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CENTAURUS JOURNAL OF THE EUROPEAN SOCIETY FOR THE HISTORY OF SCIENCE

CHRISTIÁN C. CARMAN

On Tycho's Calculation of the Coordinates of Hamal, the Fundamental Star of Tycho's Catalog

▼ **ARTICLE**

▼ **ABSTRACT** Tycho's star catalog enjoyed enormous prestige for centuries due to its accuracy. The entire catalog depends on the coordinates of one single star, Hamal (α Arietis), which explains why Tycho was so scrupulous in determining its coordinates using two different methods applied to more than 50 observations, as he described in his *Progymnasmata*. One of them proposed an ingenious way of dealing with refraction and parallax, two factors that he knew he could not control. Selecting particular observations, he was able to cancel out the effects of both refraction and parallax. Still, the entire calculation starts from the coordinates of the Sun calculated from his solar model. But Tycho's solar model assumes too large of an eccentricity, producing errors in the predictions of the solar longitude that can reach up to 8'. In this paper, I analyze Tycho's method for calculating the coordinates of α Arietis and explain how the method he proposed unintentionally avoided transferring the error of his solar model to his catalog.

▼ **KEYWORDS** Star Catalog, Alpha Arietis, Solar Theory, Tycho Brahe

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1. Introduction

That the so-called "fixed stars" were fixed, that is, that they kept their relative angular separation over time, was not a matter of dogma for ancient astronomers. Instead,

Christián C. Carman • Universidad Nacional de Quilmes, Buenos Aires, Argentina, correspondence: Christián C. Carman, Centro de Estudios de Filosofía e Historia de la Ciencia (CEFHIC), Universidad Nacional de Quilmes (UNQ), Roque Sáenz Peña 352, B1876BXD Bernal, Buenos Aires, Argentina. ccarman@gmail.com

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this statement was subjected to empirical scrutiny several times throughout history. For example, in the first chapter of Book 7 of the *Almagest*, when Ptolemy starts his analysis of the stars, he says that he compared the positions of several stars with those reported by Hipparchus, and admits that he could not perceive any change in them. Nonetheless, he included his new data so that future astronomers could continue assessing this statement.[1] His effort was not in vain because, 16 centuries later, Edmund Halley used Ptolemaic observations to detect the proper motion of some stars for the first time in history. In 1717, he presented this discovery for consideration at the Royal Society in a brief note. According to Halley, some stars had changed their position since Ptolemy's times by around half a degree. Still, he points out that the proper motion of Sirius is consistent with the value offered in Tycho's catalog (which changed only two minutes in the century separating Tycho from Halley). Nevertheless, Halley affirms, "two minutes, in such a Star as Sirius, is somewhat too much for him to be mistaken."[2] More than a century after its publication, Tycho's star catalog enjoyed enormous prestige among astronomers because of its accuracy. Indeed, Tycho's catalog was extraordinarily accurate.[3]

Traditionally, a star catalog lists the ecliptic or equatorial coordinates of a set of stars for a given epoch. One can try to determine the coordinates of each star independently. However, because the relative angular separation among the stars is assumed to be constant and it is not easy to determine the coordinates of a star independently, another approach is preferred. This different strategy involves three steps: 1) determining the relative positions of stars; 2) establishing the coordinates of only a small number of them (called "fundamental stars"); and then, 3) using the relative positions to calculate the coordinates of the other stars from those of the fundamental stars. This final step is a trivial problem of spherical trigonometry. It is possible to find the relative angular separation between stars through direct observation. As I mentioned, however, obtaining the coordinates of the fundamental stars is a more complicated issue. Ptolemy describes his method for doing so in *Almagest* 7.2.[4] His solar theory allows him to know the solar coordinates at every time. So, if he could measure the angular separation of any star from the Sun, he might also obtain the coordinates of the star. The problem, however, is that no stars are visible when the Sun is above the horizon. Therefore, Ptolemy proposes using the Moon as an intermediary, since the Moon is visible in both day and night. He measures the angular separation between the Sun and the Moon just before setting, and half an hour later—when the Sun is already below the horizon—he measures the angular separation between the Moon and the star. With minor corrections for lunar motion in elongation and parallax, Ptolemy can calculate the coordinates of the star.[5]

1 Toomer (1998, pp. 321–322).
2 Halley (1717, pp. 737–738).
3 Verbunt & van Gent (2010).
4 Toomer (1998, pp. 327–329).
5 Hipparchus also used the Moon as an intermediary going from the coordinates of the Sun to those of the stars, but in a slightly different way. Knowing that in a lunar eclipse the Moon is at opposition, he already knew the coordinates of the Moon (its longitude is that of the Sun plus 180°, and its latitude is 0°). Thus, he calculated the

The Moon, however, is not the only celestial body that can serve as an intermediary between the Sun and the stars: Venus is a better option in every respect. First, its motion relative to the Sun is slower, or even negligible when it is close to its maximum elongation. Second, the parallax of Venus is also smaller. Finally, Venus looks like a star, so it is easier to identify its center, facilitating measurment of its angular separation from another celestial body. Thus, it is not a surprise that the next serious attempt to build a star catalog, that of Walther at Nurnberg observatory, used Venus instead of the Moon as the intermediary.[6]

Still, there is one direct way of determining a star's coordinates, that is, without referring to the already known coordinates of another celestial body like the Sun. Knowing the latitude of the place, one can obtain the declination (δ) of the star by simply measuring its altitude when crossing the meridian. To obtain the right ascension (RA), a clock is needed because the exact time of the meridian transit must be known. William IV, Landgrave of Hesse-Kassel, used this direct method.[7]

Tycho, who knew very well what his predecessors had done, also attempted to use this direct method to determine the position of a dozen stars in order to calculate the position of the comet of 1577, but he realized that the clocks at his disposal were not accurate enough.[8] After some attempts to improve their accuracy and once he had a reliable solar theory, he followed a mixed method, incorporating the best of both worlds. He calculated δ by the direct method, measuring altitudes during meridian transits. But, to obtain RA, he returned to Walther's idea of using Venus as an intermediary between the star and the Sun.[9]

When measuring the angular separation between Venus and the Sun, or Venus and a star, however, Tycho needed to account for refraction and parallax. Refraction is a real problem, reaching values of around half a degree when the celestial body is at the horizon. Parallax, though, was only a problem for the Moon, but not for the Sun, Venus, or the stars because they are sufficiently far from Earth that the parallax is far beyond the limit of accuracy of Tycho's observations. Still, even if Tycho knew that the parallax of the stars was undetectable, he thought that this was not the case for the parallax of Venus and the Sun. Tycho assumed as correct the traditional value for the solar distance (around 1,140 terrestrial radii), which involves maximum parallaxes of approximately 3', a value he could not ignore. Because in his geo-heliocentric model Venus revolves around the Sun, the parallax of Venus is sometimes smaller but sometimes greater than that of the Sun. Accordingly, when building his star catalog, Tycho considered that he should somehow deal with both refraction and parallax, but that he was not able to calculate them with enough accuracy.

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angular separation between the Moon and the stars in a lunar eclipse and added the lunar coordinates. See Evans (1998, pp. 250–251).

6 Dreyer (1890, p. 348); Kremer (1980).

7 Thoren (1990, Ch. 5); TBOO (2, 156–158). TBOO refers to Dreyer (1913–1929), and is followed by book and page numbers.

8 Tycho's *De Nova Stella* of 1572 is full of explicit references to Ptolemy and even Hipparchus (quoted mainly from Pliny). See, for example, TBOO (1, 16–17).

