



## On favoritism in auctions with entry

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### ABSTRACT

We examine the problem of endogenous entry in a single-unit auction when the seller's welfare depends positively on the utility of a subset of potential bidders. We show that, unless the seller values those bidders' welfare more than her own "private" utility, a nondiscriminatory auction is optimal.

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### 1. Introduction

The most important results in auction theory hold under the assumption of a fixed number of bidders. However, as [Klemperer \(2004\)](#) claims, one of the key issues regarding auction design is how to encourage entry. For instance, [Engelbrecht-Wiggans \(1987\)](#) suggests that New York consolidated as the main port of the US East Coast (over Philadelphia and Boston) after passing legislation (with the express intention of attracting bidders) that required that all goods offered at auction at the New York port should be sold without reservation price to the highest bidder. Another issue that arises quite frequently when auctions are employed, especially in procurement, is whether some of the bidders should be given preferential treatment, e.g. in public procurement when domestic firms have some form of advantage in the bidding process over their foreign rivals. One reason – although not the only possible one – for awarding such a preference is favoritism, i.e. that favored bidders' welfare positively influences the seller's welfare. In our previous example, it may be the case that domestic firms generate more tax revenue than their foreign rivals. Alternatively, the seller and some of the bidders may be firms in the same conglomerate.

Some papers, which we briefly discuss below, study auctions with endogenous entry, but in all of them the seller's utility is not affected by the utility of any of the bidders. Another branch of the literature, which we also discuss below, analyzes auctions where the seller's welfare depends on the utility of some of the bidders, but all of them study the case of a fixed number of bidders. Here, we examine the problem of endogenous entry when the seller's welfare depends on the utility of a subset of potential bidders. To the best of our knowledge, this note is the first to deal with this problem.

The literature on endogenous entry in auctions can be divided into two groups. First, there are models in which the entry decision is made after each bidder knows her valuation. This set of papers starts with [Samuelson \(1985\)](#) and includes [Stegeman \(1996\)](#), [Menezes and Monteiro \(2000\)](#) and [Celik and Yilankaya \(2009\)](#), among others. Alternatively, there are models where the decision is made before the valuation is realized, which is the assumption we adopt. These models start with [McAfee and McMillan \(1987\)](#) and [Engelbrecht-Wiggans \(1987\)](#) and include [Engelbrecht-Wiggans \(1993\)](#), [Levin and Smith \(1994\)](#) and [Ye \(2004\)](#).<sup>2</sup>

As for the issue of favoritism in auctions, [Laffont and Tirole \(1991\)](#) and [Vagstad \(1995\)](#) study the case of multidimensional auctions, where favoritism may appear when the auctioneer assesses product quality. [McAfee and McMillan \(1989\)](#), [Branco \(1994\)](#) and [Naegelen and Mougeot \(1998\)](#) examine single-dimensional auctions, where price-

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<sup>2</sup> [Li and Zheng \(2009\)](#) study different entry models that fall into the two branches of the literature that we have described.

preferences may be used. The basic result is that the optimal allocation rule follows from comparing the maximum valuation of the preferred bidders with the maximum “virtual” valuation of the non preferred bidders. A usual way to discriminate, known as *right of first refusal*, is giving one of the preferred bidders the right to match the highest bid that any of her rivals may submit. This right has been studied in Walker (1999), Bikhchandani et al. (2005), Arozamena and Weinschelbaum (2006), Lee (2008), Choi (2009) and Burguet and Perry (2009).

In what follows, we allow for the possibility that the seller attaches a positive weight to some bidders’ utility when computing her own welfare. Once we endogenize entry, we show that, unless that positive weight is larger than one (i.e. unless the seller values those bidders’ welfare more than her own “private” utility), there is an optimal auction design which is nondiscriminatory. The intuition is simple: whenever the seller is able to appropriate the whole surplus in an efficient auction, the joint surplus of the seller and the preferred buyers cannot be improved, and it will certainly decrease whenever discrimination introduces some inefficiencies into the auction. In our setup with an endogenous number of bidders, there are nondiscriminatory auctions, which are efficient, that allow the seller to appropriate the whole surplus. There could be some way to discriminate without reducing the seller’s welfare, but it is not possible to increase it. As a corollary, we show that awarding a right of first refusal to a preferred bidder generally reduces welfare.

It should be noted that the justification for using discriminatory auctions that we examine, which derives from the fact that the seller values some of the bidders’ utilities, is not the only possible one. With a fixed number of bidders, discriminating against strong bidders raises revenue, as shown by optimal auction theory. With endogenous entry, giving preferential treatment to weak bidders (which might otherwise not enter) may also be optimal for the seller. Those arguments require asymmetry among bidders, while our model is (but for the unequal consideration of bidders’ utilities by the seller) symmetric. In sticking to symmetry, we concentrate on the reason for discrimination that we intend to question.

In the following section we present the model and the basic results.

