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Gravitational waves from the birth of the universe with extended General Relativity

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ABSTRACT

During preinflation, the Hubble parameter increases from an initial null value and the expansion of the universe is produced by the effect of back-reaction effects. This fact is viewed in the global dynamics as a cosmological parameter which induces the initial expansion in a non-conservative General Relativity theory. In this framework, we study the production and evolution of primordial gravitational waves during preinflation using extended General Relativity with boundary terms included, that describe the sources of these waves.

1. Introduction

Inflation has become the standard paradigm for explaining the homogeneity and the isotropy of our observed Universe [1–5]. During this epoch the energy density of the Universe was dominated by some scalar field (the inflaton), with negligible kinetic energy density, in such a way that its corresponding vacuum energy density was responsible for the exponential growth of the scale factor of the universe. Along this phase a small and smooth region of the order of size of the Hubble radius, grew so large that it easily encompassed the comoving volume of the entire presently observed Universe, and consequently the observable universe become very spatially homogeneous and isotropic. Moreover, it is now clear that the structure in the Universe has its origin primarily with an almost scale-invariant super-horizon curvature perturbation. Inflationary cosmology explains how the universe suffered an early quasi-exponential expansion and how the initially quantum fluctuations of the inflaton field become classical at large scales [6,7].

However, during inflation, and all the further expansion, the Hubble parameter was decreasing until reach the present day value of $H = 70^{+12}_{-8}\,(\mathrm{km/seg})\,\mathrm{Mpc}$ [9], from a maximum value considered at the beginning of the inflationary epoch. The question of how the universe could have been reach such that maximum Hubble value from a null initial value, has been studied recently in a preinflation model [16]. During preinflation the Hubble parameter increases and the kinetic energy density remains with the same proportion of the potential contribution.

On the other hand, exists a consensus that the theory of General Relativity cannot be the ultimate theory of gravitation since it has a well defined regime of validity. In particular, a definitive description of relativistic dynamics must be possible to include boundary terms when the action is minimized. This fact was emphasized by York, Gibbons and Hawking [10,11] in the 70's decade. However, this is not the only manner to study this problem. As was demonstrated in [12], there is another way to include the flux around a hypersurface that encloses a physical source without the inclusion of another term in the Einstein-Hilbert (EH) action, but by making a constraint on the first variation of the EH action by including the non-zero flux of the vector metric fluctuations through the $3d$-closed hypersurface when the action is minimized.

This letter is organised as follows: In Sect. 2 we revisit the extended General Relativistic formalism with boundary terms included in the background dynamics, such that the boundary terms are described on an extended manifold and the background dynamics of system is studied as a Riemann one. In Sect. 3 we explain how we introduce the extended manifold and the nature of the new covariant derivatives on this manifold. In Sect. 4 we describe the dynamics (with self-interactions included) of the generator of the extended manifold, which is

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a scalar field named σ, and the gravitational waves that can be defined from σ: $\sigma = g^{\alpha\beta}\delta\Psi_{\alpha\beta}$. In Sect. 5 we revisit the idea of preinflation. In Sect. 6 we study the dynamics for the components of gravitational waves with sources during preinflation. Finally, in Sect. 7 we develop some final comments and conclusions.

2. Extended General Relativity revisited

We consider the Einstein-Hilbert action $\mathcal{I}$, which describes gravitation and matter

$$\mathcal{I} = \int_V d^4x \sqrt{-g}\left[\frac{R}{2\kappa} + \mathcal{L}_m\right], \tag{1}$$

where $\kappa = 8\pi G/c^4$, $\mathcal{L}_m$ is the Lagrangian density that describes the background physical dynamics and R is the background scalar curvature, and g is the determinant of the background metric tensor $g_{\alpha\beta}$, such that the line element is defined by

$$dS^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta. \tag{2}$$

We are aimed to study the flux $\delta\Phi$ originated by a gravitational source, after considering the variation of the Einstein-Hilbert action with boundary terms included:

$$\delta\mathcal{I} = \int d^4x\sqrt{-g}\left[\delta g^{\alpha\beta}\left(G_{\alpha\beta} + \kappa T_{\alpha\beta}\right) + g^{\alpha\beta}\delta R_{\alpha\beta}\right] = 0, \tag{3}$$

the last terms in (3) are very important and cannot be neglected in the dynamics of the system. We are interested in the case where $\delta R_{\alpha\beta}$ is given by

