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Transport properties and current inversion by white Gaussian noise in a coupled ratchet model

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ABSTRACT

We have found, by varying two parameters, several stationary trajectories in a system consisting in many elastically coupled particles that are placed in a periodic ratchet potential on a ring. The system is assumed to be over-damped and driven by an external potential that is periodic both in space and time. The transport properties of these orbits are quite different and their values are quantified. The symmetries allow us to study the orbits with and without the presence of thermal fluctuations and there is found current inversions due to the addition of white Gaussian noise.

1. Introduction

It is well known that a ratchet system is a set of particles (interacting or not) far from the equilibrium and due to their spatial-temporal symmetries may present a directional current even when the acting forces have zero mean value. They may be classified as deterministic [1], stochastic [2] (Brownian), and inertial [3] (under damped [4] or over damped). From early works, this kind of system [5,6] (for comprehensive reviews see refs in [7] and [8]) have been the object of interest in many fields [9], since it provides not only the understanding of the transport phenomena in biological systems [10–12] but has quite an importance in the design and construction of artificial devices (classical or quantum, [13] and the references therein).

One of the properties in these systems is the inversion of the current with the variation of some system parameters that modify the dynamical symmetry of the potential and therefore the current direction [14–18]. On the other hand, the addition of noise in moderate quantities in a deterministic system contributes to the order of the system rather than the disorder as was shown in stochastic resonators or in Brownian ratchets. These are deterministic systems with no net transport of particles but for an optimal level of noise a directional current arises. It is worthwhile to mention that under appropriate conditions (i.e. multi-stability) the inclusion of noise leads to diminishing the Lyapunov exponent of a deterministic ratchet [3] or can enhance the stability of metastable orbits [19].

Hence, a new question arises regarding if it is possible to reverse the current by the simple addition of noise in a deterministic ratchet. Recently some work in that direction was performed, using colored noise or cross-correlated noise sources [20–22].

In the present work we studied the dynamics in a deterministic coupled ratchet (the same as in [23]) but with more particles. The system is inspired in the Frenkel–Kontorowa model (FKM) [24]. More precisely it is an underdamped, driven FKM. This kind of models were extensively used to describe many physical phenomena and systems such as dislocation and other lattice defects (crownions, self-interstitials or vacancies) dynamics, magnetic chains, Josephson junction arrays,

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nonlinear DNA dynamics, interfacial slip and microscopic models of friction (nanotribology) [25]. The main characteristic of this system is the huge number of stationary orbits present as the parameters change. These orbits may be quite different among them, in particular the flow can change their direction and it is quantified. Each stationary orbit defines different regions in the parameter space separated by well-defined curves (separatrices). Since the deterministic dynamic is robust under the presence of a moderate quantity of non-correlated noise, this can induce transitions among the different types of stationary orbits. That means that, while the stationary orbits of the system is deterministic for a given set of parameters (and therefore with a certain current directional), the inclusion of uncorrelated white noise can change or mix the stationary orbits corresponding to another region of parameter space.

This paper is organized as follows. In Section 2 a description of the model is shown. Section 3 is devoted to the description and discussion of the stationary orbits in parameter space for the deterministic system. Section 4 shows the behavior when the uncorrelated Gaussian noise is added and in Section 5 a summary and conclusions are drawn.

2. The model

The system under study consists in N interacting particles placed in a one-dimensional ring of length L subject to a periodic potential $V_\alpha(x)$ and driven by an external force in the over-damped regime.

The time evolution of the i-th particle is given by the equation:

$$\dot{x}_i(t) + \frac{\partial V_\alpha(x_i)}{\partial x_i} + \frac{\partial V^{osc}(x_{i-1}, x_i, x_{i+1})}{\partial x_i} = F^{\mathrm{dr}}(x_i, t) + \sqrt{2k_B T}\xi_i(t). \tag{1}$$

In Eq. (1), $x_i(t)$ represents the coordinate of the $i-$th particle, $i = 1, 2, \ldots, N$ with the conventions $x_{N+1} = x_1$ and $x_0 = x_N$. These are measured in the counterclockwise direction along a ring of length L, therefore fulfills the periodicity condition $x_i + L = x_i$. $V_\alpha(x)$ is a one-dimensional, asymmetric periodic potential given by

$$V_\alpha(x) = \begin{cases} V_o \cos\left[\frac{\pi}{\alpha}\left(\frac{(\alpha+1)x}{d}\right)\right], & \text{for } 0 \le \frac{x}{d} \le \frac{\alpha}{(\alpha+1)} \\ -V_o \cos\left[\pi(\alpha+1)\frac{x}{d} - \alpha\right], & \text{for } \frac{\alpha}{(\alpha+1)} \le \frac{x}{d} \le 1. \end{cases} \tag{2}$$