9 TBOO (2, 159, ll. 25–31).

Both refraction and parallax depend on the altitude of the body (increasing as it approaches the horizon). Therefore, one obvious way to reduce the error would be to measure the angular separation between the bodies when the two involved bodies are so high in the sky that the effect of both refraction and parallax is negligible for observational accuracy. Unfortunately, given that Tycho measured the angular separation between the Sun and Venus, this was not possible. The Sun is necessarily close to the horizon when Venus becomes visible. Typically, in the observations that Tycho used, the Sun is around 10° above the horizon. This altitude implies, according to Tycho, a parallax correction of approximately 3', and a refraction correction of 10' (which is actually about 4'). As I show later, in Tycho's observations, even if Venus is higher in the sky, it is usually closer to Earth. Therefore, its refraction is smaller, but its supposed parallax is usually greater.

Tycho devoted the 1580s to collecting observations for his star catalog. But it was only in 1598, a few months after the traumatic departure from Uraniborg, that his catalog began to circulate as a manuscript.[10] In 1602, a few months after his death, a shorter but better presented version of 777 stars was published in the first part of Tycho's *Progymnasmata*.[11] Finally, Kepler published the whole catalog in his *Rudolphine Tables*, with 1,004 stars.[12]

At the beginning of Chapter 2 of the first part of *Progymnasmata*, Tycho explains the methods he employed to build his star catalog.[13] The whole catalog depends on one—and only one—star, which he selected as the fundamental star: Hamal (α Arietis).[14] Consequently, the coordinates of α Arietis needed to be determined with great care. Tycho directly obtains δ of α Arietis by measuring the altitude of the star at meridian transits. As I mentioned above, this is not problematic. Still, to obtain the RA, Tycho analyzes 54 different observations (27 observations of the Sun–Venus angular separation and 27 of Venus–star angular separation) linking the RA of α Arietis to that of the Sun, utilizing Venus and several secondary stars as intermediaries. He uses two different methods. In the first one, Tycho does his best to introduce corrections for refraction and parallax. In the second, he ignores the effects of refraction and parallax. The RAs obtained are paired so that the effects of refraction and parallax cancel each other out. Both methods obtain for him very consistent values for the RA of α Arietis, with a maximum scatter of around 40". Tycho proposes a rounded average as a final value, which disagrees with the modern value by no more than 15". Once the coordinates of α Arietis are securely established, Tycho establishes the coordinates of the other 20 reference stars and calculates all the others from them. Thus, the accuracy of the entire catalog rests on the coordinates of α Arietis.

In this paper, I analyze the method that Tycho used. In doing so, I show that there is an error in the input values related to his solar model that Tycho canceled by

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10 TBOO (3, 331–377).

11 TBOO (2, 258–280).

12 Kepler (1627).

13 TBOO (2, 150–161).

14 While Copernicus preferred γ Arietis as his fundamental star because it is at the beginning of Aries, Tycho preferred the brighter α Arietis because it is easy to observe. See Thoren (1990, p. 288).

chance. In Section 2, I describe the method that he used to obtain the RA of α Arietis in a general way. In Section 3, I analyze in detail the second method that pairs two observations to cancel out the effect of parallax and refraction. In Section 4, I do the same with the first method, introducing corrections due to parallax and refraction. In Section 5, I offer some concluding remarks.

2. General Description of the Method

In the first step, Tycho obtains the RA of Venus, starting from that of the Sun. He measures the angular separation between the Sun and Venus at a given time. Then, using his solar model, he calculates the true longitude of the Sun and, knowing the angle between the horizon and the ecliptic at that time, obtains δ and the RA of the Sun. He finds δ of Venus by measuring its altitude at the meridian transit. Sometimes, Tycho takes the angular separation between the Sun and Venus at the moment of the meridian transit of Venus. If not, he slightly corrects the value of δ obtained at the moment of transit to get δ at the moment of the observation. Having δ of the Sun and of Venus, plus the angular separation between them, Tycho calculates the difference in RA between the Sun and Venus using spherical trigonometry. Then, adding the RA of the Sun, he obtains the RA of Venus.

On the left side of Figure 1, the horizontal line represents the horizon, and the dotted line represents the equator. W is the west and D represents the intersection of the ecliptic (not drawn in the diagram) and the equator at the vernal equinox. The Sun is at S, and Venus at V. The star, still not visible because of the glare of the Sun, is T. Accordingly, DN is the RA of the Sun, and SN is δ of the Sun. LV represents δ of Venus. SV is the angular separation between Venus and the Sun. Knowing SN, LV, and SV, Tycho calculates NL, the difference in RA between Venus and the Sun. Then, adding DN to this angular separation, he obtains DL, the RA of Venus.

In the second step, Tycho obtains the RA of a star, departing from that of Venus. After sunset or before sunrise (depending on whether Venus is an evening or a morning star) on the day of the first observation, Tycho takes the angular separation of Venus and a given star. Tycho already knows the RA and δ of Venus a few hours after or before, so he corrects the values for the time of the new observation. He also knows δ of the star (having measured his altitude at meridian transit several times). So, again, having δ of Venus and of the star, and the angular separation between them, he calculates the difference in RA between the star and Venus. Then, adding the RA of Venus, he gets the RA of the star. One more step is necessary if the star is not α Arietis. He knows the difference in RA between the star and α Arietis. So, he adds this difference to the obtained RA of the star and gets the RA of α Arietis. The precession of the equinoxes slowly changes the stellar coordinates over time and the observations that Tycho uses are distributed over several years. Therefore, in the final step, Tycho, applies the precession rate and gets the RA of α Arietis at a given epoch, the end of 1585.

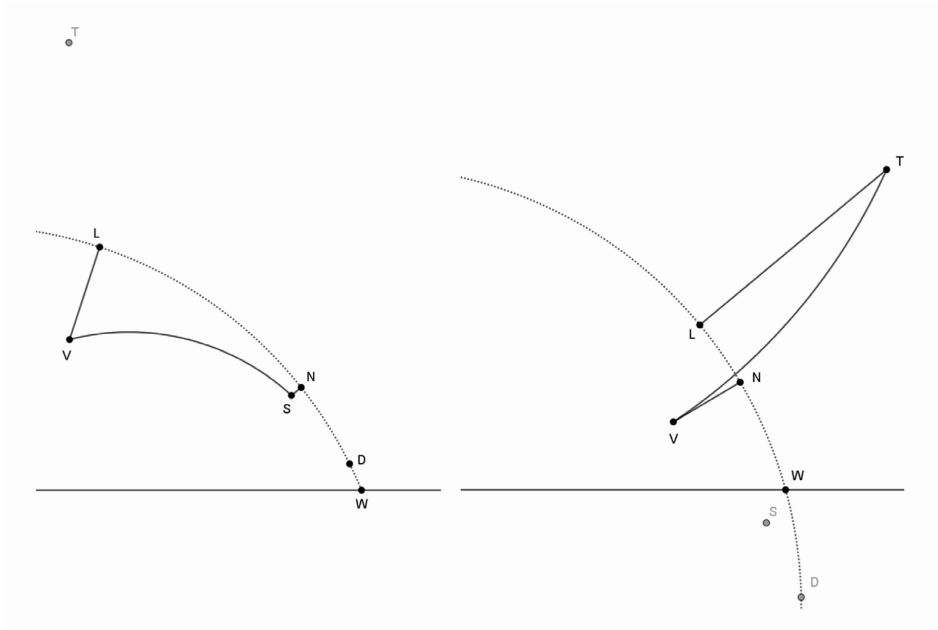

Figure 1. Representation of the two observations that Tycho used in his calculation. In both figures, the horizontal line is the horizon, and the dotted line is the equator. W is the west, and D is the intersection of the ecliptic and the equator at the vernal equinox. The Sun is at S, Venus at V, and the star at T. The figure on the left represents the calculation of the solar–Venus angular separation when the Sun is above the horizon: LV is δ of Venus, SN is δ of the Sun, DN is the RA of the Sun, and DL is the RA of Venus. SV is the angular separation between Venus and the Sun, and NL the difference in RA between the Sun and Venus. The figure on the right shows the calculation of the Venus–star angular separation when the Sun is below the horizon: NV is δ of Venus, LT is δ of the star, DN is the RA of Venus, and DL is the RA of the star. TV is the angular separation between Venus and the star, and NL the difference in RA between the star and Venus.