## 2. The model

We start with the setup in McAfee and McMillan (1987). Specifically, the owner of a single, indivisible object is selling it through an auction.<sup>3</sup> For simplicity, we assume the seller attaches no value to the object. There are  $N$  potential bidders, any of whom can enter the auction upon paying a fixed cost  $k \geq 0$ . We assume that  $N$  is arbitrarily large, so that if all potential bidders entered they could not recover the entry cost. After paying her entry cost, bidder  $i$  learns her valuation for the object,  $v_i$ , which is distributed according to a c.d.f.  $F$  with support on the interval  $[\underline{v}, \bar{v}]$  and a density  $f$  that is positive and bounded on the whole support. Bidders’ valuations are independent.

The seller’s utility depends on the surplus of one of the bidders, the “preferred” bidder. We assume that there is only one preferred bidder, although our arguments would generalize to the case of any favored subset of  $\{1, \dots, N\}$ . We want to characterize a selling mechanism that maximizes the sum of the expected utilities of the seller and the preferred bidder – we assume that the latter is bidder 1.<sup>4</sup> Our problem is a modification of the standard optimal auction problem with independent private values.<sup>5</sup> The seller and all bidders are risk neutral.

Let  $\mathbf{B}$  be the set of bidders that enter the auction. The timing is as follows. First, the seller announces the auctioning mechanism. In

principle, she could condition that mechanism on the subset of entering bidders, so she has to announce functions  $H_i^{\mathbf{B}}((v_i)_{i \in \mathbf{B}})$ ,  $P_i^{\mathbf{B}}((v_i)_{i \in \mathbf{B}})$ ,  $\mathbf{B} \in 2^{\{1, \dots, N\}}$ .<sup>6</sup>  $H_i^{\mathbf{B}}((v_i)_{i \in \mathbf{B}})$  ( $P_i^{\mathbf{B}}((v_i)_{i \in \mathbf{B}})$ ) is the probability that bidder  $i$  gets the object (respectively, the price bidder  $i$  has to pay to the seller) if  $\mathbf{B}$  is the set of bidders that enter the auction and their valuations are given by  $(v_i)_{i \in \mathbf{B}}$ . Once that announcement is made, the  $N$  potential bidders simultaneously make their entry decisions. We focus on pure-strategy equilibria of the entry game. Those bidders who have entered learn their valuations and take part in the announced direct mechanism that corresponds to the actual set of entrants.

Bidder  $i$ ’s expected utility when she belongs to  $\mathbf{B}$ , her valuation is  $v_i$ , and she decides to enter the auction expecting that all the other members of  $\mathbf{B}$  will enter as well, is

$$U_i(v_i, \mathbf{B}) = h_i(v_i, \mathbf{B})v_i - p_i(v_i, \mathbf{B}),$$

where  $h_i(v_i, \mathbf{B}) = E_{v_{-i}}[H_i^{\mathbf{B}}((v_j)_{j \in \mathbf{B}})]$  and  $p_i(v_i, \mathbf{B}) = E_{v_{-i}}[P_i^{\mathbf{B}}((v_j)_{j \in \mathbf{B}})]$ ,  $i \in \mathbf{B}$ .

The mechanism announced by the seller has to satisfy the corresponding incentive compatibility

$$U_i(v_i, \mathbf{B}) \geq h_i(v'_i, \mathbf{B})v_i - p_i(v'_i, \mathbf{B}) \text{ for all } \mathbf{B}, \text{ for all } i \in \mathbf{B}, \text{ for all } v_i, v'_i \quad (1)$$

and participation constraints<sup>7</sup>

$$U_i(v_i, \mathbf{B}) \geq 0 \quad \text{for all } \mathbf{B}, \text{ for all } i \in \mathbf{B}, \text{ for all } v_i \quad (2)$$

Given the seller’s announcement, the equilibrium set of bidders,  $\mathbf{B}^*$ , will be such that

$$\int_{\underline{v}}^{\bar{v}} U_i(v_i, \mathbf{B}^*) f(v_i) dv_i \geq k, \forall i \in \mathbf{B}^* \quad (3)$$

$$\int_{\underline{v}}^{\bar{v}} U_j(v_j, \mathbf{B}^* \cup \{j\}) f(v_j) dv_j \leq k, \forall j \notin \mathbf{B}^*$$

The seller’s problem is then<sup>8</sup>

$$\max_{i \in \mathbf{B}, \mathbf{B} \in 2^{\{1, \dots, N\}}} \sum_{i \in \mathbf{B}^*} \int_{\underline{v}}^{\bar{v}} p_i(v_i, \mathbf{B}^*) f(v_i) dv_i + \int_{\underline{v}}^{\bar{v}} U_i(v_i, \mathbf{B}^*) f(v_i) dv_i - k$$

subject to Eqs. (1), (2) and (3).

