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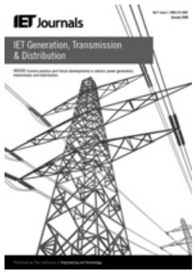

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Harmonic load-flow approach based on the possibility theory

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Abstract: Harmonics in power systems are responsible for several technical problems that justify the development of models to study them. Well-established models exist to analyse the harmonic load-flow (HLF) from a deterministic point of view. Moreover, models based on the probability theory have been developed to deal with the inherent variability and random nature of loads, network configuration etc. In the last few years, possibility theory has arisen as an alternative tool that in many cases could be better suited to describe and quantify the real nature of the uncertainty involved in harmonic studies. In this study a methodology for HLF calculation based on the possibility theory is presented. Possibility distributions instead of probabilities are the input used to describe the uncertainty in the magnitude and composition of the loads. Tests presented shows that the results of the proposed model are consistent with those obtained with a probabilistic method, and that both models lead to the same ranking of the risk that the bus harmonic voltages exceed a given level. Independent possibility distributions are assumed at the development stage reported here; research is being carried out in order to overcome this constraint.

Nomenclature

x	real scalar
$\tilde{x}$	complex number, phasor
$\boldsymbol{V}$	real vector or matrix
$\tilde{\boldsymbol{V}}$	vector or matrix of complex numbers, phasors
X	classical set
X^{C}	complement of the classical set X
$\hat{X}$	fuzzy set or fuzzy number
$\hat{\boldsymbol{x}}$	fuzzy vector
$\mu_{\hat{X}}$	membership function of the fuzzy set $\hat{X}$
α	α-value of a fuzzy set, membership degree
$X^{(\alpha)}$	α-cut set of the fuzzy set $\hat{X}$
$\underline{x}^{(\alpha)}$	lower boundary of an α-cut set
$\bar{x}^{(\alpha)}$	upper boundary of an α-cut set
$v_{(h)}$	hth harmonic bus voltage magnitude

1 Introduction

Harmonics distortion is a growing problem in power systems because of the increasing use of non-linear loads [1]. Mathematical models have been developed to study problems related to harmonics [2]. Harmonic load-flow (HLF) calculations can be performed on a deterministic basis, assuming that all the relevant parameters are well known and non-random. However, such studies provide a static and certain image of a varying and uncertain situation. In fact, the network configuration usually changes and its linear and non-linear loads vary all the time; in addition, even if they were constant their parameters are not usually well known. All these features make harmonic distortion a phenomenon involving uncertainty.

Methodologies based on the probability theory, with different degrees of sophistication, have been developed to deal with these uncertainties [3–9] (Romero *et al.* [8] and Ribeiro [9] provide a complete review of the state of the art on this subject). Practical application of them, however, often has to face the lack of information to describe in probabilistic terms the amount and type of medium-sized and small distributed non-linear loads (NLLs), as well as the composition of linear loads (LLs). This suggests that in many practical cases, the available information for HLF can be described better through fuzzy measures like possibility distributions.

Possibility measures are particularly well suited to integrate the judgment of experts regarding the uncertainty or likelihood. For example, an expert may describe a load saying that it could be between 80 and 100 MVA, 90 MVA being the most 'possible' value. He might estimate the power factor between 0.87 and 0.9. In addition, he could guess that between 70 and 80% of the total load is linear and of this percentage, about 30–40% may be because of the induction motors and so on. In general, such information is incomplete, imprecise, even contradictory or deficient in some other way. Possibility theory is ideal to model this kind of fuzzy information.

Like models based on probability, those based on possibility rely on measures that quantify uncertainties or likelihood and

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allow calculating how these propagate from the inputs or the parameters of a system into its outputs. Possibilistic models are usually simpler than their probabilistic counterparts, but the key feature certainly is their ability to model expert knowledge, opinions, incomplete information and other kinds of evidence, very common in the area of harmonic studies, which are difficult to handle through probabilistic models.

The first methodology in the area of fuzzy HLF has been proposed by Hong *et al.* in [10]. An analysis of this proposal reveals the following features:

1. The methodology relies on the classic fuzzy solution, which could overestimate or underestimate the uncertainty in the harmonic voltages.
2. Even though linear loads could have a major influence on the HLF, [11], uncertainty in their composition is not modelled and cannot be handled by the formulation.
3. Primary results are the fuzzy real and imaginary components of harmonic voltages from which the more useful fuzzy magnitudes have to be calculated. A measure of the possibilistic dependence (An analogous to the probabilistic dependence in the possibility theory.) between the components should be known in order to not overestimate the uncertainty, but the methodology does not provide this figure.
4. Possibilistic dependencies between uncertain parameters describing the loads are not considered.

A recent paper by the authors [12] outlines an alternative approach for a possibilistic HLF aimed at overcoming the first three aforementioned drawbacks. After comparing the classic fuzzy solution with other alternatives, a methodology based on the marginal joint solution is proposed that avoids uncertainty overestimation and/or underestimation and allows the direct calculation of fuzzy harmonic voltage magnitudes.

A major weakness of this proposal is that the uncertainties regarding the loads refer to the parameters of their equivalent circuits (e.g. resistances and reactance modelling linear loads). In the present paper whose basic proposal is substantially improved by moving the uncertainties to a higher level; for example, the uncertainty in the active and reactive bus load, its percentage because of induction motors etc.

The basic objectives and hypotheses of the developed methodology can be summarised as follows:

1. The configuration of the network does not vary and its parameters are deterministic (certain).
2. The fuzzy load models are intended to describe the uncertainties in the context of a specific load state of the system (minimum or maximum daily load, for example). In this context active and reactive bus loads, although uncertain, vary in a narrow range (results obtained for different network configuration and load levels can however be post-processed, taking into account their relative possibility of obtaining global figures).
3. Harmonic currents injected by non-linear loads are assumed independent of the voltage waveform (harmonic penetration approach).
4. Injected harmonic currents as well as admittances to ground that model linear loads are functions of uncertain parameters (more precisely defined below) such as the total active and reactive bus load, the percentage of the active and reactive bus load because of the ac/dc converters etc.
5. The uncertain parameters referred above are non-interacting; roughly speaking, a concept analogous to independent in probability theory.
6. Node voltages are given by continuous functions of the uncertain parameters, that is, no infinite voltage at the analysed harmonic frequencies occurs when the loads vary within the limits allowed by the possibility distributions. Notice that owing to losses this is actually the case.
7. The paper has been organised as follows: in Section 2, fundamental concepts of possibility theory are briefly reviewed. In Section 3 the possibilistic HLF formulation is developed. In Section 4 the load models are exposed: (i) the circuit model of the loads is described; (ii) a set of fuzzy parameters regarding the load magnitude and composition is defined; and (iii) relationships between the fuzzy parameters characterising the non-linear loads and their circuit models are described, analogous relationships for linear loads are summarised in an appendix. In Section 5, the possibilistic HLF is numerically compared with Monte Carlo simulations (MCS) using the IEEE 14-bus test system for harmonic analysis of reference [13]. Finally, conclusions are given.

2 Key concepts regarding possibility theory

Possibility theory is a powerful mathematical tool which is ideal for formalising incomplete information expressed in terms of fuzzy propositions (which is usually the case for data involved in HLF calculations), [14].

In possibility theory, available information is modelled through possibility distributions which in some extent are analogous to probability distributions in probability theory. Moreover, possibility distributions can be suitably formulated in terms of fuzzy sets (FS), hence taking advantage of many of the developments in this area. In what follows, key concepts of the possibility theory will be presented briefly.

