

Imposible, pero no problemático: comprendiendo la adopción con teoría dialógica de tipos

*Impossible, yet not problematic: Making sense of adoption with
dialogic type theory*

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Resumen

Las leyes de la lógica no pueden adoptarse, tal como sostiene el *problema de la adopción* de Kripke y Padró. Su argumento puede interpretarse como una invitación a revisar la forma en que relacionamos la lógica con la práctica inferencial: la primera no viene antes, sino después de la segunda. En este artículo profundizo en esta conclusión mostrando cómo la imposibilidad de la adopción puede ser asociada muy naturalmente con algunas características de *immanent reasoning*, un cruce entre la lógica de diálogos y la teoría intuicionista de tipos que incorpora elementos pragmáticos en el corazón de su formalismo. La observación más importante de esta aproximación es que la adopción, aunque todavía imposible, ya no es necesaria; y, por lo tanto, el “problema” deja de ser “problemático”. Esto ilustra algunas de las ventajas de favorecer una aproximación lúdico-teórica a la semántica de la lógica filosófica.

Palabras clave: problema de la adopción, dialógica, teoría constructiva de tipos, anti-excepcionalismo, semánticas lúdico-teóricas

Abstract

Logical laws cannot be adopted —so goes Kripke and Padró’s *Adoption Problem*. Their argument can be seen as an invitation to revisit the way in which we relate Logic and inferential practice. For them, the former does not come before, but after the latter. In this paper I delve into this conclusion by showing how the impossibility of adoption can be naturally associated with some features of *Immanent Reasoning*, which is a mix between Dialogic and Constructive Type Theory that incorporates the pragmatics of inference at its core. The major insight to be drawn from this approximation is that adoption, although not possible, is no longer needed. Subsequently, the ‘problem’ loses its ‘problematic’ character. This conclusion illustrates some of the advantages of favoring the game-theoretic approach to semantics in philosophical logic.

Keywords: adoption problem, dialogic, constructive type theory, anti-exceptionalism, game-theoretic semantics

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1. Introduction

In 1974, at a seminar held at Princeton University, Saul Kripke made a series of observations on the possibility of revising logic. The argument remains unpublished by the author, but thanks to those who had access to the transcriptions of the seminar, the philosophical community became aware of the challenge posed by Kripke (Berger, 2011; Padró, 2015). Recently, based on Kripke's original ideas, Padró proposed *the Adoption Problem* (AP). Two online seminars in 2020, one held by the University of Buenos Aires and the other by the Saul Kripke Center at CUNY, and a two-day session of the X Workshop on Philosophical Logic held by the Buenos Aires Logic Group in 2021, all showed the growth of the interest in this topic.

In a nutshell, AP establishes that one cannot adopt a logical law; all we can do is agree with the laws with which we already reason. Roughly speaking, this is because under certain conditions, the adoption process presupposes the very law that is supposed to be adopted. Logical laws are different than other scientific laws.

For Padró, the AP is a consequence of neglecting the pragmatics of inference on our standard accounts of logic. This is reminiscent —and Padró is well-aware of this— of the Intuitionist claim that logic is not prior to the rational practices but rather stems from it. The purpose of this paper is to exploit this reminiscence. To do so, I will express the AP into a system called *immanent reasoning* (IR), a Dialogue Logic (*Dialogic*) that incorporates features of Constructive Type Theory (CTT). One main conclusion will be drawn: the AP is no longer a “problem” within IR. A secondary conclusion comes as an invitation: that a philosopher truly interested in the AP should consider working in such a dialogical framework. The exposition will proceed as follows. I deal with IR and its peculiarities right away in section 2, for it is the most technical part and somewhat the most difficult. In section 3 I sketch the AP as depicted in Padró (2015). Section 4 is devoted to how the AP can be naturally linked to the notion of *strategic reason*, based on which I sustain the first of the above-mentioned main conclusions of

this work. The second conclusion is exposed and defended in section 5. Section 6 summarizes the conclusions.

2. Immanent Reasoning

Game-theoretical approaches to semantics in Logic are intended to recover certain pragmatic aspects of human reasoning. In this context, Immanent Reasoning (IR) is an improvement upon the standard Dialogical framework that incorporates some features of Constructive Type Theory, inspired by Brandom's idea of reasoning as "games for asking and giving reasons". In this section I will explain how the formal system works clearly and briefly. A complete exposition of the formalism can be found in Rahman *et al.* (2018).

