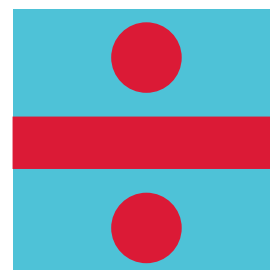
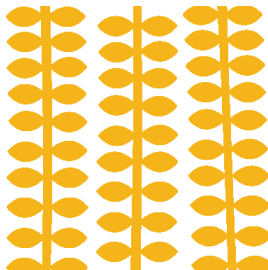
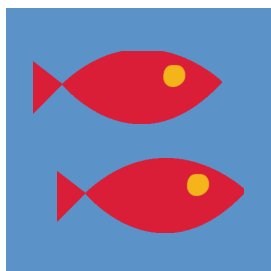
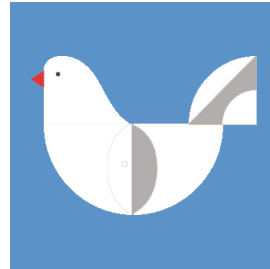




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# MATHEMATICS FOR ACTION

Supporting Science-Based Decision-Making

Published in 2022 by the United Nations Educational, Scientific and Cultural Organization, 7, place de Fontenoy, 75352 Paris 07 SP, France

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## Mathematics empowers sustainable development

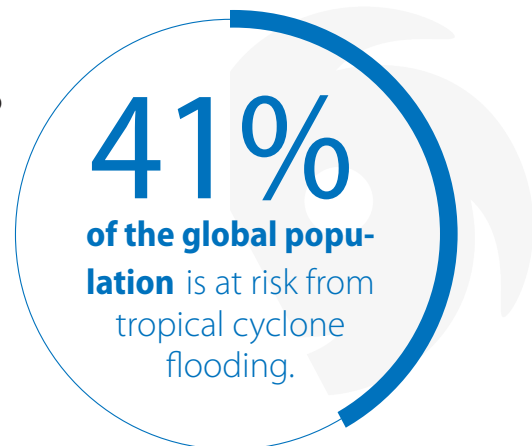
Everything we do is based on some mathematical structure, and although mathematics is often considered abstract, it is fundamental to how we understand nature, the larger universe, with its time and space dimensions and a myriad of uncertainties.

The Covid-19 pandemic brought mathematical modeling to the forefront of public attention and debate. Vocabulary such as ‘flattening the curve’ has become part of the collective lexicon. Governments all over the world rely on mathematics not only to forecast the epidemic but also to understand social issues like vaccine hesitancy.

Mathematics has allowed for pivotal improvements in weather prediction and has applications in agriculture and fisheries. **With new mathematical approaches, a tropical cyclone’s track can now be predicted up to 1 week in advance giving communities time to evacuate, and potentially saving lives and reducing economic losses.**

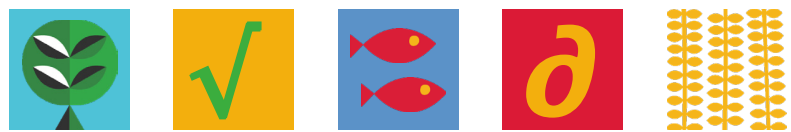
The *Mathematics for Action* toolkit focuses on engaging stories of mathematics *in action*. Written by mathematicians and thought leaders from across the globe, it presents fascinating research of how mathematics is addressing the world’s most pressing challenges.

The toolkit provides insightful information for decision-makers and for all those who seek proofs to challenging questions and it presents new avenues for scientific research.