2.1 Fuzzy sets

In classical sets, the characteristic function assigns a value of either 1 or 0 to each individual x in the universal set X, thereby discriminating members and non-members of the set. This function can be generalised assigning values within the unit interval [0, 1] to the elements of X, thus indicating the membership grade of each element in the set, [15]. This function, called the FS membership function, is denoted as

$$\mu_{\hat{A}}: X \rightarrow [0, 1] \qquad (1)$$

Therefore an FS is a pair usually noted as

$$\hat{A} = \{(x, \mu_{\hat{A}}(x)): x \in X\} \qquad (2)$$

The FS, $\hat{A}$, can also be defined in terms of its α-cuts being the subset

$$A^{(\alpha)} = \{x | \mu_{\hat{A}}(x) \geq \alpha\} \qquad (3)$$

We are particularly interested in fuzzy real numbers and fuzzy real vectors. That is, fuzzy sets with $X \subseteq \mathbb{R}$ or $X \subseteq \mathbb{R}^n$ and $\mu_{\hat{A}}$ convex, at least segmentally continuous, and with $\mu_{\hat{A}}(x) = 1$ for at least one element of X, that is, normal. Fig. 1 shows a generic FS in order to introduce its basic relation with possibility theory as well as the notation used in what follows. It can be seen in the figure that the α-cuts of an FS are intervals in the real axis.

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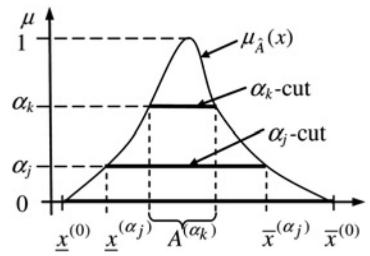

Fig. 1 *Representations of the membership function of a fuzzy number $\hat{A}$*

2.2 Possibility distributions and necessity and possibility measures

A possibility distribution can be conveniently seen as a sequence of nested confidence intervals, coincident with the α-cuts $A^{(\alpha)} = [\underline{x}^{(\alpha)}, \bar{x}^{(\alpha)}]$ of fuzzy set $\hat{A}$; each interval has an associated confidence level regarding the truth of the statement 'the actual value of the uncertain magnitude belongs to interval $A^{(\alpha)}$', [16]. This belief measure, called necessity (nec), is maximum for $A^{(0)}$ (i.e. nec$(A^{(0)}) = 1$) because it is completely certain that the value of the magnitude is within the 0-cut, and decreases as α increases (shorter α-cuts), that is, nec$(A^{(\alpha)}) = 1 - \alpha$.

Necessity is not defined only for the α-cuts of A, else for any subset, S, of A. Then a second fuzzy measure, called Possibility, which is related to Necessity can be defined through the following relationships

$$\text{pos}(S) = 1 - \text{nec}(S^c) \quad (4)$$

$$\text{nec}(S) = 1 - \text{pos}(S^c) \quad (5)$$

Furthermore, possibility theory states that for any pair of subsets S and T, it is

$$\text{pos}(S \cup T) = \max(\text{pos}(S), \text{pos}(T)) \quad (6)$$

$$\text{nec}(S \cap T) = \min(\text{nec}(S), \text{nec}(T)) \quad (7)$$

In particular, for any interval $S = \{x | \underline{x} \le x \le \bar{x}\}$, it can be written as

$$\text{pos}(S) = \max_{x \in S}(\text{pos}(x)) \quad (8)$$

From these basic definitions, it follows that (see Fig. 1)

$$\text{pos}(x) = \mu_{\hat{A}}(x) \quad (9)$$

Therefore possibility and necessity of any interval S can be evaluated from the membership function as

$$\text{pos}(S) = \max_{x \in S}(\mu_{\hat{A}}(x)) \quad (10)$$

$$\text{nec}(S) = 1 - \max_{x \notin S}(\mu_{\hat{A}}(x)) \quad (11)$$

Analogous concepts apply to fuzzy vectors. The membership function $\mu_{\hat{\boldsymbol{P}}}([x_1, x_2, \ldots, x_n])$ is a convex function that maps vectors in $\mathbb{R}^n$ to the interval [0, 1]; α-cuts are nested regions in $\mathbb{R}^n$, defined through $\boldsymbol{P}^{(\alpha)} = \{\boldsymbol{X} | \mu_{\hat{\boldsymbol{P}}}(\boldsymbol{X}) \ge \alpha\}$; the degree of belief (Necessity) that the actual vector belongs to a specified α-cut is 1-α, and the possibility of a vector $\boldsymbol{X} = [x_1, x_2, \ldots, x_n]$ is just its membership degree $\mu_{\hat{\boldsymbol{P}}}(\boldsymbol{X})$.

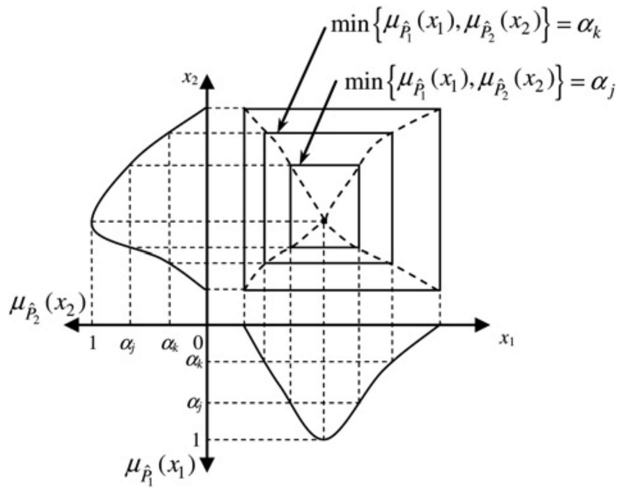

Fig. 2 *Fuzzy Cartesian product defined on $\mathbb{R}^2$*

For the methodology described here a specific fuzzy vector is of interest: let $\boldsymbol{X} = [x_1, x_2, \ldots, x_n]$ be a vector of uncertain magnitudes with possibility distributions, pos$(x_i) = \mu_{\hat{P}_i}(x_i)$, $i = 1\ldots n$, then, if the variables are non-interacting, according to the possibility theory it is

$$\text{pos}(\boldsymbol{X}) = \min\{\text{pos}(x_1), \text{pos}(x_2), \ldots, \text{pos}(x_n)\} \quad (12)$$

or in terms of the membership functions

$$\mu_{\hat{\boldsymbol{P}}}(\boldsymbol{X}) = \min\{\mu_{\hat{P}_1}(x_1), \mu_{\hat{P}_2}(x_2), \ldots, \mu_{\hat{P}_n}(x_n)\} \quad (13)$$

The fuzzy vector with membership functions defined by (13) is called the fuzzy Cartesian product of $\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_n$ and is often denoted as $\hat{\boldsymbol{P}} = \hat{P}_1 \times \hat{P}_2 \times, \ldots, \hat{P}_n$. An important feature of the fuzzy Cartesian product is that its α-cut $\boldsymbol{P}^{(\alpha)}$ is the classic Cartesian product of the corresponding α-cuts $P_i^{(\alpha)}(i = 1, \ldots, n)$. Fig. 2 illustrates this relationship with a two-dimensional example.

Finally, both fuzzy measures of uncertainty, possibility and necessity, are the basis of several techniques developed in areas such as decision-making under uncertainties. To this purpose, different ranking methods have been proposed for comparing and ordering obtained FS, that is, for different scenarios [17].

3 Proposed possibilistic HLF

3.1 General formulation

The physical model of the network is based on the deterministic harmonic penetration method, [18] (NLLs are modelled as injected harmonic currents independent of the voltage waveform). Harmonic voltages and injected currents are related through the nodal admittance matrix calculated at the corresponding harmonic frequency, that is

$$\tilde{\boldsymbol{V}}_{(h)} = [\tilde{\boldsymbol{Y}}_{(h)}]^{-1}\tilde{\boldsymbol{I}}_{(h)} \quad (14)$$

where h stands for the hth harmonic order and will be omitted from here on for notational simplicity.