$$\delta R_{\alpha\beta} = \lambda(x)\ \delta g_{\alpha\beta}. \tag{4}$$

Here, $\lambda(x)$ is a function of all the coordinates x^α. In this case, the boundary terms for the varied action

$$g^{\alpha\beta}\delta R_{\alpha\beta} = \delta\Phi, \tag{5}$$

describes the flux of the 4-vector $\delta W^\alpha = \delta\Gamma^\epsilon_{\beta\epsilon} g^{\beta\alpha} - \delta\Gamma^\alpha_{\beta\gamma} g^{\beta\gamma}$, through the 3D closed hypersurface ∂M. The covariant derivative of $g_{\alpha\beta}$ on the extended manifold is nonzero: $\nabla_\nu g_{\alpha\beta} = 0$. Therefore, using the propose (4), in this work we shall consider a flow given by

$$\delta\Phi = \lambda(x)\ g^{\alpha\beta}\,\delta g_{\alpha\beta}. \tag{6}$$

Using the fact that $\delta\left[g_{\alpha\beta}\, g^{\alpha\beta}\right] = 0$, we obtain that $\delta g^{\alpha\beta}\, g_{\alpha\beta} = -\delta g_{\alpha\beta}\, g^{\alpha\beta}$, and the varied action $\delta\mathcal{I}$ in (3), results

$$\delta\mathcal{I} = \int d^4x\sqrt{-g}\left[\delta g^{\alpha\beta}\left(G_{\alpha\beta} - \lambda(x)\ g_{\alpha\beta} + \kappa T_{\alpha\beta}\right)\right] = 0, \tag{7}$$

such that $T_{\alpha\beta}$ is the background stress tensor

$$T_{\alpha\beta} = 2\frac{\delta\mathcal{L}_m}{\delta g^{\alpha\beta}} - g_{\alpha\beta}\mathcal{L}_m. \tag{8}$$

Therefore, the dynamics of the system with boundary conditions included will be given by the Einstein equations with the boundary terms assimilated, that now takes the form

$$G_{\alpha\beta} - \lambda(x)\ g_{\alpha\beta} = -\kappa\, T_{\alpha\beta}. \tag{9}$$

The boundary terms with $\lambda(x)$ in (9) can be assimilated to the Einstein tensor, or the stress tensor. In the first case the boundary additional terms in the Einstein's equations should be considered as geometrical sources, but in the second one, they are of physical nature and the redefined stress tensor is given by

$$\bar{T}_{\alpha\beta} = T_{\alpha\beta} - \frac{1}{\kappa}\lambda(x)\, g_{\alpha\beta}, \tag{10}$$

and the dynamics for the physical fields is given by the equations

$$\nabla_\beta T^{\alpha\beta} = \frac{1}{\kappa}\, g^{\alpha\beta}\,\frac{\partial\lambda(x)}{\partial x^\beta}, \tag{11}$$

where $T_{\alpha\beta}$ is given by (8) and the geometric dynamics being given by the equation $\nabla_\beta G^{\alpha\beta} = 0$. This means that the flux due to the boundary terms in the minimized action will be the source for the dynamics of the physical fields.

In order for describe the background relativistic velocities we can assume a stress tensor that describe a perfect fluid

$$T^{\alpha\beta} = (P + \rho)\, U^\alpha U^\beta - P\, g^{\alpha\beta}, \tag{12}$$

where P is the background pressure and ρ is the background energy density of the system. The equation (11), with (12) provide the geodesic equation for a perfect fluid with arbitrary P and ρ, when the classical flux along the 3d-hypersurface is given by its expectation value calculated on the background Riemann manifold, which always is considered as classical[1]:

$$\delta\Theta = \lambda(x)\, g^{\alpha\beta}\delta g_{\alpha\beta} \equiv \langle B|\, \delta\hat{\Theta}\, |B\rangle\,. \tag{13}$$

Notice that equation of state for the physical system $P/\rho = \omega$, does not is necessary constant.