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*"Since wars begin in the minds of men and women it is in the minds of men and women that the defences of peace must be constructed"*



# MATHEMATICS FOR ACTION

Supporting Science-Based  
Decision-Making

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(eds)

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European Mathematical Society (EMS)

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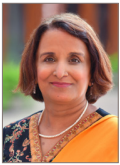
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# TABLE OF CONTENTS

FOREWORD ..... xv

INTRODUCTION ..... xvi



## GOAL 1

### End poverty in all its forms everywhere

Visualizing Poverty: AI-Powered Maps Improve Estimates and Predictions ..... 1  
*Christiane Rousseau*



## GOAL 2

### End hunger, achieve food security and improved nutrition and promote sustainable agriculture

Strengthening Food Security: Food System Resilience and Sustainability ..... 3  
*Hans Kaper and Hans Engler*



## GOAL 3

### Ensure healthy lives and promote well-being for all at all ages

Modeling Infectious Diseases: Forecasting the Spread of an Epidemic ..... 5  
*Christiane Rousseau*

Harnessing the Power of Data: Artificial Intelligence-Based Pandemic Support ..... 7  
*Jude Kong, Bruce Mellado, and Jianhong Wu*

Improving Pandemic Forecasts: Assimilating Observations and Simulations ..... 9  
*Geir Evensen, Javier Amezcua, Alberto Carrassi, and Alison Fowler*

Enhancing Vaccine Design: Mathematics Accelerates Medical Innovations ..... 11  
*Wilfred Ndifon*

Modeling Vaccine Hesitancy: Understanding the Free Riders of Herd Immunity ..... 13  
*Chris Bauch*



## GOAL 4

**Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all**

Teaching Mathematics: Mathematics Education for Sustainable Development ..... 15  
*Merrilyn Goos and Anjum Halai*



## GOAL 5

**Achieve gender equality and empower all women and girls**

Tracking Gender Parity: Mathematical Foundations of Gender Equality Indicators .... 17  
*Barbara Cozzens, Helen Roberts, and Fred Roberts*



## GOAL 6

**Ensure availability and sustainable management of water and sanitation for all**

Managing Water Resources: Probabilistic Tools for Water Resources ..... 19  
Vulnerability  
*Helen Roberts and Fred Roberts*

Shifting Lake Turbidity: Alternative States in Shallow Lakes ..... 21  
*Christiane Rousseau*



## GOAL 9

**Build resilient infrastructure, promote inclusive and sustainable industrialization and foster innovation**

Reckoning with Uncertainty: Social Sciences Lessons for Mathematical ..... 23  
Modeling  
*Andrea Satelli*



## GOAL 11

**Make cities and human settlements inclusive, safe, resilient and sustainable**

Preparing for a Crisis: Improving the Resilience of Digitized Complex Systems ..... 25  
*Hans Kaper, Fred Roberts, Igor Sheremet*



## GOAL 12

### Ensure sustainable consumption and production patterns

- Valuing Natural Capital: Quantifying the Value of Ecosystem Services ..... 27  
*Hans Kaper*
- Allocating Scarce Resources: Modeling to Support Food-Energy-Water Sustainability ..... 29  
*Elena Rovenskaya*



## GOAL 13

### Take urgent action to combat climate change and its impacts

- Modeling Climate: The Energetics of a Warming World ..... 31  
*Hans Kaper*
- Facing Future Climates: Climate Modeling for Improved Decision-Making ..... 33  
*Mouhamadou Bamba Sylla*
- Forecasting Cyclones: The Mathematics of Tropical Cyclone Prediction ..... 35  
*Marc Sedjro and Mike Cullen*
- Attributing Extreme Weather: The New Science of Extreme Event Attribution ..... 37  
*Michael F. Wehner*
- Pinpointing the Indian Monsoon: Improving Predictability of the Annual Summer Rains ..... 39  
*Amit Apte*



## GOAL 14

### Conserve and sustainably use the oceans, seas and marine resources for sustainable development

- Sustaining Fisheries: Bioeconomic Models for Fisheries Management ..... 41  
*Nadia Raissi*



## GOAL 15

### Protect, restore and promote sustainable use of terrestrial ecosystems, sustainably manage forests, combat desertification, and halt and reverse land degradation and halt biodiversity loss

