
	<p>Contents lists available at www.eelet.org.uk/ShowPage.aspx?ID=7</p> <p>European Economics Letters</p> <p><i>Semi Annual Online Journal</i></p> <p>ISSN 2323-5233, ISSN-L 2323-5233 Journal homepage: www.eelet.org.uk</p>	
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Economic Examples of Latent Budget Analysis

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ARTICLE INFO

Article history:

Received 1 July 2013

Accepted 7 October 2013

Available online December 2013

JEL classification:

C140

L660

R530

E220

H540

Keywords:

Compositional data

Latent budget model

Capital composition data

Food composition data

ABSTRACT

Latent budget analysis is a classification technique that allows clustering identification by using compositional data. This paper presents examples of how this technique deals with the unit-sum constraint by establishing an initial independence model to which subsequent models are compared in terms of their relative fitness degree. In fact, latent budget analysis does not impose linearity, homogeneity, or even specific distributions on data. Examples analyzed in this work help to understand some important relationships between capital stock composition and income or food diet composition in a heterogeneous sample of countries.

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1. Introduction

Compositional data have restrictions for applying traditional multivariate analysis due to, among other things, the absence of an interpretable covariance structure (Aitchison, 1986). The latent budget analysis approach of compositional data is a real alternative in terms of efficiency and versatility for dealing with the analysis of this constrained data even though the sensitive advances made it from the log-ratio approach of the CODA school research contributions. Latent budget analysis is a mixture model for the analysis of two-way tables with frequency data where the purpose is to analyze the relationship between the rows and the columns. The frequencies are replaced by conditional proportions. Hence each row in the table has nonnegative entries that add up to 1. Such rows are called budgets and each budget forms the observed distribution of the column variable. The latent budget model assumes that a small number of latent distributions (latent budgets) have generated the observed distributions (observed budgets) in the two-way table.

The goal of this contribution is to briefly introduce this methodology for classification, particularly focused in economic examples. Results are easy to grasp and promising in the facilitating the identification of cluster in data that is highly difficult to direct statistical analysis.

The description of the model is heavily based on van den Ark (1999) and tries to wide the knowledge of latent budget methodology for economic research application. The interesting point is that compositional data abound as a part of raw or transformed economic data so this brief contribution focus on the better use of this kind of information. The work follows with section 2 where the latent budget model is described, section 3 with examples from economics and section 4 ends the paper with short conclusions.

2. The Latent Budget Model¹

The Latent Budget Model (LBM) is a mixture model for compositional data and enables us to obtain insights in a compositional data set without the worries of a troubled covariance matrix. By performing latent budget analysis (LBA) we approximate I observed budgets, which may represent persons, groups or objects, by a small number of latent budgets, consisting of typical characteristics of the sample. Such approximation could be used for classification, for example.

Consider and $I \times J$ compositional data matrix P , consisting of I observed budgets p_i , with components p_{ji} . In the LBM the observed budgets p_i 's are approximated by expected budgets π_i , which are mixtures of K ($K = \min(I, J)$) typical compositions or latent budgets. The latent budgets are denoted by β_k , $k = (1, \dots, K)$, and the model can be written as

$$(1) \quad \pi_i = \alpha_{i1}\beta_1 + \dots + \alpha_{ki}\beta_k + \dots + \alpha_{Ki}\beta_K \quad (i=1, \dots, I)$$

where α_{kij} ($i=1, \dots, I; k=1, \dots, K$) are mixing parameters.

The elements of π_i and $\pi_{j|i}$ are called expected components. The elements of β_k are $\beta_{j|k}$ ($j=1, \dots, J$) and are called latent components. An alternative notation for (1) is the scalar notation

$$(2) \quad \pi_{j|i} = \sum_{k=1}^K \alpha_{kij} \beta_{j|k} \quad (i=1, \dots, I; j=1, \dots, J)$$

and the matrix notation

$$(3) \quad \Pi = AB^T$$

In (3) Π is a $I \times J$ matrix whose rows are the respected budgets; A is an $I \times K$ matrix of mixing parameters, and B is a $J \times K$ matrix whose columns are the latent budgets. The latent budget model with K latent budgets is denoted as LBM (K). Similar to the observed components, the parameters of the LBM are subject to the sum constraints