3. Extended manifold and new covariant derivatives

The varied Ricci tensor can be considered using an extension of the Palatini expression [8]

$$\delta R^{\mu}_{\alpha\beta\mu} = b^{-1}\left[\left(\delta\Gamma^{\mu}_{\alpha\mu}\right)_{\|\beta} - \left(\delta\Gamma^{\mu}_{\alpha\beta}\right)_{\|\mu}\right], \tag{14}$$

such that

$$\delta\Gamma^{\mu}_{\alpha\beta} = b\,\sigma^{\mu}\, g_{\alpha\beta}, \tag{15}$$

describes an extended manifold that takes into account the perturbed geometry of space-time with respect to the Riemann one, which is described by the Levi-Civita connections

$$\Gamma^{\alpha}_{\beta\nu} = \left\{ \begin{array}{c} \alpha \\ \beta\,\nu \end{array} \right\} + \delta\Gamma^{\alpha}_{\beta\nu}. \tag{16}$$

Here, σ is a scalar field that describes the scalar back-reaction of geometry due to the perturbations of the scalar field ϕ that drives preinflation. We shall denote the partial derivative $\sigma_{,\alpha} \equiv \sigma_{\alpha} = \frac{\partial\sigma}{\partial x^{\alpha}}$. The perturbations will be considered finite, but they can be large, because our formalism is non-perturbative. The closed 3d-hypersurface is finite and can be defined on any region of the background manifold. If that background is spatially isotropic and homogeneous, as is the case to be considered in this work, the results obtained on a given region of space-time will be valid for anyone region.

Now, we shall consider that gravitational waves are related to σ by

$$\sigma = g^{\alpha\beta}\,\delta\Psi_{\alpha\beta}, \tag{17}$$

so that the varied connection can be written in terms of the space-time waves components

$$\delta\Gamma^{\mu}_{\theta\epsilon} = b\,\left(g^{\alpha\beta}\nabla^{\mu}\delta\Psi_{\alpha\beta}\right) g_{\theta\epsilon}. \tag{18}$$

We define the covariant derivative of the metric tensor on the extended manifold with self-interactions included, as [14]

$$g_{\alpha\beta\|\mu} = \nabla_{\mu} g_{\alpha\beta} - \delta\Gamma^{\nu}_{\alpha\mu}\, g_{\nu\beta} - \delta\Gamma^{\nu}_{\beta\mu}\, g_{\alpha\nu} + 2(1-\xi^2)\, g_{\alpha\beta}\,\sigma_{\mu}, \tag{19}$$

where $\nabla_{\mu} g_{\alpha\beta} = 0$ is the covariant derivative of the metric tensor on the Riemann manifold. Therefore, we can define the variation of the metric tensor on the extended manifold, as[2]

$$\delta g_{\alpha\beta} = g_{\alpha\beta\|\mu}\, U^{\mu}, \tag{20}$$

where $U^{\mu} = \frac{dx^{\alpha}}{dS}$ are the components of the relativistic observers that moves on the Riemann manifold. The variation of the metric tensor (20) can be written in terms of the space-time wave components, $\delta\Psi_{\sigma\delta}$:

$$\delta g_{\alpha\beta} = g^{\sigma\delta}\left\{2(1-\xi^2) g_{\alpha\beta} U^{\mu}\nabla_{\mu}\delta\Psi_{\sigma\delta} - b\left[U_{\alpha}\nabla_{\beta}\delta\Psi_{\sigma\delta} + U_{\beta}\nabla_{\alpha}\delta\Psi_{\sigma\delta}\right]\right\}, \tag{21}$$

where $\delta\Psi_{\alpha\beta}$ is a symmetric 2-rank tensor.

4. Dynamics of σ and $\delta\Psi_{\alpha\beta}$

In order for calculate the dynamic equations for σ, we must use the equation (3), from which we obtain

$$\left[b\,\left(\sigma^{\beta}\, U^{\alpha} + \sigma^{\alpha} U^{\beta}\right) - 2(1-\xi^2) g^{\alpha\beta}\sigma_{\mu}\, U^{\mu}\right]\left[G_{\alpha\beta} + \kappa\, T_{\alpha\beta}\right] = 3\left[\Box\sigma + \left[2b + (1-\xi^2)\right]\sigma_{\mu}\,\sigma^{\mu}\right], \tag{22}$$

where we have used the expression

$$\delta g^{\alpha\beta} = g^{\alpha\beta}{}_{\|\nu}\, U^{\nu} = b\,\left(\sigma^{\alpha}\, U^{\beta} + \sigma^{\beta}\, U^{\alpha}\right) - 2\,\left(1-\xi^2\right)\, g^{\alpha\beta}\,\sigma_{\mu}\, U^{\mu}. \tag{23}$$

Because we are interested to describe a linear wave equation for σ, we shall use the gauge $\left(1-\xi^2\right) = -2b$. If we take into account the equations (4) and (9), we obtain

[1] We shall use the Heisenberg representation for the quantum states $|B\rangle$, where operators are evolving and states are squeezed.

