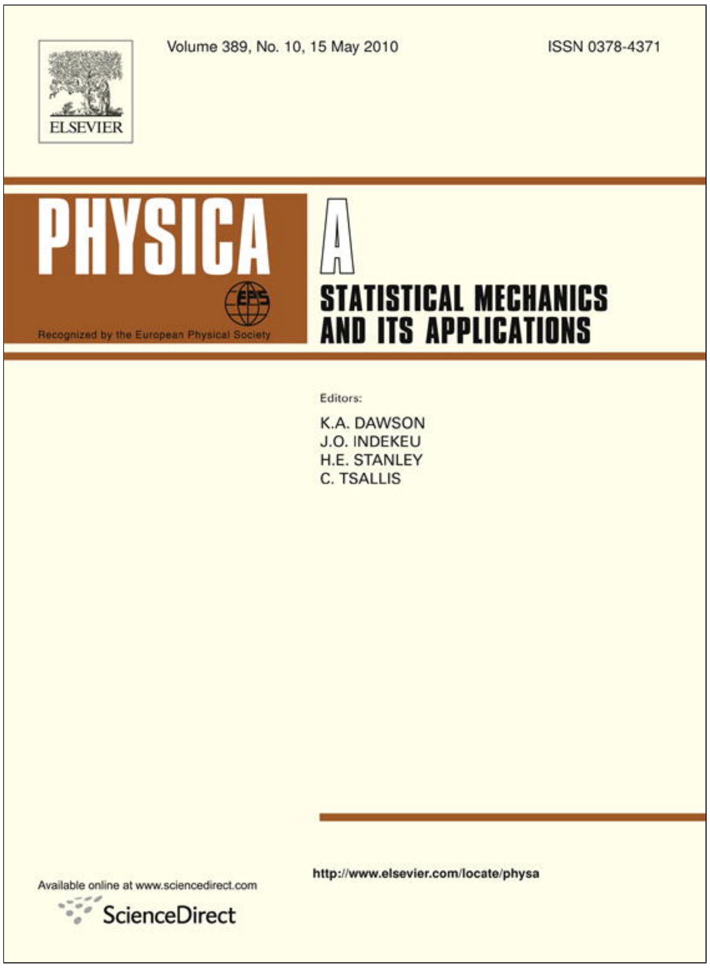
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Missing ordinal patterns in correlated noises

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A B S T R A C T

Recent research aiming at the distinction between deterministic or stochastic behavior in observational time series has looked into the properties of the "ordinal patterns" [C. Bandt, B. Pompe, Phys. Rev. Lett. 88 (2002) 174102]. In particular, new insight has been obtained considering the emergence of the so-called "forbidden ordinal patterns" [J.M. Amigó, S. Zambrano, M.A. F Sanjuán, Europhys. Lett. 79 (2007) 50001]. It was shown that deterministic one-dimensional maps always have forbidden ordinal patterns, in contrast with time series generated by an unconstrained stochastic process in which all the patterns appear with probability one. Techniques based on the comparison of this property in an observational time series and in white Gaussian noise were implemented. However, the comparison with correlated stochastic processes was not considered. In this paper we used the concept of "missing ordinal patterns" to study their decay rate as a function of the time series length in three stochastic processes with different degrees of correlation: fractional Brownian motion, fractional Gaussian noise and, noises with f^{-k} power spectrum. We show that the decay rate of "missing ordinal patterns" in these processes depend on their correlation structures. We finally discuss the implications of the present results for the use of these properties as a tool for distinguishing deterministic from stochastic processes.

1. Introduction

In the study of dynamical systems the corresponding underlying equations are, in general, not known. In fact quite often the starting point to study these systems is a set of measurements of some representative variable, X, at discrete time intervals (time series) given by the set $\mathcal{S} = \{x_t, t = 1, \ldots, N\}$, with N the number of observations. An important problem is to determine whether an observed time series is deterministic, contains a deterministic component, or is purely stochastic. Clearly, whether a time series has deterministic components or not dictates what approaches are appropriate for its characterization and for the generation of a dynamic system model.

Several methods of nonlinear dynamical systems have been developed to detect determinism in time series; see i.e. Refs. [1–13]. These methods are all based on properties of trajectories from the reconstructed phase space of the time series and require a large number of data points. An important assumption that sometimes constrains the use of these methods is that the time series analyzed need to be stationary. We must note that, acquiring a stationary time series of adequate length for phase space reconstruction is almost impossible when working with real data, particularly if the series are from natural or biological systems.