On the right side of Figure 1, the Sun, S, is now below the horizon. The star, now visible, is at T, so that LT is δ of the star and VT is the observed angular separation between Venus and the star. Knowing NV, LT, and VT, Tycho calculates NL, the difference in RA between Venus and the star. Then, adding DN (the RA of Venus) to NL, he obtains DL, the RA of the star.

If one ignores the effects of refraction and parallax, the calculation would be straightforward—but unfortunately, one cannot ignore them. Both affect the accuracy of each step of the calculations. Refer to Figure 2. While the apparent position of the Sun is still at S, the value given by Tycho's model obtains the true Sun at Z. The observed angular separation between Venus and the Sun is still SV, and the declination of Venus is LV, but now the solar δ is AZ and the solar RA is DA. Consequently, the calculated difference in RA between the Sun and Venus is AB,

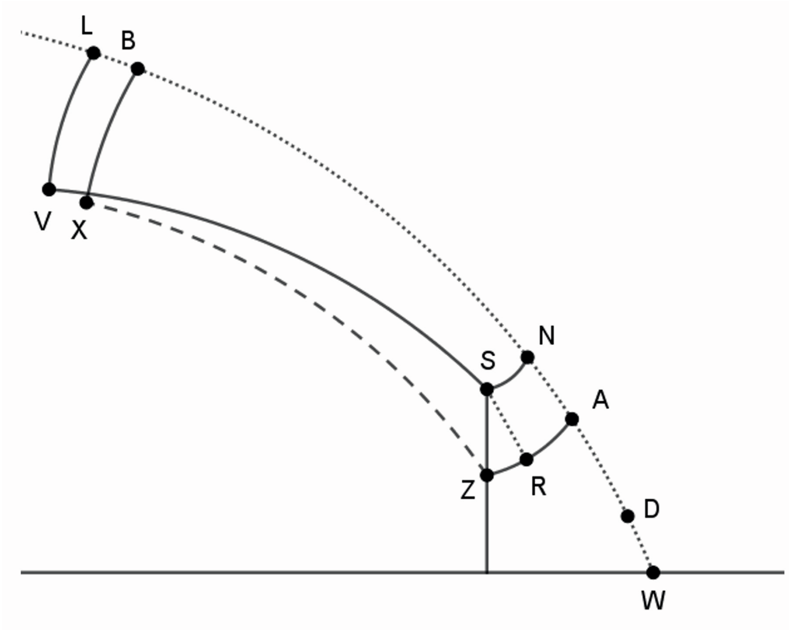

Figure 2. Representation of the influence of parallax and refraction in the calculation of the RA of Venus. The true Sun is at Z, but it is seen at S; Venus is seen at V. SV is the apparent Venus–Sun angular separation. DA is the true RA of the Sun and DN is the RA of the apparent position of the Sun. LV = BX is the δ of the apparent position of Venus. ZX = SV. Therefore, from the Sun–Venus angular separation and assuming that the Sun is at its true position, it is inferred that Venus is at X and, therefore, its RA is DB.

because the angles used are AZ, BX (= LV), and ZX (= SV). The RA of Venus is therefore DB, obtained by adding DA to AB. The difference BL reflects the effects of parallax and refraction on the calculation.[15] Similar problems arise in the calculations of the second observation as it again mixes real and apparent input values. Tycho is aware of these problems, so he offers two methods to deal with them. In the first, he attempts to incorporate the effects of parallax and refraction at each step. In the second, he devises a method for canceling out the effect of parallax and refraction in pairs of observations. I will start by analyzing the second and simpler method.

15 δ of Venus are also affected by refraction and parallax, for they have been obtained directly by the measurement of the altitude. This fact is ignored in this explanation.

3. The Method of Pairing Observations

3.1. Description

The calculations of the method of pairing observations are developed in TBOO (2, 170–197). Even if Tycho ignores the exact correction that parallax and refraction would imply, he does know the variables on which refraction and parallax depend. While parallax depends on the distance of the body and altitude, refraction depends (mainly but not exclusively) on the altitude of the body.[16] Parallax makes the apparent altitude smaller than the true one, while refraction produces the opposite. If the distance from the Earth and the altitude are the same in two different observations of the same body, the total change in altitude produced by refraction and parallax will also be the same. If the angle between the horizon and equator is also the same as well as one of the two coordinates, then the change in the coordinates will be the same, too. Refer to Figure 3. Right ascension is measured from D, the vernal equinox, and increases from west to east, that is, counterclockwise. Therefore, in the evening, when the true Sun is at Z and the apparent Sun at S, the right ascension of the apparent position (DN) is greater than of the true one (DA). In the morning, the true Sun is at B and the apparent Sun at L. Therefore, the RA of the true position is DP and of the apparent one is DO. In this case, the RA of the apparent position is smaller than of the true one. If, as in the figure, the altitudes in the true and apparent positions are the same for both observations, the corrections (arcs NA and OP) are equal in magnitude but with different directions.

As I have already mentioned, for the determination of the RA of α Arietis, two observations are involved: 1) the angular separation between Venus and the Sun when the Sun is above the horizon (cf. Figure 1, left), and 2) the angular separation between Venus and a star when the Sun is below the horizon (cf. Figure 1, right). Therefore, suppose one takes two observations of the first type in which the altitude of the Sun and Venus and their distances from the Earth are the same, but the first at morning and the second at evening. Then, refraction and parallax would produce an unknown error in the values of the RA of the Sun that will be translated to an unknown error in the RA of Venus. These two errors in the RA of Venus, however, are equal in magnitude and opposite in direction. Consequently, one can take the average of both values as the RA of Venus unaffected by refraction and parallax. The same can be done with two observations of the second type, with Venus at the same distance and both the star and Venus at the same altitude in the two observations. One would obtain two different values for the RA of the star affected by refraction and parallax in an unknown quantity. Again, however, one knows that the magnitude is the same and the sign opposite in both. Consequently, the average gives us the true RA, unaffected by refraction and parallax. This is the core of Tycho's method.

16 Tycho suspected that refraction varies from one place to another, according to the time of the year and the vagaries of the weather. Furthermore, he treats differently the refraction of the Sun, the stars, and the planets. See Thoren (1990, p. 235).

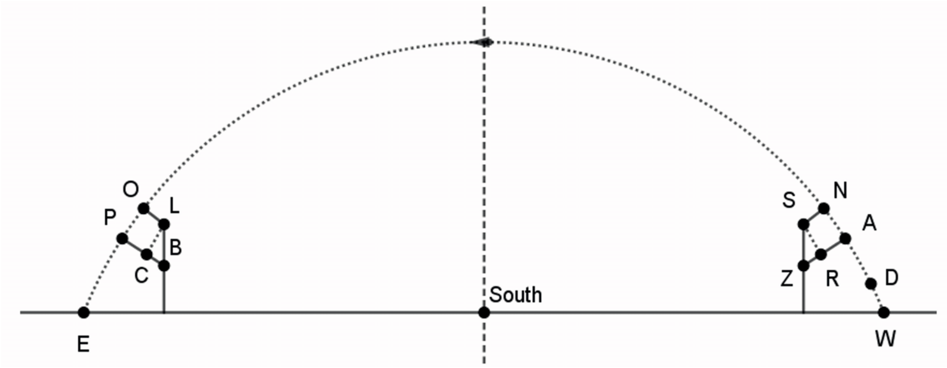

Figure 3. The correction due to parallax and refraction in RA is equal in magnitude, but opposite in sign if the body is at the same altitude and same declination, though in one case close to east, and in the other close to west. If at afternoon, the true Sun is at Z but it is seen at S, then the correction in RA is AN, that is added to the true RA, DA. If at morning, the true Sun is at B, at the same altitude that Z, it will be seen at L, and the correction in RA is OP that, in this case, must be subtracted from the true RA, DP.