- Measuring Biodiversity: A Data-Driven Approach to Global Nature Conservation ..... 43  
*Fred Roberts*

Listening in on Wildlife: Animal Sound Classification with Deep Learning ..... 45  
*Emmanuel Dufourq and Ian Durbach*

Battling Invaders: Quantifying the Impacts of Invasive Species..... 47  
*Christopher M. Baker and Michael Bode*



## GOAL 16

**Promote peaceful and inclusive societies for sustainable development, provide access to justice for all and build effective, accountable and inclusive institutions at all levels**

Preserving Privacy: The Power of Many with Collaborative Machine Learning..... 49  
*Laura Wynter*

Finding the Missing: Complex Networks Enable Searches for Missing People..... 51  
*Inés Caridi*

# FOREWORD

This toolkit showcasing *Mathematics for Action* comes at a time when mathematics is becoming an increasingly precious tool for decision-makers. A growing range of mathematical models are enabling us to analyse the extent to which natural phenomena and those we have engendered ourselves will affect how we live and whether we manage to sustain our increasingly fragile environment. This toolkit is UNESCO's way of drawing global attention to the need for public policies to be based on evidence which, increasingly, will stem from basic research.

But why should UNESCO be producing such a publication? Simply because UNESCO is the only United Nations agency with a mandate for mathematics. This mandate for mathematics is as old as the S for science in UNESCO's name, which dates from the Organization's founding in 1946.

In 1962, UNESCO founded the Latin American Centre for Mathematics (CLAM) in Buenos Aires, Argentina, in recognition of the need for a solid grounding in mathematics in the developing world. This was an exciting time for mathematics. A few years earlier, the first artificial satellite had been sent into space. A few years later, human beings would walk on the Moon for the very first time. The term artificial intelligence was coined in 1956, three years before the first microchip was patented. Over the coming decades, the miniaturization of integrated circuits would make it possible to manufacture ever-smaller mechanical, electronic and optical devices. Today's smartphones use millions of minuscule transistors to perform complex processes. These smartphones wouldn't have been possible without mathematicians.

UNESCO was also behind the establishment of another institution which has trained mathematicians and fostered research in national institutions around the world over the past few decades. I am referring to the International Centre for Pure and Applied Mathematics in Nice, France, established in 1978.

UNESCO has devoted a great deal of its work to improving the quality of mathematics education and research but remains something of an enigma to the person in the street. Everyone recognizes, for example, that mathematics is omnipresent in today's world – notably in the technological items all around us and in exchange and communication processes – but this presence is generally not in evidence. This makes it difficult for some to see the point of developing a mathematics culture beyond basic skills in numeracy, measurements and calculation.

This is why it is important for basic education to bring mathematics to the fore. This is especially vital because 'mathematical literacy' requirements far exceed needs traditionally associated with basic computational knowledge. Mathematics is still often perceived as an almost exclusively solitary activity, cut off from the problems of the real world and independent of technology. Furthermore, mathematics is often still seen as a purely deductive activity in which perfectly rigorous formal proofs are used to produce theorem after theorem. These many misunderstandings affect the teaching of mathematics by raising barriers to quality mathematics education for all.

This is why UNESCO supported the World Mathematical Year in 2000, in order to familiarize people around the world with the impact of mathematics on their daily lives. This is why a team led by UNESCO designed a travelling exhibition for the general public in 2004 called Experiencing Mathematics.

This is why the present toolkit has been designed for policy-makers. *Mathematics for Action* demonstrates how mathematics lies at the heart of the evidence-based policies that governments around the world adopt on a regular basis to tackle a particular socio-economic or environmental issue.

**Shamila Nair-Bedouelle**

Assistant Director-General for Natural Sciences



# INTRODUCTION

Today's world faces a daunting array of complex and interconnected challenges. Issues such as food insecurity, inequality, infectious agents, climate change, land degradation, biodiversity loss, mass migration, conflict, and political unrest pose obstacles to development and put societies at risk worldwide. Moreover, projected population growth patterns and climate change impacts will intensify these challenges.