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Methods based on Information Theory constitute another research branch with the objective of detecting determinism in time series [14–19]. When using Information Theory methods, it is necessary to associate a probability distribution function (PDF) to the time series under analysis. The determination of the most adequate PDF is a fundamental problem. Bandt and Pompe introduced a successful methodology for the evaluation of the PDF associated to scalar time series data using a symbolization technique [14,20]. The symbolic data is created by ranking the values of the series and defined by reordering the embedded data in ascending order, which is reconstructed with embedding dimension D (see definition and methodological details below). In this way it is possible to quantify the diversity of the ordering symbols (patterns) derived from a scalar time series, evaluating the so-called permutation entropy (the Shannon entropy corresponding to the Bandt and Pompe PDF). The Bandt and Pompe technique is computationally fast and does not require the reconstruction of an attractor in phase space. This technique, as opposed to most of those currently used, takes into account the temporal structure of the time series generated by the physical process under study. An additional advantage of using this technique is that it is based on a very weak stationarity assumption. This characteristic allows us to uncover important details concerning the ordinal structure of the time series [17,21–29], and can also uncover information about the temporal correlation [30,31].

Rosso et al. [17] have recently shown the advantage of incorporating the time causality, naturally present in the time series, into the associated PDF. Specifically, they found that different Information Theory based measures (normalized Shannon entropy and statistical complexity) allow a better distinction between deterministic chaotic and stochastic dynamics when causal information is incorporated using the Bandt and Pompe methodology [14]. New insight into the characterization of theoretical and observational time series, which has also been developed using the Band and Pompe's methodology, considers the emergence of the so-called "forbidden patterns" [18,19].

1.1. *Forbidden versus missing ordinal patterns*

As shown recently by Amigó et al. [18,19,32], in the case of deterministic one-dimensional maps not all the possible ordinal patterns (as defined below using the Bandt and Pompe methodology) can be effectively materialized into orbits, which in a sense makes these patterns "forbidden". Indeed, the existence of these *forbidden ordinal patterns* becomes a persistent dynamical property. That is, for a fixed pattern length (embedding dimension) D the number of forbidden patterns present in the time series (unobserved patterns) is independent of the series length N. It is interesting to remark that, this persistence is in opposition to other properties such as proximity and correlation, which die out with time in a chaotic (deterministic) dynamics [18]. For example, in the time series generated by the logistic map $x_{k+1} = 4x_k(1-x_k)$ if we consider patterns of length $D = 3$, the pattern [2, 1, 0] is forbidden. That is, the pattern $x_{k+2} < x_{k+1} < x_k$ never appears [18].

In the case of time series generated by an *unconstrained stochastic process* (uncorrelated process) every ordinal pattern has the same probability of appearance [18,19,32]. That is, if the data set is long enough, all the ordinal patterns will appear. In this case, when the number of observations in the time series $N \to \infty$, the associated probability distribution function should be a uniform distribution, and the number of observed patterns will depend only, on the length N of the time series under study.

For correlated stochastic processes it is reasonable to assume that the probability of pattern observation will depend not only on the time series length N but also on the correlation structure. The existence of a non-observed ordinal pattern does not qualify it as "forbidden", only as "*missing*" due to the time series finite length.

A similar observation also holds for the case of real data that always possess a stochastic component due to the omnipresence of dynamical noise [33–35]. Wold proved [33] that any (stationary) time series can be decomposed into two parts. The first part (deterministic) can be exactly described by a linear combination of its own past; the second part is a moving average component of an infinite order.

Considering all the cases mentioned above the existence of "missing ordinal patterns" could be either related to stochastic processes (correlated or uncorrelated) or deterministic noisy processes which is the case of observational time series.

The concept of forbidden/missing ordinal patterns has been recently used as a tool for the discrimination between deterministic and stochastic behavior in observational time series [18,19,29,32,36,37]. Zanin [37] and Zunino et al. [29] have recently studied the appearance of missing ordinal patterns in financial time series. They found evidence of deterministic forces in the medium and long term dynamics. Moreover, they propose that the evolution of the number of missing patterns could be an appropriate tool to quantify the randomness of certain time periods within the financial series. Also, missing ordinal patterns have been proposed recently as evidence of deterministic dynamics during epileptic states. In Ref. [36], it is suggested that the measure of missing patterns corresponding to EEG time series could be considered as a predictor of epileptic absence seizures. It is important to remark that in the research mentioned above, only non-correlated noise (white noise) has been considered.

Amigó et al. [18,32] proposed a test that uses missing ordinal patterns to distinguish determinism (chaos) from randomness in finite time series contaminated with observational white noise (uncorrelated noise), based in two important practical properties: their finiteness and noise contamination. These two properties are important because finiteness produces missing patterns in a random sequence without constrains, whereas noise blurs the difference between deterministic and random time series. The methodology proposed by Amigó et al. [18] consists of a graphic comparison between the decay of the missing ordinal patterns (of length D) of the time series under analysis as a function of the series length N, and the decay corresponding to white Gaussian noise. They presented numerical evidence that missing patterns persist in noisy deterministic data, even when the contamination with white Gaussian noise is high. However, the decay of the number of missing ordinal patterns with N for correlated stochastic and non-Gaussian processes was not analyzed, and the authors explicitly stated the need for further research in this direction.