Injected harmonic currents as well as the diagonal elements of the admittance matrix in (14) depend on the harmonic order, the model of the LLs and NLLs and on a number of parameters describing their magnitudes and compositions. For the sake of generality, in this section no hypotheses

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will be made regarding these aspects as the development of a specific model is postponed to the next section.

Let us assume, however, that a generic vector $\boldsymbol{P} = (p_1, p_2, \ldots, p_{\text{np}})$ of np real parameters completely describes the loads of the system, determining accordingly all the injected currents and the bus admittance matrix in (14)

$$\tilde{\boldsymbol{V}}(\boldsymbol{P}) = [\tilde{\boldsymbol{Y}}(\boldsymbol{P})]^{-1}\tilde{\boldsymbol{I}}(\boldsymbol{P}) \tag{15}$$

By denoting $\tilde{\boldsymbol{Z}}_j(\boldsymbol{P})$ the jth row of $\tilde{\boldsymbol{Z}}(\boldsymbol{P}) = [\tilde{\boldsymbol{Y}}(\boldsymbol{P})]^{-1}$, the amplitude of the harmonic voltage at a generic bus j, can be written as

$$v_j(\boldsymbol{P}) = \text{abs}(\tilde{\boldsymbol{Z}}_j(\boldsymbol{P})\tilde{\boldsymbol{I}}(\boldsymbol{P})) \tag{16}$$

or using a simpler notation

$$v_j(\boldsymbol{P}) = f_j(\boldsymbol{P}) \tag{17}$$

with $f_j(\boldsymbol{P}) = |\Sigma_k \tilde{z}_{j,k}(\boldsymbol{P})\tilde{i}_k(\boldsymbol{P})|$.

When the components of $\boldsymbol{P}$, parameters $(p_1, p_2, \ldots, p_{\text{np}})$, are uncertain and described through possibility distribution associated with fuzzy sets $\hat{P}_1, \hat{P}_2, \ldots, \hat{P}_{\text{np}}$, the harmonic voltage amplitude v_j in (17) becomes a possibilistic variable too.

If there were only one vector $\boldsymbol{P}$ for which v_j attains a specific value, it is clear that the possibility of this value of v_j is the same as possibility of that particular vector. In general, however, there is a set of vectors that yields the same voltage and the possibility of the voltage is the possibility that the actual vector of parameters belongs to that set; thus, according to (6), the highest among the possibilities of the vectors in that set is

$$\text{pos}(v_j) = \max_{f_j(\boldsymbol{P})=v_j}\{\text{pos}(\boldsymbol{P})\} \tag{18}$$

On the other hand, since according to the hypothesis stated above the uncertain parameters are non-interacting, then

$$\text{pos}(\boldsymbol{P}) = \min\{\text{pos}(p_1), \text{pos}(p_2), \ldots, \text{pos}(p_{\text{np}})\} \tag{19}$$

and therefore

$$\text{pos}(v_j) = \max_{f_j([p_1,p_2,\ldots,p_{\text{np}}])=v_j}\{\min\{\text{pos}(p_1), \text{pos}(p_1), \ldots, \text{pos}(p_{\text{np}})\}\} \tag{20}$$

or in terms of membership functions

$$\mu_{\hat{V}_j}(v_j) = \max_{f_j([p_1,p_2,\ldots,p_{\text{np}}])=v_j}\{\min\{\mu_{\hat{P}_1}(p_1), \mu_{\hat{P}_2}(p_2), \ldots, \mu_{\hat{P}_{\text{np}}}(p_{\text{np}})\}\} \tag{21}$$

Equation (21) provides an expression of the membership function associated to the voltage at node j at a certain harmonic order omitted for the sake of simplicity (notice that $f_j(p_1, p_2, \ldots, p_{\text{np}})$ in (21) depend on the harmonic order as it affects the injected harmonic currents and the impedances to ground modelling the LLs).

Two well-known approaches can be applied to obtain $\mu_{\hat{V}_j}$. The first one, based on the 'extension principle', is to solve directly the maximisation problem stated by (21). The second one, based on the 'resolution principle', entails solving the dual-optimisation problem of finding a set of α-cuts of m from which the membership function can be obtained. The authors follow the second approach which is $\hat{V}_j$ better explained with Fig. 3 and refers to a simple case with two parameters.

The rectangle shown in Fig. 3 encloses the α_0-cut (for a generic α_0) of $\hat{\boldsymbol{P}}$ (see Fig. 2). Also the contour level C_1: $f_j([p_1, p_2]) = v_{j_1}$ with points inside the rectangle has been drawn. The function f_j maps the points of the rectangle within the corresponding node voltages $v_j = f_j([p_1, p_2])$. Since the node voltages are given by continuous functions of the parameters (see hypotheses 6 in Section 1), the entire α_0-cut is mapped into a single interval of the v_j axis, which clearly should contain v_{j_1}.

Now we ask about the possibility of the specific voltage v_{j_1}; according to (20) it is

$$\text{pos}(v_{j_1}) = \max_{[p_1,p_2]\in C_1}\{\min\{\text{pos}(p_1), \text{pos}(p_2)\}\} \tag{22}$$

but inside and over the rectangle it is $\text{pos}(p_1) \geq \alpha_0$ and $\text{pos}(p_2) \geq \alpha_0$, and thus $\min\{\text{pos}(p_1), \text{pos}(p_2)\} \geq \alpha_0$ and since there are points of C_1 then it follows that $\text{pos}(v_{j_1}) = \max_{[p_1,p_2]\in C_1}\{\min\{\text{pos}(p_1), \text{pos}(p_2)\}\} \geq \alpha_0$.

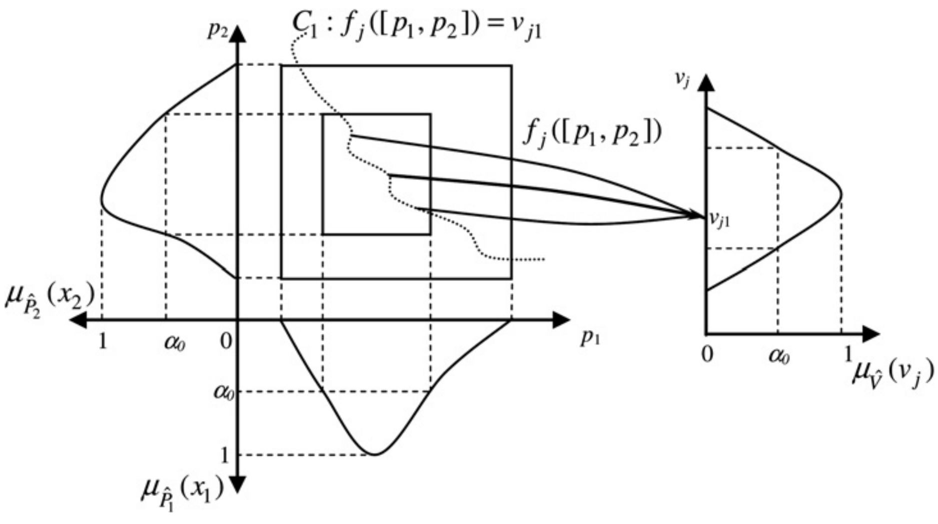

Fig. 3 *Extension principle for a continuous function defined on* $\mathbb{R}^2$

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Hence v_{j_1} belongs to the α_0-cut of $\hat{V}_j$ and the same happens with any voltage whose contour level pass through the rectangle or touch its boundary. Conversely, a voltage whose contour level does not pass through or touch the rectangle does not belong to the α_0-cut, because $\min\{\text{pos}(p_1), \text{pos}(p_2)\} \geq \alpha_0$ is not true at any point of this contour level.