[2] We shall denote as $\delta g_{\alpha\beta}$ the variations of $g_{\alpha\beta}$ on the extended manifold, and the variations on the semi-Riemannian manifold is $\Delta g_{\alpha\beta} = 0$. In general, this notation will be used for any tensor along the work.

$$\Box\sigma = 6\,b\,\lambda(x)\,\sigma_\mu U^\mu, \tag{24}$$

where $\Box\sigma \equiv g^{\alpha\beta}\left(\nabla_\alpha\nabla_\beta\sigma\right)$. Here, the covariant derivatives $\nabla_\alpha\nabla_\beta\sigma$, must be understood as the covariant derivative ∇_α of a covariant vector σ_β: $\nabla_\alpha\sigma_\beta$. Notice that the right hand of (24) is the source of σ, which also depends on σ, due to self-interaction of back-reaction effects. Furthermore, the source is viewed for different observers in a different manners, depending of how the observer is moving with respect to the source. This is evident in the expression (24) by the components of the relativistic velocity, U^μ. On the other hand, making use of the expressions (17) and (24), we obtain the exact equation for the gravitational waves

$$\Box\delta\Psi_{\alpha\beta} = 6\,b\,\lambda(x)\,\left[U^\epsilon\,\nabla_\epsilon\left(\delta\Psi_{\alpha\beta}\right)\right], \tag{25}$$

which is a wave differential equation for $\delta\Psi_{\alpha\beta}$, where the source (which is originated by the flow through the $3d$-closed hypersurface when de action is varied), is described by the right side. The equation (25) is valid on an arbitrary curved background space-time. Here, it is important to notice that

$$\Box\delta\Psi_{\alpha\beta} \equiv \left[g^{\mu\nu}\,\nabla_\mu\nabla_\nu\right]\delta\Psi_{\alpha\beta}, \tag{26}$$

where $\nabla_\mu\nabla_\nu\,\delta\Psi_{\alpha\beta}$ should be though as a covariant derivative ∇_μ of a 3-rank tensor: $\nabla_\nu\,\delta\Psi_{\alpha\beta}$. The equations (24) and (25) were obtained in a recent work [14], and describe respectively the dynamics for σ and for the gravitational waves $\delta\Psi_{\alpha\beta}$.

5. Preinflation and the birth of the universe

We can consider the model [16] that describes the birth of the universe with a null initial Hubble parameter. In that model the global expansion of the universe is driven by a single scalar field ϕ, which is minimally coupled to gravity and drives the expansion of the universe. The Lagrangian density is

$$\mathcal{L}_m = -\left[\frac{1}{2}g^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} - V(\phi)\right], \tag{27}$$

where we are considering natural units: $c=\hbar=1$. In order for describe the background dynamics with a variable time scale, we shall consider the line element [13]

$$dl^2 = e^{-2\int\gamma(t)\,dt}dt^2 - a_0^2\,e^{2\int H(t)dt}\,\delta_{ij}\,dx^i dx^j, \tag{28}$$

such that $H(t)$ is the Hubble parameter on the background metric and $\gamma(t)$ describes the time scale of the background metric. This should be the case in an emergent accelerated universe in which the time scale can be considered variable with the expansion.

The dynamics of the scalar field ϕ is given by

$$\ddot{\phi} + [3\,H + \gamma]\,\dot{\phi} + \frac{\delta\bar{\Upsilon}}{\delta\phi} = 0, \tag{29}$$

where the redefined potential with back-reaction contributions is given by

$$\bar{\Upsilon}(\phi) = \left[V(\phi)\,e^{-2\int\gamma(t)\,dt} + b\,\frac{\langle B|\,\delta\hat{\Theta}\,|B\rangle}{(8\pi\,G)}\right] = \bar{V}(\phi) + b\,\frac{\langle B|\,\delta\hat{\Theta}\,|B\rangle}{(8\pi\,G)}. \tag{30}$$

We are dealing with an emergent universe with Planck-scale energy density. Therefore, it is reasonable to assume that

$$b\,\langle B|\,\delta\hat{\Theta}\,|B\rangle = \lambda(x), \tag{31}$$

with $b=1/M_p$. In order for describe an emergent universe which starts from a null Hubble parameter to reach its maximum value at the end of preinflation, we shall propose a Hubble parameter H, which is related with the parameter γ by

$$3\,H\,[\phi(t)] + \gamma\,[\phi(t)] = \epsilon\,H_0, \tag{32}$$

where H_0 is constant and $\epsilon = \frac{2+N}{1+N}$. From the equations (9), we obtain that

$$6\,H^2 + 2\left[\dot{\phi}\,H' + \epsilon\,H_0 - 3\,H^2\right] - 16\pi\,G\,\bar{\Upsilon}\,[\phi(t)] = 0. \tag{33}$$