In the present work we extend the analysis of missing ordinal patterns to stochastic processes with different degrees of correlation: fractional Brownian motion (fBm), fractional Gaussian noise (fGn) and, noises with f^{-k} power spectrum (PS) and ($k \geq 0$). These three types of stochastic processes were chosen because they are frequently used to simulate and/or characterize time series of natural phenomena (see i.e. [38–40]). Specifically, we analyze the decay rate of missing ordinal patterns as a function of the pattern length D (embedding dimension) and number of time series data points N.

2. Methodology

2.1. Bandt and Pompe — Ordinal patterns

The Bandt and Pompe [14] methodology is used to determine the probability distribution P associated to a given time series (dynamical system). The first step is to consider partitions of the pertinent D-dimensional space that will can potentially relevant details of the ordinal structure of the one-dimensional time series $\mathcal{S} = \{x_t : t = 1, \ldots, N\}$ with embedding dimension $D > 1$. The approach analyzes the "ordinal patterns" of order D [14,21] generated by

$$(s) \mapsto \left(x_{s-(D-1)}, x_{s-(D-2)}, \ldots, x_{s-1}, x_s\right), \tag{1}$$

which assigns to each time s the D-dimensional vector of values at times $s, s-1, \ldots, s-(D-1)$. Clearly, for greater D—values there will be more "past" information incorporated into the vectors and more possible patterns are considered. An "ordinal pattern" related to a time (s) corresponds to the permutation $\pi = (r_0, r_1, \ldots, r_{D-1})$ of $[0, 1, \ldots, D-1]$ defined by

$$x_{s-r_{D-1}} \leq x_{s-r_{D-2}} \leq \cdots \leq x_{s-r_1} \leq x_{s-r_0} . \tag{2}$$

In order to get a unique result we set $r_i < r_{i-1}$ if $x_{s-r_i} = x_{s-r_{i-1}}$. Thus, for all the $D!$ possible permutations π of order D, the probability distribution $P = \{p(\pi)\}$ is defined by

$$p(\pi) = \frac{\sharp\{s|s \leq N-D+1; (s), \texttt{has type}\, \pi\}}{N-D+1}. \tag{3}$$

In this expression, the symbol $\sharp$ stands for "number".

The Bandt and Pompe methodology is not restricted to time series representative of low-dimensional dynamical systems but can also be applied to any type of time series (regular, chaotic, noisy, or observational), with a very weak stationary assumption (for $\kappa \leq D$, the probability for $x_t < x_{t+\kappa}$ should not depend on t [14]). It is implicitly assumed that enough data are available for a representative delay reconstruction. Of course, the embedding dimension D plays an important role in the evaluation of the appropriate probability distribution because D determines the number of accessible states $D!$. It also conditions the minimum acceptable length N of the time series that is needed in order to work with reliable statistics ($N \gg D!$). In particular, Bandt and Pompe suggest for practical purposes working with $3 \leq D \leq 7$, and this is what we do here (in the present work we used $D = 4$, 5 and 6).

2.2. Stochastic processes considered

The following stochastic processes are considered in the present study:

(a) Noises with f^{-k} power spectrum:

The corresponding time series are generated as follows [17]: (1) The MATLAB© *RAND* function is used to produce pseudo-random numbers in the interval $(-0.5, 0.5)$ with an (a) flat power spectrum (PS), (b) uniform probability distribution function (PDF), and (c) zero mean value, x_i. (2) Then, the Fast Fourier Transform (FFT) y_k^1 of the time series is obtained and multiplied by $f^{-k/2}$ ($k > 0$), yielding y_k^2; (3) Now, y_k^2 is symmetrized so as to obtain a real function and then the pertinent inverse FFT x_i is obtained, after discarding the small imaginary components produced by our numerical approximations. The ensuing time series x_i has the desired power spectrum properties and, by construction, is representative of non-Gaussian noises.