From this reasoning it is clear that the upper and lower limits of the α_0-cut of $\hat{V}_j$, are the maximum and minimum voltages mapped by f_j from the α_0-cut of $\hat{\boldsymbol{P}}$, and thus

$$V_j^{(\alpha)} = [\underline{v}_j^{(\alpha)}, \overline{v}_j^{(\alpha)}] = \left[\min_{\substack{p_1 \in P_1^{(\alpha)} \\ \vdots \\ p_{\text{np}} \in P_{\text{np}}^{(\alpha)}}} f_j([p_1, \ldots, p_{\text{np}}]), \max_{\substack{p_1 \in P_1^{(\alpha)} \\ \vdots \\ p_{\text{np}} \in P_{\text{np}}^{(\alpha)}}} f_j([p_1, \ldots, p_{\text{np}}]) \right] \tag{23}$$

3.2 Possibilistic HLF algorithm

Equations (21) and (23) are equivalent, if the first is based on the extension principle and directly provide the membership function or possibility distribution $\mu_{\hat{V}_j}(v_j)$, the second provides its α-cuts. The developed methodology is based on calculating sets of α-cuts of the fuzzy node voltages according to the second approach.

The basic algorithmic sequence is as shown in Fig. 4.

Optimisation problems involved have been implemented in TOMLAB®, [19]. The optimisation algorithm needs the values of the function and its partial derivatives with respect to the uncertain parameters (gradient). Irrespective of the set of parameters chosen, there are, in general, m parameters associated with the load of each bus, that is, total $np = n \cdot m$ parameters in a network with n buses. Let then $p_{k,j}$ be the ith parameters associated with the load of bus k. It can be shown that

$$\frac{\partial |\tilde{v}_{(h)j}|}{\partial p_{k,i}} = \text{Re}\left(\frac{\tilde{\boldsymbol{v}}^*_{(h)j}}{|\tilde{v}_{(h)j}|} \tilde{\boldsymbol{z}}_{(h),j,k} \left(\frac{\partial \tilde{\boldsymbol{i}}_{(h)k}}{\partial p_{k,i}} - \frac{\partial \tilde{\boldsymbol{y}}_{(h)k}}{\partial p_{k,i}} \tilde{\boldsymbol{v}}_{(h)k} \right) \right) \tag{24}$$

where $\tilde{\boldsymbol{v}}_{(h)j}$ is the hth harmonic voltage at node j; $\tilde{\boldsymbol{z}}_{(h),j,k}$ is the transfer impedance from bus j and k at the harmonic order h;

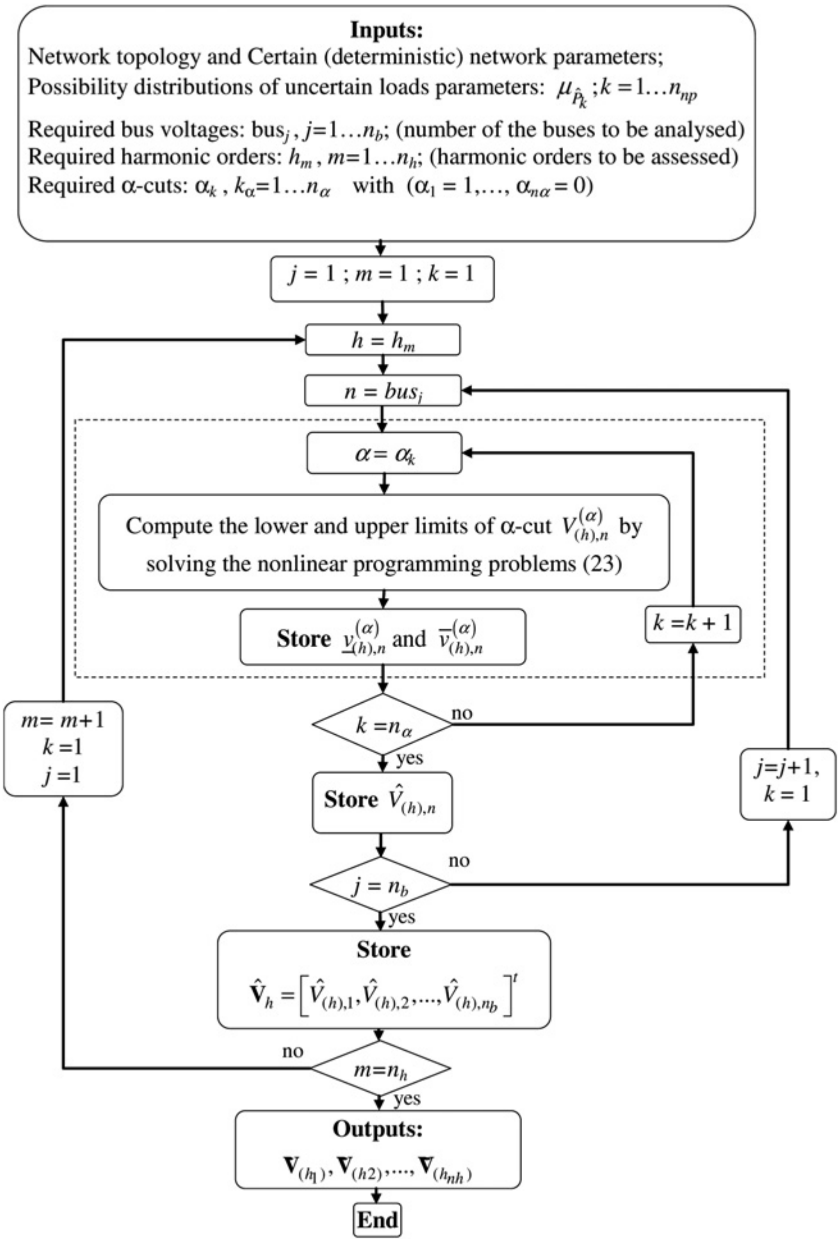

Fig. 4 *Algorithm for the new fuzzy harmonic load-flow approach*

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$\tilde{i}_{(h)k}$ is the hth harmonic current injected at node k; $\tilde{y}_{(h)k}$ is the hth harmonic admittance to ground modelling the LLs connected at node k.

Clearly, derivatives on the right-hand side of (24) depend on the models implemented for the LLs and NLLs which are developed in Section 4.3 and Appendix.

4 Fuzzy modelling of linear and non-linear harmonic loads

4.1 Deterministic aggregate model for bus loads

Standard load models for HLF calculations are parallel connections of impedances and harmonic injected currents, [11, 18, 20–23]. Results of different studies suggest the deterministic load model shown in Fig. 5, where each branch represents an aggregate of homogeneous loads connected to a bus (Section IV in [11]).

Series resistance $r_{(h)p}$ and reactance $x_{(h)p}$ model aggregate passive loads and small motors. The $r_{(h)m}$ and $x_{(h)m}$ model large motors used in industrial applications (without power electronic devices for control). Capacitive reactance $x_{(h)c}$ models the capacitors for power factor correction and cable capacitances. Finally, $\tilde{z}_{(h)\text{filter}}$ represents the impedance of the filters for harmonic distortion mitigation.

The standard aggregate model of the NLLs connected to a bus is a set of harmonic current sources with different amplitudes and phases $\tilde{i}_{(h)\text{nl}}$. Under the assumption of harmonic non-interaction, these currents do not depend on the voltage wave shape [18].