In standard Dialogic, formal arguments are games where two players, the Proponent and the Opponent (by convention, a He and a She), take turns to challenge and defend their claims. Whoever manages to have the last word wins. Validity is captured by the notion of *winning strategy*: a formal argument is valid if and only if the Proponent can defend the thesis against every possible challenge of the Opponent, provided she concedes first. These games can be played under different sets of rules, corresponding to different logical systems.

These games have two kinds of rules: *local* (operational) *rules* and *structural rules*. The former rules define the meaning of logic operators in terms of how a proposition can be challenged and defended. The latter rules provide general guidelines for the game: what is considered a move, in which order these moves can be made, who can do what in each turn and who wins.

A special feature of this framework is that it does not require model-theoretic notions for the decision of material formulas; all the information relevant for the dialogue is given within the dialogue itself. A further step in this direction was to allow the players to ask and give *reasons* for their claims. This led to IR, nourishing the formalism with some main features of Constructive Type Theory (CTT)¹. CTT arises from the endeavor of systematically blurring the distinction between form and content. The result is a mathematical environment that is at the same time a logical system, a programming language, and a foundational theory. Subsequently, the distinction between sets and propositions, which is ubiquitous in model-theoretic semantics, disappears in favor of what is known as the PAT-interpretation (*proofs-as-terms/ propositions-as-types*), where the concepts of set, proposition and data type are identified. Thus, the judgement

a: A

Reads equally as: “ is an element of the set ”, “ is a proof of the proposition ” and “ is a program for computing a datum of type ”.

IR provides a dialogical interpretation to this doctrine of judgements, thus reading as “ is a reason for holding ”. These reasons may be written in canonical or non-canonical form, and the process of reducing the latter to the former is handled dialogically through games involving *instructions*.

Let us look at the formalism itself. Identify the players through the labels **P** and **O**. Their claims in a play must correspond to one of three speech acts: *assertion*, *request*, and *judgement*. Where is one of the two players, a move is expressed as:

$X !A$ $X ?a$ $X a:A$

Read respectively as: “**X** asserts that (s)he has a reason for holding \mathcal{A} ”, “**X** requests for the reason ” and “**X** judges that is a reason for holding ”. is a judgement with an *implicit* local reason.

Local rules of IR are given in a twofold way: one for the *synthesis* and one for the *analysis* of local reasons for holding complex formulas. Table 1 displays these rules for conjunctions (\wedge), conditionals (\supset) and universal quantification (\forall)².

SYNTHESIS	Move	Challenge	Defense
Conjunction	$X !A \wedge B$	$Y ? L^\wedge$ or $Y ? R^\wedge$	$X p_1 : A$ resp. $X p_2 : B$
Conditional	$X !A \supset B$	$Y p_1 : A$	$X p_2 : B$
Universal Quantification	$X !(\forall x : A)B(x)$	$Y p_1 : A$	$X p_2 : B(p_1)$
ANALYSIS	Move	Challenge	Defense
Conjunction	$X p : A \wedge B$	$Y ? L^\wedge$ or $Y ? R^\wedge$	$X L^\wedge(p)^X : A$ resp. $X R^\wedge(p)^X : B$
Conditional	$X p : A \supset B$	$Y L^\supset(p)^Y : A$	$X R^\supset(p)^X : B$
Universal Quantification	$X p : (\forall x : A)B(x)$	$Y L^\forall(p)^Y : A$	$X R^\forall(p)^X : B(L^\supset(p)^Y)$

Table 1
(Some) rules for local reasons

The structural rules for IR are the following³: **SR0 [Starting Rule]** The start of a *formal dialogue of IR* is a move where **P** states the *thesis*. The thesis can be stated under the condition that **O** commits herself to certain other statements called *initial concessions*; in this case the thesis has the form , where *A* is a statement with implicit local reasons and are statements with or without implicit local reasons. **O** accepts the conditions by stating the initial concessions in moves numbered . Then each player chooses in turn a positive integer called their *repetition rank*, which determines the upper boundary for the number of attacks and of defenses each player can make in response to each move during play.

- **SR1 [Development Rule]** Players play alternately. Any move after the choice of repetition ranks is either an attack or a defense according to the rules of synthesis and of analysis, in accordance with the rest of the structural rules.
 - *Last Duty First*. Players can only answer against the *last non-answered* challenge of the adversary⁴.
- **SR3 [Resolution of Instructions]** A player may ask his/her adversary to carry out the prescribed instruction and thus bring forward a suitable local reason in defense of the proposition at stake. This is symbolized as:

X ...?/reason

And the corresponding answer as:

Y reason = reason: type

Once the defender has replaced the instruction with the required local reason, we say that the instruction has been resolved.