(b) Fractional Brownian motion (fBm) and fractional Gaussian noise (fGn):

fBm is the only family of processes which is (a) Gaussian, *(b)* self-similar, and *(c)* endowed with stationary increments (see Ref. [41] and references therein). The normalized family of these Gaussian processes, $\{B^{\mathcal{H}}(t), t > 0\}$, is endowed with these properties: (i) $B^{\mathcal{H}}(0) = 0$ almost surely (a.s.), i.e., with probability 1, (ii) $\mathbb{E}[B^{\mathcal{H}}(t)] = 0$ (zero mean), and (iii) covariance given by

$$\mathbb{E}[B^{\mathcal{H}}(t)B^{\mathcal{H}}(s)] = (t^{2\mathcal{H}} + s^{2\mathcal{H}} - |t-s|^{2\mathcal{H}})/2, \tag{4}$$

for $s, t \in \mathbb{R}$. Here $\mathbb{E}[\cdot]$ refers to the average computed with a Gaussian probability density. The power exponent $0 < \mathcal{H} < 1$ is commonly known as the Hurst parameter or Hurst exponent. These processes exhibit "*memory*" for any Hurst parameter *except for* $\mathcal{H} = 1/2$, as one realises from Eq. (4). The $\mathcal{H} = 1/2$—case corresponds to classical Brownian motion and successive motion increments are as likely to have the same sign as the opposite (there is no correlation among them). Thus, Hurst's parameter defines two distinct regions in the interval $(0, 1)$. When $\mathcal{H} > 1/2$, consecutive increments tend to have the same sign so that these processes are *persistent*. For $\mathcal{H} < 1/2$, on the other hand, consecutive increments are more likely to have opposite signs, and we say that they are *anti-persistent*.

Let us introduce the quantity $\{W^{\mathcal{H}}(t), t > 0\}$ (fBm-"increments")

$$W^{\mathcal{H}}(t) = B^{\mathcal{H}}(t+1) - B^{\mathcal{H}}(t), \tag{5}$$

so as to express our Gaussian noise in the fashion

$$\begin{aligned}\rho(k) &= \mathbb{E}[W^{\mathcal{H}}(t)W^{\mathcal{H}}(t+k)] \\ &= \frac{1}{2}\left[(k+1)^{2\mathcal{H}} - 2k^{2\mathcal{H}} + |k-1|^{2\mathcal{H}}\right], \quad k > 0.\end{aligned} \tag{6}$$

Note that for $\mathcal{H} = 1/2$ all correlations at nonzero lags vanish and $\{W^{1/2}(t), t > 0\}$ thus represents *white noise*.

The fBm and fGn are continuous but non-differentiable processes (in the classical sense). As a non-stationary process, they do not possess a spectrum defined in the usual sense; however, it is possible to define a *generalized power spectrum* of the form:

$$\Phi \propto |f|^{-\alpha}, \tag{7}$$

with $\alpha = 2\mathcal{H} + 1$, $1 < \alpha < 3$ for fBm and, $\alpha = 2\mathcal{H} - 1$, $-1 < \alpha < 1$, for fGn.

Due to their Gaussian nature, and other characteristics enumerated above, the Bandt–Pompe ideas are applicable to the fBn and fGn dynamical process [14,42]. For simulating the fBm and fGn time series we adopt the Davies–Harte algorithm [43], as recently improved by Wood and Chan [44], which is both exact and fast.

2.3. Decay rate of missing ordinal patterns

As explained earlier, Amigó's technique for investigating a possible deterministic structure in an observational time series is based on a graphical comparison between the decay of the missing ordinal patterns in the analyzed time series, with one corresponding to white Gaussian noise (random sequence without constrains). Amigo et al. [19] state that for white noise generated by N independent and identically distributed random variables, the probability of having missing ordinal patterns goes to 0 exponentially as N grows. In the present work, and with the aim of making the discrimination between deterministic and stochastic behavior more thorough, we extend the analysis of missing ordinal patterns to stochastic processes with different degrees of correlation. Specifically, our hypothesis is that noises with f^{-k} PS, fBm and fGn, the number of missing ordinal patterns goes to 0 exponentially as N grows. Though an analytical proof is not yet available, we present below numerical evidence that supports the validity of our hypothesis.

We call $\mathcal{M}(N, D)$ the number of missing ordinal patterns. That is, the number of ordinal patterns of length D *not* observed in a time series with N data values. In all cases, as the time series length N increases, the number of "missing ordinal patterns" decreases and eventually becomes zero.

In order to evaluate the decay rate of missing ordinal patterns we implement the following procedure:

- *Step 1:* We generate a long time series $\mathcal{S}$ of total length N_T. For fixed D, we compute subseries of varying length in the range $D \le N \le N_T$ using a small length increment δN.
- *Step 2:* We model the decay of $\mathcal{M}(N, D)$ as a function of N using an exponential function, as proposed by Amigó et al. [18,19]:

$$\mathcal{M}(N, D) = A \cdot \exp\{R \cdot N\}, \tag{8}$$

  where R is the characteristic decay, and A a constant.