4.2 Fuzzy model parameters

The circuit parameters in the load model of Fig. 5 are uncertain and in principle they could be described through possibility distribution functions. In practice, however, the available information usually refers to other characteristics of the load, such as its total active and reactive power, the percentage of it owing to non-linear devices etc. Different sets of parameters could be used to describe the loads; the model implemented by the authors is based on the parameters listed in Table 1 although other choice could be accommodated in the general formulation of Section 3. Possibility distribution functions of these parameters are the data and they are usually obtained on the basis of typical information regarding the active and reactive power of the loads and the expert judgment regarding their composition.

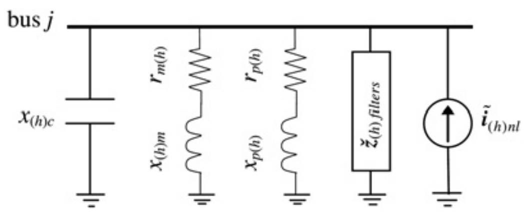

Fig. 5 *Electrical model of the harmonic loads connected to a PS's node*

4.3 Relationships between model parameters and aggregate load model

4.3.1 Some remarks: Relationships between model parameters in Table 1 and circuit parameters of Fig. 5 link the model parameters with harmonic voltage (23) through the circuit admittances and injected currents.

Models of NLLs will be analysed in this section while analogous relationships for the models of LLs are summarised in the Appendix. Some previous remarks related to hypotheses 1 and 2 stated on Section 1 are, however, in the following order:

1. There are no parameters in Table 1 referring to the system load as a whole or stating relationships between loads connected at different buses, for example, loads in different buses are independent of each other. This seems to be a reasonable assumption for calculating the harmonic voltage for a specific load state and well-known network configuration, since, in this context, uncertainties mainly refer to each bus load and do not influence the amount and characteristics of the loads connected to other buses.
2. For a specific load state the active and reactive power consumption varies within narrow limits. Thus, the amplitude and phase of the power frequency bus voltages are almost certain and can be calculated only once at the beginning, for the most possible value of the bus loads and by means a standard power frequency ac load flow. Notice that this is a key feature because the phase angles of the injected harmonic currents modelling the NLLs depend on the phase angle of the power frequency component.

4.3.2 Model of NLLs: NLLs are usually classified into two groups according to their prevailing application: residential–commercial and industrial. Residential and commercial

Table 1 Fuzzy model parameters for the harmonic load-flow calculation

Model parameter	Description
p_t	active bus load
q_t	reactive bus load
k_p	composition factor of passive loads (fraction of the passive loads in the total active bus load)
pf_p	power factor of passive loads
k_m	composition factor of induction motors (fraction of the induction motors in the total active bus load)
pf_m	power factor of the induction motors
k_{nl}	composition factor of NLLs (fraction of the NLL in the total active bus load)
pf_{nl}	power factor of NLLs
$ang_{(h)}$	phase angles of the harmonic currents injected (for the aggregate model of several small and medium power devices)
x_{LR}	locked rotor reactance in pu (x_{LR} typically ranges from 0.15 to 0.25 pu)
fq_{LR}	quality factor of the locked rotor circuit. From experience, this value is approximately 8
q_c	reactive power because of the capacitors for power factor correction and cables

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NLLs are low-power devices, distributed in the low-voltage networks. Individually they do not represent major problems for the network but, acting together, they can become an important source of harmonic distortion. Typical NLLs of this kind are: devices fed through single-phase diode bridge rectifiers or small switched power supplies (personal computers, television sets etc.), discharge lamps etc.

The group of industrial NLLs is mainly composed of static rectifiers and power converters, electronic power devices at transmission voltage level, arc furnaces and so on.

Accurate modelling of NLLs is a difficult issue because of their inherent complexity and diversity. However, in general, harmonic currents injected by a specific device can be expressed as

$$\tilde{\boldsymbol{i}}_{(h)\mathrm{nl}} = \boldsymbol{g}(h)|\tilde{\boldsymbol{i}}_{(1)\mathrm{nl}}| \exp(\mathrm{j}a(h)\mathrm{angle}(\tilde{\boldsymbol{i}}_{(1)\mathrm{nl}})) \tag{25}$$

where $\boldsymbol{g}(h)$ harmonic order h and $\tilde{\boldsymbol{i}}_{(h)\mathrm{nl}}$ is the phasor of the hth harmonic order current modelling the NLL.

These relationships are not always well known, except for some especially important kind of apparatus.

At present only models for six and 12 pulse bridges have been implemented in the methodology, for large NLLs. Detailed models of other devices are described in [1, 18, 20–24], which in principle can be implemented in the present proposal by means expressions like (25).

In particular, for six and 12 pulse bridges $a(h) = h$ and, respectively

$$\boldsymbol{g}(h) = \begin{cases} \pm 1/h & \text{where } h = 6n \pm 1, \quad \text{with} \quad n = 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

and

$$\boldsymbol{g}(h) = \begin{cases} \pm 1/h & \text{where } h = 12n \pm 1, \quad \text{with} \quad n = 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases} \tag{27}$$

with the power frequency current: $\tilde{\boldsymbol{i}}_{(1)\mathrm{nl}} = (p_{\mathrm{nl}} + \mathrm{j}q_{\mathrm{nl}}/\tilde{\boldsymbol{v}}_{(1)})^*$ (per unit values); or in terms of model parameters at bus j

$$\tilde{\boldsymbol{i}}_{(1)\mathrm{nl}} = \frac{p_t \cdot k_{\mathrm{nl}}}{pf_{\mathrm{nl}}} \left(\frac{\left(pf_{\mathrm{nl}} - \mathrm{j}\sqrt{1 - pf_{\mathrm{nl}}^2}\right)}{\tilde{\boldsymbol{v}}_{(1)}^*} \right) \tag{28}$$

The phases of the currents in (25) are finally shifted to take into account the winding connection of the transformers between the power electronic apparatus and the bus load.

On the other hand, expressions like (25) or similar ones provide accepted estimates of the amplitude of the total harmonic current injected by several small non-linear devices. It is rather difficult, however, to obtain a good approximation of its phase. In the developed methodology, the aggregate harmonic currents are first computed as in the case of the six pulse bridges (although other expressions for $g(h)$ could be easily implemented) and then the resulting phases are shifted by the fuzzy parameter $\mathrm{ang}_{(h)\mathrm{nl}}$, in order to take this additional uncertainty into account.

In practice for those buses with large connected non-linear devices, the effect of small NLLs can usually be neglected and very narrow limits in the uncertainty of $\mathrm{ang}_{(h)\mathrm{nl}}$ are adopted.

5 HLF calculation in a 14-bus test system

In this section the proposed possibilistic HLF method will be tested on the well-known IEEE 14-bus harmonic test system [13]. Results of MCS with probability distribution consistent with the possibility distribution assigned to the uncertain variables will be used to validate the possibilistic proposal. The following remarks may clarify some aspects of this validation:

In a broad sense, a probability distribution contains more information and needs more information to be properly defined than a possibility distribution. Despite this fact, and in order to use available probabilistic information in possibilistic models, several techniques have been developed to transform probability distributions in consistent possibility distributions. Although various consistency criteria may be stated, the weakest and usually imposed to such transformations is

$$\mathrm{pos}(\Lambda) \geq p(\Lambda) \geq \mathrm{nec}(\Lambda) \tag{29}$$

where $p(\Lambda)$ is the probability of the set Λ.

Equation (29) states that if it is probable to some degree that the uncertain variable belongs to set Λ, then it must also be possible at least to the same degree, [15].

