PHYSICAL REVIEW PHYSICS EDUCATION RESEARCH **18,** 010127 (2022)

Learning entropy among peers through the lens of coordination class theory

Juan Velasco, Laura Buteler,* Carlos Briozzo, and Enrique Coleoni
Instituto de Física Enrique Gaviola, Universidad Nacional de Córdoba—Consejo Nacional de Investigaciones Científicas y Técnicas, Córdoba 5000, Argentina

(Received 28 September 2021; accepted 1 March 2022; published 7 April 2022)

Coordination class theory has proven to be a useful theoretical framework for describing processes of conceptual change in certain physical and mathematical concepts. Its development throughout different studies has allowed us to understand numerous mechanisms of conceptual learning by individual subjects. There have been attempts to implement this theory to collective-learning environments. Although there are valuable contributions in this sense, there are still details at the fine-grain level to be unveiled in relation to how conceptual change proceeds within small groups. For this purpose, the development of a coordination class is analyzed in the case of an interview with two students addressing a problem-solving task about entropy, focusing on the role that interactions between participants play in the evolution of the entropy concept. A microcomplementary analysis is proposed by bringing in elements from a sociocultural approach of learning. Thus, we were able to identify different forms of codevelopment of a coordination class. Students exhibiting central as well as peripheral participation benefit from each other during the conceptual learning process of the small group, constituting a "virtuous" interaction for learning.

DOI: 10.1103/PhysRevPhysEducRes.18.010127

I. INTRODUCTION

Research on conceptual change is a multidisciplinary endeavor that examines how conceptual knowledge in specific domains is transformed. The literature on student conceptions and their change is vast, as evidenced by a periodically updated bibliography containing thousands of publications [1–4]. No single review can do justice to it all. Despite the large volume of work devoted to conceptual change, almost all of these studies have approached this phenomenon as an individual one and very few have addressed it from a sociocultural view of learning. The present work attempts to bring both perspectives together into the analysis of conceptual change.

Coordination class theory (CCT) constitutes a particularly significant contribution to the field of research on conceptual change The theory proposes a mechanism of how knowledge is organized and reorganized over time. Unlike other theoretical proposals to describe conceptual change, CCT explicitly discusses what a concept is and how it comes up in students' reasoning, which is particularly helpful for keeping track of its changes. The simultaneous occurrence of both these traits (defining what a concept is and how it changes) is what makes this theory stand out among others. Used to study problem-solving situations, it can provide information at a sufficiently fine-grained level which can afford tracing changes in conceptual knowledge. This theory has been successfully implemented to describe processes of conceptual change for some particular concepts. Several papers have presented valuable contributions for physics concepts: force [5], frequency and velocity [6], motion [7], buoyancy [8], mechanical waves [9], proper time [10], energy [11], and heat [12]. Other work has also addressed contexts such as computer science [13] and algebra [14].

DiSessa and Sherin [5] contributed with theoretical development (CCT) and with empirical evidence of different ways in which a student understands the concept of force during a problem-solving task. An interview setting with a university student, J, let them infer the changes in her concept of force. In certain particular situations she favored her intuitive knowledge and decided that the equation, $F = ma$, did not apply. Witmann [11] used CCT to understand student reasoning on wave physics during problem solving. He reported cases in which students' reasoning resources were inappropriately linked to objectlike models for waves and he modeled different kinds of conceptual change. Wagner [15] addressed the transfer problem and contributed with details of conceptual development during problem-solving tasks. The analysis was oriented to one university student, Maria, throughout eight weekly interviews. From the beginning of these interview sessions, several probability problems were considered and she perceived them as different. The author

---

*Corresponding author.
laura.buteler@unc.edu.ar

showed how Maria eventually came to address them as similar problems [15,16]. Thaden-Koch *et al.*[17] interviewed students individually, and in each interview presented a series of simulations for the motion of a ball along a straight-line track. Students were asked to judge the realism of the simulated motions. The analysis reports elements of coordination classes (readout strategies and causal net) useful for understanding students' decision-making processes. Buteler and Coleoni [8] revealed how conceptual development took place during problem solving for the case of buoyancy. In particular, they showed the role of epistemic resources within this approach. In a small-group interview setting, they identified how these resources work during problem solving. Parnafes [6] studied how students' ideas of frequency and velocity evolved through the use of computational representations during problem solving and was able to understand their role as a support for conceptual learning. The simulation setting enabled students to distinguish between velocity and frequency as they defined their strategies for determining the relevant information and refined their knowledge. Sengupta *et al.* [18] addressed a similar question for the case of linear momentum but using a video game. They showed how conceptual understanding evolves as students play a video game throughout an interview session. In a similar way, Kluge [12] investigated how interactive representations can be used to enhance conceptual learning. In a naturalistic study with four groups of students, those interactive representations are used to negotiate a meeting point between theory, previous experience, and knowledge, and they are instrumental to conceptual sense making.

The studies mentioned so far address learning as an individual phenomenon and do not probe into the influence of the interaction context on the process. There have been attempts to implement this framework to classroom contexts [10,11]. Levrini and diSessa [10] used CCT for the analysis of an entire class along an instructional sequence of several sessions on special relativity concepts. In this pioneering case they assigned conceptual learning to the entire group of students. Although this allowed them to describe the conceptual evolution of the group and its difficulties, the analysis does not emphasize the role that interactions between participants played in that conceptual development. Barth-Cohen and Wittman [11] made a valuable contribution by considering the different interpretations of students as different *contexts* of the coordination class. Even though this work makes progress in extending the definition of context to consider disagreements, many processes by which a group may or may not construct knowledge in a shared way remain open questions: How do students develop conceptual knowledge together? How do each student's contributions work collaboratively for conceptual development? What forms of collaboration are established? The importance of addressing these questions lies in the fact that in educational settings, physics learning is almost always a collective phenomenon. At the university level, problem-solving sessions are mostly organized in small working groups, as is also usually the case for study groups. In high schools, student-centered teaching calls for collaborative group efforts as students work on problem-solving tasks. All these support the need to probe into the details of students' interactions as they learn concepts during problem solving.

Across the previous research described above, which employed CCT, the common focus was the analysis of conceptual development without detailing how this knowledge was changing through the interaction between participants. Although CCT does not actually assume that projections occur in one individual, it does not specify anything about how elementary pieces of knowledge are distributed among them, nor how each individual's knowledge gets changed during interaction. Given the success of CCT in describing learning processes in detail and given the gap in the theory to account for collective conceptual change, the aim of this paper is to go further and to study how the process of conceptual change develops in a collaborative manner. We intend to use the theory to pursue an in-depth analysis of how elements of knowledge are shared, reorganized, discarded, and confronted in a collective environment. Thus, a relevant feature of the present work is that it aims at better understanding the role of interaction between students in the conceptual learning process.

II. THEORETICAL FRAMEWORK

A. Coordination class theory

The main pursuit of CCT was that of adding precision to the meaning of the term "concept" and to focus on the process of conceptual change [19,5]. Within this perspective, "having" a concept consists of knowing how to get relevant information from the world, across varied situations [20]. Many studies using CCT have focused on the process of conceptual development and the learning difficulties that may arise as students work to determine relevant information across varied contexts in the world.

A *coordination class* is a model for particular kinds of concepts, among which are physics concepts. The main function of a coordination class is to allow people to read a particular kind of information out of situations in the world. This reading takes place through specific processes and strategies and many of the difficulties people have are related to the context and circumstances in which they carry out those particular strategies and processes. Entropy is a concept with many different facets and contexts of applicability. It can, in fact, be defined macro- or microscopically. Thus, it requires different strategies to extract the relevant information from each context. Also, it applies to a particularly wide span of situations (biological phenomena, chemical phenomena, engineering, and mathematical

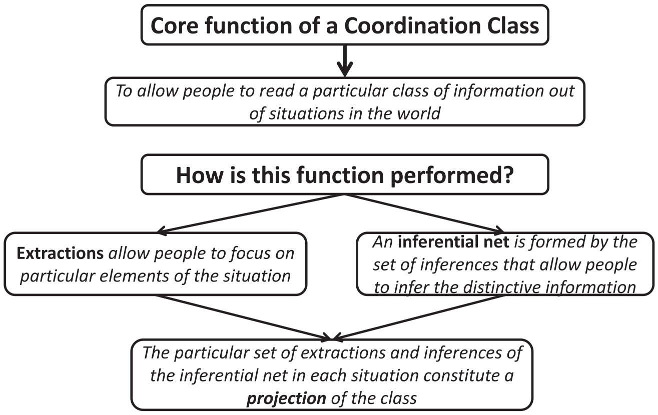

FIG. 1. Schematic representation of theoretical elements that constitute a coordination class.

problems). All these traits turn the concept into a particularly suitable one to be regarded as a coordination class.