3.2. Analysis

One can test Tycho's method by performing all the calculations that his method dictates but introducing modern values as input, thus avoiding errors due to inaccuracies in the models or observations and isolating the errors that come from the method itself. A close inspection of the observations that Tycho used shows that he was meticulous in maintaining the restrictions he imposed: the altitude and the distances of the bodies from the Earth are the same in the paired observations. So, I will test his method using the times he selected but with modern values. Like Tycho, I will use the true position of the Sun to calculate the solar RA and δ and the apparent angular separations between the Sun and Venus first, and then between Venus and the star (because these angular distances are obtained by observation). In this way, I will obtain a set of values for RA of α Arietis. The value of each calculation should be wrong because it mixes true and apparent positions. But the average of each pair should be close to the correct one, if the method works. Figure 4 (top) shows the error in the value of the RA of α Arietis obtained from the evening and morning observations and the error of the average of each pair. Because I am using modern values, the only relevant factor is refraction; parallax is negligible. This explains why the RA found in all the morning observations is greater than the correct one and smaller in the evening observations. The error in each observation can reach 15' in some cases. Nevertheless, the error in the average of each pair is always smaller than 1'. Moreover, the error of the average of all the single averages is only around 10" when compared with modern values. The method works very well.

The lower part of Figure 4 plots the same as the upper part of Figure 4 using Tycho's values. Again, the average of each pair is close to zero, and the total average is smaller

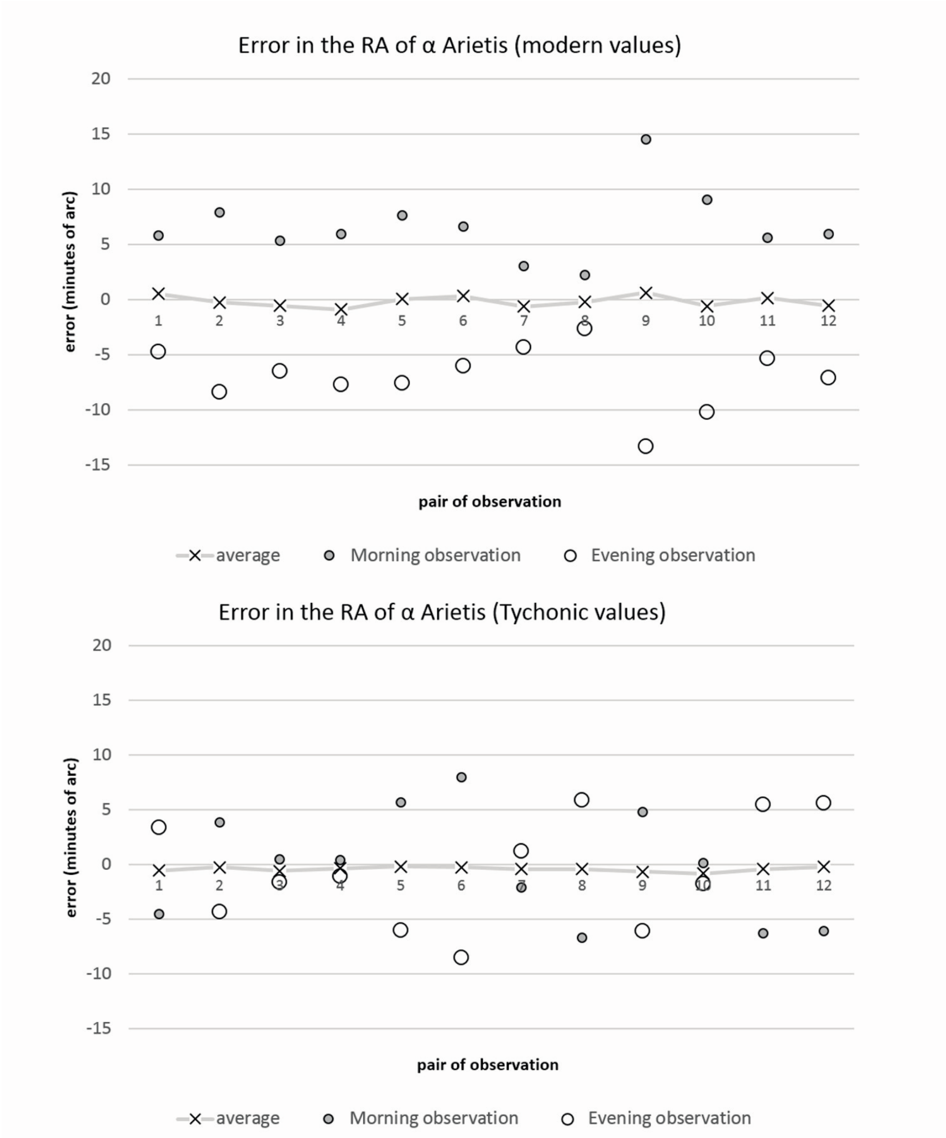

Figure 4. Error in the RA of α Arietis applying the second method. Above using modern values, below using Tycho's values.

than a quarter of a minute. The method also works pretty well with the Tychonic values. Nevertheless, a close comparison of the two figures shows that something is wrong with Tycho's calculations. The averages are always close to zero and, therefore, similar in both figures. Still, this is not the case for the error of the individual calculations. In some cases, even the signs are inverted. Take, for example, the errors of pair one. While the correct value for the morning observations is +6', Tycho's value is almost −5' (a difference of 11'); for the evening observation, the correct value is -5', and Tycho's is +4' (−9' of difference). The differences are far too great.

One possible explanation is that, for some reason, the calculations are wrong. Outside of some minor errors, however, the calculations are correct. They are certainly not responsible for such a big difference. The values for the RA for each calculation are the values that follow from the input data. One cannot explain either of these differences with reference to parallax. Tycho indeed thought that parallax affected the values in a non-negligible magnitude. In his calculations, however, he does not introduce parallax, and, of course, the bodies in the observations that he uses are only negligibly affected by parallax. The parallax effect is thus negligible in both figures. The error must therefore be in the input data. There are two sets of input data: the observations used and the calculated (not observed) RA and δ of the Sun. I examined all the observations: those used for determining the declination of Venus at meridian transit, those measuring Venus–Sun angular separation, and those measuring the Venus–star angular separation.[17] All the observations are as good as the typical Tychonic observations: usually a few seconds of error, sometimes 1 or 2 minutes.[18] There is no way to attribute this difference of about 10' to the observations.

The only remaining candidate is the solar RA and δ, from which the whole calculation departs. Again, the RA and δ that Tycho calculated are consistent with the true longitude obtained from this solar theory. The problem must be in his solar theory—and it is. Tycho's solar model assumes a somewhat larger eccentricity than the correct one, producing errors in true longitude that can reach 8'.[19] These errors are translated with similar values to the solar RA, introducing an error at the beginning of each calculation that explains the differences between Tycho's values and the correct ones. Figure 5 plots the error in the RA of the Sun for the observations of each pair. The error of each observation explains the differences between the graphs in Figure 4. The fact that the error in the RA of the Sun is also symmetrical in the observations of each pair (with minor differences in pairs 5 and 7), explains why, when Tycho applies his method even with a wrong solar RA, he still finds averages close to zero. Looking for observations with the bodies at the same altitude and distance, Tycho intends to cancel out the effects of refraction and parallax. But somehow, he also cancels out the effect that his incorrect solar model implied. How is that possible?