In this sense, in [25], it is shown that the triangular possibility distribution is a legitimate transformation of bounded symmetric unimodal probability distributions, with the same support. This consistency criterion can also be used to compare the results of possibilistic and probabilistic models when their inputs are related in this way.

On the basis of these remarks the following criteria will be applied in the validation to be carried out:

1. There is enough information to characterise the uncertain parameters by means of probability density distributions, and normal distributions truncated at two standard deviations (bounded symmetric unimodal probability distributions) fit that information.
2. Triangular possibility distributions are used as the legitimate probabilistic → possibilistic transformation for modelling the same uncertainties in the possibilistic case. (It should be remarked however, that the proposed possibilistic HLF methodology can deal with any kind of possibility distributions e.g. triangular, trapezoidal, normal, rectangular etc).
3. In validating the methodology it will be shown on an empirical base that the possibilities of the harmonic voltages obtained with the proposed methodology and their probabilities, calculated by MCS, fulfil inequalities (29), thus providing consistent information regarding the phenomenon.
4. Although the assessment of consistency is a sound validation from the theoretical point of view, it does not warrant that the results of the possibility-based methodology provide enough information to make good decisions. This is the subject of a second validation where it will be shown that both results lead to the same rankings of the bus harmonic voltages, and thus also to the same decision, for example, the buses where measurement equipment should be installed for monitoring purpose.

5.1 Description and modelling of the 14-bus harmonic test system

Parameters of the test network, shown in Fig. 6, are defined in [13]. Generators, lines and transformers have been modelled according to the recommendation in reference

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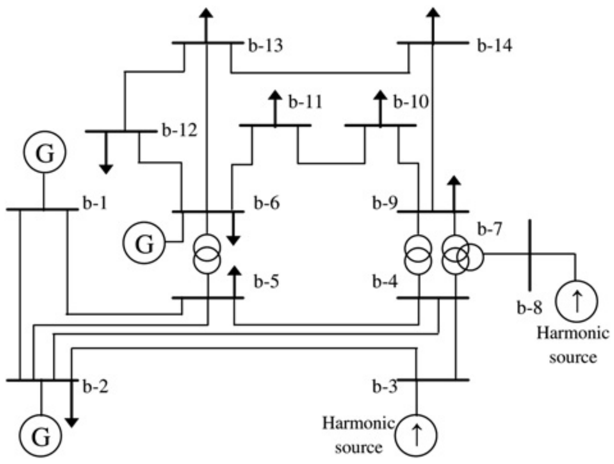

Fig. 6 *IEEE 14-bus standard test system for harmonic analysis*

[22]. Harmonic filters (all single–tuned) have been modelled as shunt harmonic impedances.

The deterministic values of active and reactive power specified in [13] for the LLs and NLLs have been assigned to their means (μ) and to the most possible values (m). The uncertainties of these parameters have been set in 5%, that is, the 0-cuts of the possibility distributions are the intervals $[lb,\ \ ub] = [(1-0.05)m,\ \ (1+0.05)m]$ and the standard deviation of the probability distributions are $\sigma = 0.025\ \mu$.

The test system has two six-pulse converters, one at bus 3 and the other at bus 8. No other loads are connected to these buses. At buses 4, 5, 9 10, 11, 12, 13 and 14 the percentage of induction motors and other linear loads have been chosen quite arbitrarily. Fig. 7 shows the parameters describing the uncertain LLs and NLLs.

5.2 Possibility and probability distributions of fifth harmonic bus voltages

Possibility and probability distribution of the node voltages magnitude at different harmonic order have been obtained. To this purpose, 11 α-cuts and 10 000 shots in MCS have been computed for each harmonic order. Results for the fifth harmonic will be shown here although the same conclusions can be drawn from the analysis of the other harmonics.

Fig. 8 shows the obtained possibility distributions together with the histograms of the corresponding MCS (dark areas).

From these results cumulative probability, possibility and necessity measures for increasingly long intervals enclosing the fifth-order harmonic voltage magnitudes have been calculated and plotted in Fig. 9.

Uncertain measures for a specific interval, in fact, fifth-order harmonic voltages magnitudes taking values higher than 0.03 pu, have also been set down in Table 2, whose rows have been sorted in descending order of probability and in descending order of possibility and necessity (when possibilities match). This ordering is a typical criterion for ranking possibilistic variables in decision-making under uncertainty, [15].

5.3 Discussion

Results of Fig. 8 show at first glance a qualitative agreement between results obtained with the possibilistic HLF and MCS.

A deeper analysis shows that all the results of the MCS are within the 0-cuts of the possibility distributions; this is clearly an expected result since harmonic voltages outside that interval have null possibility, thus null probability, according to (29).

Bus	m / lb ub	Fuzzy model parameters									
		$\hat{p}_t$	$\hat{q}_t$	$\hat{k}_p$	$\hat{pf}_p$	$\hat{k}_m$	$\hat{pf}_m$	$\hat{k}_{nl}$	$\hat{pf}_{nl}$	$\hat{x}_{LR}$	$\hat{q}_c$
3	*lb*	1.131	0.064	0		0		1	0.8		0.660
	m	1.190	0.067	0		0		1	0.8		0.825
	ub	1.250	0.071	0		0		1	0.8		0.990
4	*lb*	0.454	-0.037	0.5	0.97	0.2	0.7	0		0.15	0.163
	m	0.478	-0.039	0.7	0.975	0.3	0.8	0		0.2	0.204
	ub	0.502	-0.041	0.9	0.98	0.4	0.9	0		0.25	0.245
5	*lb*	0.072	0.015	0.55	0.97	0.2	0.8	0		0.15	0.008
	m	0.076	0.016	0.75	0.975	0.25	0.825	0		0.2	0.010
	ub	0.080	0.017	0.95	0.98	0.3	0.85	0		0.25	0.012
8	*lb*	0.124	0.000	0		0		1	0.8		0.078
	m	0.130	0.000	0		0		1	0.8		0.098
	ub	0.137	0.000	0		0		1	0.8		0.117
9	*lb*	0.280	0.158	0.5	0.8	0.2	0.7	0		0.15	0.023
	m	0.295	0.166	0.7	0.85	0.3	0.8	0		0.2	0.028
	ub	0.310	0.174	0.9	0.9	0.4	0.9	0		0.25	0.034
10	*lb*	0.086	0.054	0.5	0.8	0.2	0.8	0		0.15	.00015
	m	0.090	0.057	0.7	0.85	0.3	0.83	0		0.2	.00019
	ub	0.095	0.060	0.9	0.9	0.4	0.86	0		0.25	.00023
11	*lb*	0.033	0.017	0.5	0.8	0.2	0.8	0		0.15	0.003
	m	0.035	0.018	0.7	0.85	0.3	0.85	0		0.2	0.004
	ub	0.037	0.019	0.9	0.9	0.4	0.9	0		0.25	0.004
12	*lb*	0.058	0.014	0.5	0.8	0.2	0.7	0		0.15	0.020
	m	0.061	0.015	0.7	0.85	0.3	0.8	0		0.2	0.025
	ub	0.064	0.016	0.9	0.9	0.4	0.9	0		0.25	0.030
13	*lb*	0.128	0.055	0.5	0.8	0.2	0.8	0		0.15	0.021
	m	0.135	0.058	0.7	0.85	0.3	0.85	0		0.2	0.026
	ub	0.142	0.061	0.9	0.9	0.4	0.9	0		0.25	0.031
14	*lb*	0.142	0.048	0.3	0.9	0.5	0.8	0		0.15	0.022
	m	0.149	0.050	0.4	0.94	0.6	0.85	0		0.2	0.027
	ub	0.156	0.053	0.5	0.98	0.7	0.9	0		0.25	0.032