## MATHEMATICS OF PLANET EARTH

Nearly a decade ago, the mathematics community launched the Mathematics of Planet Earth 2013 (MPE2013), a year-long initiative to showcase the ways in which mathematical sciences can be useful in tackling these global problems. Over the year, MPE2013 grew into an international partnership of more than 150 scientific societies, universities, research institutes, and professional organizations. MPE2013 underscored the multidisciplinary nature of the problems facing the planet and emphasized multidisciplinary partnerships to address these problems.

At the end of 2013, a new structure was designed to support ongoing research efforts and maintain the initiative's momentum. This publication — *Mathematics for Action: Supporting Science-Based Decision-Making* — is one of the many outcomes of these efforts.

*Mathematics for Action* is a collection of briefs highlighting the role of mathematics in addressing issues of global relevance. Written by 32 mathematicians and thought leaders from across the world, the 26 briefs showcase three types of topics:

- **Success Stories** — Mathematical concepts and tools that advance solutions to everyday problems, such as monitoring and predicting the spread of epidemics;
- **Mathematics Illuminated** — Mathematical concepts that help us understand and describe real-world processes;
- **Grand Challenges and Opportunities for Mathematics** — Pressing problems that mathematics can help solve, from food system resilience to climate change.

While many of the briefs share common themes or concepts, each brief can be read independently or out of sequence. Together the collection emphasizes the strength and potential of the mathematical sciences to meet global challenges, and highlights opportunities and innovative approaches that may have broader applicability to science-based decision-making.

## TOOLKIT ROADMAP

The 2030 Agenda for Sustainable Development, adopted by all United Nations Member States in 2015, provides a shared plan of action for people, the planet, prosperity, and peace. The 17 Sustainable Development Goals stimulate action over the next 15 years in areas of critical importance for humanity. The 26 briefs are organized by these goals and address 11 of the 17 goals:



### Goal 1: End poverty in all its forms everywhere.

*Visualizing Poverty* details mathematical techniques for collecting and mapping poverty data — techniques as accurate and more efficient than traditional survey-based methods.



### Goal 2: End hunger, achieve food security and improved nutrition and promote sustainable agriculture.

*Strengthening Food Security* describes mathematical approaches that can help identify shocks and design optimal mitigation and adaptation strategies for building food-system resilience.



### Goal 3. Ensure healthy lives and promote well-being for all at all ages.

Five briefs address topics related to the Sars-CoV-2 pandemic. *Modeling Infectious Diseases* provides the foundation for how infectious diseases are mathematically modeled and what can be learned from these models. *Harnessing the Power of Data* details some of the mathematically-grounded and locally-nuanced pandemic response efforts underway in Africa. *Improving Pandemic Forecasts* describes how a state-of-the-art technique from weather forecasting was used to enhance the prediction accuracy of



COVID-19 models. *Enhancing Vaccine Design* describes the novel ways mathematics has helped accelerate the design, testing, and monitoring of new vaccines, including the Sars-CoV-2 vaccines. Finally, *Modeling Vaccine Hesitancy* considers the phenomenon known as the free-rider problem in the context of vaccine hesitancy, and what this means for decision-makers.



**Goal 4: Ensure inclusive and equitable quality education and promote lifelong learning opportunities for all.**

*Teaching Mathematics* reflects on the importance of mathematics education and the role of mathematics teachers in improving students' learning outcomes and socioeconomic mobility.



**Goal 5. Achieve gender equality and empower all women and girls.**

*Tracking Gender Parity* examines the mathematical and statistical underpinnings of indicators used to measure and track the legal, economic, social, and cultural factors contributing to a gender gap.