Given that an incorrect eccentricity causes the error in the solar model, the error depends on the mean anomaly, that is, on the angular separation between the mean position of the Sun and its apsidal line. The observations of each pair are symmet-

17 The observations of 1582 used in the calculations are in TBOO (10, 156–166); those of 1585 in TBOO (10, 402–405); those of 1586 in TBOO (11, 73–74); those of 1587 in TBOO (11, 189–194, 202–203); and those of 1588 in TBOO (11, 278–290).

18 To measure Venus's declination at the meridian, Tycho uses the obliquity of the ecliptic and the latitude of the place. The value of the obliquity is not particularly good. Nevertheless, when compared with modern values, the average error in the declinations that Tycho found are only −15" and a standard deviation of around 1;15'. This produces negligible errors in the final RA found.

19 TBOO (2, 19–23). See Swerdlow (2010, p. 155).

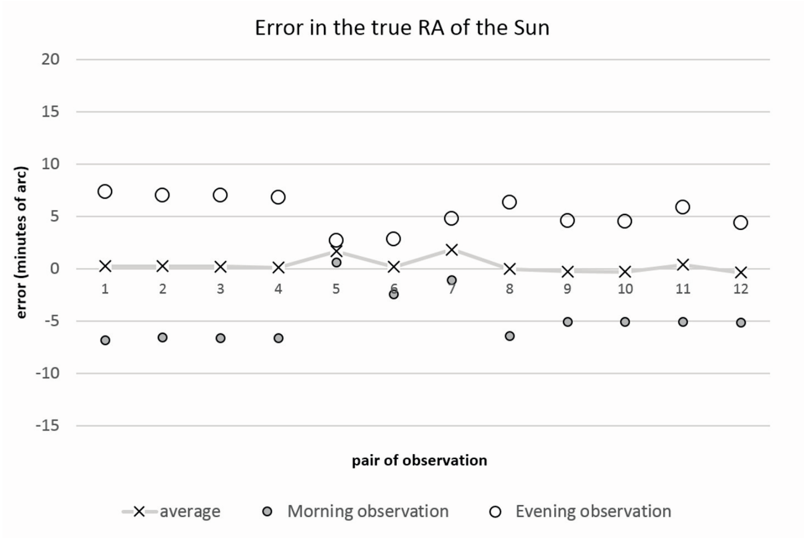

Figure 5. Error in the value of the RA that Tycho used in the second method.

rically distributed from the apsidal line, as Figure 6 shows. This can hardly have occurred by chance. Tycho somehow must have selected them on purpose.

It is out of the question that Tycho was aware of the too-large eccentricity of his model, because his solar observations are consistent with his wrong value for the eccentricity. This is so because his solar theory (like any solar theory) makes predictions from the center of the Earth. But, because the observations are taken not from the center of the Earth, but rather from the surface, the predictions have to be corrected for parallax and refraction. Now, Tycho assumes the traditional value of around 1200 terrestrial radii for the Earth–Sun distance, coming from Ptolemy, which produces an enormous parallax. This error is partially compensated by an exaggerated value of refraction. The value of the obliquity of the ecliptic and the latitude of the place also play a role here. When all are factored in and the inaccurate value predicted by his solar theory is compensated by the wrong values for parallax, refraction, obliquity, and latitude, the final value is very, very close to the observed value. There is no miracle here, because, in the end, the parameters were selected to make good apparent predictions that coincide with observations. So, it is senseless to assume that Tycho explicitly selected the observations to cancel out the error of his solar model, an error of which, I insist, he was completely unaware. The explanation must reside elsewhere.[20]

20 For the intricate relations between the wrong value assumed for Solar parallax and the other solar parameters, like refraction, obliquity of the ecliptic, and mainly, the big eccentricity, see Thoren (1990, pp. 230–235), which follows Maeyama (1974). See also Carman (2020, pp. 163–164) in which I analyzed the error of Longomontanus's solar theory, which has practically the same eccentricity as Tycho's.

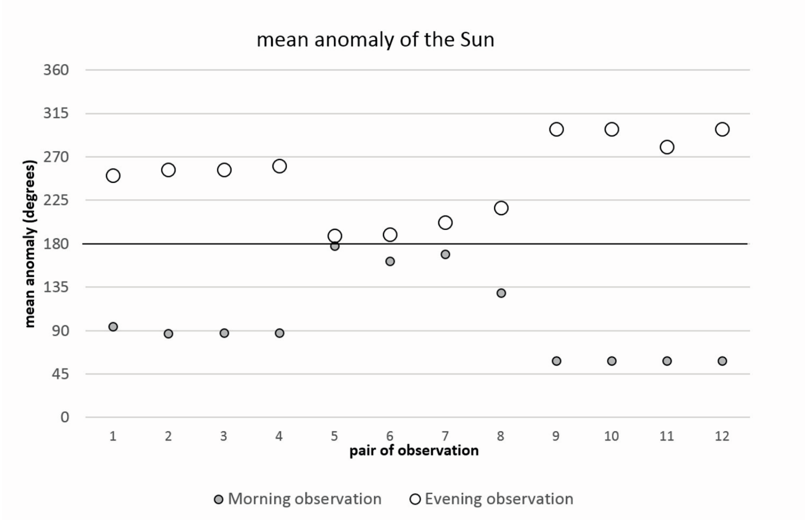

Figure 6. Mean anomaly of the Sun in each observation that Tycho uses when applying the second method.

Tycho required that the Sun be at the same distance from the Earth in the two observations of each pair to cancel out the parallax effect. Now, in an eccentric model, the distance depends on the mean anomaly. But the error in the Tychonic solar model also depends on the mean anomaly, since it is caused by a too-large eccentricity. Consequently, by requiring the Sun to be at the same distance at each pair, Tycho cancels out the error produced by his solar model. I have already mentioned that the Sun is too far from the Earth to produce a parallax that Tycho's instruments could detect. It is interesting to notice, however, that, even assuming the Tychonic solar distance (1150 terrestrial radii), the maximum difference that parallax can produce due to a change in the solar distance is no more than 12.5".[21] Consequently, aiming to cancel a maximum difference of only 12.5" between the two observations of each pair, he also canceled an effect of which he was unaware—and this effect can be around 80 times larger (reaching 16'). Tycho was undoubtedly lucky!

Had Tycho wanted to take observations at the same solar distance, given that the solar anomalistic period is 1 year, he could have simply taken both morning and evening observations on the same date—not on the same day, of course, because Venus is hardly ever visible at morning and evening, and also because on the same day Venus would be in both observations below or above the Sun, whereas Tycho

21 According to Tycho, the mean distance of the Sun is 1150 terrestrial radii (TBOO, 2, 421), and the maximum and minimum are 1190 and 1110 terrestrial radii, respectively (TBOO, 2, 424).

needed one above and one below.[22] Still, on the same date in different years one could find Venus above and below the Sun and it would keep the same distance. In that case, however, even if he had kept the same distance for the Sun, the errors in the solar model of both observations would not have canceled each other out but would instead have accumulated. Accordingly, to keep the same distance, Tycho had two choices: 1) to take observations symmetrically distributed along the apsidal line, or 2) to take observations on the same date in different years. Only the first option cancels out the error in his solar theory. And Tycho always chose this first option. So, the requirement of keeping the Sun at the same distance is not enough to explain his choice. Again, the explanation must be different.

The explanation that I find most persuasive is related to the following fact. Taking two observations on the same date, one in the morning and the other in the evening, would conflict with the constraints of Tycho's method. Tycho required not only the Sun but also Venus to be at the same distance in each pair of observations. This particular constraint is not too problematic. Because the variation of the solar distance is really small and (according to Tycho) Venus revolves around the Sun, if the distance from Venus to the Earth is the same, then the elongation from the Sun would also be the same. So, in each pair of observations, the elongation between Venus and the Sun is approximately the same. Furthermore, Tycho usually chooses observations where Venus is close to its maximum elongation. By doing so, Venus remains at a reasonable altitude when the Sun is already below the horizon.