  Other functional forms (i.e.: power laws, N^{-q}) were also considered, however, the exponential functional form was selected because it rendered the best fit.
- *Step 3:* The characteristic decay rate R is determined by fitting Eq. (8) to the numerically generated valued $\mathcal{M}(N, D)$ using least square method. The mean value of the r^2 coefficient of the exponential fit, (the goodness of the fit) for all the stochastic processes considered is $(0.9328 \pm 0.0227) \le r^2 \le (0.9994 \pm 0.0003)$. Similar values were found for $D = 4$ and $D = 5$.

In the present work we study the decay rates for length patterns (embedding dimensions) $D = 4$, 5 and 6.

3. Results and discussion

Fig. 1 illustrates a few typical examples of the three type of stochastic correlated processes considered in our analysis. The behavior of the missing ordinal patterns for these stochastic processes, is shown in Figs. 2 and 3, for selected values of the power spectrum exponents (α and k).

Fig. 2 shows the averaged missing ordinal patterns, $\langle \mathcal{M}(N, D) \rangle$, as a function of the time series length N and for $D = 6$. We average the $\mathcal{M}(N, D)$ obtained for M numerically generated time series with different initial conditions (see Step 1 of the procedure). Fig. 2(a) shows the average number of missing ordinal patterns for three different k values of f^{-k} PS process. Higher k values have lower decays rates, and thus the minimum length N for which $\langle \mathcal{M}(N, D) \rangle$ becomes zero increases as k increases. A similar trend is observed for fBm that displays the highest decay rate for $\alpha = 1.2$ and the lowest for $\alpha = 2.8$. A different trend is observed in Fig. 2(c) for the fGn processes. In this case the fastest decay corresponds to $\alpha = 0$, and decreases for both higher and lower values of α.

The corresponding averaged values $\langle \mathcal{M}(N, D) \rangle$ for the three pattern lengths (embedding dimensions) considered, for fixed values of k and α, are illustrated in Fig. 3. It can be observed from this figure that the decay rates increase rapidly as pattern length D decreases.

Fig. 4 displays the mean decay rate of missing ordinal patterns and their standard deviation ($\overline{R} \pm SD$) as a function of power spectrum exponent values (α for fBm and fGn and, k for f^{-k} PS noise). Each point in these curves ($\overline{R}$ and SD) is estimated from M values of R, obtained following the procedure described in previous section for M numerically simulated time series with different initial conditions. The trends for the mean decay rate, corresponding to each pattern length (embedding dimension) D, can be very well described by a polynomial function of order 2 for fGn and order 3 for both the fBm and f^{-k} PS noise. These trends are shown as dotted lines in the corresponding figures.

Fig. 4 shows that as the correlations of the fBm process ($1 < \alpha < 3$) and f^{-k} PS noises ($0 < k < 3$) become stronger the mean decay rate $\overline{R}$ decreases. This indicates that the rate of appearance of ordinal patterns depends on the strength of the correlations. The less correlated, the faster the patterns appear. In contrast, the fGn ($-1 < \alpha < 1$) exhibits a symmetric behavior, with a minimum $\overline{R}$ for $\alpha = 0$ (white noise), and increasing values of $\overline{R}$ for both higher and lower α's. This indicates that persistent and anti-persistent fGns have very similar values of $\overline{R}$.

Fig. 4 also shows that the standard deviation of the estimated R values decreases with increasing D. This is due to the fact that longer patterns contain more temporal information and are therefore more effective in capturing the dynamics of time series with correlation structures, which allows for a better discrimination between Gaussian and non-Gaussian stochastic processes. In other words, the characterization of the stochastic process is more accurately determined with longer pattern lengths. It can be observed from Fig. 4c (for $D = 6$) that f^{-k} PS noises and fGn with similar values of the power spectrum exponent (α or k) have distinctly different decay rates except those corresponding to the higher correlations (note the proximity of the values of $\overline{R}$ for $\alpha = 2.8$ and $k = 2.75$). In contrast, the decay rates for f^{-k} PS noises and fGn with power spectrum exponent in the interval $(0, 1)$ are not distinctly different, that is the standard deviation for the decay rates of both processes overlaps within this range. Moreover, white Gaussian noise (represented by fGn with $\alpha = 0$) and non-Gaussian white noise (f^{-k} PS noise with $k = 0$) have the same mean decay rate ($\overline{R}$).

Finally, a gap in the values of $\overline{R}$ between the fBm and its associated noise, the fGn can be observed in Fig. 4. This gap is similar to that found by Rosso et al. [23] for other Information Theory quantifiers (entropy and statistical complexity) suggesting that it is an intrinsic characteristic of these processes.