- **SR5 [Socratic Rule or Definitional Equality]** The following points are all part of the Socratic rule, they all apply.
 - *SR5.1 [Restriction of P Statements]* **P** cannot make an elementary statement if **O** has not stated it before, except in the thesis. An elementary statement is either an elementary proposition with implicit local reasons, or an elementary proposition and its local reason (it cannot be an instruction).
 - *SR5.2 [Challenging Elementary Statements in Formal Dialogues]* Challenges of elementary statements with implicit local reasons take the form:

X! A
 Y? reason
 X a: A

Where A is an elementary proposition and a is a local reason. \mathbf{P} cannot challenge \mathbf{O} 's elementary statements, except if \mathbf{O} provides an elementary initial concession with implicit local reasons, in which case \mathbf{P} can ask for a local reason.

- *SR5.3 [Definitional Equality]* \mathbf{O} may challenge elementary \mathbf{P} -statements; \mathbf{P} then answers by stating a definitional equality expressing the equality between a local reason and an instruction both introduced by \mathbf{O} , or a reflexive equality of the local reason introduced by \mathbf{O} .
- **SR7 [Winning Rule for Plays]** The player who makes the last move wins.

This is not the complete list of rules for IR, nor are complete themselves in most cases; but for present purpose this is enough.

Here is an example. The thesis is:

$$(\forall x:D)Q(x)[c_1:(\forall x:D)P(x),c_2:(\forall x:D)(P(x)\supset Q(x))]$$

The formal dialogue is depicted in table 2 and shall be read in the following way: the outer columns record the moves of the players; the inner ones record when a move is a challenge to a previous statement. Defenses are placed in front of the corresponding challenge.

	\mathbf{O}			\mathbf{P}	
0.1	$c_1 : (\forall x : D)P(x)$				
0.2	$c_2 : (\forall x : D)(P(x) \supset Q(x))$			$!(\forall x : D)Q(x)$	0
1	$m := 1$			$n := 2$	2
3	$a : D$	0		$f : Q(a)$	16 ++
7	$d : P(a)$		0.1	$L^\forall(c_1) : D$	4
5	? ... / $L^\forall(c_1)$	4		$L^\forall(c_1) = a : D$	6
11	$e : (P(a) \supset Q(a))$		0.2	$L^\forall(c_2) : D$	8
9	? ... / $L^\forall(c_2)$	10		$L^\forall(c_2) = a : D$	10
15	$f : Q(a)$		11	$L^\supset(e) : P(a)$	12
13	? ... / $L^\supset(e)$	12		$L^\supset(e) = d : P(a)$	14

Table 2

A formal dialogue of IR with concessions

In accordance with rule **SR0**, the game begins with the Opponent stating the concessions and the Proponent asserting the thesis, *with an implicit local reason*. Then the players choose repetition ranks, 1 for the Opponent and 2 for the Proponent (see Clerbout [2014] for the rationale behind this choice).

In move 3 the Opponent challenges the thesis by providing a suitable element of type α (a set in this context). Notice that, as the Opponent is not constrained by the Socratic rule, she can give direct local reasons for her claims, but the Proponent must wait until she provides the reasons to properly resolve the instructions. For instance in move 4, the Proponent invokes α as a reason the instruction α , which will be immediately counterchallenged by the Opponent (move 5) by a request for its resolution. In this case the Proponent can answer appropriately, for α is of type α , and the Opponent has already provided such an element: α . The Proponent then resolves the instruction by claiming α (move 6). Once the substitution has been conceded, in the next move of the Opponent (7) α appears in the place of α . The rest of the play should now be self-explanatory.

In this play, the Proponent wins; but he can also win *no matter what the Opponent does*, as it is easy to verify. Therefore, he has a *winning strategy*, and the game determines a *valid* argument.

We shall finish the exposition of the final key element of IR, *strategic reasons*, in section 4, for I would like to explain their connection with the Adoption Problem right away.

3. The Adoption Problem

We now turn to the Philosophy of Logic. Anti-exceptionalism is the claim that Logic is not different from other sciences. If this picture is correct, then logical laws (interpreted as general claims about the validity of arguments) are like any other scientific claims.

The AP is a challenge to this stance. In a nutshell, it says that the laws of logic are not subject to the same treatment of other empirical hypothesis, because they are supposed to regulate the process of drawing conclusions itself, even when—and this is the crucial point—logic alone is considered.