The potential fulfills the periodicity condition $V_\alpha(x + d, t) = V_\alpha(x, t)$ and therefore the total length of the circle is $L = nd$ where n is the number of minima and d is the linear distance between consecutive minima of the ratchet. α ($\alpha > 0$) controls the left–right asymmetry. Solutions for $\alpha > 1$, in which the minima of the wells of the ratchet are displaced in the counterclockwise direction, are equal to the time reversed solutions with $\alpha < 1$ in which the minima are displaced in the opposite (clockwise) direction.

The coupling potential is $V^{osc}(x_{i-1}, x_i, x_{i+1}) = \frac{1}{2}k[(x_{i-1} - x_i)^2 + (x_i - x_{i+1})^2]$. In addition these are driven by an external periodic driving force $F^{\mathrm{dr}}(x_i, t)$ given by the gradient of a time-dependent potential with a spatial periodicity that is twice the one of V_α

$$F^{\mathrm{dr}}(x_i, t) = -\varepsilon \frac{\partial V^{\mathrm{dr}}(x_i, t)}{\partial x_i} = -\varepsilon \sin(\omega t)\frac{\partial \sin(\pi x_i/d)}{\partial x_i}. \tag{3}$$

With this choice, consecutive wells alternate in time as absolute minima and are driven in opposite phases. Consequently the number of minima of the ratchet is restricted to be even.

The last term in Eq. (1) represents independent Gaussian white noise sources fulfilling $\langle \xi_i(t)\xi_i(t')\rangle = \delta(t - t')$. The factor $\sqrt{2k_B T}$ (where k_B is the Boltzmann constant) insures that the last term in the equation corresponds to a thermal bath of temperature T.

Time is measured in units of the period $\tau = 2\pi/\omega$ of the external driving and ε is the coupling strength of the external driving. All the calculations that we report were made for $d = 1$, $V_o = 5$, $\alpha = 1/3$ and $\omega = 0.2$. The space spanned by the other parameters (mainly k, ε and the temperature T) is explored in order to have a general picture of the dynamics of the system.

Since as was discussed in a previous paper [23], hereafter called CM, four constants in the model have the dimensions of an energy, therefore it is possible to construct with them three dimensionless quantities. We define $\Pi_k = k(nd/N)^2/2V_o$; $\Pi_\varepsilon = \varepsilon/2V_o$ and $\Pi_T = k_B T/2V_o$ where N is the number of particles of the system and n is the (even) number of minima of the circular ratchet. Π_k is a measure of the average potential elastic energy per particle in units of the depth of the periodic potential. The parameter Π_ε compares the energy provided by the external driving with the depth of the ratchet potential $2V_o$. The last parameter Π_T compares the energy delivered by the thermal bath also with the depth of the ratchet potential.

3. The stationary trajectories

The Langevin equations given by Eq. (1) were integrated by reducing the derivatives to first order finite differences. The numerical integration times change between $t = 0$ and 100 in units of $\tau = 2\pi/\omega$. The collisions (elastic) between particles

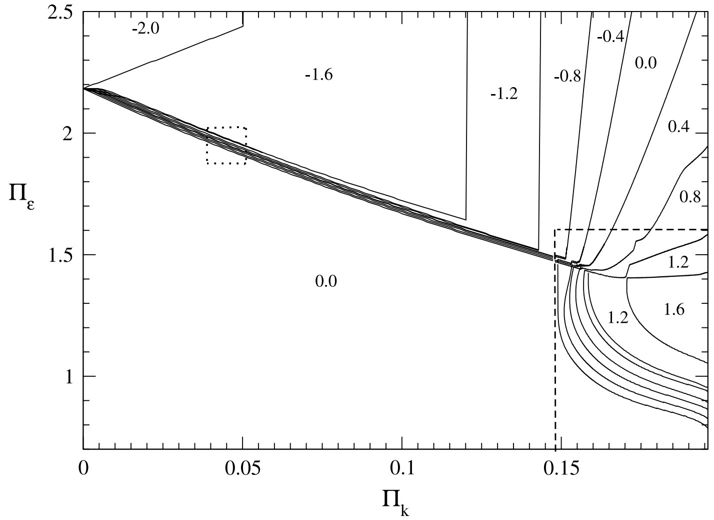

Fig. 1. (a) Phase diagram of the dynamics in the plane Π_ε vs. Π_k for $\Pi_T = 0$ in the case of 20 particles and 4 wells. The numbers on each region indicate the mean (temporal) velocity $\bar{v}$.