The architecture of a coordination class includes two elements: *extractions* and inferential *nets* [21]. *Extractions* allow people to focus their attention on certain information of the phenomenon at hand. The inferential net is the total set of inferences people make to turn those extractions into the required relevant information.

According to this theory, implementing a concept in different contexts may well imply retrieving different pieces of knowledge and/or articulating them in different ways. The particular knowledge and the particular way it is coordinated in specific applications of the concept is called a *concept projection*.

Typically, students exhibit two characteristic difficulties in creating a new coordination class: the problem of *span*, and the problem of *alignment*. Span refers to the ability (or lack thereof) to recruit and coordinate the elements of the class in a sufficiently large set of contexts in which the concept is relevant. Alignment refers to the possibility of obtaining the same relevant information by means of different projections of the concept.

The theory also establishes a stronger form of alignment: *articulate alignment*, or *articulation*. Articulation happens when students are not only able to determine the relevant information in different circumstances, but can also explicitly relate those different projections, noting differences and similarities between them. This stronger form of alignment is a metaconceptual process which is a natural extension of the theory in its original form.

Figure 1 shows a schematic diagram of these elements [8] and operational definitions of some elements of the theory are presented in Table I for the case of entropy. Please, refer to diSessa and Wagner [16] for more details.

B. Learning in groups: A scenario where individual and sociocultural views meet

Addressing group-learning environments necessarily requires considering contributions from theories within a sociocultural approach to learning which can illuminate collaborative aspects that CCT by itself may not be able to fully address. In what follows we describe aspects of recent dialogue between individual and sociocultural approaches to cognition.

These approaches start from different conceptions of what learning is. Within the sociocultural approach, learning is best understood as changes in practices. This approach focuses on similarities and differences between practices in different human activities and how they are adopted and adapted. Instead, from an individual-cognition approach, to understand learning means to focus on how knowledge is represented in the mind and how those representations are elaborated and modified [22]. These differences impact on the type of research questions, methodological aspects, and the scope of the results obtained. In other words, each approach has different things to say about learning.

Many researchers consider that building bridges between both approaches is both important and fruitful [22–25], and that this is a difficult but feasible task [23]. In fact, some

TABLE I. Operational definitions for extractions, elements of the inferential net, and projection.

Operational criteria to interpret data in students' verbalizations		
Knowledge element	Operational criteria	Some examples
Extractions	- *They refer to specific traits in objects in the situation*	- *Gas does not interact with environment during irreversible process*
	- *They are directly read from the context or statement*	- *First transformation is irreversible* - *Temperature remains constant*
Elements of the inferential net	They involve abstract elements such as concepts or physical laws.	- *It [Transformation] is reversible, so universe entropy does not change*
	There are usually expressed in the form of if-then statement	- *Here delta Q is negative! Thus delta Q between B and A is less than zero*
Projection	It is directed to producing a concept-distinctive information	*From here to here (from point A and B) gas entropy increase, and from here to there it remains constant*

attempts were made in the recent past. We believe that these bridges can help improve our understanding of learning by bringing together fruitful resources from both theoretical perspectives.

From a sociocultural point of view, learning happens through negotiation of discourse meaning. Participation within a community takes central attention in this conception of learning. Thus, learning is indicated by shifts in a person's speech and behavior [26–28]. Changes in discourse engagement, participation, argumentations, questioning, deciding, among others, are indicators of learning processes. In order to go deeper into the collaborative construction of a coordination class, we will bring in some specific elements from the sociocultural perspective.

We call on Wenger's [29] definition of *communities of learning and practice*. This concept, originally from the field of anthropology, has had a strong impact in the field of education. The concept of community of practice (COP) derives from a view of learning as a social and situated process rather than an individual and decontextualized one. A community of practice is defined by Lave and Wenger [30] as a *group of individuals with different sets of knowledge, skills, and experiences, who actively participate in collaborative processes, in which information, ideas, interests, resources, perspectives, activities, and above all, practices, are exchanged in order to build both personal and collective knowledge*. For this exchange and collaboration to take place, a climate of trust must be generated among its members, trust that allows them to exchange the particular knowledge that each one possesses. When a COP is working effectively, it generates and appropriates a shared repertoire of ideas, goals, memories, values, attitudes, from which all participating members benefit. In addition, it develops its own resources, such as tools, documents, routines, vocabularies and symbols which, to some extent, bring with them the accumulated knowledge of the community. In other words, a COP implies common ways of doing and approaching things that are of interest to the people who constitute it. One of the most notable differences between an ordinary group and a COP lies in the very sense of community, in which all members invest and to which they contribute shared ideas and values [31]. The three dimensions that differentiate a COP from an ordinary group are the mutual commitment that generates bonds of coresponsibility among practitioners, the joint practice itself, and the shared repertoire mentioned.

Why do we consider that the concept of community of practice can be adopted and adapted in the context of our research? Because this concept aligns with the characteristics of a group of students (a community) addressing a problem-solving task (practice). The practice of problem solving, central to any physics student, is a practice situated in the context of formal education. Many of the things students learn in a physics course, they learn by trying to solve problems, by giving, asking for, and receiving help from the people around them. The skills acquired during this task are developed indirectly and disseminated through discussion activities with peers interested in the development of similar tasks. Most of the skills acquired during problem solving in physics are not found in texts, but are learned as one participates in certain groups that share a given problem. In the present study, the group of students are interested in solving the same problem, and to do that they get involved in discussions that help them advance a common, deepened understanding of the situation. They are committed to finding a solution that is satisfactory for both, and which is arrived at through interaction, so they can be considered a COP.

From within the overall idea of community of practice, we also retrieve the ideas of *peripheral* and *central participation*. These refer to different subject-matter mastery, or skills, and different degrees of legitimacy that individuals may have within the community. According to this idea, newcomers peripherally begin to participate in practices while more experienced individuals participate centrally. Learning is understood as a process that involves a shift from peripheral to central participation. Although both of the students in the present study can be considered equally legitimized, different characteristics can actually be observed in their mastery of subject matter and in the skills they exhibit in their use. As we will describe in Sec. IV B, the group of students we will actually work with locally exhibit these different degrees of skills and knowledge mastery in particular times within the interview. For this reason, inspired by the classification proposed by Lave and Wenger [30] for the participants of a community of practice, we adopted the same terms to characterize the two types of participation locally observed along the interview.

III. RESEARCH QUESTION

We assume that CCT can provide important insight to the details of the process of conceptual development. Furthermore, we believe that in combination with CCT, the theoretical input provided by the characterization of central and peripheral participation within COP's can help boost our understanding of how students, as they interact while addressing a shared problem-solving task, advance their conceptual understanding. The research question we will be addressing is the following:

How do students progress in the codevelopment of the entropy class, as they interact to address a problem-solving task?

In this way, the main goal of this work is to describe the different ways in which interaction allows students to codevelop a coordination class.

IV. RESEARCH CONTEXT AND METHODS

Since the aim of this research is to elucidate the process of codevelopment of a coordination class, we decided to take the simplest case available of a collaborative environment in which we could analyze, in detail, participants' contributions and the dynamics of their collaboration. In this way, we designed a two-hour interview with two students who were presented with a problem-solving situation involving entropy and asked to solve it together.

A. Data collection

Participants were video recorded while they addressed the problem-solving task. Videos are useful for eliciting detailed information about students' reasonings. This experimental setting provides different features at the same time. The interactive dynamics between students enables us to monitor how their participation evolves throughout the solving process. The reduced number of students allows us to infer how conceptual codevelopment is happening.