$$(4) \quad \sum_{j=1}^J \pi_{j|i} = \sum_{k=1}^K \alpha_{k|i} = \sum_{j=1}^J \beta_{j|k} = 1$$

and the nonnegativity constraint

$$(5) \quad \begin{aligned} 0 &\leq \pi_{j|i} \leq 1, \\ 0 &\leq \alpha_{k|i} \leq 1, \\ 0 &\leq \beta_{j|k} \leq 1. \end{aligned}$$

Thus, all the parameters are proportions and this facilitates the interpretation of the model. In fact, it has been argued that its ease of interpretation is one of the main reasons to use LBA (for example, de Leeuw and van der Heijden, 1988; de Leeuw et al., 1990; van der Ark and van der Heijden, 1998).

If the data have a product-multinomial distribution, we can compute the unconditional expected probabilities π_{ij} from the expected components. The following properties hold for the expected components and the corresponding unconditional probabilities:

$$\pi_{ij} = \pi_{ij} / p_{i+},$$

$$(6) \quad p_{i+} = \pi_{i+},$$

$$p_{+j} = \pi_{+j}$$

(see de Leeuw *et al.* (1990)).

Van der Heijden, Mooijaart, and de Leeuw (1992) proposed two ways to interpret the model, which we will call the mixture model interpretation and the MIMIC-model interpretation (Multiple Indicator Multiple Cause-model). Up to now we have treated the LBM as a mixture model and the interpretation given earlier is: the LBM writes the expected budgets as a mixture of a small number of typical, or latent, budgets. Hence, each expected budget is built up out of the K latent budgets, and the mixing parameters determine to what extent. The latent budgets can be characterized by comparing them with the latent budget LBM(1). LBM(1) is the independence model, with $\alpha_{1|i} = 1$ ($i=1, \dots, I$), and $\beta_{j|1} = p_{+j}$ ($j=1, \dots, J$), in this case $\pi_i = \beta_1$ for $i=1, \dots, I$. Hence, if latent component $\beta_{j|k}$ is greater than that component in the independence model, p_{+j} , then β_k is characterized by the j -th category.

On the other hand, if a $\beta_{j|k}$ is less than p_{+j} then the j -th category is of lesser importance. The budget proportions $\pi_k = \sum_i p_{i+} \alpha_{k|i}$ express the relative importance of each latent budget, in terms of how much of the expected data they account for. At the same time, π_k , ($k=1, \dots, K$) denotes the probability of latent budget k when there is no information about the level of the row variable. To understand how the expected budgets are constructed from the latent budgets we must compare the mixing parameters to π_k . If $\alpha_{k|i} > \pi_k$ then the expected budget π_i is characterized more than average by latent budget β_k , and if $\alpha_{k|i} < \pi_k$ then the expected budget π_i is characterized less than average by latent budget β_k . In practice the mixture model interpretation is carried out most easily when we first characterize the latent budgets and then interpret the expected budgets in terms of the latent budgets.

Now, for interpreting the LBM as a MIMIC-model, we look at the observed components as conditional proportions of the row variable X , with I categories, and the column variable Y , with J categories. If we assume that the row variable and the column variable are independent given some latent variable Z with K categories, then the LBM describes the relationship between the row variable and the column variable in an asymmetric way, i.e. $\pi_{j|i} = P(Y = j | X = i)$ denotes the probability to respond to category j of Y , given that one belongs to the i -th category of X ; these probabilities are explained by $\alpha_{k|i} = P(Z = k | X = i)$ which is the probability that row category i belongs to latent category k , and $\beta_{j|k} = P(Y = j | Z = k)$ which is the probability that a member of latent category k responds to the j -th category of Y .

¹ See van den Ark (1998: 8-13)

If the compositional data do not have a product multinomial distribution then the *MIMIC*-model interpretation may be troublesome: for example, if each observed budgets represents a multivariate observation on a single subject, then it is unclear what $P(Y = j | Z = k)$ means. If the rows of the compositional data are not independent, for example if they denote groups, and people may belong to more than one group, then $P(Z = k | Y = i)$ is not well defined.