are allowed and the particles are assumed to be identical. The dynamics of the deterministic coupled ratchet model have been widely discussed in CM where we have defined for a non-vanishing value of k, the static solution requires that each well should be occupied by the same number of particles $n_s = \rho = N/n,\ \forall s, s = 1, 2, \ldots$ labeling the minima of the ratchet. The external driving causes that consecutive wells alternate in time as absolute minima. Its periodicity forces the dynamics to be invariant under rotations of $2\pi/(n/2)$ (or equivalently under translations of $2d$). Thus, the total number of particles in every *pair* of wells is 2ρ, and it is a constant of motion.

As long as thermal fluctuations are small and the external driving force is weak enough, orbits are bound, i.e. particles oscillate around their equilibrium positions without leaving the well in which they are placed. As increasing Π_ε may give rise to transport solutions. In this case the transport of particles is a highly ordered, collective motion in which the net current scales as the density of particles of the system, that is $j = \rho\bar{v}$ where $\bar{v}$ is the mean (temporal) velocity of the particles in the stationary orbit and due to be an integer multiple of $1/\rho$. Therefore the current is quantified as $j = m$, $m = 0, \pm 1, \pm 2 \cdots$. It is worthwhile to recall that the different values of the current correspond to the flow produced by jumps of clusters with different numbers of particles per period of the driving force in counterclockwise $(+)$ or clockwise $(-)$ directions.

In Fig. 1 a phase diagram in the plane $(\Pi_k - \Pi_\varepsilon)$ for $\rho = 5$ without noise is displayed. The corresponding phase diagram for $\rho = 2$ was shown in Fig. 1 of CM. It is worth noting the huge complexity in the present case. The regions are labeled with the values of $\bar{v}$ of the corresponding stationary orbit. The separatrices have an intricate behavior, therefore for the sake of clarity we have amplified two areas (they are shown in Figs. 2 and 3). We also perform two blow ups of Fig. 2 depicted in Fig. 4 (a) and (b).

Starting of $\Pi_k = 0$ the separatrices are born of a common point. One of them (which separates the region $\bar{v} = -2, j = -10$ from the region $\bar{v} = -1.6, j = -8$) is increasing and at $\Pi_k \sim 0.05$ breaks being almost vertical. The other separatrices are decreasing as almost a paralel beam separating regions of $\bar{v} = 0, j = 0$; $\bar{v} = -0.2, j = -1$; $\bar{v} = -0.4, j = -2$; $\bar{v} = -0.6, j = -3$; $\bar{v} = -0.8, j = -4$; $\bar{v} = -1, j = -5$; $\bar{v} = -1.2, j = 6$; $\bar{v} = -1.4, j = -7$; $\bar{v} = -1.6, j = -8$ (see Fig. 3). Some regions disappear (because some lines stick together) and at a determined point the lines also break and become almost vertical. Near to $\Pi_k = 0.15$ regions with $\bar{v} > 0, j > 0$ appear upwards and downwards (see Fig. 2). Except in the vicinity to $\Pi_k = 0.15$ (see Fig. 1), the regions with $\bar{v} < 0$ and the region with $\bar{v} > 0$ are quite separate by a region in which there is no current ($\bar{v} = 0, j = 0$).

By control of the parameters Π_k and Π_ε it is possible to vary the current. For example, maintaining $\Pi_k = 0.05$ and varying Π_ε from 1.9 to 1.96, the current takes different values (0,-1,-2,-3,-4,-5,-6,-7,-8). Or by keeping constant $\Pi_k = 0.19$ and varying Π_ε from 0.7 to 1.1 we will obtain reverse currents (0,1,2,3,4,5,6,8). Taking $\Pi_\varepsilon = 2$ and varying Π_k from 0.152 to 0.158, we will be able to vary the current of -2 to 2, etc.