Fig. 7 *Fuzzy parameters modelling linear and non-linear loads connected to the IEEE 14-bus power system*

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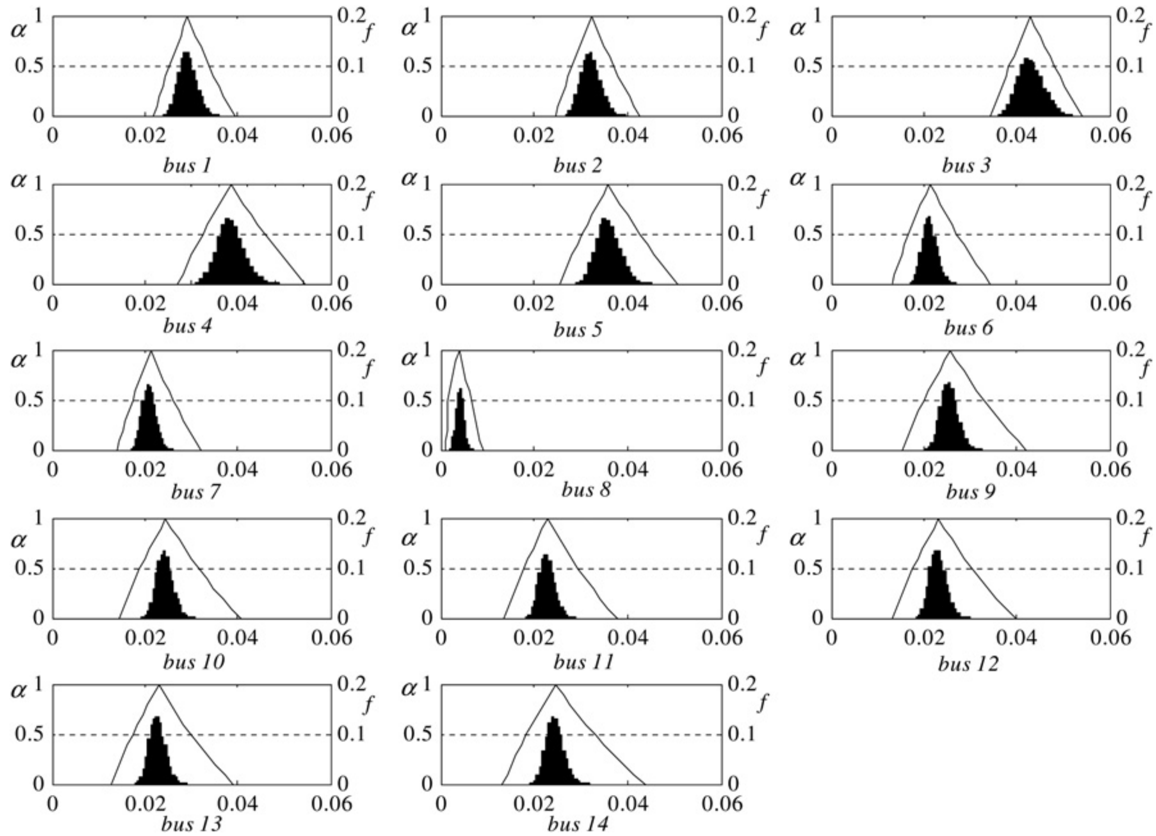

Fig. 8 *Per unit fifth-order harmonic voltage magnitude at 14-buses of the power system*

Right axes correspond to the relative frequency of occurrence (f) of the histograms from MCS and left axes correspond to the α values of the fuzzy voltages from the possibilistic HLF

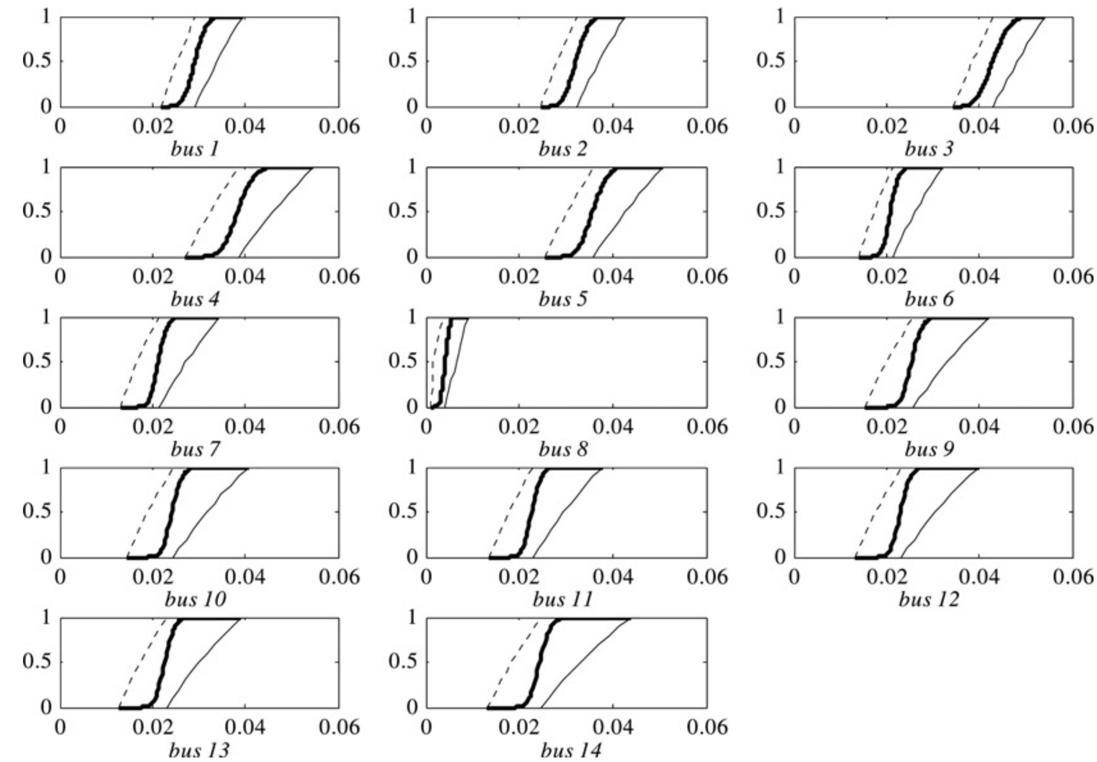

Fig. 9 *Cumulative probability (bold continuous line), possibility (dotted line) and necessity (continuous line) measures for increasingly long intervals enclosing the fifth-order harmonic voltages magnitudes at 14-buses of the power system*

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Table 2 Measures of uncertainty for the question: 'may the fifth-order harmonic voltage magnitude take values higher than 3%?'

MCS			Possibilistic HLF			
Ranking	Bus	$p(v_j \geq 0.03)$	Ranking	Bus	$pos(v_j \geq 0.03)$	$nec(v_j \geq 0.03)$
1	3	1	1	3	1	1
2	4	1	2	4	1	0.7184
3	5	0.9976	3	5	1	0.5217
4	2	0.8574	4	2	1	0.2628
5	1	0.3282	5	1	0.9187	0
6	9	0.0118	6	9	0.6983	0
7	14	0.0028	7	14	0.6701	0
8	10	0.0014	8	10	0.6068	0
10	6	0	9	12	0.5308	0
10	7	0	10	13	0.5021	0
10	8	0	11	11	0.4676	0
10	11	0	12	6	0.2938	0
10	12	0	13	7	0.1699	0
10	13	0	14	8	0	0

Fulfilment of inequalities (29) for the set of increasingly long intervals of harmonic voltage level is shown in Fig. 9 for all buses.

