambda_k \rightarrow 0$, then $|\Theta\rangle \rightarrow |0\rangle$ and both predictions for the scalar power spectrum (the traditional and the CSL one of Refs. [47–49]), are the same. Since we know that the traditional prediction for the scalar spectrum fits very well the observational data, it is plausible to think that if $\lambda_k \ll 1$, then the scalar spectrum of Refs. [47–49] will be consistent with the observations too.

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