4. The effect of noise

By inspection of Figs. 2 and 4, it is feasible to expect, for certain values of the parameters Π_k and Π_ε, that the addition of an adequate amount of noise in the deterministic system generates different transport properties than those. This assertion is confirmed by Fig. 5 where we show the average $\langle\bar{v}\rangle$ over 20 realizations of noise and initial conditions as a function of the noise intensity Π_T. Panels (a), (b) and (c) correspond to deterministic orbits with $\bar{v} = 0.2, j = 1$; $\bar{v} = 0.6, j = 3$ and $\bar{v} = 1, j = 5$. In all the cases, an inversion of current is clearly obtained by the addition of noise.

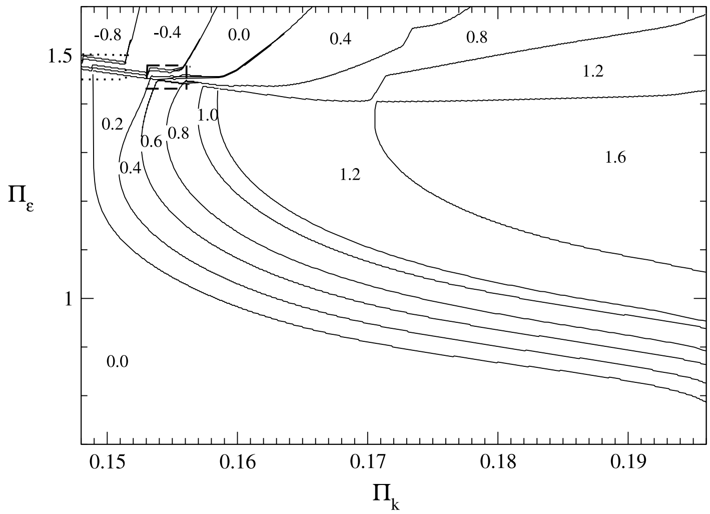

Fig. 2. Blow up of the phase diagram of the dynamics in the plane Π_ε vs. Π_k corresponding to the dashed frame shown in Fig. 1.

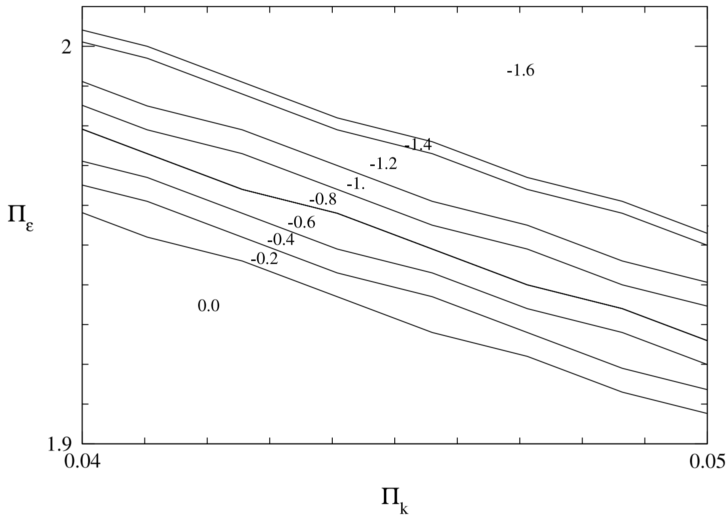

Fig. 3. Blow up of the phase diagram of the dynamics in the plane Π_ε vs. Π_k corresponding to the dotted frame shown in Fig. 1.

In these cases, the current inversion is produced by the mix of stationary trajectories of different regions. Fig. 6 is an example of these facts. Panel (a), (b), (c) and (d) show the position of one particle (the particle labeled as 11) on the ring as a function of time t/τ corresponding to four different deterministic stationary trajectories,$\bar{v} = 1, j = 5$; $\bar{v} = 0.4, j = 2$; $\bar{v} = 0, j = 0$; $\bar{v} = -0.4, j = 2$ respectively. Although there are 20 particles, they are all equivalent and it is sufficient to study the trajectory of one of them. The parameters Π_k and Π_ε are very close for the four trajectories displayed.

The addition of noise in the orbit of Fig. 6 (a) leads to the velocity of Fig. 5 (c). When the intensity of the noise is indicated by the arrow (labeled (e) in Fig. 5 (c)), trajectories are typified by those shown in Fig. 6 (e): it is a mix of the deterministic trajectories (b) and (c). The noise of the arrow (f) corresponds to the trajectory of Fig. 6 (f): it is very similar to the deterministic orbit (c). The typical orbit when the noise is indicated by the arrow (g) is displayed in Fig. 6 (g): a mix of the deterministic orbits (c) and (d). When the noise is sufficiently intense (arrow h), so the motion becomes Brownian (Fig. 6 (h)). Similarly one can see that typical orbits displayed in noisy systems that give rise to the curves of Fig. 5 (a) and (b) are different mixtures (as noise intensity increases) from those that appear in the regions shown in Fig. 4.