The parameters estimates of the LBM model should be identified before the latent budget solution can be analyzed. We follow with some examples from economics.

3. Examples from Economics

In this section we will present examples of application of LBA to economic data. We will define some economic problem to identify through LBA. When feasible, raw data will be presented and estimations will be interpreted. Following Ark (1998: 164) we will identify latent budgets by following three rules-of-thumb:

- Selecting the latent budget whose proportion of lack of fit is as large from baseline model as possible.
- The improving of adding an extra latent budget should be large enough to identify the extra effort of interpreting a larger set of parameter estimates. Ark (1998) proposes that the average improvement of fit per degree of freedom should be at least as great as the difference between the weighted Residual Sum of Squares (wRSS) of the baseline model less the observed wRSS divided by the degrees of freedom.
- Finally, the result should be interpretable.

For a matter of exposition we are going to follow clause c with more emphasis than advisable. We follow with the first example.

3.1 Capital composition and income classification

The first application of the latent budget analysis for a classification example is on capital composition and income. In Table 2 we see the average participation of each capital component for the 1960-1990 time periods (data were used previously in Larrosa (2005)). The definition of the components is presented in Table 1.

Table 1 – Major destinations for international tourist's arrival and tourism receipt

Code	Description
<i>KDUR</i>	Percentage of capital per worker allocated in durable production assets (machinery and equipment).
<i>KOTHR</i>	Percentage of capital per worker allocated in other buildings.
<i>KNRES</i>	Percentage of capital per worker allocated in non-residential building.
<i>KRES</i>	Percentage of capital per worker allocated in residential building.
<i>KTRAN</i>	Percentage of capital per worker allocated in transportation equipment.

We average and closed each capital component by income category. Data are presented in Table 2 and it shed light in the long run average capital composition of the sample of countries.

Notice that poorer countries possess a lower than average transportation equipment participation on capital stock and that a high-income-OECD country and a lower-middle-income country have above average residential building capital participation. Besides, poorer countries have an above from average participation of other buildings. There

are 56 countries in the sample. As the sample is not large and data have not additive lognormal distribution we could not rely on maximum likelihood estimates. We use least squares for estimating latent budgets on the model (Table 2).

Table 2 – Capital composition by income category (n=56)

	KTRAN	KOTHR	KDUR	KRES	KNRES
low income [9]	0.0092	0.3204	0.1467	0.2230	0.3008
lower middle income [13]	0.0292	0.3308	0.1536	0.3279	0.1586
upper middle income [9]	0.0289	0.2389	0.1712	0.2521	0.3090
high income OECD [19]	0.0443	0.2100	0.1927	0.3424	0.2106
high income non-OECD [6]	0.0214	0.2255	0.1835	0.3244	0.2452

Source: column variables: Data from Penn World Table 5.6; row classification: 2000 World Development Indicators – The World Bank. Brackets indicate the number of countries that belongs to this category.

Table 2 – Estimations of mixing parameters and identified latent budgets

Identified Mixing Parameters				
	LB 1	LB 2	LB 3	independence
LowInc	0.495	0.121	0.384	1
LowMdlIn	0.512	0.285	0.203	1
UpMidIn	0.371	0.232	0.397	1
HighInc	0.326	0.402	0.272	1
HighInc	0.347	0.345	0.308	1
π_k	0.410	0.277	0.313	1
Identified Latent Budgets				
	LB 1	LB 2	LB 3	independence
Ktran	0	0.096	0	0.027
Kothr	0.646	0	0	0.265
Kdur	0.059	0.284	0.214	0.17
Kres	0.295	0.620	0.004	0.294
Knres	0	0	0.783	0.245

Interpretation requires going latent budget by latent budget. The first budget associates low-income countries with a higher presence of other capital al also residential capital. This would be the *low-middle income budget*. The second latent budget remarks the relation of residential capital and durable goods in OECD and non-OECD high income countries. This would be the *high income budget*. Finally, the third budget associates the non residential capital with a wider scope of countries, such middle-up and lower income countries. This would be the mid-low income budget. So far, estimations don't shed much light into the capital composition question. We are going to see if widening the sample helps to arrive to clearer conclusions.