It is also apparent that the supports (0-cut) of some possibility distributions are substantially wider than the intervals between the maximum and minimum voltages (v_{max} and v_{min}) obtained by MCS. Actually, the probability of voltages higher v_{max} and lower than v_{min} are not null, but negligible; there are combinations of parameters that lead to them but they are so rare that very few (or any at all) happened in 10 000 shots.

It may seem rather annoying that possibilities do not show the very low-likelihood of these events. It should however, be realised that such low likelihood is known here because much more information than that used by the possibility-based HLF has been assumed and provided to the probabilistic model. Even if probability and possibility distributions have very similar shapes (as it happens in this case with the distribution of the uncertain input parameters), the amount of information implicit in the first is notably much larger than in the second. On the other hand, it could be argued that despite the nature of the uncertainties, it could be better to postulate some 'reasonable' probability distributions for them in order to use probabilistic models that are more accurate, but in proceeding this way the results themselves are doubtful, increasing the calculated uncertainties and in non-quantified extent.

The good news is that even if the underlying probabilistic distributions of the inputs are unknown, the results obtained with the possibilistic HLF are useful enough to make good decisions, as the coincidence of the rankings that Table 2 shows.

6 Conclusions

A methodology for HLF calculation based on the possibility theory has been developed, which improves a former proposal of the authors. Possibility-based models seem to be an interesting alternative for situations where the information is too vague to define reliable probability distributions; a common situation in the context of harmonic studies.

The proposed model allows for uncertainties in the loads that directly refers to their magnitude and composition, instead of the parameters of their circuit model (resistances, inductances and amplitude of harmonic sources), as was the case in the former proposal.

The methodology has a general formulation, quite independent of the specific set of uncertain parameters chosen and of the circuit used to model the loads. A particular implementation based on standard circuit models for aggregate loads has been developed.

The resulting implementation has been tested in the standard IEEE 14-bus network for harmonic studies, by comparing the obtained possibilities of the harmonic bus voltages with their probabilities, calculated by MCS. Results fulfil the theoretical consistency inequalities, and, from a more practical point of view, it has been shown that both lead to the same ranking of the risk that harmonic voltages exceed a given level.

This suggests that the results of the possibilistic HLF can be successfully used for making decisions under uncertainty and can be applied in different areas where some measure of the risk of harmonic resonance or high harmonic level are relevant, such as in planning capacitive shunt compensation, selecting measurement points for monitoring of power quality etc.

In the developed model no interaction among the uncertain parameters has been assumed. This assumption can lead to an undesirable uncertainty overestimation. Research aimed at overcoming this constraint is underway.

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8 Appendix

8.1 Model of LLs

LLs are modelled by the parallel RL and C branches connected to ground in Fig. 5. Total admittance to ground owing to LLs can then be expressed as

$$\boldsymbol{y}_{(h)} = \tilde{\boldsymbol{y}}_{(h)\mathrm{p}} + \tilde{\boldsymbol{y}}_{(h)\mathrm{m}} + \tilde{\boldsymbol{y}}_{(h)\mathrm{c}} + \tilde{\boldsymbol{y}}_{(h)\mathrm{filters}} \tag{30}$$

where p refers to passive load (linear and non-motor); m denotes the induction motors; c denotes power factor correction and cables and filters represents harmonic filters. Expressions for the admittances in (30) in terms of the parameters listed in Table 1 will be developed in what follows.

Passive loads: in general, its impedance at fundamental frequency can be written as

$$\tilde{z}_{(1)p} = \frac{|\tilde{\boldsymbol{v}}_{(1)}|^2}{\tilde{\boldsymbol{s}}_p^*} \tag{31}$$

where $\tilde{\boldsymbol{s}}_p$ the complex apparent power because of passive loads and (*) indicates the conjugate of a complex number. Then, writing the apparent power in terms of the active power and the power factor pf_j, the following expression can easily be obtained.

$$\tilde{z}_{(1)\mathrm{p}} = \frac{(|\tilde{\boldsymbol{v}}_{(1)}|pf_{\mathrm{p}})^2}{p_t k_{\mathrm{p}}}(1 + \mathrm{j}\,\tan(\cos^{-1}(pf_{\mathrm{p}}))) \tag{32}$$

where the active power because of the passive load has been expressed in terms of the total active bus power p_t and the composition factor of passive loads k_p.

Now, in order to model the harmonic behaviour it is necessary to include the influence of the frequency in expression (32). Skin effect increases the resistance when the frequency rises; this phenomenon is usually taken into account by multiplying the resistance by the factor $\sqrt{h}$, (see chapter 6 in [20]). On the other hand, the reactance essentially varies linearly with the frequency, so that it can be written $x_{(h)\mathrm{p}} = x_{(1)\mathrm{p}}h$.

Thus, the harmonic admittance because of the passive load components will be

$$\tilde{y}_{(h)\mathrm{p}} = \frac{p_t K_{\mathrm{p}}\left(1 - \mathrm{j}\sqrt{h}(\tan(\cos^{-1}(pf_{\mathrm{p}})))\right)}{(|\tilde{\boldsymbol{v}}_{(1)}|pf_{\mathrm{p}})^2\sqrt{h}(1 + h(\tan^2(\cos^{-1}(pf_{\mathrm{p}}))))} \tag{33}$$

Induction Motors: even basic models need some information in addition to the aggregate active power and power factor. The model implemented is a simple standard model that assumes a high slip of the rotor at all harmonic orders. Under this hypothesis, the harmonic inductance of the induction motors is the locked rotor inductance x_{LR}. In addition, it is assumed that all motors are operating at full apparent power, so that their aggregate nominal power can be expressed as the quotient of the aggregate real power and the power factor of the aggregate motor load. Under these assumptions the locked rotor reactance of the aggregate motor load becomes

$$x_{\mathrm{m}} = \frac{x_{\mathrm{LR}} pf_{\mathrm{m}} |\tilde{\boldsymbol{v}}_{(1)}|^2}{p_t k_{\mathrm{m}}} \tag{34}$$

Damping effects are considered by including a resistor r_{m} in series to x_{m}. Typical figure of $x_{\mathrm{m}}/f_{\mathrm{m}}$ is $fq_{\mathrm{LR}} \cong 8$ (model 6 in [11]).

The total locked rotor impedance at harmonic frequencies becomes

$$\tilde{z}_{(h)\mathrm{m}} = x_{\mathrm{m}}(1/fq_{\mathrm{LR}} + \mathrm{j}\,h) \tag{35}$$

By replacing (34) within (35) inverting, the following expression for the harmonic admittance because of the motive load component is obtained

$$\tilde{y}_{(h)\mathrm{m}} = \frac{p_t k_{\mathrm{m}} fq_{\mathrm{LR}}}{x_{\mathrm{LR}} pf_{\mathrm{m}} |\tilde{\boldsymbol{v}}_{(1)}|^2 (1 + h^2 fq_{\mathrm{LR}}^2)}(1 - \mathrm{j}(hfq_{\mathrm{LR}})) \tag{36}$$

More sophisticated models of induction motors [20] could also be implemented in the general formulation of Section 3.1.

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Capacitive loads: Admittance modelling capacitors for power factor improvement is

$$\tilde{y}_{(h)\mathrm{pfc}} = \mathrm{j}\frac{q_\mathrm{c}}{|\tilde{v}_{(1)}|^2}h \tag{37}$$

Passive filters: Passive filters are commonly associated with medium- and large-power electronic devices in order to provide a low-impedance path for harmonic currents to avoid their flow into the supply network. Filters are designed for a single harmonic or for a broad band depending on requirements and also supply part of the reactive power consumed by the apparatus. Usually, configuration and parameters of large harmonic filters are well known and in the developed methodology they are assumed as a certain (non-fuzzy) part of the network.

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