5. Summary and conclusions

We have studied the dynamics of a deterministic ratchet system consisting of many interacting particles in the damped regime. By varying the system parameters a bunch of stationary orbits appear with different currents, not only in magnitude but also in direction. This allows us to control the current by varying the parameters mentioned above. So we have found the boundaries (separatrices) among different regions in the parameter space, in which every one of these orbits laid. Moreover,

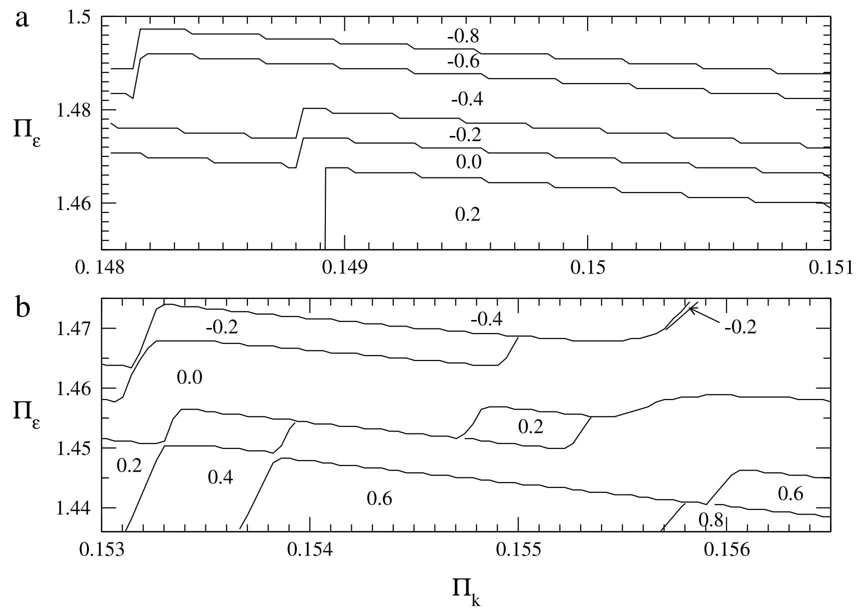

Fig. 4. (a) Blow up of the phase diagram of the dynamics in the plane Π_ε vs. Π_k corresponding to the dotted frame shown in Fig. 2. (b) Blow up of the Phase diagram of the dynamics in the plane Π_ε vs. Π_k corresponding to the dashed frame shown in Fig. 2.

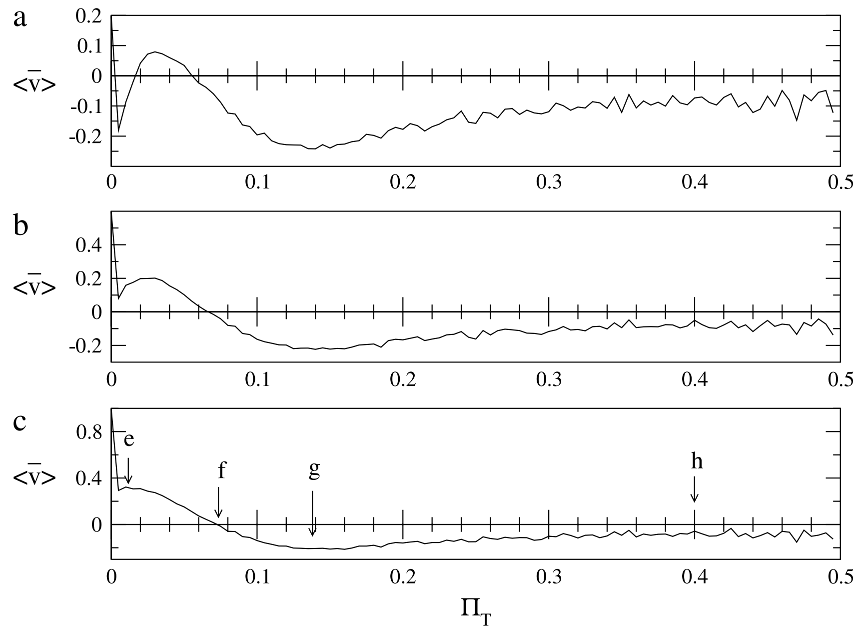

Fig. 5. Average mean velocity $\langle\bar{v}\rangle$ vs. noise intensity Π_T. The parameters are (a) $\Pi_k = 0.15$, $\Pi_\varepsilon = 1.425$; (b) $\Pi_k = 0.155$ and $\Pi_\varepsilon = 1.430$ and (c) $\Pi_k = 0.158$ and $\Pi_\varepsilon = 1.422$, the arrows (labeled by e, f, g and h) indicate the values of noise corresponding to the (typical) orbits displayed in Fig. 6 e, f, g and h respectively (see the text).