The analysis was carried out on the audio-video data obtained during the interviews. It involved two distinct methodological instances. A first stage consisted of an individual (one single researcher) revision of the videos as they were transcribed. Each researcher's individual interpretations are made explicit, compared, and contrasted with those of their peers. In a second instance, all differences are put to the consideration of the group. In this second stage, coding schemes were reviewed by a research team (in our case the authors) as proposed by Jordan and Henderson [32]. This collaborative viewing is powerful for neutralizing preconceived notions of individual researchers and discourages the tendency to see in the interaction what one is conditioned to see or even wants to see. Collaborative viewing is an effective antidote to what Hutchins [33] refers to as "confirmation bias," and explains it as the propensity to affirm prior interpretations while discounting or even ignoring counterevidence. Collaborative video analysis allows us to revisit them and question a given interpretation, regardless of whether or not it is the predominant one within the group. When this work is iterative and continues until a consensus is reached, results are more reliable than, for example, those obtained by reaching high percentages of agreement between individual researchers' interpretations [high interrater reliability (IRR)]. Recent literature argues that high IRR does not necessarily mean validity [34]. Coding schemes presented in this paper emerged from the process described above.

In order to study how the entropy (coordination) class was developed, certain operational criteria were defined. They were used to identify the different elements that intervene in the class, and are displayed in Table I along with examples for each case. (for similar criteria, used for the concept of buoyancy, please refer to Ref. [8]

B. Participants

The study was carried out in a school of physics in a public university in Argentina. The two students interviewed volunteered to participate. Both of them had completed and passed a thermal physics course three months before the interview with similar grades. The course pertains to the second year of a physics degree program and deals with topics of heat, temperature, thermodynamics laws, and macroscopic description of entropy, as typically presented in commonly used textbooks [35,36].

C. The interviewer

The interview was a semistructured one. It was conducted by a researcher (first author) who was not the students' instructor. The interviewer's role was to propose the task and, from then on, to follow students' reasoning. Elements proposed by Halldén [37] were considered: the interviewer's mission was to follow student's ideas and not to conduct their reasoning; interviewer's interventions were oriented at asking for deeper explanations, checking understanding, or highlighting differences between students' reasonings. Any type of evaluation, or opinion was strictly avoided with the purpose of distributing authority among the students interviewed and of fostering their participation as much as possible [38].

D. The task

Our study focuses on the concept of entropy. Entropy features fit with those that a concept must meet to be well described by CCT. The problem posed to students was designed with the purpose that they could make explicit their ideas about entropy, its different representations, its properties (e.g., that it is a state function), and its relation to reversible-irreversible processes, among others. This problem presents an irreversible transformation that students have analyzed as part of the content covered in their thermodynamics course. This analysis includes the consideration that this, as any irreversible process, cannot be represented in P-V diagrams that can only account for equilibrium states. The problem was expected to evoke some previous student ideas about entropy reported in the literature [39–50]. Options were designed to elicit those ideas and for students to make them explicit. This would enable us to monitor the way those ideas evolved as the class was codeveloped. Students were required to justify any particular choice and, most importantly, they were required to *agree* on their choice. This was often achieved after having initially picked different choices.

Options play a crucial role in data collection. First, they offer the possibility of making explicit students' previous knowledge in regards to entropy. Furthermore, when picking among the available choices, students seldom coincide from the beginning: options are designed on

Problem

A monoatomic ideal gas completes de cycle A-B-A. The process from A to B (not shown in the figure) consists of a free (irreversible) expansión in which the gas doubles its volumen without heat exchange with the environment. From B to A the gas undergoes a reversible isothermal compression. Considering that in state A the gas has a temperature T_0 and volumen V_0:

- Choose the correct answer(s) regarding the gas
  - (a) Gas entropy increases after the cycle
  - (b) Gas entropy decreases after the cycle
  - (c) Gas entropy does not change after the cycle
- Choose the correct answer(s) regarding the environment
  - (a) The entropy of the environment decreases after the complete cycle
  - (b) The entropy of the environment does not change after the complete cycle
  - (c) The entropy of the environment increases after the complete cycle, and it does so during the irreversible transformation of the gas
  - (d) The entropy of the environment increases after the complete cycle, and it does so during the reversible transformation of the gas
  - (e) Other ………………………………………………………………………………

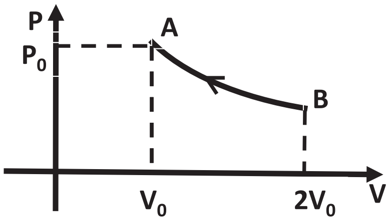

FIG. 2. Problem given to students.

the basis of likely previous ideas that can make several of them equally eligible. Thus, students are nudged into discussions and argumentations in order to agree on their final choice. Therefore, the value of those options lies in the profound discussions they promote, rather than on which one they actually pick. Those discussions are a rich data source that allow us to "see" how their ideas change and evolve during the problem-solving session.

The task consisted of analyzing the entropy of a system and that of its environment as it undergoes a particular process. Figure 2 shows the problem statement.

Since entropy is a state function, after completing the whole cycle, the entropy of the gas is the same as it was at the beginning. The environment increases its entropy and it does so during the reversible compression, during which the gas provides heat to it. A counterintuitive trait of the situation is that the net entropy of the system actually increases during the reversible process, and not during the irreversible one. The association of reversible transformations to zero entropy increases does hold, but for the "universe" (system + environment). The entropy of the universe increases during the free expansion from A to B (not representable in P-V diagrams). When the system transforms reversibly from B to A, the universe maintains its entropy, since the increase in environment entropy is equal to the decrease in system entropy.

V. RESULTS AND ANALYSIS

The analysis is carried out over four segments (S1, S2, S3, and S4), which are excerpts from students' dialogues. In order to be selected, any excerpt had to meet one or more of the following criteria: (i) to be clear in illustrating the conceptual progress on entropy, (ii) to be clear in showing how that progress stems from the interaction between students, and (iii) to be a good depiction of the different types of interactions between students throughout the interview. The different dimensions of the analysis are described in Secs. V A, V B, and V C, respectively.

A. Coordination of the class

In this dimension of our analysis we take CCT to study how the entropy class develops. In S1, students conclude that gas entropy increases after the complete cycle because it increases during the irreversible transformation and remains the same during the reversible one. In S2 they review their answer and use a mathematical formulation. Thus, they compute entropy changes and obtain zero for the first transformation and a negative value for the second one. As an overall result, they conclude that gas entropy decreases after the cycle. In a third stage (S3), they reconcile these ideas. Finally (S4), they solve for the entropy changes in the environment, considering elements discussed before.

B. Determination of codevelopment

Besides analyzing how the class is coordinated, we also focused on the coconstruction of those coordinations. This dimension of our analysis deals with the details of how the projections are constructed with the participation of each student. We are interested in understanding how the contributions from each student do or do not contribute to the development of the class. The criterion that guides this dimension of the analysis is to determine if the projections are being codeveloped (i.e., both students are actually building the same projection) or not (i.e., they take two different ways of reasoning).

We define that a projection is codeveloped when the elements (extractions or inferences) of a projection are provided by both students. This means that each student must contribute with at least one extraction or inference. If all the elements in a projection are provided by only one student, then that projection is not considered to be a codeveloped one.

C. Types of codevelopment

In order to analyze types of codevelopment, we first turn to types of participation. Our data reveal changes in the interaction throughout the interview. Two categories were defined for the type of participation.

- *Central participation*.—Interventions that can serve to bring new or divergent positions in relation to what is being discussed. These interventions are clear, express certainty, propose arguments or challenge an idea.
- *Peripheral participation*.—interventions that do not bring new information in relation to what is being discussed. Rather, they express doubt or inquire about their peers' opinions.

After defining categories for students' types of participation, we define two types of codevelopment: *symmetrical* and *asymmetrical*.

- *Symmetrical codevelopment*.—We call a codevelopment symmetrical when two conditions are met: projection's elements (extractions and inferences) are evenly distributed among the participants and the participations of all students are majorly of a *central type*.
- *Asymmetrical codevelopment*.—This occurs when the elements of the projection are not contributed evenly by both students or when the participations of at least one student are majorly of a peripheral type.

In the first segment, students incorrectly evoke ideas on reversibility to answer about the gas's entropy. In the second one, they appeal to mathematical expressions and produce another different projection. In the third segment, they match both projections in an aligned one throughout an articulation process. Finally, in segment 4, we can see them switch to a different (auxiliary?) context that allows them to project the class in an aligned way with respect to disciplinary knowledge.

1. Segment 1: "Gas entropy increases after the complete cycle"

Coordination of the class.—The evolution of students' projection in this segment can be synthesized as follows: they started by evoking that during a reversible process, the universe's entropy remains constant. After that, they checked the summative property of entropy. Then they noticed the irreversibility of the free expansion, and finally they concluded that gas entropy over the complete cycle increases.

The characteristics of the projection in this segment are consistent with misconceptions reported in the literature. Namely, misuse of entropy as a state function [43,39] and failing to realize that gas entropy can actually decrease during reversible processes [44].