The original insight from Kripke is illustrated in the following words:

You see, one might think, “Look, logic’s got to be different from geometry, because we need to use logic to draw conclusions from any

hypothesis whatsoever.” Well, now, this is supposed to have been answered with the notion of a formal system. Formal systems can be followed blindly; a machine can check the proof; you don’t have to do anything at all; etcetera, etcetera. Nothing [...] is more erroneous. Of course, a machine can be programmed to check the proof. But we are not machines. [...] We are given a set of directions, that is, any statement of the following form is an axiom: if these premises are accepted any conclusion of this form must be accepted. These directions themselves use logical particles, as Quine has rightly pointed out. Because of this use of logical particles, understanding what follows in a certain formal system itself presupposes a certain understanding of logic in advance, and cannot be done blindly (Kripke quoted in Padró [2015, p. 82], transcription from the 1974 seminar).

Kripke’s understanding of “adoption” was somewhat loose, but precisely because he believed that none of the authors he criticized at that time (mainly Putnam and Quine at some places of his work) had any more of a precise idea, nor proposal, about what does it mean to “adopt” a logical law. Padró’s contribution in this sense was precisely to make such a precision, to have, in turn, a neater understanding of the problem.

Following Padró (2015), we define the *adoption* of a logical law in the following terms:

A subject *adopts* the logical law in time if and only if the following three conditions hold:

- (A1) does not reason according to before
- (A2) accepts (propositionally) in time
- (A3) reasons according to after *in virtue of* (A2)

Note that these criteria may serve to regulate the adoption of other principles as well, and for these, everything seems to be working fine. For instance, let be the following physical principle, call it G (after *Galileo*):

Velocity in a free fall does not depend on the mass of the falling body

It seems unproblematic to depict the adoption of it by as this three-stage process: Before , has the natural intuition that heavier bodies fall faster than lighter

ones and reasons accordingly. One day she learns G (i.e., she comes to accept G propositionally); and from this moment on, she reasons differently. For instance, before she may predict the outcome of the thought experiment of the cannonball and the feather incorrectly; but after , she manages to get the experiment right. It is important to stress that it does not matter if she *sees* that G is true or believes in it sincerely; nor are we interested in the *process* that led to her propositional acceptance of the principle. For our current purposes, all that matters is that all (A1-3) seem to be fulfillable without problem.

In the case of Logic, however, there seems to be a problem with this picture. In fact, a previous acknowledgement of itself will be needed to account for her change in behavior, if this is supposed to happen *because of* (A2).

Specifically, the AP may be stated as follows: *every rational subject that fulfills condition (A2) will fail in fulfilling either condition (A1) or (A3).*

The case where validates (A2) and (A3) but not (A1) cannot be said to be a genuine adoption; so, the most interesting case is when validates (A1) and (A2) but not (A3). And the reason in this case is the following: for to recognize as valid an instance of an -argument, she needs to apply the propositional content of . In certain cases, this application will be governed by itself; as does not have in her former (inner) logical theory, it follows that she will not be able to reason according to in or after.

Padró illustrates this situation with Harry, a rational agent that has never reasoned according to Universal Instantiation (UI):

From a universal statement all its instances follow

If Harry is unable to see that “this is a black raven” is a valid consequence of accepting that “All ravens are black”, then it is unlikely that accepting UI will make him change his mind. The reason is that to recognize the following argument:

“All ravens are black” “This [raven] is black”

As an instance of UI, he needs to already reason according to UI⁵. First, we see that:

[“All ravens are black” is a universal statement]
From a universal statement all its instances follow
All instances of “All ravens are black” follows from it

And then we accept that:

[“This [raven] is black” is an instance of “All ravens are black”]

All instances of “All ravens are black” follows from it

“This [raven] is black” follows from “All ravens are black”

Therefore, to accept or acquire the propositional knowledge of UI will not make Harry reason according to it.

The difference between UI and G is evident. G is a claim about physical processes. The inferential processes involved in physical reasoning are not, in turn, physical processes such as the ones G speaks about. Therefore, to draw conclusions from it, G itself is not needed. But in the case of UI, the step from propositional acceptance to proper performance involves a step where the subject is supposed to already perform properly, turning the overall process of adoption into something either circular or ungrounded.

Padró’s preliminary conclusion from this is that the laws of logic are not like hypotheses that can be freely adopted: “[b]asic rules of inference do not play a fundamental role in shaping actual inferential uses” (Padró, 2015, p. 194). This is a controversial statement, and some philosophers may consider it incorrect or at least dubious. In any case, I am not concerned with that discussion here: my interest concerns only the problem as it is formulated. In section 5 I shall briefly return to the philosophical conclusions of the AP, but first, what I will show is that the argument itself may be found quite naturally at the core of IR, in relation to the distinction between *local* and *strategic reasons*. That is the purpose of the following section.