Because we are aimed to describe the birth of the universe, we must consider that the Hubble parameter is initially null: $H(t=0)=0$. For the choice $\phi(t) = N\,\phi_0\left[1-e^{-H_0\,t}\right]$, we obtain that the effective potential is

$$\bar{\Upsilon}\,[\phi(t)] = \frac{H_0^2}{8\pi\,G}\left[\frac{1}{2(N+1)}\left(\frac{\phi(t)}{\phi_0}\right)^2 - \frac{N}{N+1}\left(\frac{\phi(t)}{\phi_0}\right) + N\right]. \tag{34}$$

Here, N is a dimensionless natural number that give us the scale of the energy for different epochs in the evolution of the universe. Furthermore, from the equation of motion (29), we obtain that

$$-H_0^2\,(N\,\phi_0 - \phi) + 3\,\epsilon\,H_0^2\,(N\,\phi_0 - \phi) + \frac{\delta\bar{\Upsilon}}{\delta\,\phi} = 0, \tag{35}$$

where ϕ_0 is asymptotic maximum value for $\phi(t)$, which is an increasing function of t and always takes sub-Planck values: $0 \leq \phi(t) < N\phi_0 < M_p$, and $\bar{V}(\phi)$ is[3]

$$\bar{V}[\phi(t)] = \frac{H_0^2}{8\pi G}\left[\frac{1}{2(N+1)}\left(\frac{\phi(t)}{\phi_0}\right)^2 - \frac{N}{N+1}\left(\frac{\phi(t)}{\phi_0}\right) + \frac{[6N(N+1)-1]}{6(N+1)}\right]. \tag{39}$$

Therefore, the cosmological parameter during preinflation results to be constant: $\lambda(x) \equiv \lambda_0 = \frac{1}{6}\frac{\delta^2\bar{V}}{\delta\phi^2} = \frac{H_0^2}{6(N+1)}$. Hence, since $\lambda_0 > 0$, for $b > 0$ we obtain a positive flow due to the expectation value of back-reaction effects:

$$\langle B|\,\delta\hat{\Theta}\,|B\rangle = \frac{H_0^2}{6b\,(N+1)}, \tag{40}$$

for $\phi_0^2 = \frac{1}{8\pi G}$. Therefore, if we take $b = 1/M_p = G^{1/2}$, by using the equation (3), we obtain

$$\langle B|\,\delta g^{\alpha\beta}\,g_{\alpha\beta}\,|B\rangle = 18\,b\,\langle B|\,\sigma_\mu\,U^\mu\,|B\rangle = M_p, \tag{41}$$

where

$$\langle B|\,\sigma_\mu\,U^\mu\,|B\rangle = \frac{M_p^2}{18}. \tag{42}$$

The background geodesic dynamics described by (11) is given by

$$2\,\rho\,(1+\omega)\,\nabla_0 U^0 + \frac{\partial\,(\rho\,(1+\omega))}{\partial x^0}\,U^0 + \frac{\partial P}{\partial x^0}\,U^0 = 0, \tag{43}$$

with

$$\omega = \frac{\frac{\dot{\phi}^2}{2} - \bar{\Upsilon}}{\frac{\dot{\phi}^2}{2} + \bar{\Upsilon}}, \tag{44}$$

such that the background energy density and pressure, are respectively given by

$$\rho = \left(\frac{\dot{\phi}^2}{2} + \bar{\Upsilon}\right)e^{2\int \Gamma(t)\,dt}, \qquad P = \left(\frac{\dot{\phi}^2}{2} - \bar{\Upsilon}\right)e^{2\int \Gamma(t)\,dt}. \tag{45}$$

In the model here worked for preinflation $\omega \gtrsim -1$, and $\dot{\omega} < 0$ along the evolution of this emergent stage of the universe. For a co-moving observer we must set $U^i = 0$, so that the solution of the equation (43), results

$$U^0 = e^{\frac{1}{2}\int\left[\frac{2\gamma[\rho(1+\omega)] - \dot{\rho}}{\rho(1+\omega)}\right]dt}. \tag{46}$$

We must remember that we are considering natural unities $c = \hbar = 1$.