3.1.1 Example 1 revisited

The former example was completed by sample averaging on World Bank's income categories. We review the example by estimating latent variables that reveals capital composition among countries. So, this time, LBA is related to specific country data. This would help us to see any latent variable that conditioning capital composition in this sample. Since it would make no sense to assume a multinomial distribution and to apply a maximum likelihood estimation procedure, our weighted least squares approach seems reasonable. The weights used are $1/p_j^{1/2}$ for the columns.

I estimate latent budgets that determine unobserved patterns in capital composition in the sample. As the sample is not

large and data have not additive lognormal distribution² we could not rely on maximum likelihood estimates. We use instead weighted least squares. First, we try to identify the model by analyzing the dissimilarity index and the mean angular deviation (van den Ark, 1999: 126-127).

Table 3 – Identification of model 2

Latent Budgets	LBM(K)	
	<i>l - D</i>	<i>m.a.d.</i>
1/J	71.2739	0.5915
K=1	80.4209	0.3945
K=2	86.809	0.2635
K=3	90.6867	0.1955
K=4	98.4739	0.0314

As observed, as the number of latent budgets increases, the goodness-of-fit statistics (GFS) also increases. We decide for three latent budgets because of ease of interpretation and explanation of results.

The parameters can be interpreted as follows. First, we interpret the latent budgets by comparing them to the independent model. The first latent budget has greater proportions of other kinds of buildings in the capital stock. We could call it the “other buildings budget”. The second latent budget has greater values of transportation equipment and durable goods than the independent model. We could call this budget as the “durable goods budget”. Finally, residential and nonresidential buildings have the greater values of the third latent budget that is a sensitive preeminence of buildings. This would be the “building budget”. This finding is congruent with Larrosa (2003) results, but in that case I utilized the log-ratio approach. The interpretation of these latent budgets con each country in particular could be difficult, because quite different individuals are in the same budget.

Another way of looking at the findings is through plotting the latent budgets in ternary diagrams. Figures 1 and 2 show the same data that Table 4. Figure 1 presents latent components estimates and Figure 2 displays the calculated LBM (3). As observed, latent budget 3 (LB 3) groups most data from countries. That is, the building budget. At the same time, one can see that the durable goods budget is what discriminates the sample.

3.3 Food expenditure

Food expenditure represents the money individuals from an economy spend in feeding. This is related with socioeconomic conditions but also for cultural and religious items. I have data for the 1970 total expenditure on 32 food components from Belgium (BEL), Colombia (COL), Germany (DEU), France (FRA), U.K. (GBR), Hungary (HUN), Iran (IRN), India (IND), Italy (ITA), Japan (JAP), Kenya (KEN), South Korea (KOR), Malaysia (MYS), Netherlands (NLD), Philippines (PHI), and USA (USA).

I will try to find the latent diet of these countries by calculating the first three (unconstrained) latent budgets. Goodness-of-fit statistics show that four latent budgets could better identify the model but for ease of explanations and results visualization I stick on three. Table 6 shows the parameter estimates of the identified mixing parameters with the name of the 32 food components.

