Determination of nongeometric effects: equivalence between Artmann's and Tamir's generalized methods

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This work shows that all first- and second-order nongeometric effects on propagation, total or partial reflection, and transmission can be understood and evaluated considering the superposition of two plane waves. It also shows that this description yields results that are qualitatively and quantitatively compatible with those obtained by Fourier analysis of beams with Gaussian intensity distribution in any type of interface. In order to show this equivalence, we start by describing the first- and second-order nongeometric effects, and we calculate them analytically by superposing two plane waves. Finally, these results are compared with those obtained for the nongeometric effects of Gaussian beams in isotropic interfaces and are applied to different types of interfaces. A simple analytical expression for the angular shift is obtained considering the transmission of an extraordinary beam in a uniaxial–isotropic interface.

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1. INTRODUCTION

When a finite beam impinges on the surface of separation between two media of any type (linear or nonlinear, isotropic or anisotropic, right- or left-handed), the reflected and transmitted beams may present some peculiarities from the point of view of geometric optics. These peculiarities, known as nongeometric effects, depend on the media involved and the characteristics of the incident beam (shape, angle of incidence, polarization, and frequency). In the sixteenth century, Newton [1] had anticipated that the total reflection could not occur at the interface. He suspected that light beams followed parabolic trajectories whose vertices were located in the least dense medium.

The laws of reflection and refraction and the amplitudes of the reflected and refracted waves in the isotropic interfaces (Fresnel formulas) were determined in the early nineteenth century. By the end of that century, it was possible to use Maxwell's equations to show that, when the angle of incidence of a plane wave is larger than the angle of total reflection, the continuity of the electromagnetic field along an isotropic interface requires an evanescent wave in the second medium, which is optically less dense. It was then that a new idea emerged: in order to formally demonstrate the penetration of the wave in the least dense medium, the finiteness of the beam needed to be considered. Finally, between 1925 and 1929, the problem of total reflection was formally solved by considering a transversely limited beam (or bidimensional beam). However, these papers by Picht [2,3], which are based on energetic considerations, are not clear as to what the physical process involved is. Numerous experiences were performed in order to determine the characteristics of the evanescent wave, but all attempts failed until 1941. Between 1947 and 1949, Goos and Hänchen [4,5] performed the first experiments in conditions of total reflection where the energy flux the least dense medium was not perturbed. Goos and Hänchen's idea consisted of measuring the longitudinal displacement of the reflected beam instead of measuring the penetration in the least dense medium. This is why the longitudinal lateral displacement was named "Goos–Hänchen effect." This effect was satisfactorily explained for the first time by Artmann [6] in 1948. Artmann applied theories of physical optics to very simple bidimensional beams (in addition to monochromatic and linearly *s*- or *p*-polarized beams) in linear, isotropic, and homogeneous interfaces. He obtained expressions for the lateral displacement based on the phase difference between the incident beam (consisting of two plane waves of the same amplitude) and the reflected beam. The method applied is called the stationary phase method. In 1977, McGuirk and Carniglia [7] extended Artmann's method by considering the incident beam as a superposition of infinite propagating plane waves (i.e., with an angular spectrum such that the evanescent components were excluded) which constituted a bidimensional, monochromatic, and linearly *s*- or *p*-polarized beam. This procedure, when applied to total internal reflection, anticipated the existence of a second-order effect called longitudinal focal shift of the reflected beam. They obtained, from the first and second derivatives of the phase of the

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reflection coefficient with respect to the angle of incidence, the magnitudes of the lateral displacement (which coincided with the ones obtained by Artmann) and the longitudinal focal shift for both polarization modes and in conditions of total reflection. That same year, Carniglia and Brownstein showed that the focal shift of the reflected beam in conditions of total reflection could be regarded as a consequence of the Goos–Hänchen effect if a geometric model was used [8]. That is, we can describe, in conditions of total reflection, the first-order nongeometric effect (longitudinal lateral displacement) based on the phase difference (which is obtained by methods used in physical optics). In turn, we can describe the second-order effect (focal shift) based on the first-order effect and geometric optics considerations (ray model).

In 1977, White *et al.* [9] explained the existence of a third effect (of the first order) in conditions of partial reflection. This effect was called longitudinal angular shift and had been identified by Ra *et al.* in 1973 [10]. From a bidimensional incident beam with Gaussian intensity distribution and by considering Fresnel's coefficients for each polarization mode, they showed that partial reflection alters the intensity distribution of the reflected beam. In 1986, while studying the nonspecular effects in bidimensional Gaussian beams reflected in multilayers, Tamir [11] found another second-order effect, which consisted in widening or narrowing of the beam waist in conditions of partial reflection. These papers consider the particular situation in which the waves constituting the beam do not undergo a phase shift upon reflection.