**Goal 6. Ensure availability and sustainable management of water and sanitation for all.**

*Managing Water Resources* highlights how a statistical tool known as Bayes Theorem can be used to quantify risks and identify appropriate options for the management of water supplies. *Shifting Lake Turbidity* highlights how mathematics can provide insights into the mechanisms that drive shallow lakes from clear to turbid and support effective, cost-efficient, and sustained approaches to restore clarity.



**Goal 9. Build resilient infrastructure, promote inclusive and sustainable industrialization and foster innovation.**

*Reckoning with Uncertainty* shares lessons for responsible mathematical modeling that can help society demand the quality it needs from mathematical modeling.



**Goal 11. Make cities and human settlements inclusive, safe, resilient and sustainable.**

*Preparing for a Crisis* deals with the resilience of digitized systems and describes some of the mathematical methods and tools that have proven invaluable for addressing vulnerabilities in critical systems and processes and building more resilient systems and societies.



**Goal 12. Ensure sustainable consumption and production patterns.**

*Valuing Natural Capital* considers efforts to integrate the value of

ecosystem services in national development frameworks and describes the role of mathematics in strengthening these efforts. *Allocating Scarce Resources* illustrates how mathematics can support integrated approaches to food-energy-water nexus management and decision-making, including model-based solutions to prioritize and optimize investments.



**Goal 13. Take urgent action to combat climate change and its impacts.**

Five briefs address climate change, many aspects of which are intertwined in mathematics. *Modeling Climate* highlights the energy-balance model, a simple but powerful mathematical model that can help governments, policymakers, and the public understand Earth's past, present, and future climates. *Facing Future Climates* describes how mathematical models can provide information for policy-relevant and regionally-specific decision-making, allowing countries to scale up and accelerate adaptation and disaster reduction activities. *Forecasting Cyclones* describes how mathematical models are used to predict the path and intensity of tropical cyclones and their projected impacts. *Attributing Extreme Weather* details the new science of event attribution, which has enabled scientists to make quantitative statements about the influence of human-induced global warming on specific individual extreme weather events. Finally, *Pinpointing the Indian Monsoon* describes how mathematical models are used to predict the arrival, intensity, and duration of the Indian Summer Monsoon, a phenomenon vital to Indian society, agriculture, tourism, and economic development.



**Goal 14. Conserve and sustainably use the oceans, seas and marine resources for sustainable development.**

*Sustaining Fisheries* describes newer integrated mathematical models that capture the economic, social, and ecological drivers of fisheries and promise improved support for sustainable fisheries management and decision-making.



**Goal 15. Protect, restore and promote sustainable use of terrestrial ecosystems, sustainably manage forests, combat desertification, and halt and reverse land degradation and halt biodiversity loss.**

*Measuring Biodiversity* looks at quantitative diversity indices and describes innovative mathematical tools for choosing these measures and gathering and processing biodiversity data. *Listening in on Wildlife* describes innovative mathematical techniques that can provide a rapid, efficient way

to process wildlife sound data and ultimately better support biodiversity conservation efforts. *Battling Invaders* highlights mathematical models that can help scientists predict the impact of invasive species on native species and quantify efforts required to control and eradicate damaging invasive populations.



**Goal 16. Promote peaceful and inclusive societies for sustainable development, provide access to justice for all and build effective, accountable and inclusive institutions at all levels.** *Preserving Privacy* deals with federated learning, a new mathematical technique that supports building models trained in a distributed fashion such that private data never leaves a given participant or institution. This advancement will have significant ramifications for medicine, banking, and other areas where data privacy is paramount. Finally, *Finding the Missing* describes how complex networks can help support searches for people who go missing in connection with armed conflicts, other situations of violence, migration, and natural disasters.

While the 26 briefs cover a variety of mathematical applications, the collection is by no means exhaustive. But it does give an indication of the many diverse ways that mathematics can empower sustainable development.