Figure 1. Graphic display of latent components

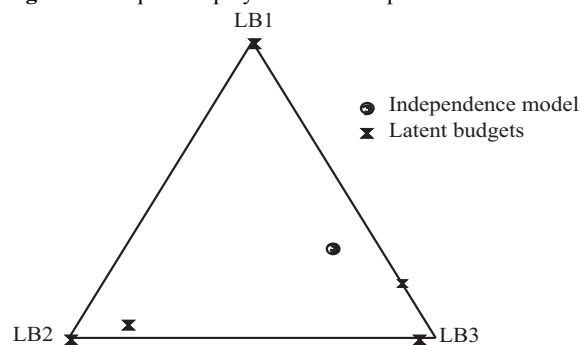
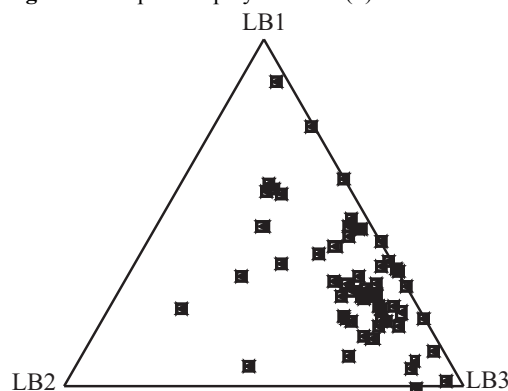


Figure 2. Graphic display of LBM (3)



Interestingly, the interpretation begins with latent budget one that represents all Western countries in the sample. This would be the *Western diet budget*. Related to this culture pattern I could match a basket of food items as represented by the mixing parameters relatively to the budgets proportions: So, Western diet budget includes meat preparations, coffee, beef and veal, chocolates, cereals, butter, other fresh meats, mineral water and other food components. The second latent budget reflects a higher value for countries other than Western. The majority are Asian countries, so this would be the *Asian diet budget*. This basket reflects higher than average presence of rice, salt and spices, fresh and processed fish, flour and cereals, potatoes, jams and jellies, pastas and sugar. Finally, the third latent budget is highly represented by a Muslim country like Iran, with low participation of other Western and African countries. The most important food component is tea. So, this would be the *tea diet budget* and indicates a higher presence of, of course, tea, cheese and preserved milk, fresh fruits, flour and cereals, bread, margarine and oils and other foods components. The picture as described is quite easy to grasp and this is what is really attractive about this technique.

The communication ends with the conclusions.

² Tests results are available upon request.

Table 4 – Parameter estimates of the identified mixing parameters

Country	LB 1	LB 2	LB 3	Country	1	LB 2	LB 3	
ARG	0.199	0.003	0.797	NOR	0.325	0.399	0.276	
AUS	0.201	0.202	0.597	PAN	0.569	0.213	0.218	
OST	0.282	0.141	0.577	POR	0.271	0.086	0.643	
BEL	0.269	0.176	0.555	SLE	0.468	0.275	0.257	
BOL	0.887	0.030	0.083	SWE	0.229	0.095	0.676	
BOT	0.149	0.182	0.669	SWI	0.177	0.081	0.741	
CAN	0.351	0.036	0.613	SYR	0.234	0.066	0.701	
CHL	0.457	0.036	0.507	TAI	0.312	0.175	0.513	
COL	0.604	0.002	0.394	THAI	0.411	0.122	0.467	
DEN	0.24	0.100	0.660	TUR	0.269	0.129	0.602	
DOM	0.365	0.011	0.624	UK	0.091	0.244	0.665	
ECU	0.758	0.008	0.234	USA	0.194	0.111	0.694	
FIN	0.266	0.092	0.642	VEN	0	0.123	0.877	
FRA	0.21	0.198	0.592	GUA	0.592	0.197	0.211	
GER	0.257	0.124	0.620	KEN	0.299	0.074	0.627	
GRE	0.47	0.055	0.475	MAD	0.562	0.181	0.257	
HON	0.227	0.596	0.176	MAL	0.179	0.128	0.693	
HKG	0.061	0.511	0.428	MOR	0.342	0	0.658	
ICE	0.108	0.028	0.863	NIA	0.459	0.029	0.512	
IND	0.437	0.072	0.491	PAR	0.055	0.108	0.837	
IRE	0.142	0.162	0.696	PER	0.487	0.042	0.472	
ISR	0.078	0.087	0.835	PHI	0.022	0.04	0.939	
ITA	0.194	0.1	0.705	SPA	0.336	0	0.664	
IVC	0.284	0.109	0.607	SRL	0.426	0	0.574	
JAM	0.356	0.281	0.362	ZIM	0.293	0	0.707	
JAP	0.391	0.173	0.436	π_k	0.312	0.127	0.561	
KOR	0.219	0.05	0.731	Latent budgets			p_{-i}	
LUX	0.327	0.105	0.568	KTRAN	0	0.252	0	0.031
MEX	0.302	0.146	0.553	KOTHR	0.845	0	0	0.264
NET	0.195	0.187	0.619	KDUR	0.045	0.727	0.12	0.174
NZL	0.578	0.19	0.232	KRES	0.111	0	0.475	0.301
				KNRES	0	0.021	0.405	0.23