S_1 [3:02 min to 5:40 min]

1. *D*: *Entropy was…*?
2. *F*: *Differential of Q over T*
3. *F*: *It is reversible, so the universe's entropy does not change*
4. *D*: *But it is the gas's…*
5. *F*: *Yes, the gas's does…*
6. *D*: *Does Entropy* [of the gas] *remain constant during a reversible process*?
7. *F*: *Yes, yes…mm… I don't really remember…I have a question. If I compute the entropy, gained or lost, whatever, the variation up to here* [from A to B and back] *and I add them, that is the total entropy, isn't it*?
8. *Int: Yes. Just to check I am following you. When a process is reversible there is no entropy change* [for the gas] *and when it is irreversible, then there is one*.
9. *F*: *Yes..it always increases*
10. *D*: *But the process from A to B is irreversible, isn't it?… Oh no! Here it says: it corresponds to a reversible isothermal compression*
11. *F*: *It is irreversible…. oh, ok, fine then… From here to there* [from A to B] *gas entropy increases because the process is irreversible, and from here to there* [from B to A] *it remains constant …So, it increased* (gas entropy in the whole cycle)
12. *Int: So, you're saying that gas entropy increases during the first transformation because the free expansion is an irreversible process. In the process back to A, gas entropy does not change because the process is reversible*
13. *D* & *F*: Both nod

When *D* asks about entropy (turn 1) *F* brings up the corresponding mathematical formulation or definition. *F* identifies the cycle as a reversible process and evokes a known property according to which the Universe's entropy remains constant. Identifying the reversible characteristic of the process is actually the first extraction in this projection. This enables her to make the first inference (element of the inferential net): that the universe's entropy does not change (turn 3). Then, *F* remembers the additivity of entropy and she checks for its applicability with the interviewer (Int) (turn 8). This will enable them to infer new information on the situation. Later, *D* introduces a new element, which *F* had not noticed: the irreversibility of free expansion, i.e., a new extraction (turn 10). This element lets *F* and *D*

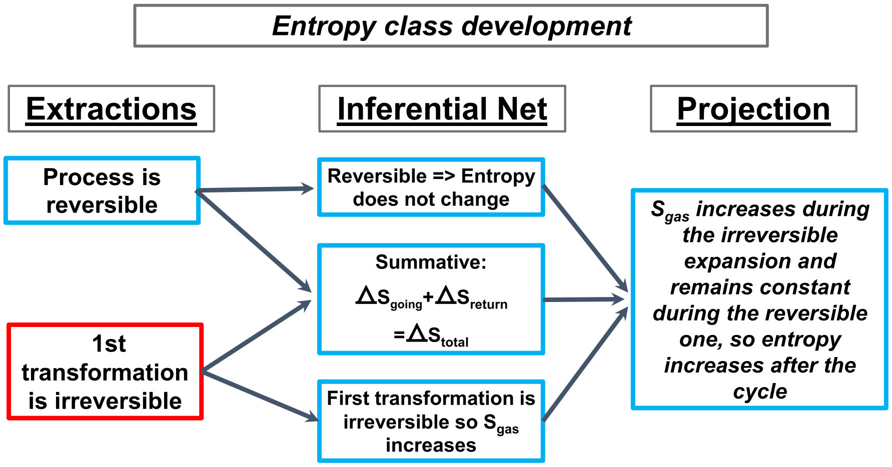

FIG. 3. Development of the entropy class during segment 1. *F*'s contributions in blue frames, *D*'s contributions in red frames, where S_{gas} stands for the entropy of the gas and ΔS the entropy variation.

complete the first projection and assert that entropy increases after the cycle (turn 13).

Determination of codevelopment.—Transcriptions during segment 1 show that there is a single projection which is built by extractions and inferences from both students. *F* brings up a property that *D* did not remember such as "entropy remains constant in reversible processes" and additivity, while *D* noticed information that *F* did not (first transformation as irreversible). Although clearly *F* is leading the reasoning process, *D*'s contribution is relevant. Figure 3 presents a diagram of the development of a class during fragment 1. This diagram is inspired by the graphical resources proposed by Wittman [51] to represent conceptual change. Although different from the representation in that work, we believe it provides a good scheme for the development of the coordination class. Each "box" indicates an element of the class, which is not literally taken from the transcriptions, but is derived from it. It can be seen that *D*'s extraction is essential to the projection.

Moreover, *F*'s inferences are based on it. It is clear that there is one projection which is built by both students. There is no doubt that there are many more contributions from *F* than from *D*. This last point is addressed in the following subsection.

Type of codevelopment.—*F*'s interventions are central (see Table II): she gives a definition ("Differential *Q* over *T*"), she applies a known property ("It is reversible, so the Universe's entropy does not change") and she answers questions. Instead, *D*'s interventions are peripheral: she expresses doubts ("Entropy was…?"); she asks her partner for information she herself can't recall ("Does entropy remain constant during a reversible process?"). Figure 3 shows that in this segment there is a codevelopment of the class. It also shows that the contributions were not balanced.

TABLE II. Examples of categorization of participations during segment 1.

	Central	Peripheral
Student D	• *D*: *But the process from A to B is irreversible, isn't it?…Oh no! Here it says: it corresponds to a reversible isothermal compression* • *D*: *To me, it's irreversible*	• *D*: *Entropy was…? (express doubt)* • *D*: *Does Entropy remain constant during a reversible process? express doubt*
Student F	• *F*: *Differential Q over T* • *F*: *It is reversible, so the universe's entropy does not change* • *F*: *Yes, the gas's does…* • *F*: *Yes, yes…mm… I do not really remember…I have a question. If I compute the entropy, gained or lost, whatever, the variation up to here [from A to B and back] and I add them, that is the total entropy, isn't it?* • *F*: *It is irreversible…oh, ok, fine then… From here to there [from A to B] gas entropy increases, and from here to there [from B to A] it remains constant..So, it increased (express doubt)*	• *F*: *Right…*

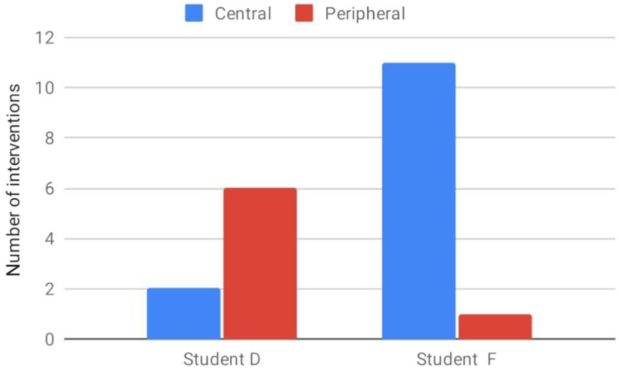

FIG. 4. Students' intervention types during segment 1.

Student F proposed most of the elements in the projection, while student D contributed with a single, although important, extraction. In addition, Fig. 4 shows that F's participation is clearly central. Most of her interventions are substantial: arguments and inferences are critical to address the solution of the problem. On the other side, D's participations are mostly of a peripheral type, i.e., her interventions were not to propose new arguments, original elements or ideas but rather to follow F's proposals.

An intervention was defined as each student's uninterrupted speaking turn. This categorization is defined by means of an iterative group discussion among researchers. As for its validation, test runs were conducted using particular examples (excerpts). A second discussion meeting was carried out when particular ambiguous cases arose. These were, however, a clear minority.

Through these examples we observe that during the first interventions student F brings ideas and properties of entropy to the discussion. Student D asks her about her arguments and tries to follow F's ideas until she realizes about the irreversibility of the free expansion (central intervention) and introduces this element to discussion. Despite its peripheral involvement, D introduces a substantial element in the development of the class. Thus, considering the irreversibility of free expansions, F projects the class in the particular way shown. So, we conclude that this segment shows an asymmetrical codevelopment of the projection.

2. *Segment 2: "Gas entropy decreases after the complete cycle"*

Coordination of the class.—Throughout this segment we will see how students evoke mathematical representations to review their own answer and compute gas entropy changes. We will show how they use the expression $dS = dQ/T$ for both reversible and irreversible processes. In this way they compute $\Delta S = 0$ for the irreversible part and $\Delta S < 0$ for the reversible one.

We focused on a part of this segment which is divided in two excerpts: in the first one, they compute the entropy variation from A to B and in the second one, the one from B to A.