If the set of bidders,  $\mathbf{B}^*$ , were exogenously fixed, following the usual steps in the auction literature, the seller’s objective function would become

$$\int_{\underline{v}}^{\bar{v}} h_1(v_1)v_1 f(v_1) dv_1 + \sum_{\substack{i \in \mathbf{B}^* \\ i \neq 1}} \int_{\underline{v}}^{\bar{v}} h_i(v_i) J(v_i) f(v_i) dv_i - k$$

where  $J(v) = v - \frac{1-F(v)}{f(v)}$ . The allocation that solves this problem is well known<sup>9</sup>

$$H_1^{\mathbf{B}^*}((v_j)_{j \in \mathbf{B}^*}) = \begin{cases} 1 & \text{if } v_1 > \max_{j \neq 1} J(v_j) \\ 0 & \text{otherwise} \end{cases}$$

$$H_i^{\mathbf{B}^*}((v_j)_{j \in \mathbf{B}^*}) = \begin{cases} 1 & \text{if } J(v_i) > \max\{v_1, \max_{j \neq i} J(v_j)\} \\ 0 & \text{otherwise} \end{cases}$$

<sup>6</sup>  $2^{\{1, \dots, N\}}$  is the power set of  $\{1, \dots, N\}$ .

<sup>7</sup> Even after having paid the entry cost  $k$ , once she knows her valuation a bidder may choose not to take part in the selling mechanism.

<sup>8</sup> We are implicitly assuming that bidder 1 always enters. If this were not the case, the second and third terms in the objective function would disappear. If bidder 1 did not enter our main result would remain valid, as we explain below.

<sup>9</sup> See for example Arozamena and Weinschelbaum (2006).

<sup>3</sup> All our results are valid as well in the case of procurement auctions.

<sup>4</sup> Our results generalize to the case where the seller attaches a weight lower than one to the preferred bidder’s utility.

<sup>5</sup> See Myerson (1981) and Riley and Samuelson (1981).

for  $i \neq 1$ . This is a discriminatory rule. However, when there is free entry of bidders the result changes substantially. The allocation rule, as we show in the following proposition, is a nondiscriminatory one.

**Proposition 1.** *The allocation rule that maximizes the seller's expected utility is a nondiscriminatory one. A first price or a second price auction that treats equally the favored and the non favored bidders maximizes the seller's expected utility.*

**Proof.** The expected value of the game (i.e. the expected total surplus in the auction) is the winning bidder's expected valuation minus the entry cost paid by entering bidders:  $Ev - \#(\mathbf{B}^*)k$ .<sup>10</sup> This, in turn, has to equal the expected revenue of the seller plus the summation of the surpluses of each of the bidders. As shown in McAfee and McMillan (1987), the allocation rule that maximizes expected total surplus by generating optimal entry awards the object to the entrant with the maximum valuation. If we ignore that the number of entrants has to be an integer, a first price or a second price auction with a reservation price equal to the seller's valuation maximizes the expected value of the game, and the seller receives the whole surplus. Considering the integer constraint, as McAfee and McMillan (1987) show, the seller can still maximize the value of the game and receive the whole surplus by charging an entry fee exactly equal to each entrant's expected utility when the number of entrants is the largest integer lower than the optimal number of entrants in a continuum.<sup>11</sup>

It just remains to prove that when the seller cares about bidder 1's surplus she cannot do better. If she could, she would be getting more than the maximal value of  $Ev - \#(\mathbf{B}^*)k$ . Thus, at least one of the bidders has to receive a negative expected surplus, which is clearly impossible.  $\square$

Note that the seller does not care if the favored bidder is in the set of entrants or not. This is because the expected surplus of each of the bidders is zero. The seller could give a subsidy to the preferred bidder to induce him to enter, but she would still get the same utility. But whenever the bidders that have entered are treated differently in the bidding process an inefficiency is created, and the seller's utility decreases.

The result holds for any number of preferred bidders whenever their surplus enters in the seller's utility function with a weight less than or equal to one. More generally it holds for any equilibrium in which the value of the game is maximized and the seller receives the whole surplus.

The result applies as well if we consider the symmetric, mixed-strategy equilibrium of the entry game studied in Levin and Smith (1994), since in that case the seller still receives the whole surplus. However, symmetric, mixed-strategy equilibria of the entry game are inefficient. The seller has incentives to induce some bidders to enter and move from the symmetric equilibrium to an asymmetric, pure-strategy, efficient one. These incentives are already present in Levin and Smith (1994). It doesn't matter if she subsidizes the preferred bidder or not: the seller is indifferent among any of the possible ways

of moving from the symmetric equilibrium to the asymmetric one studied in McAfee and McMillan (1987).

Finally, as a corollary, note that whenever the seller gives a right of first refusal, if it matters, she awards the right-holder some advantage in the bidding process. This advantage generates an inefficiency. Thus, the seller's utility is reduced.

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<sup>10</sup>  $\#(\mathbf{B}^*)$  denotes the cardinality of the set  $\mathbf{B}^*$ .

<sup>11</sup> Actually, as pointed out in Engelbrecht-Wiggans (1993), the seller may choose to subsidize entry so that the number of bidders is the lowest integer larger than the optimal number in a continuum. This way, she would still receive the whole surplus.