4. Strategic reasons and the adoption of logical principles

As we saw in section 2, dialogical validity is captured by the notion of *winning strategy*: the Proponent needs to be able to win no matter what the Opponent does. The intended philosophical interpretation of this mathematical fact is a pragmatic sense of argumentative dexterity: logical knowledge amounts to *knowing-how* to reason validly, rather than *knowing-that* an argument is valid. This is directly connected with the main conclusion of the AP: Harry needs to learn *how* to conclude an instance from a universal claim, and for doing so learning *that* the instances follow from the universal is not enough. In this section I will go deeper into this analogy, to show how close these insights are to each other.

In standard Dialogic, there is a conceptual distance between the level of plays and the level of strategies. But IR has a formal device that expresses strategic notions at the first level, thus collapsing the metalogic into one single layer, as CTT does with mathematical logic. These formal devices are *strategic reasons*.

We previously saw that a formal dialogue begins with the Proponent stating the thesis, leaving his local reason as implicit. The game is then his endeavor to make this reason become explicit. When the thesis is stated under initial concessions, he must refer to the reasons given by the Opponent: for instance, in the dialogue of table 2 he is forced to appeal to and to obtain the local reasons for the defense of the thesis. If the thesis has no initial concessions and is a logical validity, then the Proponent may be able to force the Opponent to give him the local reasons he needs, as in the example of table 3.

O		P		
			$!(\forall x : D)((P(x) \wedge Q(x)) \supset P(x))$	0
1	$m := 1$		$n := 2$	2
3	$a : D$	0	$b : (P(a) \wedge Q(a)) \supset P(a)$	4
5	$c : P(a) \wedge Q(a)$	4	$L^\supset(b) : P(a)$	8
7	$d : P(a)$		$?R^\wedge$	6
9	$? \dots / L^\supset(b)$	8	$L^\supset(b) = d : P(a)$	10 ++

Table 3

A formal dialogue of IR without concessions

This is a play where the Proponent wins, but it does not prove that the thesis is a logical validity. Sometimes it can be the case that the Proponent wins merely because the Opponent does not play intelligently. In the example of table 4, the Opponent asks for the left conjunct and her repetition rank does not allow her for a second try. Had she asked for the right conjunct, the Proponent would have lost.

	O		P	
			$!(\forall x : D)(P(x) \supset (P(x) \wedge Q(x)))$	0
1	$m := 1$		$n := 2$	2
3	$a : D$	0	$b : P(a) \supset (P(a) \wedge Q(a))$	4
5	$c : P(a)$	4	$d : P(a) \wedge Q(a)$	6
7	$?L^\wedge$	6	$L^\wedge(d) : P(a)$	8
9	$? \dots / L^\wedge(d)$	8	$L^\wedge(d) = c : P(a)$	10++

Table 4.

A formal dialogue of IR without concessions. The Proponent wins because the Opponent does not play intelligently

To prove that a thesis without concessions is valid, we must show that the Proponent has a way to succeed in every possible choice made by the Opponent. Once all choices have been considered and we know that there is, in fact, a winning strategy, we can produce a strategic reason.

Consider again the game for . It is a simple game, and there are not very interesting choices to be made by the Opponent: To challenge a universal, she must provide an element of type (set) , . To challenge the conditional, she states the antecedent, which is a conjunction; the counterattack authorizes the Proponent to choose which conjunct must be asserted. If the Proponent chooses wisely (the left one), he wins, for now he can defend the conditional by stating its consequent. This explanation summarizes the *core* of the strategy. And, as he has a winning strategy, we may say that there is a *strategic reason* for this thesis.

The strategic reason reads as a recipe for the winning strategy of the Proponent: *the sooner O gives an element of type (set) and a local reason for the conjunction in the antecedent of the universally quantified conditional, request the reason for the left conjunct and assert this as a reason for the consequent of the universally quantified conditional.*

I will not deepen in the processes of synthesis and analysis of the strategic reasons, for it is not relevant for the present objective⁶. What is important for us are the conditions under which they become available for the players and the kind of information they provide:

The equalities provided in each of the plays constituting a **P**-winning strategy, and found in the analysis of strategic reasons, convey the information required for **P** to play in the best possible way by specifying those **O**-moves necessary for **P**'s victory. This information however is not available at the very beginning of the first play [...]; this information will be available only once the whole strategy has been developed, and each possible play considered. So when a play starts, the thesis is a simple statement; it is only at the end of the construction process of the strategic reason that **P** will be able to have the knowledge required to assert the thesis, and thus provide in any new play a strategic reason for backing his thesis (Rahman *et al.*, 2018, p. 172).