From that moment on, several authors have written about the nongeometric effects of Gaussian beams. Their papers extend the method introduced by Tamir in 1986, not only to polarized and nonpolarized tridimensional Gaussian beams, but also to the reflection and transmission in different types of isotropic and anisotropic interfaces in conditions of total or partial reflection [12–28]. These and other papers showed that transmitted or reflected tridimensional Gaussian beams can not only undergo the four nongeometric effects simultaneously, but also that four other effects are possible when considering limited beams in two directions (that is, tridimensional Gaussian beams): transversal lateral displacement, focal shift, angular shift, and modification of the beam waist (even if considering isotropic interfaces when the incident beam is polarized). Although measurements may be not significantly affected by these nongeometrical effects when the beams are spatially very wide, they can be technologically important when considering very precise angular metrology, ellipsometry, or phase shifts measurements. They can also be significant when some characteristics of beams in waveguides and optical fibers must be determined.

This work shows that the four longitudinal nongeometric effects corresponding to symmetric beams can be simply analyzed by using arguments which are analogous to those of Artmann's by considering limited beams formed by only two plane waves. This procedure is so simple that it describes the effects qualitatively and leads to quantitative results which coincide with those obtained for Gaussian bidimensional beams both in reflection and transmission in isotropic and anisotropic interfaces.

2. LONGITUDINAL GOOS–HÄNCHEN EFFECT IN TOTAL REFLECTION: ARTMANN'S BEAM

The simplest bidimensional beam consists in the superposition of two plane waves of the same amplitude and frequency (referred to as Artmann's beam). As it is known, if an Artmann's beam impinges an isotropic dielectric surface in conditions of total reflection (reflection coefficient of unit magnitude), the phase difference between both reflected waves causes the lateral displacement of the interference maximum or Goos–Hänchen effect (first-order effect) of the reflected beam. The longitudinal lateral displacement of the reflected beam obtained by Artmann's method [6] is

$$L = -\frac{1}{k\cos\bar{\alpha}}\frac{d\varphi}{d\alpha}\bigg|_{\bar{\alpha}}, \tag{1}$$

where k is the magnitude of the wavenumber vector in the medium from which the light is incident, $\bar{\alpha}$ is the mean angle of reflection [and of incidence, i.e., $\bar{\alpha} = (\alpha_1 + \alpha_2)/2$] and φ is the phase shift between the reflected and the incident electric field (Fig. 1). Its value is independent of the beam aperture, and the expression is identical to the one obtained in the total reflection of beams with Gaussian intensity distribution [11] which are not excessively narrow. On the other hand, the longitudinal focal shift (second-order effect) can be geometrically deduced from the expression for lateral displacement [8,11],

$$F = -\frac{1}{k}\frac{d(L\cos\alpha)}{d\alpha}\bigg|_{\bar{\alpha}} \tag{2}$$

which also coincides with the one obtained by Tamir for the reflection of Gaussian beams in dielectric interfaces. Next, we will show that the four first- and second-order nongeometric effects can be easily obtained by the superposition of two interfering plane waves. Then, we will apply the results to reflection and transmission.

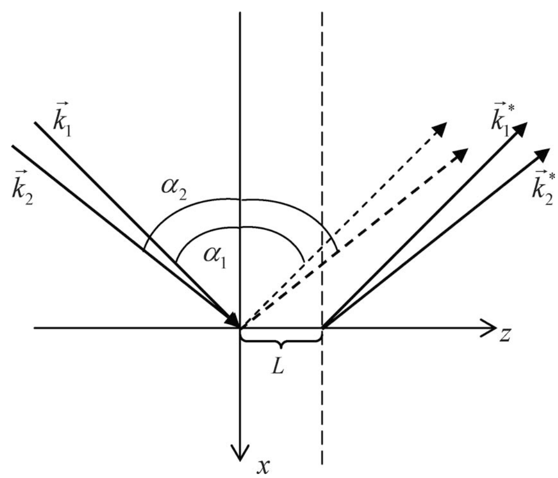

Fig. 1. Incident and reflected waves in total reflection. The dashed lines indicate the geometric reflected waves, and the solid lines indicate the nongeometric ones. L corresponds to the lateral displacement (Goos–Hänchen effect).