6. Gravitational waves from preinflation

During preinflation, the system interacts with the environment by delivering energy through of space-time waves and scalar back-reaction effects, which are described by the fields σ and $\delta\Psi_{\alpha\beta}$. We shall consider that $\delta\Psi_{0\beta} = 0$. Because the symmetry of the waves: $\delta\Psi_{\alpha\beta} = \delta\Psi_{\beta\alpha}$, the relevant components of $\delta\Psi_{\alpha\beta}$ will be $\delta\Psi_{ij}$ [i, j can take the values $1, 2, 3$], which, for the metric (28), obey the dynamics

$$\begin{aligned}\delta\ddot{\Psi}_{ij} + \left[3h\left(e^{-4\int\Gamma dt} - \frac{4}{3}\right) + \Gamma\right]\delta\dot{\Psi}_{ij} - e^{-2\int(h+\Gamma)dt}\nabla^2\delta\Psi_{ij} - \left[4h^2\left(2e^{-4\int\Gamma dt} - 1\right) + 2h\,\Gamma + 2\dot{h}\right]\delta\Psi_{ij} \\ = 6\,b\,\lambda_0 e^{-\int\Gamma dt}\,\delta\dot{\Psi}_{ij},\end{aligned} \tag{47}$$

where the right hand term is the flux for a co-moving observer that moves with $U^0 \neq 0$ and $U^i = 0$. During preinflation, the fluctuations of space-time are statistically distributed and produce gravitational space-time waves. We can consider a Fourier expansion for $\delta\Psi_{ij}$,

[3] The particular solutions for the Hubble parameter and the function $\gamma[\phi(t)]$ that comply with the dynamic equations (33) and (35), are

$$H[\phi(t)] = H_0\left[\left(\frac{\phi(t)}{\phi_0}\right) - \frac{1}{2N}\left(\frac{\phi(t)}{\phi_0}\right)^2\right]. \tag{36}$$

$$\gamma[\phi(t)] = H_0\left[\frac{2+N}{1+N} - 3\left[\left(\frac{\phi(t)}{\phi_0}\right) - \frac{1}{2N}\left(\frac{\phi(t)}{\phi_0}\right)^2\right]\right], \tag{37}$$

and the relevant slow-roll parameters are [15]

$$\varepsilon(\phi) = \frac{1}{16\pi G}\left(\frac{\bar{\Upsilon}'}{\bar{\Upsilon}}\right)^2, \qquad \eta(\phi) = -\frac{1}{8\pi G}\left(\frac{\bar{\Upsilon}''}{\bar{\Upsilon}}\right). \tag{38}$$

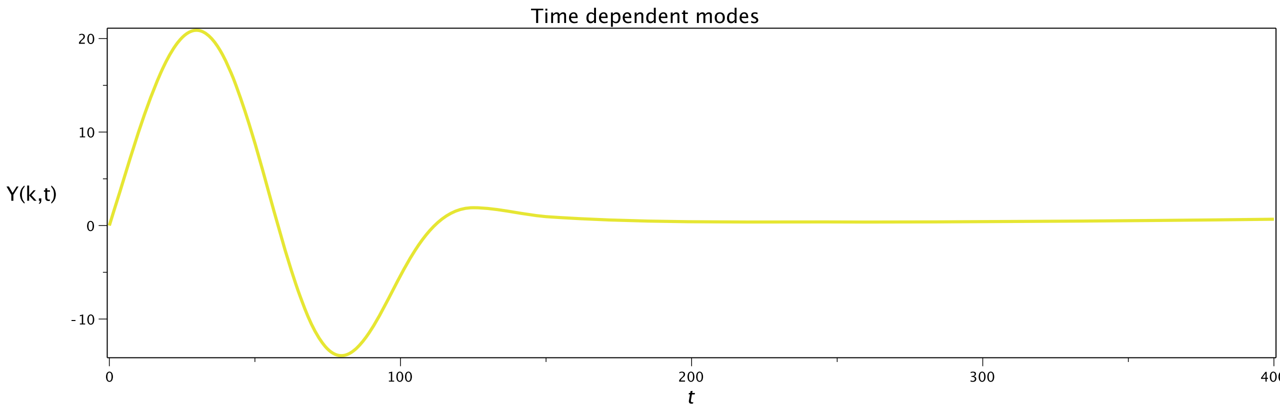

Fig. 1. Plot of time dependent modes $Y(k,t)$ for $k = 0.05\,\mathrm{M_p} \gg k_H \equiv 2\pi\, H$, $H_0 = 1/a_0 = 0.0005\,\mathrm{M_p}$ and $\lambda_0 = \frac{H_0^2}{6(N+1)}$.