Excerpt a

F brings up the macroscopic formulation of entropy. She uses the formula $dS = dQ/T$ for the irreversible process to find that entropy change for the gas is zero from A to B.

$\mathbf{S_{2a}}$ [8:03 min to 9:45 min]

1. F: *I don't know…so we can compute it and see what happens.*
2. F: *In the first one, differential Q… I don't remember what was the letter for entropy… W*?
3. *Int*: *S*
4. F: *S… ok, well delta S is equal to the integral of dQ over T from state A to state B in the first transformation… So… differential Q here is zero* [for the gas], *over T… hmm…ohh*!
5. D: *Hmmm, it does not exchange heat with the environment*
6. F: *What's going on?….hmm…And now, delta S2 would be from B to A, differential Q over T*
7. D: *Is this like a total entropy computation*?
8. F: *Hm…no*
9. D: *Is this the entropy* [change] *for the system* [gas] *that you are looking at*?
10. F: *Yes…*
11. D: *Ok…*
12. F: *Differential Q over T*

Here we see how students realize that this result differs from their prior consideration about the irreversible expansion. They first found that entropy of the gas increases from A to B because it is an irreversible process, but mathematically they compute this entropy change as zero. At this point, they misuse the formula, considering the equal sign for an irreversible process. This idea was also identified in previous research [42,49].

Excerpt b

In this excerpt, the entropy change for the gas is computed by students for the transformation from B to A. They recognize that during this reversible isothermal compression the gas delivers heat to the environment and thus its entropy decreases. This idea contrasts with what they thought before when they assumed that gas entropy did not change during the reversible process. Table II synthesizes this projection.

$\mathbf{S_{2b}}$ [12:02 min to 13:03 min]

1. F: *I think isothermal lines cut through adiabatic ones, no sorry, adiabatics cut through isothermals…* [students sketch the diagram shown in Fig. 5]
2. F: *So, the gas is doing this… It is going from a point* [B] *of less heat to one* [A] *with more heat*.[1] *So here*

---

[1] They are regarding adiabatic curves in a P-V diagram in a way similar to how isothermal curves are considered: each one corresponds to a particular T value. Thus, adiabatic curves also are assigned a unique "Q" value (see Fig. 5).

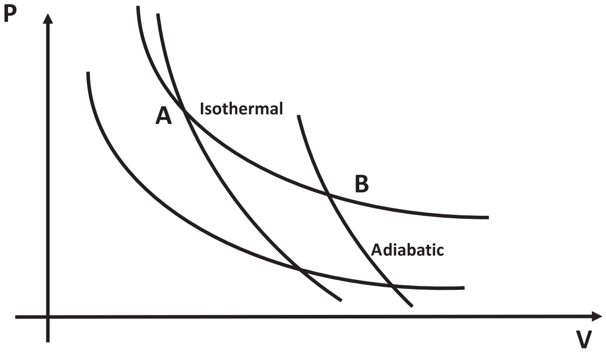

FIG. 5. Students' sketch during segment 2.

delta Q is negative! Thus delta Q between B and A is less than zero. So entropy here…

3. *Int: Sorry…that means…*
4. *F* and *D*: *Gas losses heat*
5. *F*: *So, entropy here…if I have zero in the first case… which I am not sure is ok, but the computation gives us that*
6. *D*: *Ahh…oh no…*
7. *D*: *The universe's entropy increases…*
8. *F*: *Right, it increases. Gas entropy does not necessarily increase in an irreversible process.*
9. *F*: *Well, here we have that heat is lost, so gas entropy decreased in this reversible transformation*

Here CCT provides insight into students' conceptual path, as it helps to understand how they arrive at this different projection. During the first part, *F* extracts from the context that there is no heat exchange during the free expansion (turn 4). This element lets her infer that the gas entropy change is zero after the first transformation. As we have mentioned before, the macroscopic formula is misapplied in this context.

F noticed that temperature remains constant throughout the reversible isothermal compression (extraction). By means of a graphical scheme, she infers that dQ is negative and, consequently, she claims that the entropy of the gas decreases during the compression. Bringing the two reasonings together, she concludes that after the complete cycle, entropy of the gas decreases (projection 2). This projection is misaligned with respect to the first one.

Two aspects are worth noting in this segment. First, neither of the students was convinced about their first answer so they looked for other entropy representations. Among these, the use of the mathematical formulation (which holds for macroscopic processes) had flawed or incomplete aspects. Second, and more important, they recognized these different results and they put them at the same hierarchical level. Later we will show how they articulate these two misaligned projections.

Determination of codevelopment.—We have defined that there is codevelopment if there is a single projection and it is done with elements proposed by more than one student. Figure 6 shows a scheme of the class development and the contribution by each student. The transcription, as well as the projection scheme in the figure, both show that there is a unique projection and that all of its elements are provided by one only student. Student *F* made the two extractions, both inferences and projected the entropy class. *D* does not contribute elements to the projection. As Fig. 7 shows, her interventions were almost entirely peripheral as opposed to those of *F*, who again held central-type participations for the most part. For these reasons we conclude that there is no codevelopment of the projection during segment two.

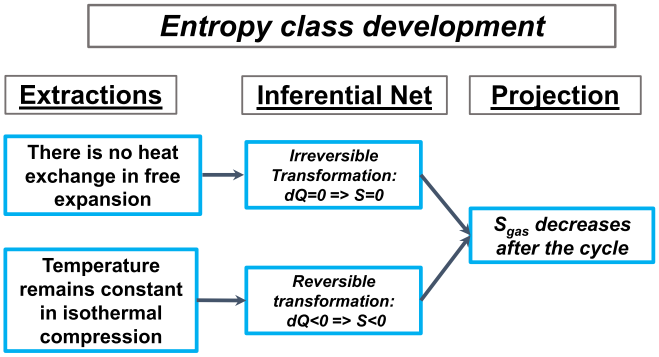

FIG. 6. Diagram of the projection development during segment 2. In blue, *F*'s contributions.

3. *Segment 3: "Gas entropy increases during the irreversible process and decreases during the reversible one"*

Coordination of the class.—In this segment, we will see the interviewer review the logic of previous misaligned projections and how that allows students to articulate them. They first doubt the mathematical expression and they think that something is wrong with it. After discussions, students align both projections and reformulate their answers.

$\mathbf{S_3}$ [18:07 min to 21: 16]

1. *Int: So, we still have two options, don't we? Either Entropy* [of the gas] *increases during the first transformation because it's an irreversible process or it doesn't change because delta Q is zero…*

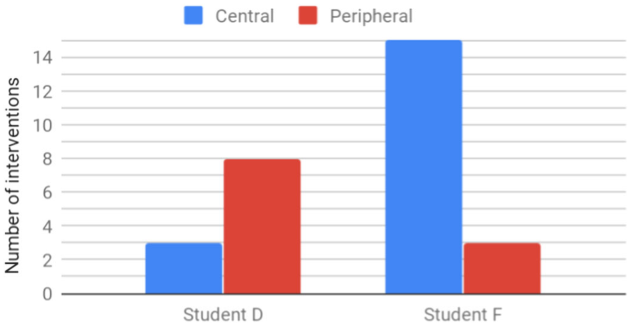

FIG. 7. Students interventions during the whole segment 2.

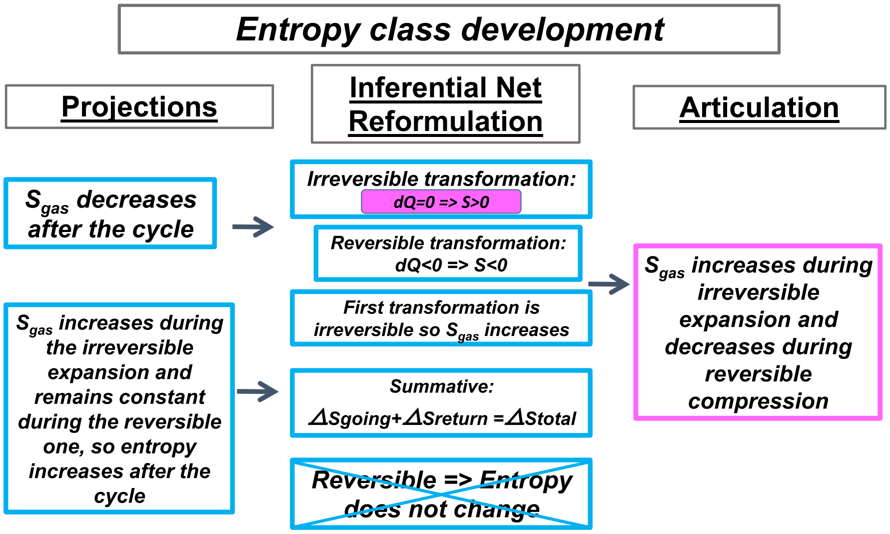

FIG. 8. Articulation diagram during the third segment. Purple highlight is used for contributions or changes proposed by both students.