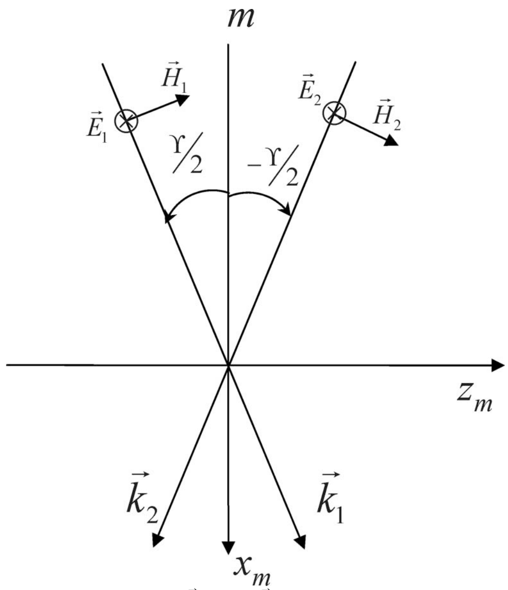

Fig. 2. Artmann's beam. $\vec{k}_1$ and $\vec{k}_2$ are the normals to the wavefronts of each component. Υ is the aperture of the beam, and m its mean direction. The purpose of the z_m axis is to set the origin of coordinates.

3. SUPERPOSITION OF TWO PLANE WAVES: GENERALIZATION OF ARTMANN'S METHOD

We will calculate the field resulting from two waves with different direction of propagation, amplitude, and phase. We will consider, in general terms for isotropic media, that the waves are perpendicularly polarized to the plane of incidence and they have the same phase velocity.

Figure 2 represents two plane waves with mean direction corresponding to the x_m axis. These make angles $\Upsilon/2$ and $-\Upsilon/2$. Υ denotes Artmann's beam aperture. The electric field corresponding to each wave (they have a geometric mean direction given by the straight line m in Fig. 2) is

$$\vec{E}_1(x_m, z_m, t) = A_1 e^{ik(\cos\Upsilon/2 x_m + \sin\Upsilon/2 z_m)} e^{-i\omega t} e^{i\xi_1} \breve{y}$$
$$\vec{E}_2(x_m, z_m, t) = A_2 e^{ik(\cos\Upsilon/2 x_m - \sin\Upsilon/2 z_m)} e^{-i\omega t} e^{i\xi_2} \breve{y}, \tag{3}$$

where ξ_1 and ξ_2 represent the initial phases, respectively.

By applying the principle of superposition [29], the temporal average of the Poynting vector associated with Artmann's beam is

$$\langle\vec{S}\rangle = \frac{1}{2}k(\breve{x}_m \cos\Upsilon/2[(A_1^2 + A_2^2) + 2A_1A_2\cos(2kz_m\sin\Upsilon/2 + \xi_1 - \xi_2)] + \breve{z}_m \sin\Upsilon/2(A_1^2 - A_2^2)). \tag{4}$$

As expected, the first maximum does not coincide with the origin of coordinates when there is an initial phase shift between the waves. It is located in a point that does not depend on their amplitudes, where

$$z_m = -\frac{\xi_1 - \xi_2}{2k\sin\Upsilon/2}. \tag{5}$$

On the other hand, the quotient between the components z_m and x_m in the maximums allows us to obtain the direction of the ray, that is, the angle formed by the Poynting vector and the x_m axis, which is independent of the phase shift of the waves constituting the beam. The quotient is

$$\tan\beta_S = \tan\Upsilon/2\frac{(A_1 - A_2)}{(A_1 + A_2)}. \tag{6}$$

If we consider that the amplitudes of the waves constituting Artmann's beam are not very different and that it is valid to make a second-order approximation,

$$A_1 = A(\Upsilon/2) \approx A|_{\beta=0} + \frac{dA}{d\beta}\bigg|_{\beta=0}\Upsilon/2 + \frac{1}{2}\frac{d^2A}{d\beta^2}\bigg|_{\beta=0}(\Upsilon/2)^2$$
$$A_2 = A(-\Upsilon/2) \approx A|_{\beta=0} - \frac{dA}{d\beta}\bigg|_{\beta=0}\Upsilon/2 + \frac{1}{2}\frac{d^2A}{d\beta^2}\bigg|_{\beta=0}(\Upsilon/2)^2. \tag{7}$$

Then, by replacing in Eq. (6),

$$\tan\beta_S \approx \tan\Upsilon/2\frac{\frac{dA}{d\beta}\Big|_{\beta=0}\Upsilon/2}{A|_{\beta=0} + \frac{1}{2}\frac{d^2A}{d\beta^2}\Big|_{\beta=0}(\Upsilon/2)^2}. \tag{8}$$

If we also consider that the beam aperture is not excessively large, we can write to the second order the angular shift from the geometric mean direction:

$$\beta_S \approx (\Upsilon/2)^2\frac{\frac{1}{A|_{\beta=0}}\frac{dA}{d\beta}\Big|_{\beta=0}}{1 + \frac{1}{2A|_{\beta=0}}\frac{d^2A}{d\beta^2}\Big|_{\beta=0}(\Upsilon/2)^2}. \tag{9}$$

Equation (9) clearly shows that both the first- and second-order expressions strongly depend on the aperture of the beam.