currents are quantified since their magnitude is associated with jumps of clusters with different numbers of particles between adjacent minima of the potential ratchet. Thus, the disposition of the separatrices in some places of parameter space implies that continuous variation of the parameters leads to a discrete change in the current.

In addition we have included uncorrelated white noise in the system to verify the robustness of the deterministic dynamics. We have recognized, in the noisy trajectories, structures that are related to deterministic trajectories for different regions mentioned above. Before reaching the Brownian regime the noise produces a mix of different trajectories. Especially when the deterministic system parameters to which is added the noise are close to another region in which the deterministic stationary orbit has opposite current, a current inversion assisted by noise can occur.

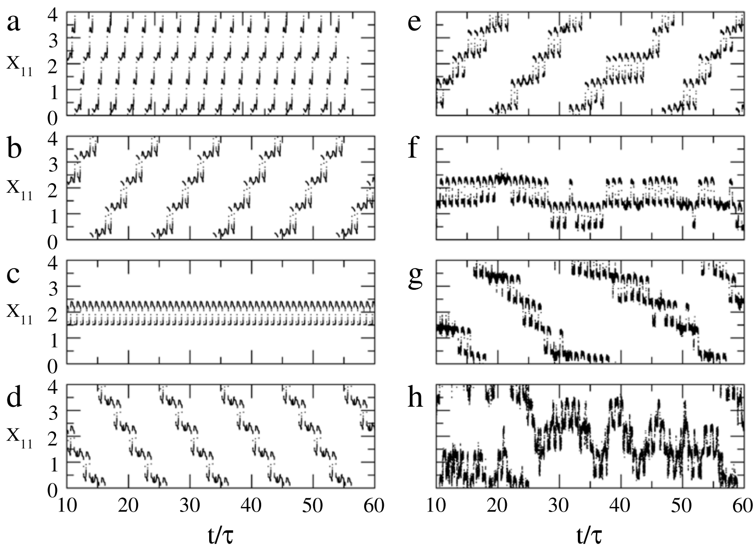

Fig. 6. Position on the ring of particle 11, X_{11} as a function of time t/τ. Deterministic trajectories, $\Pi_k = 0.158$: (a) $\Pi_\varepsilon = 1.422$ and $\bar{v} = 1$; (b) $\Pi_k = 0.158$, $\Pi_\varepsilon = 1.4448$ and $\bar{v} = 0.4$; (c) $\Pi_k = 0.158$, $\Pi_\varepsilon = 1.5$ and $\bar{v} = 0$; (d) $\Pi_k = 0.158$, $\Pi_\varepsilon = 1.6$ and $\bar{v} = -0.4$. For $\Pi_k = 0.158$ and $\Pi_\varepsilon = 1.422$. Noisy trajectories: (e) $\Pi_T = 0.01032$; (f) $\Pi_T = 0.0725$; (g) $\Pi_T = 0.1386$; (h) $\Pi_T = 0.4$.

The existence of the regions in the space of parameters with different stationary orbits and separatrices between them does not depend on the type of interaction between the particles. However, rings with another kind of interaction (for example $(x_i - x_j)^\alpha$, $\alpha > 0$) or the inclusion of second or more neighbor interactions will have another distribution of regions and separatrices. This fact might be used for designing the separatrices' layout, thus the characteristic of the controlled current by the parameters.

Up to now, we have worked with integer densities $\rho = n_s$, that is an integer number n_s of particles per site (or a minimum of the ratchet potential). These configurations are equivalent to a perfect lattice and the transport effects are collective and extended phenomena. However, for non-integer densities the system will have some disorder. This type of disorder is quite different to that described in ref. [26] where this is introduced in the ratchet potential. For instance, in systems with $\rho = n_s \pm (k/n)$, $k < n/2$ (n is the number of sites), they will have k self-interstitials or vacancies. In such systems we expect a non-collective and localized transport regime associated with the migration of the defects.

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