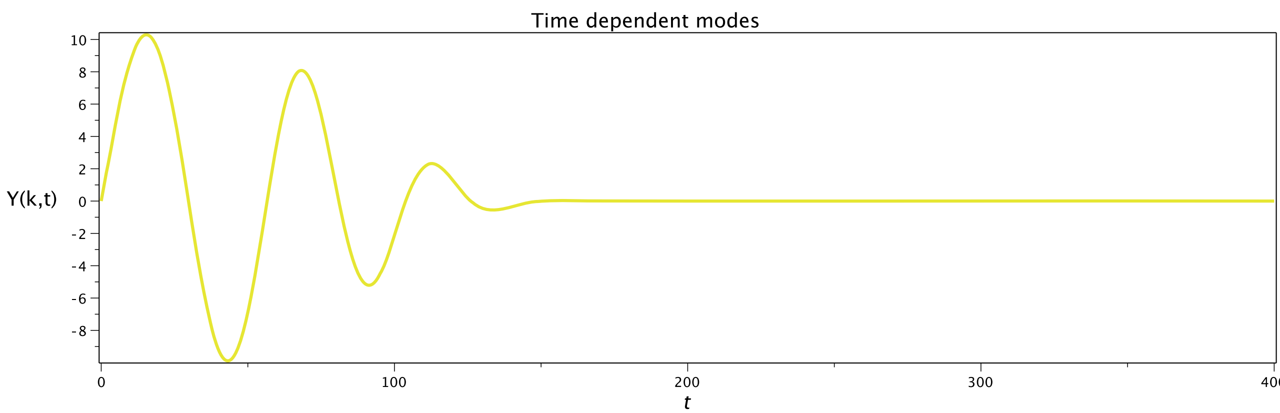

Fig. 2. Plot of the modes $Y(k,t)$ for $k = 0.1\,\mathrm{M_p} \gg k_H \equiv 2\pi\, H$, $H_0 = 1/a_0 = 0.0005\,\mathrm{M_p}$ and $\lambda_0 = \frac{H_0^2}{6(N+1)}$.

which propagates in an arbitrary spatial direction, which we can make coincident with z. When sources are considered, there are three transversal modes of the gravitational waves: $+$, $\times$ and b, where the b-mode denotes the breathing of the wave. In other words, this mode takes into account the transversal amplitude for the wave's oscillations. Therefore, the possible transversal components of the wave will be xx, xy, yx, and yy, which we shall denote with the letters m, n:

$$\delta\Psi_{mn}\left(t, \vec{r}(x,y,z)\right) = \frac{1}{(2\pi)^{3/2}} \int d^3k \sum_{\iota=+,\times,b} {}^{(\iota)}E_{mn} \left[{}^{(\iota)}A_k\, e^{i\,|k|\vec{r}\cdot\hat{e}_z}\, Y(k,t) + {}^{(\iota)}A_k^{\dagger}\, e^{-i\,|k|\vec{r}\cdot\hat{e}_z}\, Y^*(k,t) \right], \tag{48}$$

where ι takes into account the degree of freedom of polarizations $+, \times, b$ and ${}^{(\iota)}E_{mn}$ is the polarization tensor. Here, ${}^{(\iota)}A_k^{\dagger}$ and ${}^{(\iota)}A_k$ are the creation and the destruction operators for a given (ι)–polarization. The transversal plane xy, can be generated by the orthogonal vectors $\vec{u} = (p(t), 0)$ and $\vec{v} = (0, q(t))$. Therefore, the components for the polarization tensor ${}^{(\iota)}E_{mn}$, generated by $\vec{u}$ and $\vec{v}$, are:

$${}^{(+)}E_{mn} = u_m\, u_n - v_m\, v_n, \qquad {}^{(\times)}E_{mn} = u_m\, v_n + v_m\, u_n, \qquad {}^{(b)}E_{mn} = u_m\, u_n + v_m\, v_n, \tag{49}$$

where m, n can take the values 1 and 2. The components ${}^{(\iota)}E_{mn}$, can be explicitly written with 2×2-matrices

$${}^{(+)}E_{mn} = \begin{pmatrix} p^2 & 0 \\ 0 & -q^2 \end{pmatrix}, \quad {}^{(\times)}E_{mn} = \begin{pmatrix} p\,q & 0 \\ 0 & p\,q \end{pmatrix}, \quad {}^{(b)}E_{mn} = \begin{pmatrix} p^2 & 0 \\ 0 & q^2 \end{pmatrix}. \tag{50}$$