2. *F*: *Yes…that is the problem… I'm not sure now*
3. *Int: Your thoughts on irreversibility and entropy don't completely fit that equation, is that it*?
4. *F*: *Yes… they don't fit!*.
5. *F*: *Wait…this equation…Does it always work? Now I'm not sure… Is it only for reversible processes? I don't remember that*
6. *D*: *But if the cycle is reversible it* [entropy change for the system] *would always be zero*
7. *F*: *If it's a cycle, but not a transformation. If I compute from A to A it will be always zero, but here on the way back we even got a negative value*
8. *D*: *Right…*
9. *Int: Well, in fact in this* [mathematical] *expression the equal holds only for reversible transformations. Otherwise, the "less than" sign holds.* $[dQ/T < dS]$
10. *D*: *ahh…* (expressing understanding)
11. *F*: *That's true, you're right! You can't compute entropy changes in irreversible processes. I remember that now*
12. *Int: Did that solve the problem*?
13. *D*: *Yes…gas entropy increases*
14. *F*: *Yes, it always increases*

Students articulate their projections after the interviewer answers their questions about the limits and contexts in which the mathematical formulation works. The following considerations are in order here:

- Students discard the idea that gas entropy remains constant during reversible processes. Computations convince them about this.
- One could consider that it is the interviewer who articulates the entropy projections for them, but it is actually not the case. Students identified discrepancies between these projections from the beginning. They are the ones who actually doubt about the equation (turn 5). They not only spontaneously notice there is a problem, but they also identify where that problem could reside. All the interviewer does is to bring these discrepancies back to the discussion (S_2, turn 4).

In sum, at this stage, students think that gas entropy increases during the irreversible transformation and decreases during the reversible isothermal compression. They still need to decide whether, after the complete cycle, gas entropy remains unchanged, increases, or decreases.

From CCT we can infer that in this segment an articulation process occurred. Students realized that the two projections are different and they try to make them fit. They modify their extractions and inferences of projection 1 and projection 2 in order to articulate them. F doubts the use they made of the equation (turn 5). Incorporating inequality into the equation, they manage to reconcile their idea of irreversibility and increase of entropy (turn 10 and 11). Students also refine the relationship between reversibility and change of entropy, concluding that the entropy of the gas decreases on the reversible isothermal compression (turn 7). Figure 8 synthesizes this articulation process.

Determination of codevelopment.—During the third segment students are working to reconcile their two projections from segments 1 and 2. The articulation process involves both students. Thus, we claim that there is codevelopment. Although *F*'s participation seems to prevail in the overall process, *D*'s contributions provide relevant information to the projection. In the next section we will go deeper into this analysis.

Type of codevelopment.—Although more equally distributed between students as compared to segment 1 and 2, contributions are still uneven. While *D*'s interventions are equally distributed between central and peripheral, *F*'s interventions are almost entirely central (see Fig. 9). On this basis, we claim that the codevelopment is asymmetric.

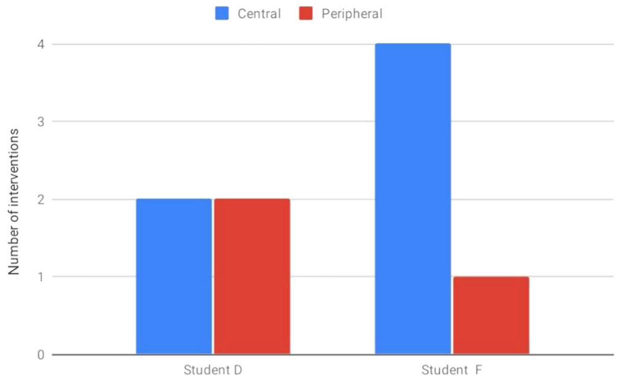

FIG. 9. Types of interventions during segment 3.

Before moving to the next item in the problem, students discuss the properties of entropy as a state function in order to decide what its change actually is. After a 20-min discussion, they arrive at an answer and they are able to integrate representations and properties of the concept.

4. Segment 4: “Enviroment’s entropy increases and it does so during the reversible compression”

Coordination of the class.—Once the discussion on gas entropy is over, students shift their attention towards the entropy of the environment. They quickly recall ideas on reversible-irreversible processes and conclude that during the reversible transformation its entropy increases. After a couple of minutes, they notice that the environment does not interact with the gas during the free expansion, so they state that the entropy of the environment does not change during that irreversible process.

In the following segment students recover, from previous discussions, elements about entropy changes, reversibility, and first law.

$\mathbf{S_4}$ [45:03 min to 48:15 min]

1. *D*: *During the reversible transformation, it increases…*
2. *F*: *Yes…*
3. *D*: *During the irreversible one… I do not know*
4. *F*: *Me neither*
5. *F*: *Right, on the reversible transformation, it* [environment entropy] *increases*
6. *D*: *and on the irreversible one…why would it decrease*?
7. *F*: *It doesn’t have to decrease. There is no reason why.*
8. *F*: *We know that it* [for the gas entropy] *decreases on the way back and the entropy of the environment remains constant* [in the irreversible expansion] …
9. *D*: *Ah! The environment’s! I meant the universe… Right… So, during the irreversible transformation maybe nothing happens.*
10. *F*: *You mean the Universe*?
11. *D*: *No, because…. why does the entropy of the environment increase? For me, during the irreversible process it doesn’t change.*
12. *F*: *Right, gas does not interact with the environment during the irreversible process. I mean, it’s not doing any work…. it’s not exchanging heat either… Right, I think it remains constant during the first process and in the second one it must increase. There’s no other option.*
13. *D*: *Yes. So, option d.*
14. *F*: *Yes. Moreover, if you consider the entropy of the Universe, i.e., the gas plus environment, you already know that its entropy must increase and the gas has already increased (during the irreversible process).*
15. *D*: *Right*
16. *F*: *I think that the entropy of the environment increases and it does so during the reversible process. More than that, the entropy increase of the universe happens on the irreversible transformation. In the first stage, entropy increases and during the second one it remains constant because the gas entropy decreases just as much as the environment’s entropy increases. All the increase of entropy [Universe] happens during the gas’s irreversible process.*

Students realize that the entropy of the environment increases during the reversible process due to heat provided by the gas as it is compressed at constant temperature (first element of the inferential net, turns 1 and 2). After that, they notice that during the irreversible transformation the environment does not interact with the gas, so they infer that its entropy [environment] remains constant (second element, turn 12) and they completed the projection finding that the entropy of the environment increases (turn 16). *F*’s explanation reveals a deep understanding of the process. Figure 10 synthesizes this projection.

Throughout this single interview, we were able to observe a wide variety of projections, which are indicative of how the entropy class evolves. The first two projections, although misaligned, were a fundamental starting point for

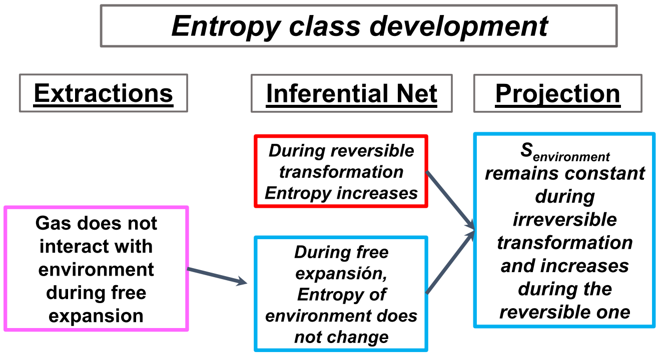

FIG. 10. Class development during segment 4. Purple highlight is used for contributions or changes proposed by both students.