An analogous analysis can be performed if the phase shifts between the waves are not excessively large (which can be guaranteed if the aperture is not excessively large and if there are no phase jumps). In such a case, the phases can be expanded around the mean direction of incidence x_m, obtaining from Eq. (5):

$$\xi_1 \approx \xi|_{\beta=0} + \frac{d\xi}{d\beta}\bigg|_{\beta=0}\Upsilon/2 + \frac{1}{2}\frac{d^2\xi}{d\beta^2}\bigg|_{\beta=0}(\Upsilon/2)^2$$
$$\xi_2 \approx \xi|_{\beta=0} - \frac{d\xi}{d\beta}\bigg|_{\beta=0}\Upsilon/2 + \frac{1}{2}\frac{d^2\xi}{d\beta^2}\bigg|_{\beta=0}(\Upsilon/2)^2. \tag{10}$$

By replacing Eq. (10) into Eq. (5), we obtain that the displacement of the first interference maximum from the origin of coordinates can be written as

$$z_m|_{\max} = -\frac{1}{k}\frac{d\xi}{d\beta}\bigg|_{\beta=0}. \tag{11}$$

Thus, both first- and second-order longitudinal lateral displacements are identical and independent of the aperture of the beam.

We will apply the results obtained in this section to determine the angular shift in two different situations. First, we will consider the reflection of a beam in an isotropic dielectric interface. Since the nonspecular effects for beams with Gaussian distribution in isotropic interfaces were calculated by Tamir and are well known, we will show the equivalence between their results and the ones obtained by the generalization of Artmann's method. Then, we will apply this method to determine the nongeometric effects in the transmission of a limited beam in a uniaxial medium–isotropic medium interface for the case in which the incident beam is constituted by extraordinary waves.

4. NONSPECULAR EFFECTS IN ISOTROPIC DIELECTRIC INTERFACES

By determining the first-order nongeometric effects in the reflection of a bidimensional Gaussian beam (Fig. 3) incident on an isotropic interface, we obtain a complex expression which is only valid for beams that are not excessively narrow. The real part determines the longitudinal lateral displacement, and the imaginary part determines the angular shift [11]. This complex lateral displacement is different for each polarization state. Therefore, a nonpolarized beam gives rise to two reflected beams (*s*- and *p*-polarized). The expression for the complex lateral displacement obtained for Gaussian beams is

$$L = i\frac{d(\ln R)}{dk^*_{z_r}}\bigg|_{k^*_{z_r}=0} \equiv L' + iL'', \tag{12}$$

whereas the complex lateral displacement corresponds to

$$F = -2i\sqrt{\mu\omega^2\varepsilon_1}\,\frac{\partial^2 \ln R}{\partial k^{*2}_{z_r}}\bigg]_{k^*_{z_r}=0}, \tag{13}$$

where R corresponds to the reflection coefficient for the polarization mode considered and $k^*_{z_r}$ is the component of the wavenumber vector reflected in the direction that is perpendicular to the specular mean direction.

From Eq. (12), we can easily obtain that, if two isotropic and dielectric interfaces are considered and the mean angle of incidence is larger than the angle of total reflection, the lateral displacement L is real

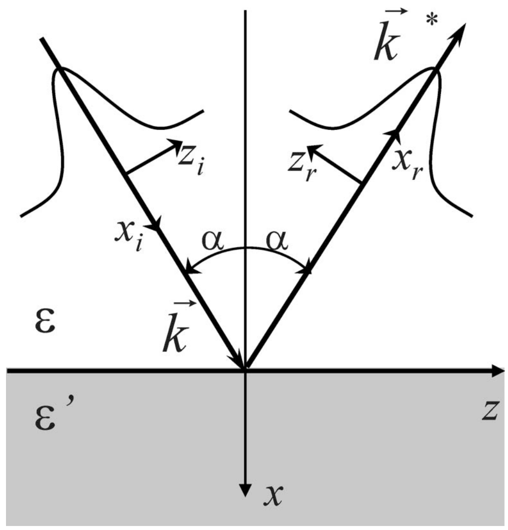

Fig. 3. Incident and geometrical reflected Gaussian beam on an isotropic dielectric interface ($x = 0$ corresponds to the interface); (x_i, z_i) and (x_r, z_r) indicate the incident and reflected coordinate systems; $\vec{k}$ and $\vec{k}^*$ correspond to the mean wave directions.

$$L' = -\frac{1}{k\cos\bar{\alpha}}\frac{d\varphi}{d\alpha}\bigg|_{\bar{\alpha}}, \tag{14}$$

where φ is the phase shift between the reflected and the incident waves. This expression is independent of the beam width and coincides with the one obtained by Artmann, considering a reflected bidimensional beam constituted by two waves with a mean direction of specular reflection $\bar{\alpha}$.