4. Conclusions

In this study we analyze the rate of decay (R) as a function of the time series length (N) and pattern length (D), of the missing ordinal patterns for stochastic processes with different degree of correlation: fractional Brownian motion (fBm), fractional Gaussian noise (fGn) and, noises with f^{-k} power spectrum (PS), $k \geq 0$. As a general behavior we find that, for a fixed pattern length, the decay rates are much lower for processes with higher correlation structures. It is possible to conclude from these results that the decay of ordinal missing patters in correlated stochastic processes depends not only on the series length but also on their correlation structures. In other words, missing ordinal patters persist in the time series depending on how strong their correlation structures are.

As mentioned in the introduction, Amigó et al. [18] presented numerical evidence that "forbidden patterns" persist in noisy deterministic data, even when the contamination is high. Considering that in noisy data these patterns are not really forbidden, this assertion only means that they have just not appeared yet. The persistence of "missing patterns" in noisy deterministic data only reveals that longer time series are needed for all patterns to appear and this persistence depends in the noise level (see Fig. 2 in Ref. [18] for the case of deterministic dynamics with additive Gaussian noise).

In this work we find that stochastic time series with long term correlation structures (i.e, fractional Brownian motion with high values of the Hurst Exponent or f^{-k} noises with high k) also display "very high persistence of missing ordinal patterns" as shown by the low decay rates R values in Fig. 4. In other words, the number of missing patterns decreases very slowly with series length in a similar fashion to that of noisy deterministic data. Therefore, we conclude that the persistence of missing patterns is not necessarily a signature of underlying determinism, as it was argued in the literature, because this same persistence is found in stochastic time series with long term correlation structures.

