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Corrigendum

Classification of abelian complex structures on six-dimensional Lie algebras

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It has been pointed to us by E. Rodríguez Valencia that the complex structures J_t^1 and J_t^2 on the Lie algebra $\mathfrak{n}_4$, appearing in Theorem 3.3, are in fact equivalent. These structures are introduced in the proof of Theorem 3.2, which is used later in the paper to determine the moduli spaces of abelian complex structures. However, in that proof, statement (J^2) is in fact impossible, since it implies that the Lie algebra $\mathfrak{g}$ is abelian. Indeed, if $\ker(\operatorname{ad}_x|_{\mathfrak{v}})$ were J-stable for any $x \in \mathfrak{v}$, then we would have $[x, Jx] = 0$ for any $x \in \mathfrak{g}$. Therefore, for any $x, y \in \mathfrak{g}$,

$$0 = [x + y, J(x + y)] = [x, Jy] + [y, Jx] = 2[x, Jy],$$

and this implies that $\mathfrak{g}$ is abelian, which is a contradiction. So, we are reduced to case (J^1), and as a result, one obtains only one family of abelian complex structures on $\mathfrak{n}_4$, up to equivalence.

Therefore, the correct statement of Theorem 3.3 is as follows, where the complex structure J_t in (iv) below stands for J_t^1 of the previous version.

Theorem 0.1. *Let $\mathfrak{n}$ be a six-dimensional nilpotent Lie algebra with an abelian complex structure J. Then $(\mathfrak{n}, J)$ is holomorphically isomorphic to one and only one of the following:*

(i) $(\mathfrak{n}_1, J)$, *with its unique complex structure:* $Je_1 = e_2,\ Je_3 = e_4,\ Je_5 = e_6$;
(ii) $(\mathfrak{n}_2, J_\pm)$, *with* $J_\pm e_1 = e_2,\ J_\pm e_3 = \pm e_4,\ J_\pm e_5 = e_6$;
(iii) $(\mathfrak{n}_3, J_s)$, *with* $J_s e_1 = e_2,\ J_s e_3 = e_4,\ J_s e_5 = se_5 + e_6,\ s \in \mathbb{R}$;
(iv) $(\mathfrak{n}_4, J_t)$ *with* $J_t e_1 = e_3,\ J_t e_2 = e_4,\ J_t e_5 = te_6,\ t \in (0, 1]$;
(v) $(\mathfrak{n}_5, J)$ *with* $Je_1 = e_2,\ Je_3 = -e_4,\ Je_5 = e_6$;
(vi) $(\mathfrak{n}_6, J)$, *with* $Je_1 = e_2,\ Je_3 = -e_6,\ Je_4 = e_5$;
(vii) $(\mathfrak{n}_7, J_t)$ *with* $J_t e_1 = e_2,\ J_t e_3 = -e_4,\ J_t e_5 = te_6$, $0 < |t| \leqslant 1$.

Accordingly, the moduli space of abelian complex structures on $\mathfrak{n}_4$ is connected and given by

$$\mathcal{C}_a(\mathfrak{n}_4)/\operatorname{Aut}(\mathfrak{n}_4) \cong (0, 1].$$

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