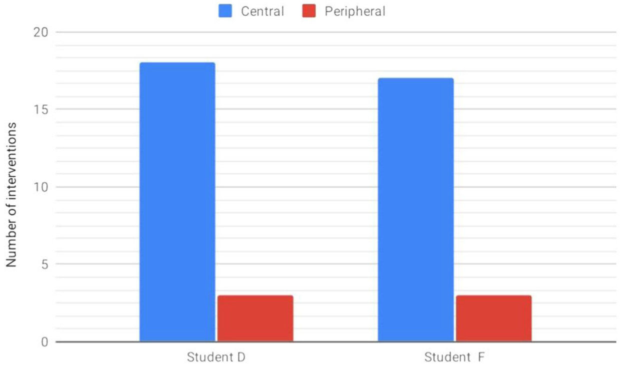

FIG. 11. Type of interventions during segment 4.

a productive process of conceptual change. In fact, it is from the discussion on these projections, and through their attempt to articulate them, that they recover essential elements that allow them to project the class in an aligned way with disciplinary knowledge. Students' final explanations reveal that they reached a deeper understanding of entropy.

Determination of codevelopment.—As in the entire interview, students work together in the same projection. Also, as detailed in Fig. 6, both students contributed elements to the class projection. Each student contributed with one inference and the projection is agreed on by both. It is trivial to define this segment as codeveloped projection.

Type of codevelopment.—In this segment, the elements of the class are distributed evenly among both students. In Fig. 11 we can see that *D*'s participations are now mostly of a central type, just as those of student *F*. Our claim is that *D*'s involvement in discussions changed during this segment as compared to the previous ones. Given that both students contributed elements to the class in an even form and that they had a central type of participation, we can say that the codevelopment is symmetrical.

Throughout the whole interview, different kinds of codevelopment took place. In the first segment, asymmetrical codevelopment is observed. During segment 2, there is no codevelopment as *F* contributed and developed the whole projection by herself. In segment 3, an asymmetrical type of codevelopment took place again, with *D* still contributing mostly in a peripheral way. By the end of the session, *D*'s participation changes and a symmetrical codevelopment is instantiated, as S4 reveals, both students continue developing the class but with elements evenly contributed by both. Data indicate that as the interview unfolded, students (specially *D*) improved the way they integrated their contributions.

VI. DISCUSSION

Our research interest was to explore how a coordination class is developed by two students when they address a problem-solving task involving entropy. Our study was oriented towards understanding students' building of common knowledge. CCT has been usually used, so far, to study individual conceptual development. A few studies, which have addressed collective environments, have assigned the class to the group of people as a whole, without analyzing the role of the individual contributions in the overall conceptual development. What is the point of zooming into the participation of each student during their interaction? The main reason is that it enables us to understand when and how opportunities for learning take place through students' interaction. With this motivation we addressed our research question:

How do students progress in the codevelopment of the entropy coordination class, as they interact to address a problem-solving task?

During several minutes of the interview students struggle with difficulties that have been reported as misconceptions in previous research [39,42–44,49]. CCT, as in previous studies on conceptual change, once again proved to be a fruitful theoretical approach to understand this struggle. By analyzing different projections and articulations we were able to understand students' difficulties and how they moved beyond them. In a way similar to that reported previously [8], students in the present research noticed differences between projections and doubted them. They recoordinated elements and, finally, projected the class in a way aligned with disciplinary knowledge. Once again, we encounter evidence of the importance of misaligned projections in initiating an articulation process that can lead to the coordination of the class, in this case, the entropy class. Thus far, this just extends previous results to a new case (entropy). CCT, once again proves to be a fruitful, fine-grain theory to analyze conceptual change in a student group.

However, our main interest was focused on how this class was codeveloped by the two students. To do so, the ideas of central and peripheral participation were introduced. In this way, we included an interaction-analysis approach to learning in order to account for different forms of interaction. This move goes beyond the work of Levrini and diSessa [10] and Barth-Cohen and Wittman [11], because we zoom into students' moment-to-moment interactions and their relation to conceptual progress. Therefore, the phenomenon is understood from two simultaneous, concurring perspectives of learning: individual and sociocultural. This particular approach allowed us to differentiate three types of codevelopment throughout the interview: asymmetrical codevelopment, symmetrical codevelopment, and no codevelopment.

Complementing CCT with the notions of central and peripheral participation proved to be a valuable contribution. It allowed us to challenge some common-sense ideas that, as teaching practitioners, we have been commonly seen to circulate among teachers regarding the poor contribution that less-involved students have in the progress of the ideas in the classroom, as compared to their more-involved peers. In asymmetric codevelopment conditions, even those students who have a more peripheral participation can actually be learning. Also, in asymmetrical codevelopment not only students with peripheral participations benefit from the contributions of central-type participants. Asymmetrical participation can offer learning opportunities even for those who are participating in a central way. This is the case of F, who is actually a central participant in the discussions. Her classmate offers a unique but crucial issue (the irreversibility of a free expansion) which directs the development of the class in a particular direction.

Zooming into the types of participation of these students has allowed us to understand important issues. Students with symmetrical and asymmetrical participation benefit from each other during the conceptual progress of the small group, constituting a "virtuous" interaction for learning. We believe that the substantial contribution of this study lies in considering both the conceptual progress *and* the interaction among these students in a detailed way. This allows us to better understand how the types of participation are intertwined during the conceptual development at a fine-grain level. During the coordination of the class, instances were observed in which D, although with peripheral participation, contributed elements that were the support for F to make more significant contributions. In the final phase, during the last projection, D's participations changed substantially and became central, thus showing the mutual benefit of the two types of participations during the learning process. These considerations are what lead us to conclude that we were able to observe learning at both the individual, as well as the collective level.

As we stated in Sec. II B, many researchers consider that building bridges between sociocultural and individual cognition is both important and fruitful [22–25], and that this is a difficult but feasible task [24]. We believe that the present work is a specific contribution in that sense. From a theoretical point of view, it represents a dialog between CCT and the ideas of central and peripheral participation, inspired by the work of Leave and Wenger [29,30] and adapted to our context. This complementarity between both views allowed us to understand students' learning dynamics in these small-group environments at a very fine-grain level. Each of these complementary theoretical perspectives has a critical role in identifying particular traits of the collective learning enterprise. While CCT informs on the task of reading out information, and aligning projections, differentiating central from peripheral participations gives a clearer picture of the ways in which students collaborate to carry out their collective enterprise. Both theoretical views are thus complementary in a very intimate way. This constitutes, as defined by diSessa *et al.* [21], a case of microcomplementarity between two distinct research paradigms, knowledge analysis and interaction analysis.

Finally, there are two methodological considerations that we wish to highlight. One of them concerns the limits of the current study. We have presented a single case study consisting of two students solving a problem, and this could raise questions about the validity of the results. However, we believe that particularization supersedes the validity of generalization [52]. The focus is on the in-depth study and knowledge of the case, and not on the generalization of the results over and above it [53,54]. In this paper we aim to understand how conceptual change can develop from peer interaction during problem solving, and we believe that the case study presented here makes a central contribution to this understanding. Indeed, there could be other forms of conceptual learning, as human thinking is highly creative and idiosyncratic. Our study reports on one such possible form.

The second consideration has to do with how these results contribute to understanding what might occur in a real classroom. Educational research is contextual, and context cannot be treated as just another independent variable, but is a constitutive part of the research problem. However, the current results may constitute a clue, a primary starting hypothesis, for investigating what might occur in an actual classroom. In fact, we are currently conducting an investigation of students' participations in a real physics class while solving, collectively, a problem posed by the teacher [55]. So, we are well aware that the present results are limited to the simplest collective case, and much more needs to be done in order to understand what goes on in actual physics classrooms.

ACKNOWLEDGMENTS

This work was funded by the Consejo Nacional de Investigaciones Científicas y Técnicas, PIP 2014-2016 #112-201301-00099.

---

[1] S. Vosniadou, *International Handbook of Research on Conceptual Change* (Routledge, New York and London, 2009).

[2] R. Duit and D.F Treagust, *Second International Handbook of Science Education* (Springer, Dordrecht, 2012).

[3] T. Amin, C. Smith, and M. Wiser, in *Handbook of Research in Science Education*, edited by N. Lederman and S. Abell (Routledge, New York, 2014).

[4] T. G. Amin and O. Levrini, *Converging Perspectives on Conceptual Change: Mapping an Emerging Paradigm in the Learning Sciences* (Routledge, New York, 2017).

[5] A. diSessa and B. Sherin, What changes in conceptual change?, Int. J. Sci. Educ. **20**, 1155 (1998).

[6] O. Parnafes, What does "fast" mean? Understanding the physical world through computational representations, J. Learn. Sci. **16**, 415 (2007).