Tycho, however, also requires that the two altitudes of Venus be the same for each pair, as well as the two altitudes of the Sun—and this is not easy to find. We already know that the elongation between the two bodies must be the same. For the sake of simplification, let me assume that Venus does not have latitude—that is, that, like the Sun, it travels along the ecliptic. Then, if the Sun has the same altitude in the two observations, Venus will have the same altitude if the angle between the ecliptic and the horizon is the same in both observations. Refer to Figure 7. Arc AEL represents the ecliptic. A is the intersection between the ecliptic and the horizon. Suppose that in the morning, the Sun is at E, at an altitude ET. Venus is at N, so that EN represents the elongation between Venus and the Sun. Then, the altitude of Venus will be NO. At setting, arc BSL again represents the ecliptic that crosses the horizon at B. Angles CAL and CBL representing the angles between the horizon and ecliptic are equal. Suppose that the Sun is at S so that ET = SQ, that is, that the altitude of the Sun is the same in the morning and evening. If Venus is at V and SV = EN, then VR, the altitude of Venus in the evening, is equal to NO, the altitude of Venus in the morning. Now, suppose that the ecliptic at evening is represented by arc WSP so that angle LAC ≠ PWC. Then, even if the Sun is still at S and EN = SZ, the altitude of Venus, ZX, will not be the same that NO. The only way to have both bodies at the same

22 Tycho was able to see Venus both as morning and evening star for several days from February 24, 1587: see TBOO (11, 199). He relates this extraordinary phenomenon to Peucer in a letter of September 13, 1588: TBOO (7, 129).

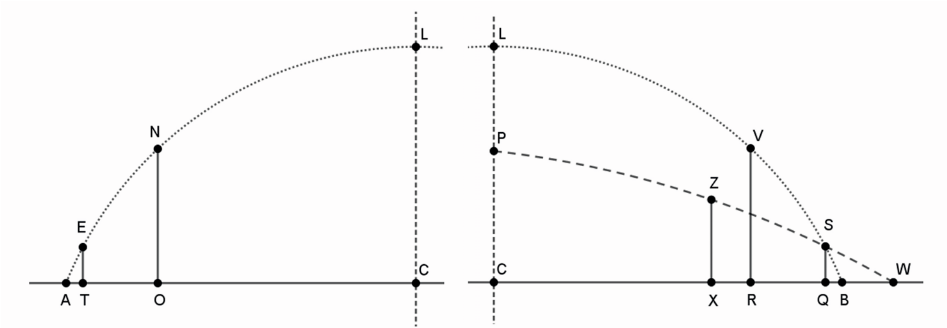

Figure 7. If the elongation between two bodies is the same, both are on the ecliptic, and the altitude of one is the same, then the altitude of the other is also the same if the angle between the horizon and the ecliptic is also the same. The left figure represents morning configuration. The horizontal line is the horizon and arc AEL is the ecliptic, so that A is the intersection between the ecliptic and the horizon. The Sun is at E, at an altitude ET. Venus is at N, so that EN is the Venus–Sun elongation. The altitude of Venus is NO. At setting (right figure), arc BSL is the ecliptic crossing the horizon at B. CAL = CBL. The Sun is at S, ET = SQ. If Venus is at V and SV = EN, then VR = NO. Now, if the ecliptic at evening is arc WSP and LAC ≠ PWC and the Sun is still at S, and EN = SZ, then ZX ≠ NO, that is, the altitude of Venus is not the same.

altitude when the elongation between them is the same is if the angle between the horizon and the ecliptic are the same at the moment of both observations.

Now, the angle between the ecliptic and the horizon changes all the time. Figure 8 plots the angle between horizon and ecliptic, at the latitude of Uraniborg, during rising and setting as a function of the angular separation of the mean Sun from the summer solstice (that is, mean longitude +90°). The figure shows that the angle is not the same at the rising and setting of the same date, except on solstices. Therefore, to keep the same solar distance, taking the same date for the two observations is not an option, since the altitudes of the two bodies would not be the same (except, of course, on solstices). This explains why, from the two options for keeping the same solar distance that Tycho had (same date and two dates equidistant from the apsidal line), he chose the second.

The figure also shows that the angle between horizon and ecliptic (and, consequently, the altitude of the two bodies) is equal at rising and setting for any pair of dates equidistant from, but on either side of, the same solstice. Accordingly, to look at two observations with the bodies at the same *altitude*, Tycho must choose between observations equidistant from the solstices. To look at two observations with the Sun at the same *distance*, however, Tycho must choose between observations equidistant from the apsidal line. These two constraints are impossible to fulfill at the same time, unless the solar apsidal line is at the solstices. This is what approximately happened in Tycho's time: the solar apogee was only 5° from the winter solstice. For example, in Ptolemy's time, the two constraints would have been impossible to fulfill, because the longitude of the solar apogee was around 71°, that is, around 20° from the solstices.

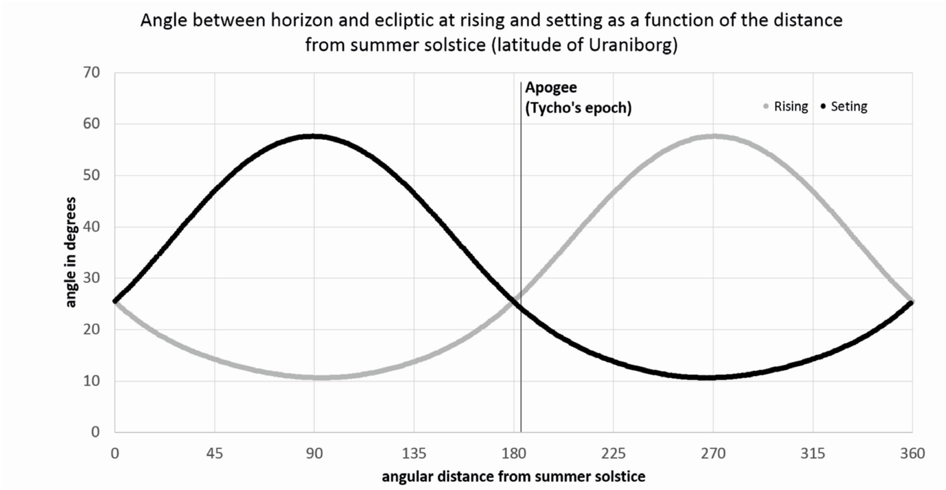

Figure 8. The angle between horizon and ecliptic at rising and setting as a function of the angular separation from the summer solstice (that is, mean longitude +90°). Only at solstices are the values of the angle at rising and setting the same on the same day. The apogee is very close to the winter solstice.

Of course, Tycho knew that the solar apsidal line was close to the solstices in his time. Although he did not explicitly mention it, he was probably aware that this coincidence made his method applicable. Tycho could not have been luckier!

In Tycho's time, by looking for observations that fulfill one of the constraints, the other would automatically be fulfilled. Suppose Tycho looked for observations in which, being at the same elongation, both the Sun and Venus were at the same altitude in the morning and evening. Then, the Sun would be at approximately the same distance from the Earth in both observations, canceling out the error in the solar model. Consequently, it would be impossible for Tycho to detect the error in his solar theory. The errors necessarily cancel each other out by requiring the bodies to be at the same altitude at each pair. Even if he had only used observations of solstices, where the error of the solar model is zero, he would have not detected it either.

4. The Method of Single Observations

I shall now analyze the first method, which is developed in TBOO (2, 162–169). As I have already mentioned, Tycho applies this method to the first three cases. According to Tycho, the method accounts for parallax and refraction of both the Sun and Venus at each step of the calculation. In fact, only parallax is taken seriously into account. While Tycho carefully and explicitly calculates parallax, refraction is typically ignored or undervalued. In these three observations, both Venus and the Sun are higher than in the typical observations of the previous method. Still, the effect is not negligible.