When the mean angle of incidence is smaller than the angle of total reflection, the reflection coefficient is real and the lateral displacement is pure imaginary, i.e.,

$$L'' = \frac{1}{k\cos\bar{\alpha}}\frac{1}{R}\frac{dR}{d\alpha}\bigg|_{\bar{\alpha}}. \tag{15}$$

This imaginary lateral displacement allows us to obtain the angular shift of the center of the reflected beam from the geometric mean direction of reflection. At the first order, the angular shift is

$$\Delta\alpha|_{\mathrm{Tamir}} \approx 2\frac{1}{k^2w^2}\frac{d(\ln R)}{d\alpha}\bigg|_{\bar{\alpha}}. \tag{16}$$

When there is an isotropic interface and the conditions are those of partial reflection, it follows from Eq. (16) that $\Delta\alpha|_{\mathrm{Tamir}}$ is positive. That is, the angle between the direction of the Poynting vector and the normal to the interface is larger than the geometric mean value of the angle of reflection.

The second-order correction to the angular shift only affects the effective width of the reflected beam. That is, to the second order

$$\Delta\alpha|_{\mathrm{Tamir}} \approx 2\frac{1}{k^2w_m^2}\frac{d(\ln R)}{d\alpha}\bigg|_{\bar{\alpha}}, \tag{17}$$

where w_m corresponds to the half-width of the Gaussian beam modified in the reflection [11]:

$$w_m^2 = w^2 + 2\frac{1}{k^2}\frac{d^2\ln R}{d\alpha^2}\bigg|_{\alpha=\bar{\alpha}}. \tag{18}$$

Next, we will show that the results obtained by Tamir's method are equivalent to those obtained by generalizing Artmann's method. In order to apply the latter to dielectric partial reflection, we must slightly modify the results for two plane waves with different amplitudes from the former section, since the mean angle of incidence is not null. By calculating the components of the Poynting vector associated to Artmann's reflected beam (Fig. 1),

$$\bar{\vec{S}} = \frac{1}{2}[\breve{e}_x(R_1+R_2)(R_1\cos\alpha_1 + R_2\cos\alpha_2) - \breve{e}_z(R_1+R_2)(R_1\sin\alpha_1 + R_2\sin\alpha_2)], \tag{19}$$

where R_1 and R_2 are the reflection coefficients associated to the angles of incidence α_1 and α_2, respectively. We obtain that the direction of $\vec{S}$ with respect to the normal is given by

$$\tan\alpha^S = \frac{(R_1\sin\alpha_1 + R_2\sin\alpha_2)}{(R_1\cos\alpha_1 + R_2\cos\alpha_2)}. \tag{20}$$

Therefore, we can calculate to the second order the difference between the geometric mean angle of reflection $\bar{\alpha}$ and the mean angle corresponding to the propagation of the energy reflected α^R. From Eqs. (19) and (20), we obtain without approximations

$$\tan(\alpha^S - \bar{\alpha}) = \sin\left(\frac{\alpha_2 - \alpha_1}{2}\right)\frac{R_2 - R_1}{R_1 + R_2}. \qquad (21)$$

By approximating the reflection coefficients of the waves constituting the beam to the second order,

$$R_1 \approx R|_{\bar{\alpha}} - \frac{dR}{d\alpha}\bigg|_{\bar{\alpha}}\left(\frac{\alpha_2 - \alpha_1}{2}\right) + \frac{1}{2}\frac{d^2R}{d\alpha^2}\left(\frac{\alpha_2 - \alpha_1}{2}\right)^2, \qquad (22)$$

$$R_2 \approx R|_{\bar{\alpha}} + \frac{dR}{d\alpha}\bigg|_{\bar{\alpha}}\left(\frac{\alpha_2 - \alpha_1}{2}\right) + \frac{1}{2}\frac{d^2R}{d\alpha^2}\left(\frac{\alpha_2 - \alpha_1}{2}\right)^2. \qquad (23)$$

Defining $\Delta\alpha|_{\text{Artmann}} = \alpha^S - \bar{\alpha}$ and by replacing in (21)–(23), we obtain to the second order

$$\Delta\alpha|_{\text{Artmann}} \approx \left(\frac{\Upsilon}{2}\right)^2 \frac{\frac{d\ln R}{d\alpha}}{1 + \frac{1}{2R}\frac{d^2R}{d\alpha^2}\left(\frac{\Upsilon}{2}\right)^2}. \qquad (24)$$

This is the expression that corresponds to Eq. (9). The derivatives are evaluated in the geometric mean angle of reflection $\bar{\alpha}$, and the aperture of the incident beam is $\alpha_2 - \alpha_1 \equiv \Upsilon$.