[7] T. Thaden-Koch *et al.*, in *Transfer of Learning: Research and Perspectives*, edited by J. Mestre (Information Age, Greenwich, 2005).

[8] L. Buteler and E. Coleoni. Solving problems to learn concepts, how does it happen? A case for buoyancy, Phys. Rev. Phys. Educ. Res. **12**, 020144 (2016).

[9] M. C. Wittmann, The object coordination class applied to wave pulses: Analysing student reasoning in wave physics, Int. J. Sci. Educ. **24**, 97 (2002).

[10] O. Levrini and A. diSessa, How students learn from multiple contexts and definitions: Proper time as a coordination class, Phys. Rev. Phys. Educ. Res. **4**, 010107 (2008).

[11] L. A. Barth-Cohen and M. C. Wittmann, Aligning coordination class theory with a new context: Applying a theory of individual learning to group learning, Sci. Educ. **101**, 333 (2017).

[12] A. Kluge, Learning science with an interactive simulator: Negotiating the practice-theory barrier, Int. J. Sci. Educ. **41**, 1071 (2019).

[13] C. Lewis, Ph.D. thesis, University of California, Berkeley, 1980.

[14] M. E. Levin, Ph.D. thesis, University of California, Berkeley, 2012.

[15] J. F. Wagner, Transfer in pieces, Cognit. Instr. **24**, 1 (2006).

[16] A. diSessa and J. Wagner, What coordination has to say about transfer, in *Transfer of Learning: From a Modern Multidisciplinary Perspective*, edited by J. Mestre (Information Age Publishing, Greenwich, 2005).

[17] T. C. Thaden-Koch, R. J. Dufresne, and J. P. Mestre, Coordination of knowledge in judging animated motion, Phys. Rev. Phys. Educ. Res. **2**, 020107 (2006).

[18] P. Sengupta, K. D. Krinks, and D. B. Clark, Learning to deflect: Conceptual change in physics during digital game play, J. Learn Sci. **24**, 4 (2015).

[19] A. diSessa (volume from the Eleventh Advanced Course), Epistemological micromodels: The case of coordination and quantities, *Psychologie génétique et sciences cognitives*, edited by J. Montangero and A. Tryphon (Archives Jean Piaget, Geneva, 1991).

[20] A. A. diSessa, in *Reconsidering Conceptual Change: Issues in Theory and Practice*, edited by M. Limon and L. Mason (Springer, Netherlands, 2002).

[21] A. A diSessa, B. L. Sherin, and M. Levin, Knowledge analysis: An introduction, in *Knowledge and Interaction: A Synthetic Agenda for the Learning Sciences* (Routledge, New York, 2016).

[22] A. A. diSessa, M. Levin, and N. J. Brown, *Knowledge and Interaction: A Synthetic Agenda for the Learning Sciences* (Routledge, New York, 2016).

[23] L. Mason, High school students' beliefs about maths, mathematical problem solving, and their achievement in maths: A cross-sectional study, Educ. Psych. **42**, 1 (2007).

[24] J. G. Greeno and C. van de Sande, Perspectival understanding of conceptions and conceptual growth in interaction, Educ. Psych. **42**, 1 (2007).

[25] P. A. Alexander, Bridging cognition and socioculturalism within conceptual change research: Unnecessary foray or unachievable feat?, Educ. Psych. **42**, 1 (2007).

[26] J. L. Lemke, *Talking Science: Language, Learning, and Values* (Ablex Publishing Corporation, Norwood, 1990).

[27] A. Sfard, On two metaphors for learning and the dangers of choosing just one, Educ. Res. **7**, 2 (1998).

[28] A. Sfard, When the rules of discourse change, but nobody tells you: Making sense of mathematics learning from a commognitive standpoint, J. Learn. Sci. **16**, 565 (2007).

[29] E. Wenger, *Communities of Practice: Learning, Meaning, and Identity (Learning in Doing: Social, Cognitive and Computational Perspectives)* (Cambridge University Press, Cambridge, England, 1999).

[30] J. Lave and E. Wenger, *Situated Learning: Legitimate Peripheral Participation* (Cambridge University Press, Cambridge, England, 1991).

[31] G. Wisker, G. Robinson, and M. Shacham, Postgraduate research success: Communities of practice involving cohorts, guardian supervisors and online communities, Innov. Educ. Teaching Int. **44**, 301 (2007).

[32] B. Jordan and A. Henderson, Interaction analysis: Foundations and practice, J. Learn. Sci. **4**, 39 (1995).

[33] E. Hutchins, The social organization of distributed cognition, in *Perspectives on Socially Shared Cognition*, edited by L. Resnick, J. M. Levine, and S. D. Teasley (American Psychological Association, Washington, DC, 1991), pp. 283–307.

[34] D. Hammer and L. K. Berland, Confusing claims for data: A critique of common practices for presenting qualitative research on learning, J. Learn. Sci. **23**, 37 (2014).

[35] R. A Serway and J. W. Jewett, *Física: Para ciencias e ingeniería con Física Moderna* (Cengage Learning, Mexico, 2009).

[36] D. Halliday, R. Resnick, and J. Walker, *Fundamentals of Physics* (Wiley, Hoboken, 2009).

[37] O. Halldén, L. Haglund, and H. Strömdahl, Conceptions and contexts: On the interpretation of interview and observational data, Educ. Psych. **42**, 1 (2007).

[38] E. A. Coleoni and L. M. Buteler, Entrevistas clínicas en la investigación educativa: Un análisis desde la progresividad discursiva, Cién & Ed. **27**, 1 (2021).

[39] M. J. Granville, Student misconceptions in thermodynamics, J. Chem. Educ. **62**, 847 (1985).

[40] C. Brosseau and J. Viard, Quelques reflexions sur le concept d'entropie issues d'un enseignement de thermodynamique, J. Ens. Cien. **10**, 1 (1992), https://raco.cat/index.php/Ensenanza/article/view/39882.

[41] M. Sözbilir, What students' understand from entropy? A review of selected literature, J. Bal. Sci. Educ. **2**, 21 (2003).

[42] S. C. Covolan and D. D. Silva, A entropia no Ensino Médio: utilizando concepções prévias dos estudantes e aspectos da evolução do conceito, Ciên. & Ed. **11**, 98 (2005).

[43] J. R. Thompson, B. R. Bucy, and D. B. Mountcastle, Assessing student understanding of partial derivatives in thermodynamics, AIP Conf. Proc. **818**, 77 (2006).

[44] M. J. Cochran and P. R. Heron, Development and assessment of research-based tutorials on heat engines and the second law of thermodynamics, Am. J. Phys. **74**, 734 (2006).

[45] M. Sözbilir and J. M. Bennett, A study of Turkish chemistry undergraduates' understanding of entropy, J. Chem. Educ. **84**, 1204 (2007).

[46] J. J. Weber, Ph.D. thesis, University of Georgia, Athens, 2009, https://getd.libs.uga.edu/pdfs/weber_john_j_200912_phd.pdf.

[47] D. Meltzer, Chemical and physical contexts, in *Proceedings of the Winter Meeting of the American Association of Physics Teachers, Jacksonville, Florida* (AAPT, College Park, MD, 2011).

[48] Y. Ayyildiz and L. Tarhan, The effective concepts on students' understanding of chemical reactions and energy, Hacettepe Univ. J. Educ. **42**, 72 (2012).

[49] F. Jeppsson, J. Haglund, J. T. G. Amin, and H. Strömdahl, Exploring the use of conceptual metaphors in solving problems on entropy, Learn. Sci. **22**, 1 (2013).

[50] T. G. Amin, F. Jeppsson, and J. Haglund, Conceptual metaphor and embodied cognition in science learning: Introduction to special issue, Int. J. Sci. Educ. **37**, 745 (2015).

[51] M. C. Wittmann, Using resource graphs to represent conceptual change, Phys. Rev. Phys. Educ. Res. **2**, 020105 (2006).

[52] R. Stake, *Investigación con estudio de casos* (Morata, Madrid 1995).

[53] X. S. Blasco, Estudio de casos, Etnografía: metodología cualitativa en la investigación sociocultural, Marcombo (1995), pp. 203–208.

[54] *Estrategias de investigación cualitativa*, edited by I. Vasilachis de Gialdino (Gedisa, Barcelona, 2006) pp. 42–50.

[55] N. Velasco, N. Gandolfo, and L. Buteler, Investigación Basada en el Diseño: Un caso de involucramiento disciplinar productivo en la escuela secundaria Argentina.