Table 5 – Mixing parameters and latent components of the LBM (3) solution

Food component	LB 1	LB 2	LB 3	Independ.	Food component	LB 1	LB 2	LB 3	Independ.
1. Rice	0	0.944	0.056	1	26. Teas	0.166	0.2	0.634	1
2. Flour and cereals	0	0.765	0.235	1	27. Cocoa	1	0	0	1
3. Bread	0.673	0.109	0.218	1	28. Raw & refined sugar	0.331	0.461	0.207	1
4. Other bakery products	0.687	0.255	0.058	1	29. Jam/jelly/honey/s	0.404	0.526	0.07	1
5. Other cereal products	0.759	0.145	0.096	1	30. Chocolate & confetti	0.898	0.067	0.035	1
6. Macaroni & similar pasta	0.489	0.511	0	1	31. Salt, spices & sauces	0.154	0.842	0.004	1
7. Beef & Veal	0.833	0.129	0.038	1	32. Mineral water	0.608	0.358	0.034	1
8. Lamb, mutton & go	0	0	1.00	1	π_k	0.423	0.449	0.128	
9. Pork	0.647	0.353	0	1	Latent budgets				
10. Poultry	0.607	0.308	0.085	1	BEL	0.093	0	0	0.042
11. Other fresh-frozen meats	0.677	0.271	0.052	1	COL	0.154	0.006	0	0.065
12. Meat preparations	0.984	0	0.016	1	DEU	0.091	0	0	0.036
13. Fresh/frozen fish	0.252	0.748	0	1	FRA	0.093	0	0.01	0.042
14. Preserved/proc/dried seafood	0.266	0.734	0	1	GBR	0.069	0	0.073	0.034
15. Fresh milk	0.502	0.408	0.09	1	HUN	0.118	0.012	0	0.057
16. Preserved milk/Other milk products/Cheese	0.589	0.109	0.302	1	IND	0	0.204	0.114	0.12
17. Eggs	0.602	0.289	0.11	1	IRN	0	0	0.688	0.085
18. Butter	0.781	0.037	0.182	1	ITA	0.099	0.027	0.013	0.06
19. Margarine, oils & fat	0.311	0.49	0.199	1	JPN	0.033	0.116	0	0.052
20. Fresh fruit	0.504	0.292	0.204	1	KEN	0.025	0.116	0.1	0.086
21. Fresh vegetables	0.428	0.433	0.139	1	KOR	0	0.201	0	0.092
22. Dried fruit & nut	0.659	0.19	0.151	1	MYS	0.043	0.107	0	0.065
23. Dried/frozen/prepared vegetables	0.505	0.391	0.104	1	NLD	0.087	0.002	0.002	0.04
24. Potatoes & other tuber /Potato products	0.381	0.518	0.1	1	PHL	0.043	0.194	0	0.098
25. Coffees	0.896	0.104	0	1	USA	0.052	0.014	0	0.027

4. Conclusions

Latent budget analysis is an attractive alternative for dealing with compositional data. As shown in the previous sections, it is easy to use and to interpret the results. LBA deals with highly problematic and widely available data. This contribution tries to be an introduction for more insightful and in-depth texts, as van den Ark (1999) is. Examples show how LBA discriminates group of countries by capital composition and levels of income and then introduces the algorithm inside a complex international food consumption compositional dataset and find interesting diet groups.

As described above, LBA seems to be ideal for statistical classification and searching for unobserved patterns in data. These kinds of problem are a common in social sciences and LBA seems a practical tool for classification. For this

to be done, however, it is necessary the use of specific software and some products are freely available.

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