The difference between the angular shift $\Delta\alpha|_{\text{Artmann}}$ obtained to the first and second order can be interpreted as a modification of the angular aperture of the reflected beam, even though the law of reflection is valid for each wave. By considering the second-order correction, the semiaperture of the reflected beam is

$$\left(\frac{\Upsilon_m}{2}\right)^2 = \frac{\left(\frac{\Upsilon}{2}\right)^2}{1 + \frac{1}{2R}\frac{d^2R}{d\alpha^2}\left(\frac{\Upsilon}{2}\right)^2}. \qquad (25)$$

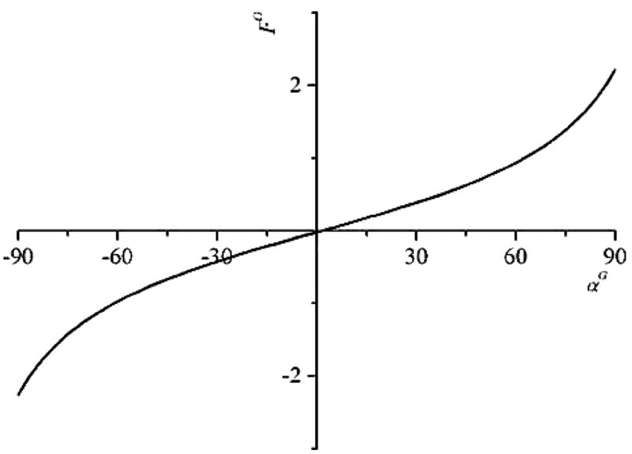

Fig. 4. F^G factor: correction to the mean angle of refraction as a function of the geometric angle of refraction for an extraordinary transmitted beam in a NeYag–air interface when the optical axis is in the plane of incidence and makes an angle of 70° with the interface. $n_o = 1.96$, $n_e = 2.17$, $\lambda = 1064$ nm.

In addition to the explicit results obtained by replacing in Eqs. (24) and (25) for the angular shift and the aperture of Artmann's reflected beam in the partial reflection, we can clearly see the equivalence between the values obtained by using two waves (Artmann) or infinite waves (Tamir).

In fact, the angular aperture Υ of Artmann's beam can be related to the aperture Θ of a Gaussian beam (far from the beam waist) and half-width w [30]. From Eqs. (17) and (18), we obtain that the angular shift of a reflected Gaussian beam can be expressed in terms of the aperture of the beam as

$$\Delta\alpha|_{\text{Tamir}} \approx \frac{1}{2}\left(\frac{\Theta}{2}\right)^2 \frac{\frac{d(\ln R)}{d\alpha}\Big|_{\bar{\alpha}}}{1 + \frac{1}{2}\frac{d^2\ln R}{d\alpha^2}\Big|_{\alpha=\bar{\alpha}}\left(\frac{\Theta}{2}\right)^2}, \qquad (26)$$

whereas from Eq. (18) we obtain that the modified aperture in the reflection is

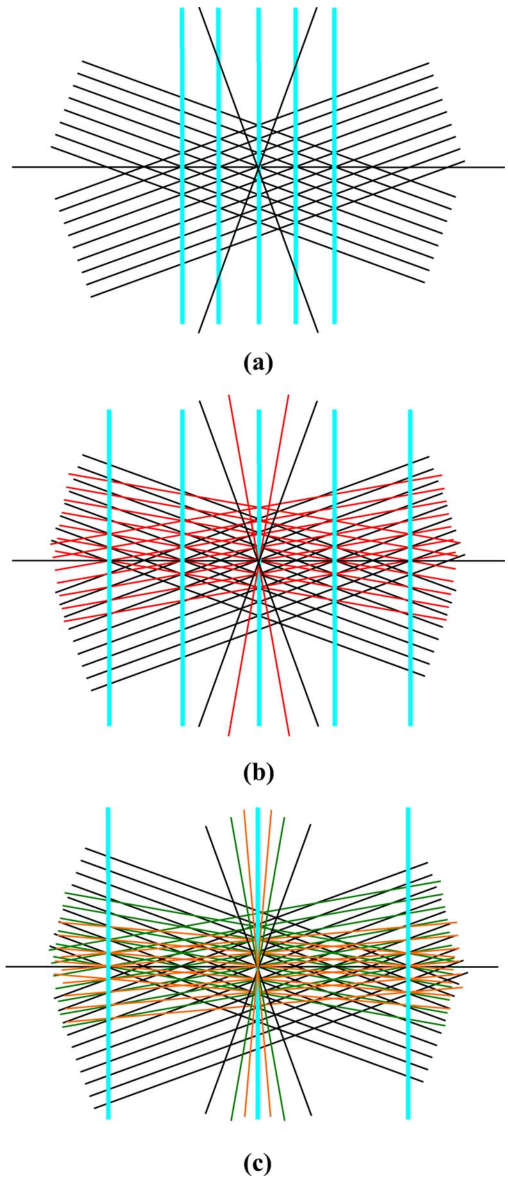

Fig. 5. (Color online) Interference of plane waves and formation of limited beams. The beams are obtained as the superposition of (a) two, (b) four, and (c) six plane waves.