The modes $Y(k,t)$ are the solution of the differential equation

$$\begin{aligned} \ddot{Y}(k,t) &+ \left[3h\left(e^{-4\int \Gamma dt} - \frac{4}{3}\right) + \Gamma(t) - 6\,b\,\lambda_0\, e^{-\int \gamma(t)\,dt}\right] \dot{Y}(k,t) + \left\{k^2\, e^{-2\int (h+\Gamma) dt} \right. \\ &\left. - \left[4h^2\left(2e^{-4\int \Gamma dt} - 1\right) + 2h\,\Gamma + 2\dot{h}\right]\right\} Y(k,t) = 0. \end{aligned} \tag{51}$$

In the Figs. 1, 2 and 3 we have plotted the modes $Y(k,t)$ for different wavenumbers $k = 0.05\,\mathrm{M_p}$, $k = 0.1\,\mathrm{M_p}$ and $k = 0.2\,\mathrm{M_p}$, which are wavenumber values that correspond to wavelength inside the horizon during preinflation.

7. Final comments

The novel idea of preinflation is that the birth of the universe can be initiated by strong quantum effects [16], which are the fuel that drives the initial global expansion of an hyperbolic region of space-time that expands in an super-exponential manner. This dynamics

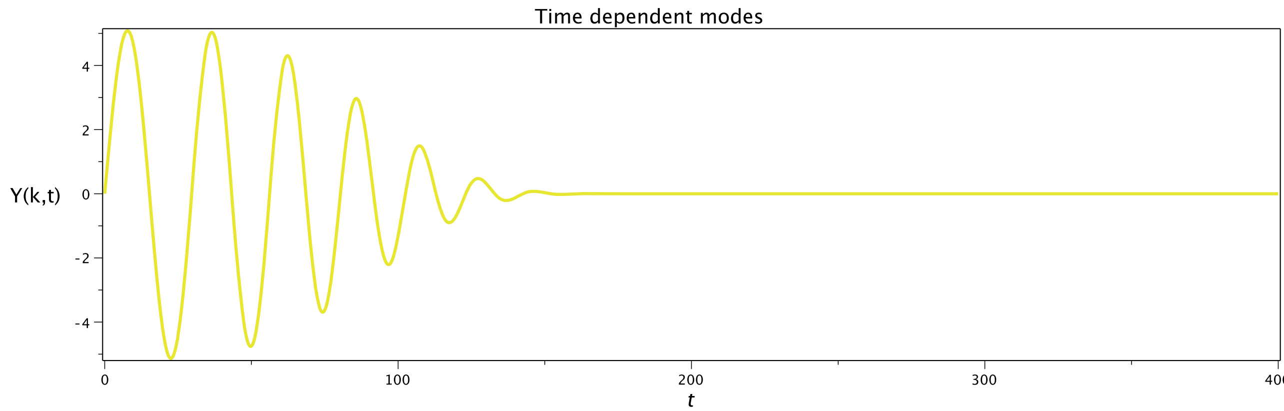

Fig. 3. Plot of the modes $Y(k,t)$ for $k = 0.2\,\mathrm{M_p} \gg k_H \equiv 2\pi\, H$, $H_0 = 1/a_0 = 0.0005\,\mathrm{M_p}$ and $\lambda_0 = \frac{H_0^2}{6(N+1)}$.

must be treated as a non-conservative physical system, where the quantum dynamics of the geometric fluctuations alters the global dynamics of the universe in a framework of an extended General Relativistic theory. In this framework we have explored a model where the sources of gravitational waves are these strong geometric fluctuations, in a non-perturbative formalism for gravitational waves with sources. A notorious fact is that during the initial expansion of the universe there is a variable time scale, so that the time in a physical scale $d\tau = U_0\, dt$, evolves is accelerated [see equation (46)]. Another remarkable fact is that quantum back-reaction effects are the cause of the universe's expansion, so that they are also the cause of gravitational waves. In our model, we understand the universe as a causal connected region of spacetime that grows since $t > 0$, which is physically manifested by the fact that $H(t) \geq 0$, for $t \geq 0$. This is because quantum fluctuations of spacetime constitute the fuel of the big bang through the flow of σ^{μ}, that injects energy in the causal connected universe [see equation (40)].

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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