When the angular separation between Venus and the Sun is measured at the first observation, Venus is high enough to have a negligible refraction (always higher than 45°). Still, the Sun is close to the horizon, with altitudes varying from 15° to 39.5°, implying refraction corrections from 15" to 7.5' (according to Tycho's table).[23] In the second observation, when Tycho measures the angular separation between Venus and a star, the refraction of the star can be ignored (with all of them around 45°). But this is not the case for the refraction of Venus. The altitudes of Venus range from 17° to 27°, implying refractions (if the star table is used) that reach 2'. Nevertheless, in the first calculation, Tycho arbitrarily adds 30" to go from the apparent to the true RA of Venus for the first observation, and in the third, he adds 15" to the apparent RA of the star. In the second, he does not include refraction corrections at all. Unfortunately, the application of the method is not as careful as one would like, particularly in the case of refraction.

In the first step, he calculates the true longitude of the Sun from his model and then corrects it for parallax, obtaining the apparent longitude of the Sun. Knowing the apparent longitude of the Sun and the angle between the ecliptic and the horizon, he calculates the solar apparent δ and RA. Knowing the angular separation between the Sun and Venus as well as the δ of Venus, he obtains the RA of Venus that is correctly interpreted as the RA of its apparent position. Assuming the distance between Venus and the Earth and the altitude of Venus, he applies a parallax correction to the RA of Venus, obtaining the true RA of Venus at the moment of the first observation. Then, knowing the total motion in RA of that day and the time between the first and the second observations, he applies a small correction to the RA of Venus at the first observation, obtaining the true RA of Venus at the second observation. Venus is now at a lower altitude, so he applies a new parallax correction to obtain the RA of the apparent position of Venus again. From the angular separation between Venus and a given star, Tycho calculates the RA of the apparent position of the star. He does not distinguish between true and apparent positions in the stars. Consequently, also adding the known difference in RA between this star and α Arietis, he obtains the apparent RA of α Arietis at the moment of the second observation. Finally, he corrects the value by precession and gets the value for the end of 1585. The values obtained are 26;0,44°, 26;0,32°, and 26;0,30°. These values are, again, very close to the real ones. The difference between them is smaller than a quarter of a minute, and on average, they are not more than a quarter of a minute from the correct value. This accuracy deserves an explanation. On the one hand, Tycho practically ignores the effect of refraction, an effect that is certainly not negligible. On the other, he does incorporate parallax. In principle the effect of parallax is negligible. Assuming that the Sun and Venus are much closer than real, however, Tycho's parallax value is not negligible. As I have already mentioned, parallax and refraction corrections go in opposite directions. Consequently, Tycho does not correct by refraction and

23 Tycho's table of solar refraction is in TBOO (2, 64), and the table for star refraction is in TBOO (2, 287). Tycho considered producing a table for planetary refraction, but he finally used the star table for them. See Thoren (1990, p. 235)

introduces a wrong correction that goes in the opposite direction. Still, he obtains correct values.

The successive modifications that parallax introduces, however, cancel the original error that Tycho's solar model introduces in the longitude of the Sun. Figure 9 plots the error at each step of the three calculations. The first value corresponds to the error of the true longitude of the Sun calculated from the Tychonic solar model. The three observations used are between the end of February and the beginning of April, when the Tychonic solar model produces errors between 6' and 7.5'. Then, he applies the solar parallax correction and calculates the apparent RA. The error diminishes because the solar parallax partially compensates for it. The RA of the apparent position of Venus is obtained from the apparent solar RA and the angular separation between the Sun and Venus. The main differences are due to errors in the measured angular separation. The error is still 5' for the first and third calculations and around 1' for the second. This magnitude does not change significantly when the parallax of Venus is applied to obtain the true RA of Venus. As I have already mentioned, in the first observation Venus is too high in the sky and, consequently, its parallax is very small. In the next step, Tycho goes from the true RA of Venus from the first observation to the true RA of Venus from the second observation, applying the time correction. These corrections are reasonably correct; therefore, again, the error does not change significantly. But then Tycho moves from the RA of the true position of Venus to the apparent one, applying parallax again. Venus is now so close to the horizon and so close to the Earth that it produces a significant correction (between 2' and 5'). The error in the RA of the apparent position of Venus is now close to zero. The original error caused by the exaggerated solar eccentricity has been canceled out. The error stays close to zero in the last two steps because the input values are good enough. There are no relevant modifications when Tycho calculates the RA of the star from the star–Venus angular separation, or when he obtains the RA of α Arietis from the RA of the star and the difference in RA from the star and α Arietis.

One may say that Tycho is extraordinarily lucky because his wrong values for the distances of the Sun and Venus from the Earth produce parallaxes that cancel out the errors caused by the exaggerated eccentricity of this solar model. Of course, refraction seems to have been intentionally excluded to obtain values close to the ones obtained using the paired method. Still, the correction introduced by the parallax of Venus, assuming that Venus revolves around the Sun, is very close to the error produced by the solar model around the vernal equinox. At the end of the list of observations in 1582, Dreyer edited an appendix made by one of Tycho's assistants, entitled *Exquisitior et verior inquisitio ascensionis rectae quarundam stellarum fixarum, ex observationibus earum a Venere, ipsa Venere interdiu, tum quo ad meridianas altitudines tum quo ad distantias a sole observata.*[24] This appendix carefully calculates the RA of several stars following the same method I describe here using 20 observations as input. For each one, he uses two possible parallaxes for Venus, depending on two possible distances: one assuming that Venus revolves around the Sun; and another assuming, like in

24 TBOO (10, 204–230).

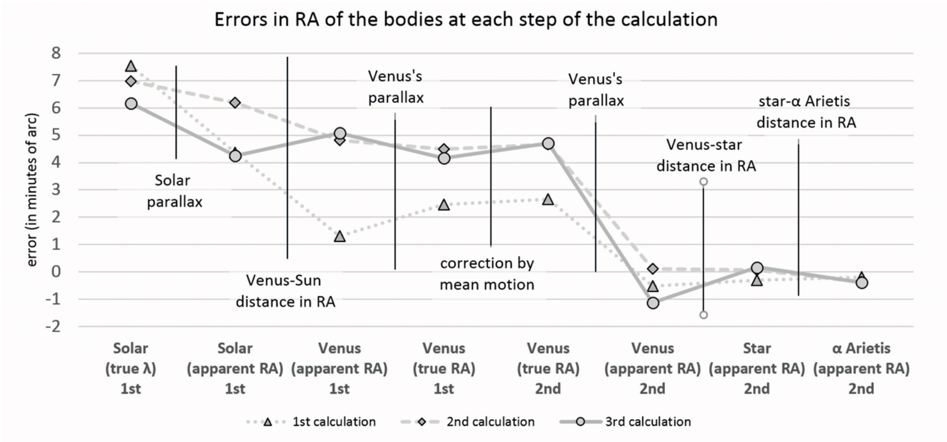

Figure 9. Errors in the longitude of the Sun and in the RA of the Sun, Venus, and the star at each step of the calculation, when applying the first method.

the Ptolemaic model, that the center of the Venus epicycle is aligned to, but closer than, the Sun in such a way that the maximum distance of Venus coincides with the minimum solar distance. The Ptolemaic model always predicts that Venus is closer to the Earth than the Copernican model, implying a larger parallax. Thus, it seems that by 1582 Tycho had not yet decided whether to follow the Copernican or the Ptolemaic model for Venus.[25] The appendix does not calculate the RA of α Arietis, but of several other stars. I took the values obtained for each star and, adding the correct difference in RA between that star and α Arietis, I obtained the RA of α Arietis that results from this calculation. Then, I added the correction by precession for getting the RA of α Arietis at the end of 1585. Finally, I obtained the difference between these values and the value that Tycho proposed (26;0,30°). Figure 10 plots the differences derived from the two sets of calculations, assuming the Copernican and Ptolemaic parallaxes. The error when assuming the Copernican parallax is very small. Again, Tycho is extraordinarily lucky: only assuming a Copernican model for Venus, the parallax cancels out the error in his solar model.