$$\left(\frac{\Theta_m}{2}\right)^2 = \frac{\left(\frac{\Theta}{2}\right)^2}{1+\frac{1}{2}\frac{d^2 \ln R}{d\alpha^2}\left(\frac{\Theta}{2}\right)^2}. \quad (27)$$

From expression (27), which was obtained by using Tamir's method, we can see that the beam waist of a Gaussian beam obtained to the second order is modified by the logarithmic derivative $\frac{d^2 \ln R}{d\alpha^2}$, whereas the beam obtained by Artmann's method is modified by $\frac{1}{R}\frac{d^2 R}{d\alpha^2}$. This is to be expected, since the amplitudes of the waves constituting the beam have a quadratic distribution, whereas the amplitudes of the waves constituting Gaussian beams have an exponential distribution. Moreover, comparing Eqs. (24) and (25), we can observe that there is a difference between them (a factor of $1/2$). This factor also appears when using the method of superposition considering intensities instead of amplitudes [28].

5. ANGULAR SHIFT IN TRANSMISSION OF AN EXTRAORDINARY BEAM

We will calculate the angular shift corresponding to an extraordinary beam which is transmitted from a uniaxial anisotropic medium to an isotropic one. We will consider that the optical axis $\breve{z}_3$ is contained in the plane of incidence but forming an arbitrary angle with the interface. With the method here suggested, the calculation is simple; considering that if the direction of the transmitted wave is given by α (in the isotropic medium), the transmission coefficient for the parallel mode (corresponding to the extraordinary waves in this geometry) can be obtained from [31]

$$T_p = 2\frac{n_o n_e \cos\rho_e}{n_o n_e \cos\alpha + n(h_x - n^2 \sin^2\alpha)^{1/2}}, \quad (28)$$

where ρ_e and α are the angles that the extraordinary incident and the transmitted rays form with the normal to the interface, n_o and n_e are the main refractive indices of the crystal, n is the index of the isotropic medium, and h_x is a characteristic parameter of uniaxial crystals,

$$h_x = n_o^2(\breve{z}\cdot\breve{z}_3)^2 + n_e^2(\breve{x}\cdot\breve{z}_3)^2, \quad (29)$$

where $(\breve{z}\cdot\breve{z}_3)$ and $(\breve{x}\cdot\breve{z}_3)$ are the projections of the optical axis on the interface and on the normal to it, respectively. The ρ_e angle can be obtained through the relationship between the extraordinary ray and wave normal

$$\tan\rho_e = \frac{n n_o n_e \sin\alpha}{(h_x - n^2\sin^2\alpha)^{1/2}} + \frac{h_{xz}}{h_x}, \quad (30)$$

where

$$h_{xz} = (n_e^2 - n_o^2)(\breve{z}\cdot\breve{z}_3)(\breve{x}\cdot\breve{z}_3). \quad (31)$$

The shift can be calculated from Eq. (9):

$$\Delta\alpha \approx (\Upsilon/2)^2 \frac{\left.\frac{1}{T_p|_{\bar{\alpha}}}\frac{dT_p}{d\alpha}\right|_{\bar{\alpha}}}{1+\left.\frac{1}{2T_p|_{\bar{\alpha}}}\frac{d^2T_p}{d\alpha^2}\right|_{\bar{\alpha}}(\Upsilon/2)^2}. \quad (32)$$