Consider the following dialogue as an example:

- Pepe:** All white poodles are white.
Olga: Here is a white poodle. Can you say that it is white?
Pepe: Is this white poodle white?
Olga: Yes, indeed.
Pepe: And on what grounds you can say that?
Olga: Because I see it.
Pepe: Well done: I can say that it is white, because you said so, and for the exact same reason: that you are seeing that it is.

This is one possible instance of the dialogue schema of table 3. The implicit reason of the Proponent (Pepe) is a local one: he will be able to defend the conditional with the same reason the Opponent (Olga) has for the defense of the conjunction. A strategic reason, on the other side, is a reflection on this particular dialogue and its possible outcomes:

- Pepe:** (thinks: “The sooner she presents a white poodle, all I have to do is to ask her for the reason for holding that it is, in particular, white; and this reason will be my own reason for stating that the dog is, indeed, white.”) All white poodles are white
Olga: Here is a..., etc.

As we can see, the strategic reason is not the local reason with which Pepe will defend his thesis (for he cannot know what the reason of Olga for the defense of the conjunction will be). Instead it expresses his confidence —his *knowledge*— that he will be able to obtain such a reason along the dialogue.

The difference between local and strategic reasons should now illuminate some aspects of the AP. Local reasons are pieces of evidence, mathematical constructions or whatever may constitute a guarantee for holding an atomic proposition; whereas the latter are reflections on ways to play—in this context, to reason.

Let us return to the case of Harry, but this time as if he were a Proponent player of IR. We should first distinguish two situations:

1. When Harry knows how to play, but he lacks all the strategic reasons for games where he must challenge a universal quantification⁷.
2. When Harry does not know how to play; in particular, he is unable to play properly in games where he must challenge a universal quantification.

In the first case, Harry reasons according to UI but does not know that this is a logical law: he is able to win games that involve universal instantiations, but he has not played enough to recover all the required information for producing the corresponding strategic reasons. This is not a legitimate case of adoption: Harry knows the rule, he has just not acknowledged that he does.

The second case is the one we are interested in. Suppose that, at a certain point of a dialogue, the Opponent states a universal quantification:

Olga: $(\forall x:D) \varphi$

Harry: ...?

When we say that Harry does not know how to challenge this proposition, we mean that he is not understanding what his Opponent is *entitling* him to do by saying .

There are ways to teach (or pretend) Harry to continue the play, but not all of these would be cases of adoption. As Padró clarifies, adoption “does not simply consist in picking up a basic inferential practice, but doing so by means of the acceptance of a logical principle” (Padró, 2015, p. 32). Adoption in the sense required by (A2) should be understood as one of the following two situations:

1. At time t , the players begin a new game, where ϕ is given as an initial concession.
2. At time t , Harry is instructed with the strategic reason for the logical validity (or ψ) and then asked to play again.

Both cases produce scenarios like the one predicted by the AP, in the sense that Harry is not yet able to behave as expected by (A3). The first case is redundant. If he is unable to challenge a universally quantified proposition in a game, he will be as equally unable to do so in the case of a concession. In the second, even if he blindly follows the instruction provided by the strategic reason, this still does not guarantee that he will be able to do the same for a new ϕ , or even a new ψ . The strategic reason is only a reason *for* a logical law after the performance of the winning strategy has been internalized.

Despite depicting similar situations, the AP and IR explain these in different ways. Most notably, the circularity flavor of AP vanishes almost entirely in the dialogical understanding: the laws seem to no longer be “self-governing” (Finn, 2019), for they no longer serve any “govern-like” role in the rational practice. This is due to two factors. First, strategic reasons are not *prior*, but come *after* the practice itself. Strategic reasons are “a kind of a recapitulation of what can happen for a given thesis and show the entire history of the play by means of the instructions”; “they are only a perspective on all the possible variants of plays for a thesis and *not an actual play*” (Rahman *et al.*, 2018, p. 168) (emphasis mine). This means that the rational agent does not need the strategic reason to play, instead she needs to play (or *know how* to play) to produce it. The rules are no longer self-governing nor self-justifying, because applying and justifying a rule are two different moments of the production of the strategic reason. Second, inference is no longer described as a single-agent rule-following process, but rather as an interaction. Therefore, Kripke’s words quoted above, i.e., “we are given a set of directions, that is, any statement of the following form is an axiom: if these premises are accepted any conclusion of this form must be accepted” no longer describe what is going on in a dialogical game.