Tycho, however, must have been aware that this method was not too convincing given its exclusion of refraction, as he offers the second that, as I already showed, cancels out the uncontrolled effects of parallax and refraction.

25 There are other attempts in the appendix: changing the latitude of Uraniborg to 34;9,50° (TBOO, 10, 221–223), assuming refraction (TBOO, 10, 223–224), and not assuming refraction and parallax (TBOO, 10, 229–230).

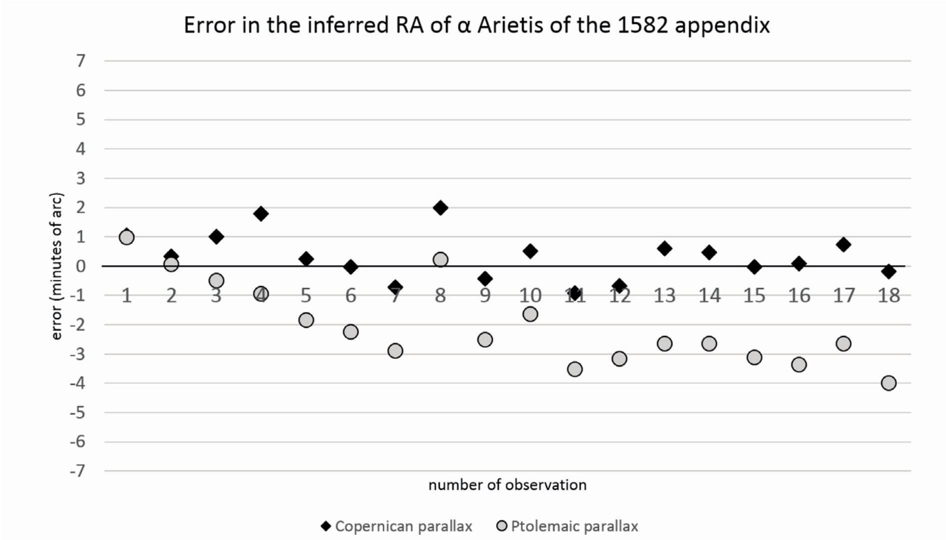

Figure 10. Error in the inferred RA of α Arietis in the Appendix *Exquisitior et verior inquisitio ascensionis rectae quarundam stellarum fixarum*: TBOO (10, 204–230).

5. Concluding Remarks

Tycho's star catalog enjoyed an enormous prestige for centuries given its accuracy, which is truly unique. The entire catalog depends on the coordinates of one single star: α Arietis. An error in its coordinates would be systematically translated to all the stars. Aware of this, Tycho was extraordinarily scrupulous in determining the equatorial coordinates of α Arietis. The declination is not a problem, but obtaining the RA is a delicate issue. Thus, to find the RA of α Arietis, Tycho uses a total of 54 independent observations (27 of the Sun–Venus angular separation, and 27 of the Venus–star angular separation). From these observations, using Venus as intermediary, he obtains 27 different values for the RA of α Arietis. Tycho also knows that he cannot control two variables to a sufficient degree of accuracy: refraction and parallax. In the first three cases, he incorporates parallax and practically ignores refraction. He obtains excellent values, but remains aware that the method is not trustworthy. As such, he proposes an ingenious way of canceling out the uncontrolled effects of refraction and parallax, averaging the results of two observations, one in the morning and the other in the evening, in which the altitudes of the bodies and the distances from the Earth are the same. He is successful with this method because, even if refraction implies corrections of several minutes and parallax would imply errors also of several minutes (assuming Tycho's incorrect distances for the Sun and Venus), the scatter of the averaged values is a slightly greater than half a minute, and the difference between the final value adopted and the modern value is no more than a quarter of a minute. Because the method begins from the calculated coordinates of the Sun, an error in the solar model would also be translated to the star coordinates and to the whole catalog.

Indeed, this is what happens in Ptolemy's star catalog, which suffers a systematic error of around 1° in longitude due to the same error in his solar model.[26] Tycho's solar model assumes too large of an eccentricity, producing errors in predictions of the solar longitude that can reach 8'. Still, his star catalog does not show this systematic error, because α Arietis does not show it either. Intending to cancel out the effects of parallax and refraction in the observations for obtaining the RA of α Arietis, Tycho also unknowingly cancels out the error that his solar model produced. Because the error is produced by too large of an eccentricity, it is symmetrical with respect to the apsidal line. In order to obtain observations with both the Sun and Venus at the same altitude, Tycho must look for observations that are symmetrical to the line connecting the solstices. Fortunately, in Tycho's time, the solar apsidal line was very close to that line. Consequently, when choosing the observations for intentionally canceling the effect of refraction and parallax, he unintentionally also canceled out the error in his solar model. Tycho was an extraordinary observer and a great astronomer. But, from time to time, he was also very lucky.

Acknowledgements

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References

Carman, C. C. (2020). Longomontanus' model for the longitudes of Mars. *Journal for the History of Astronomy, 51*(2), 162–186. https://doi.org/10.1177/0021828620923802

Dreyer, J. L. E. (1890). *Tycho Brahe: A picture of scientific life and work in the sixteenth century*. Edinburgh, UK: Adam and Charles Black.

Dreyer, J. L. E. (Ed.). (1913–1929). *Tychonis Brahe Dani opera omnia.* Copenhagen, Denmark: Libraria Gyldendaliana.

Dreyer, J. L. E. (1917). On the origin of Ptolemy's catalogue of stars. *Monthly Notices of the Royal Astronomical Society, 77*(7), 528–539. https://doi.org/10.1093/mnras/77.7.528

Dreyer, J. L. E. (1918). On the origin of Ptolemy's catalogue of stars. Second paper. *Monthly Notices of the Royal Astronomical Society, 78*(5), 343–349. https://doi.org/10.1093/mnras/78.5.343a

Evans, J. (1998). *The history and practice of ancient astronomy*. Oxford, UK: Oxford University Press.

Halley, E. (1717). Considerations on the change of the latitudes of some of the principal fixt stars. *Philosophical Transactions of the Royal Society, 29*, 736–738.

26 Dreyer (1917; 1918); Evans (1998, pp. 265–274).

Kepler, J. (1627). *Tabulae Rudolphinae, quibus astronomicae scientiae, temporum longinquitate collapsae restauratio continentur*. Ulm, Germany: Jonas Saur.

Kremer, R. (1980). Bernard Walther's astronomical observations. *Journal for the History of Astronomy, 11*, 174–191.

Maeyama, Y. (1974). The historical development of solar theories in the late sixteenth and seventeenth centuries. *Vistas in Astronomy, 16*, 35–60. https://doi.org/10.1016/0083-6656(74)90003-8

Swerdlow, N. M. (2010). Tycho, Longomontanus, and Kepler on Ptolemy's solar observations and theory. In A. Jones (Ed.), *Ptolemy in perspective: Use and criticism of his work from antiquity to the nineteenth century* (pp. 151–202). Dordrecht, The Netherlands: Springer.

Thoren, V. E. (1990). *The lord of Uraniborg: A biography of Tycho Brahe*. Cambridge, UK: Cambridge University Press.

Toomer, G. J. (1998). *Ptolemy's Almagest*. Princeton, NJ: Princeton University Press.

Verbunt, F., & van Gent, R. H. (2010). Three editions of the star catalogue of Tycho Brahe: Machine-readable versions and comparison with the modern Hipparcos catalogue. *Astronomy and Astrophysics, 516*, A28. https://doi.org/10.1051/0004-6361/201014002