**Mathematics compares the most diverse phenomena and discovers the secret analogies that unite them.**

— Joseph Fourier,  
French mathematician and physicist

# FINDING THE MISSING

## COMPLEX NETWORKS ENABLE SEARCHES FOR MISSING PEOPLE

Across the world, hundreds of thousands of people go missing in connection with human rights violations, armed conflicts, other situations of violence, migration, and mass disasters. These disappearances cause incalculable suffering for their families and communities and are an obstacle to peace. Policymakers need better data and tools to objectively address disappearances and advance humanitarian objectives. Complex network analysis can be a powerful instrument for searching for and collecting relevant information on the missing in multiple contexts. Moreover, such approaches have the potential to highlight information that otherwise would not be evident.

International humanitarian and human rights laws contain provisions to ensure the “dead are managed in a proper and dignified manner and to clarify the fate and whereabouts of missing persons.” A lack of reliable statistics on the number of missing persons as well as decentralized systems to address the problem challenge the design and implementation of policies to address disappearances. In some cases, the mathematics of complex networks combined with statistical techniques can assist the search process by making it possible to exploit clues to identify clusters of people that may share the same fate.

### THE INVISIBLE LINKS IN NETWORKS

A complex network is a set of connected nodes that interact in different ways. Systems that take the form of a network are common and wide-ranging in the world, from food webs to postal delivery routes, high-voltage transmission networks, and social networks of friends or other connections of individuals, business relationships, or organizations. Studies suggest that properties of complex networks — notably, community structure and hierarchical organization — can help explain the behavior of the underlying systems. For example, groups of strongly connected nodes often correspond to known functional units on the system.

In some cases, networks contain a lack of information, both about nodes and links. In recent years, scientists have developed mathematical techniques to predict missing network connections in real-world systems.

### KEY MESSAGES

- ✓ A complex network consists of nodes connected by links. In social networks, nodes represent individuals and the connecting links are the relationships between the respective individuals.
- ✓ Complex networks provides a powerful tool to help clarify the fate of missing people by classifying individuals sharing common properties or other similarities within groups or clusters.
- ✓ The structure of a complex network helps researchers to suggest hypotheses that can be explored later using other information and also tested statistically.
- ✓ The structure of a complex network can be further honed to refine results. This could be done, for instance, by classifying the connections into strong or weak connections, or by assigning a number to represent connection strength.