Here Υ is given by the effect of the refraction in the uniaxial–isotropic interface and must be calculated from the direction of the two extraordinary waves in the crystal that cause the two refracted waves. The angular shift is, to first order, $\Delta\alpha = (\frac{\Upsilon}{2})^2 F^G$, where

$$F^G \equiv \left(\frac{\sin\alpha^G}{(h_x - n^2\sin^2\alpha^G)^{1/2}}\frac{n^3\cos\alpha^G + n_o n_e(h_x - n^2\sin^2\alpha^G)^{1/2}}{n_o n_e\cos\alpha^G + n(h_x - n^2\sin^2\alpha^G)^{1/2}} - \frac{\cos\alpha^G}{(h_x - n^2\sin^2\alpha^G)^{3/2}} n n_o n_e \frac{\tan\rho_e}{1+\tan^2\rho_e}\right), \quad (33)$$

and α^G corresponds to the geometric mean angle of transmission. It is very interesting to observe from Eq. (33) that, when the extraordinary wave impinges normally to the interface ($\alpha = 0, \alpha^G = 0$), there is an angular displacement [that corresponds to the second term of F^G in Eq. (33)] that can be either positive or negative according to the characteristics of the crystal and that is null when the optical axis is normal or parallel to the interface:

$$\Delta\alpha|_{\alpha=0} = \left(\frac{\Upsilon}{2}\right)^2 n n_o n_e \frac{(n_e^2 - n_o^2)(\breve{z}\cdot\breve{z}_3)(\breve{x}\cdot\breve{z}_3)}{(n_o^2(\breve{z}\cdot\breve{z}_3)^2 + n_e^2(\breve{x}\cdot\breve{z}_3)^2)^{1/2}(n_o^4(\breve{z}\cdot\breve{z}_3)^2 + n_e^4(\breve{x}\cdot\breve{z}_3)^2)}. \quad (34)$$

On the other side, if the extraordinary ray impinges normal to the interface ($\rho_e = 0, \alpha^G \neq 0$), it is the first term of Eq. (33) that contributes to the angular displacement. From Eqs. (29)–(33), it follows that this displacement is positive if $h_{xz} < 0$, or negative if $h_{xz} > 0$.

Figure 4 shows the factor F^G that gives angular shift (in 1/radians) as a function of the geometric mean angle of refraction α^G for a NeYag–air interface when the optical axis and the interface make an angle of 70°. We can see that the shift is smaller for near-normal incidence and it can be of the order of some degrees (positive or negative), which corresponds to the fact that the nongeometric mean angle approaches the normal to the interface. It is possible to explicitly calculate the angular shift to the second order. However, the correction of the angular shift to the first order is negligible.

On the other hand, we can easily calculate which extraordinary beam corresponds to another beam with aperture Υ and mean angle α^G [30]. It is worth noticing that, in Eq. (30), we do not take into account the angular shift due to Snell's law. We only consider the one caused by the different amplitudes resulting from the dependence of the transmission coefficient on the angle of incidence.

6. DISCUSSION

We have shown that, from the superposition of two plane waves with different amplitudes and directions of propagation (generalized Artmann's beam), we can obtain not only the lateral displacement (Goos–Hänchen effect) and the longitudinal focal shift in total reflection in isotropic interfaces, but also the expressions for all the first- and second-order nongeometric effects in reflection and transmission through interfaces constituted by linear media. First, they allow us to calculate the angular shift and the width change of the beam in dielectric reflection, comparing the results to those obtained for a beam with Gaussian intensity distribution (Tamir's method). The results are qualitatively identical and quantitatively comparable. This equivalence can be understood by means of the concept of formation of a limited beam. In effect, the simplest bidimensional beam consists of the superposition of two plane waves with the same amplitude and frequency (Artmann's beam). The result is an interference figure as the one shown in Fig. 5(a). By superposing two Artmann's beams in such a way that the normals to the wavefronts are in the same plane and share the same mean normal, we will obtain another interference figure such that the maximums are gradually less frequent [Figs. 5(b) and 5(c)]. However, as more Artmann's beams are being superposed, the location of one of the interference maximums is not altered. This is the basis of symmetric limited beam formation propagating in isotropic dielectric media through Fourier's integral or series. Moreover, that interference maximum will determine the direction of the associated beam, that is, the direction of propagation of the beam energy.

The generality of the method suggested in this paper allowed us to easily calculate the angular shift and the width change of an extraordinary beam refracting in a uniaxial–isotropic interface. This method will simplify the task of calculating both longitudinal and transversal nongeometric effects when the analytical resolution of Fourier's integral becomes long and tedious. Moreover, we consider that our extended analytical treatment helps to clarify and to predict the existence of multiple nongeometric effects in any kind of interface and under different incidence conditions.

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29. When the waves have the same initial phase, the total electric field is $E(x_m, z_m, t) = 2e^{-i\omega t} e^{\ln A_0 + C_0(\Upsilon/2)^2} e^{ik\cos\Upsilon/2 x_m} \times \cos\left[k\sin\Upsilon/2\left(z_m - i\frac{B_0\Upsilon/2}{k\sin\Upsilon/2}\right)\right]$, with $C_0 = \frac{1}{2}\frac{d^2\ln A}{d\alpha^2}$.
30. $\theta = \text{divergence} = \frac{\lambda}{\pi w}$. The total angular spread of the beam far from the waist is then given by 2θ.
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