In the specific case considered by Padró, in the dialogic framework, UI is no longer an instance of its own major premise. Instead, it corresponds to a family of *strategic reasons* (depending on the formula ϕ) that will read as the following instructions: *give an element of the domain of quantification and ask for a local reason for the universal statement; the sooner \mathbf{O} provides this local reason, use it to defend the statement, instantiated with the element that you chose.*

And yet the main philosophical insight of the AP remains unaltered: logical knowledge is *indeed* not a case of propositional knowledge. This leads us to the second main conclusion of this work, that I will explore in the following section.

5. Logical knowledge and rational practices

Neither Kripke nor Padró claim that logic cannot be revised. They do not see the AP as a refutation of anti-exceptionalism. Rather, they see AP as a challenge to it⁸. I uphold the same idea. If logic is a science, then a more fine-grained picture of its theories and laws is needed. In particular, one in which the AP is no longer a threat.

This can be done in many ways, though generally following one of two strategies. To use a medical analogy, I would call them the *disease-treatment* and the *symptom-treatment*. In the first case, the AP is to be solved: solutions along this strategy may, for instance, dismantle Kripke-Padró's argument by showing that some concepts are unfairly used, or that some intuitions are being pushed too far; or maybe it would try to fix (or complicate) Padró's definition of Adoption in the hope of finding an alternative that is as equally intuitive as hers, yet one in which the AP does not show up. The second, instead, is to take the conclusions of the AP at face value and recognize that the real problem lies within our understanding of Logic itself. The proposal I am suggesting in this paper corresponds with this second strategy.

We are warned against these kinds of approaches. We are told that it is methodologically better to try solutions implying the lesser reforms first (Quine's famous *principle of minimal mutilation* (1986)). A symptom-treatment is only encouraged when the disease-treatment results systematically unfruitful or when the problem denounced is unsurmountable. But my proposal does not qualify as a *mutilation* in this sense. It is an incentive to work with a new framework, the game-theoretic one.

Let me illustrate this idea with a close comparison. At the beginning of the seventies, Quine challenged the sympathizers of non-classical logics by claiming that changes of logic are actually changes of subject. If the unary connective in your formalism does not behave as classical negation, then it is *tout court* not a negation; you have only defined a new operation (see Quine, 1986, ch. 6). This is an interesting observation, and most philosophers working on non-classical logics agreed that something must be said about it in the overall defense of their proposals.

Quine and his contemporaries worked mostly with axiomatic systems, which means that the meaning of the connectives was supposed to be given by the va-

lities involving them: for instance, the fact that formulas like $\neg A$ or $\neg\neg A$ were logical theorems was taken as meaning-constituent of negation. But once one moves to Sequent Calculus, this insight becomes blurred. The reason is that in this formal theory the meaning of connectives is given by their introduction rules (left and right), and you can obtain different logic systems by changing the structural rules only. As these rules are supposed to govern the relation of logical consequence, one gets a way to sustain that a change of logic may not involve a change of meaning of the logical constants. See Ferrari and Orlandelli (2021) for a fruitful application of these features of Sequent Calculus.

Even though Logical Pluralism is a thesis about which there is considerable debate today, nobody would say that the preference for Sequent over Axiomatic Calculi corresponds to a *mutilation* of a former theory. Not even may one say that it is a radical reform; it is just a technical decision, a choice over two equally respectable mathematical frameworks. But, as the example of the Quinean challenge shows, sometimes an interesting problem may dissipate by just adjusting the scope of, or changing, the formal theories under consideration; this may be a reason for doing it in the first place. For some enthusiasts of game-theoretic approaches to logic, this is indeed their motivation for moving from model- and proof-theoretic environments. Take as an example of such a spirit the following passage from Blass:

The proof-based semantics of intuitionistic logic can be viewed as a special case of game semantics, namely the case in which the opponent O has nothing to do. The ‘debate’ consists of the proponent P presenting a proof; the winner of the debate is P or O according to whether the proof is correct or not. The truth-value semantics of classical logic is the even more special case in which P has nothing to do either. The winner is P or O according to whether the sentence being debated is true or false. There is a suggestive analogy between truth-value, proof, and game semantics on the one hand and deterministic, non-deterministic, and alternating computation on the other (Blass, 1992, p. 184).