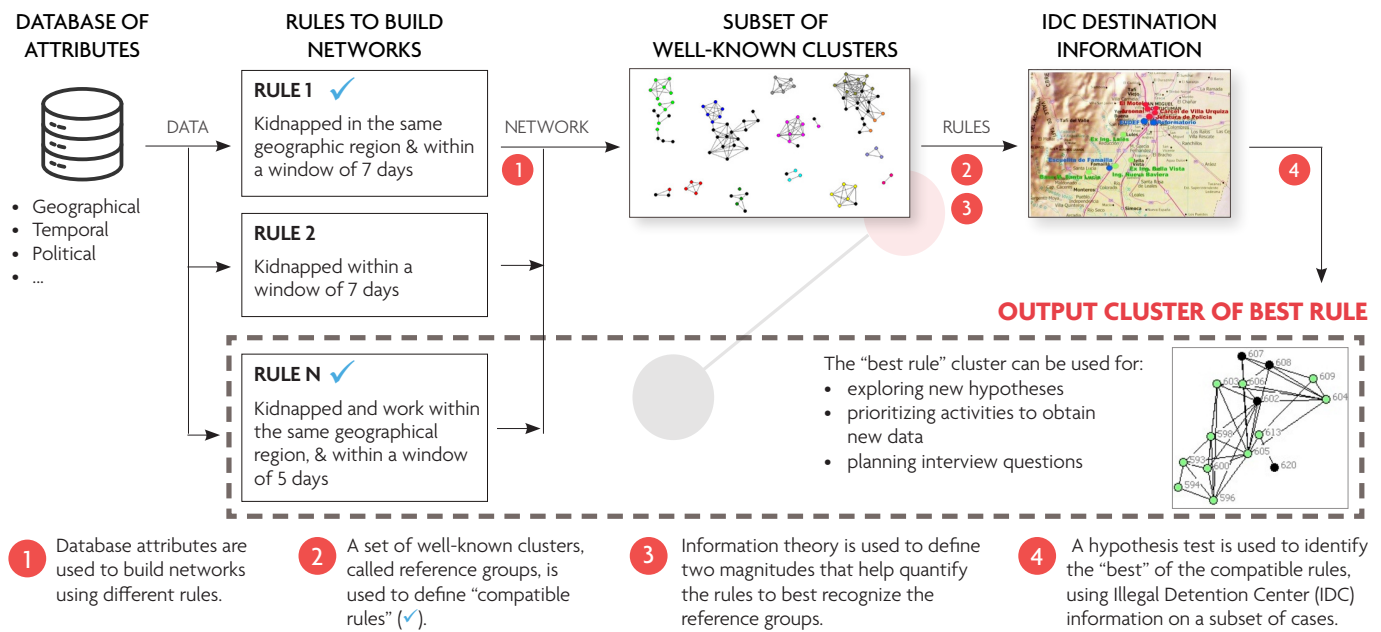
### EXPLOITING CLUES

In 2011, scientists with the Argentine Forensic Anthropology Team and Conicet set out about building a complex network reflecting people who disappeared during the country’s military dictatorship that was in power from 1976 to 1983. Nodes represent individuals, and links between the nodes represent explicit or implicit relationships between the individuals at some point. For example, the individuals may have lived in the same area, belonged to the same political group, or disappeared inside an interval of 7 days. These relationships could vary over time.

Clusters of interconnected nodes emerged within the network. The nodes were classified based on how much information was known of the individual’s fate. The team first used anthropological and forensic information to classify 64 nodes into 12 color-coded reference groups. After building the network, these nodes connect with 41 other nodes, colored black. Considering this subset of 105 nodes, quite a bit was known about 64 of them, including their fates, and less was known about the destinies of the remaining 41 nodes.

The team’s goal was to determine a structure of clusters in the complex network compatible with these reference groups. Black nodes in a cluster of colored nodes might

# CONNECTING THE DOTS TO HELP LOCATE ARGENTINA'S DISAPPEARED



suggest the individuals share the same fate. For example, they may have been held captive in the same illegal detention center. It also suggests what other information needs to be collected, who should be interviewed, and what questions need to be asked.

There are many different ways to choose relationships between individuals in these analyses, and different relationships may lead to different networks. Determining which one to choose involves testing different types of relationships to find a set of compatible rules that best discriminates the reference groups. To select the set of compatible rules that generates the best clusters, the team used Illegal Detention Center (IDC) destination data in combination with statistical inference techniques.

The method was also applied to investigate the circuit of Illegal Detention Centers, where several thousand people were held captive during the Argentinian dictatorship. Two criteria were used for relationships. Strong relationships were those between individuals with the same political affiliation and kidnapped in the same region within a window of 7 days. Weak relationships were those between individuals of the same region, which disappeared in a window of 3 days.

These techniques can be used globally to analyze and connect information concerning the fate of other missing people. Relationships between travel companions along African or Central American migratory routes, for example, might shed light on their whereabouts and assist search and information-collecting activities. In migration contexts, it could be particularly helpful to work with networks at different locations and times, for example, before the departure, during the trip, and upon arrival to the final

destination. Nodes or links may change over time, and the corresponding clusters associated with a given node may suggest a set of clues that, exploited together, reveal better information.

## CONCLUSIONS

Complex networks have been studied across a range of topics, from genetics to scalable communication networks, vaccination strategies, and ecosystem stability and function. Sophisticated mathematical techniques, combined with computational tools, can help understand and exploit a complex network's very rich structure and dynamics to help answer fundamental questions that challenge humankind. Such advances will require multi-disciplinary efforts.

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