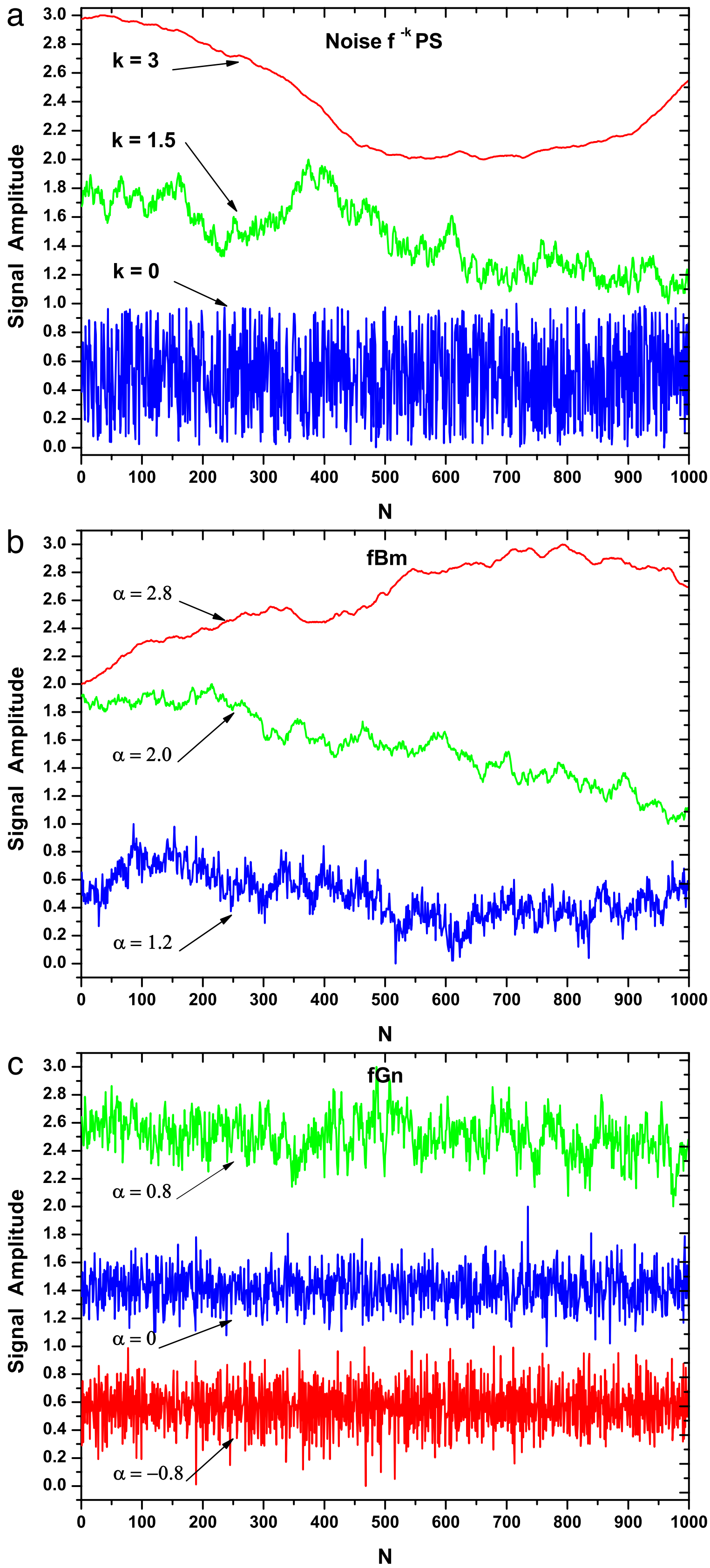

Fig. 1. Typical time series of length $N = 1000$ for selected values of power spectrum exponents (α for fBm and fGn, k for f^{-k} PS noise) and for (a) f^{-k} PS noise (b) fBm and (c) fGn.

We suggest that the use of missing ordinal patterns as a tool for distinguishing underlying determinism in time series must be reconsidered and some criteria has to be formalized. The results here presented could be used to evaluate whether a

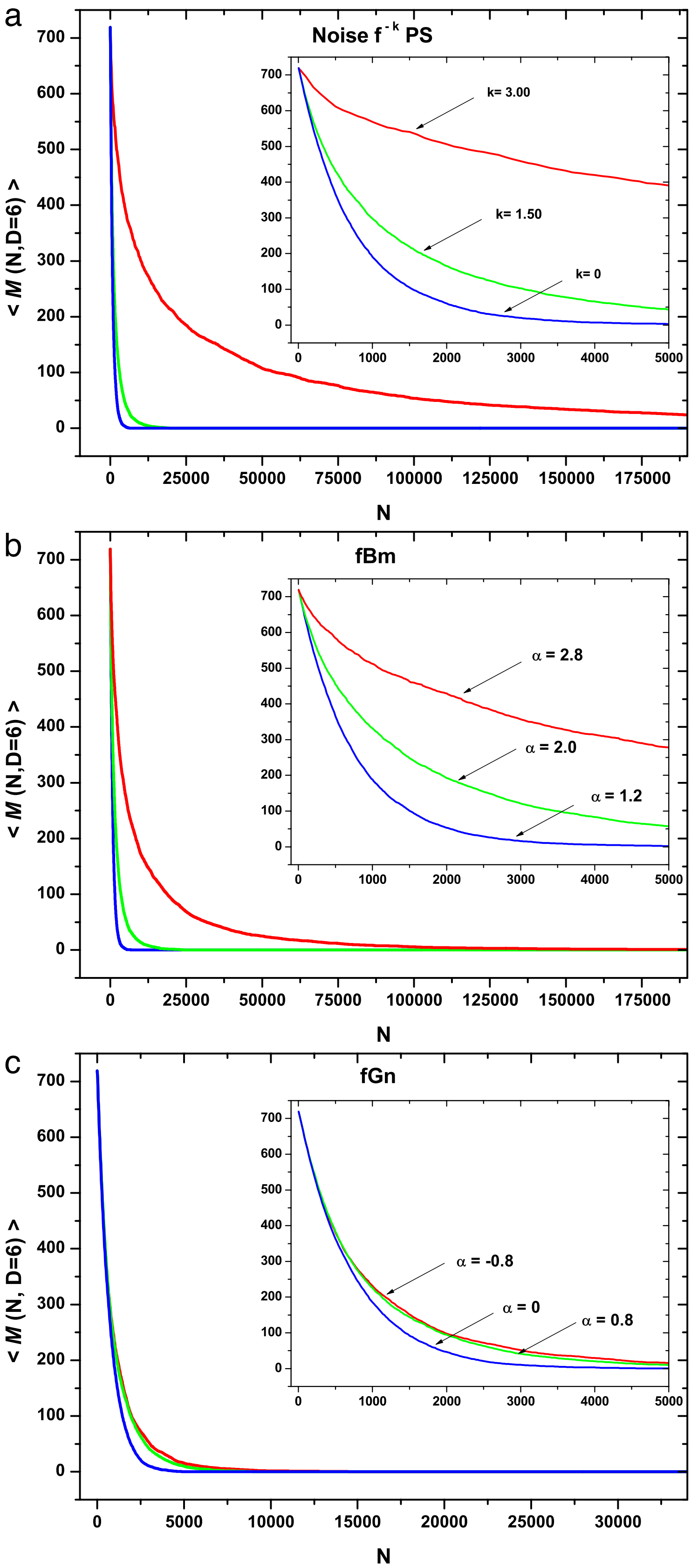

Fig. 2. Average number of missing ordinal patterns $\langle \mathcal{M}(N, D) \rangle$ as function of the time series length N, for embedding dimension $D = 6$ and for selected values of power spectrum exponents (α for fBm and fGn, k for f^{-k} PS noise). (a) f^{-k} PS noise; (b) fBm and; (c) fGn. The number of series M considered was 30.