As we saw in the last section, the AP loses its “problematic” character in the IR framework: it simply illustrates the difference between local and strategic reasons. The sympathizers of game-theoretic semantics may take this fact as a support for the mathematical tools they work with, just as logical pluralists may favor Sequent Calculus because it facilitates a natural answer to Quine’s challenge.

In conversation, Padró has recognized that her version of the AP is reminiscent of a well-known Intuitionist stance on philosophy of logic: that logical laws and theorems are not *prior*, but rather they stem from rational practices and therefore are only to be found *afterwards*. According to this, logic is not merely a description of how we reason, but rather it is a reconstruction of what we intuitively understand as reasoning correctly *once we effectively do so*. It is not surprising that the same insight was found in IR, a formalism nurtured by CTT—an overtly Intuitionistic theory⁹. As already mentioned at the beginning of section 4, this is an invitation to interpret the subject matter in epistemic terms: that is, seeing Logicality as a human activity, and Logic as involving a matter of *know-how* rather than *know-that*. On these tenets, a formal system that better expresses the performative aspects of logical behavior may be better suited for the philosophical study of Logic than others, and game-theoretic semantics are a reasonable option to this end.

To be clear, I am not necessarily defending that Logic *should* be understood as a multi-agent activity¹⁰, nor taking the AP as a mortal wound for the more traditional single-agent perspective. I am saying that a philosopher truly concerned with the AP may find my observations as an invitation for taking game-theoretic semantics more seriously.

6. Conclusions

The AP claims that Logic, unlike other sciences, is trapped in a circularity when it comes to accounting for the revisability of its principles. This is because revision itself seems to be ruled by logical principles; therefore, to adopt or revise some of these principles may presuppose them, which means that they must be “already there”.

In this article I showed that moving to the game-theoretic framework may help to dismantle this unpleasant conclusion. I worked with IR, which was a non-arbitrary choice. As a Dialogic, IR considers the pragmatic aspect of inferential practices that Padró correctly suspects to be the source of the AP; and in being an application of CTT, it blurs the distinction between form and content, logic and metalogic. All those distinctions that feed the paradoxical flavor of AP. So, it is the pedigree of IR what makes it useful to provide a non-problematic understanding of the fact that logical laws *cannot be adopted*. And this understanding states that it is not because these laws are “already there” but because they are not the kind of knowledge that can be learned propositionally.

This provides yet another reason to be interested in game-theoretic semantics when working in Philosophical Logic. Sometimes a change of perspective may provide an interesting solution to an intricate problem or it may dilute its problematic nature. This allows for a deeper and more comprehensive understanding of the original phenomenon, as is the case here.

Notes

- 1 Chapter 2 of Rahman *et al.* (2018) may also serve as a proper presentation of CTT. For a more complete and historical exposition the reader is referred to Kamareddine, Laan & Nederpelt (2004).
- 2 For the sake of simplicity, throughout this paper we will deal only with this fragment of the language.
- 3 Adapted from Rahman *et al.* (2018, sect. 7.2.1).
- 4 The rule SR1 can be followed with or without respecting the condition *Last Duty First*: in the first case, the logic captured by formal dialogues is Intuitionistic Logic; in the second, the logic captured is Classical Logic. See Keiff (2007) for more on structural variants and Logical Pluralism.
- 5 Please notice that “All ravens are black” is being formalized as $\forall x (R(x) \supset B(x))$ and not along the Łukasiewicz standard: $\forall x (R(x) \supset \neg B(x))$. Putting aside the question on which is the correct formalization of the Universal-Affirmative, the main reason for this is that we want UI to be the only rule explicitly applied. In the two arguments depicted below, the premise in brackets is implicit and corresponds to the type-match of the introduced subject with the domain of quantification.
- 6 The interested reader is referred to Rahman *et al.* (2018, sec. 7.7).
- 7 This is the dialogical equivalent of UI. See Rahman *et al.* (2018, chap. 9) on how to translate dialogues into CTT (and therefore, to some extent, also Natural Deduction) proofs.
- 8 Both Kripke and Padró are about to publish on this topic soon.
- 9 These considerations are, of course, independent of the broader Intuitionistic agenda. Nothing about constructivity, the Law of Excluded Middle or any of the most popular *dicta* of this tradition is presupposed.
- 10 Although there are defenders of this idea. Perhaps the most prominent one is Dutilh-Novaes, who has published several articles promoting the dialogic understanding of logic. Still, to be sure, the tradition I am working with is prior and somewhat independent from the trend initiated by Dutilh-Novaes, so her ideas are not being presupposed in the current exposition.

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