given time series follows or not the behavior of the processes here studied. The estimated values of both the missing ordinal pattern decay rate R and the power spectrum exponent parameter (obtained from Fourier Transform power spectral analysis

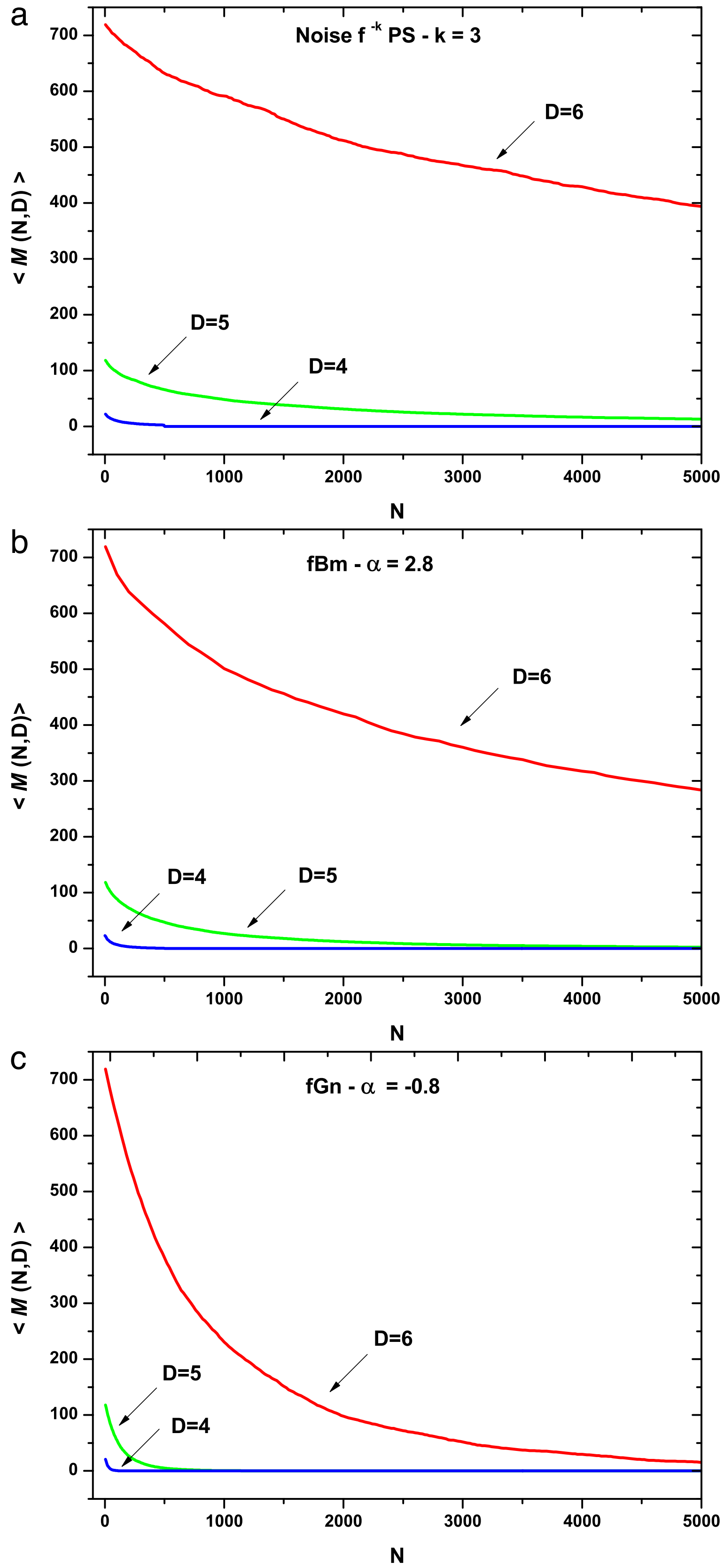

Fig. 3. Average number of missing ordinal patterns $\langle \mathcal{M}(N, D)\rangle$ as function of the time series length N, for embedding dimensions $D = 4, 5$ and 6 and for fixed values of power spectrum exponents (α for fBm and fGn, k for f^{-k} PS noise). The number of series M considered for embedding dimensions $D = 4, 5$ and 6 was 1000, 250 and 30 respectively. (a) f^{-k} PS noise; (b) fBm and; (c) fGn.

of the series) could be used to implement new tests with the objective of distinguishing deterministic from stochastic dynamics. Of course, the case of deterministic dynamics contaminated with correlated noises must be considered. Research

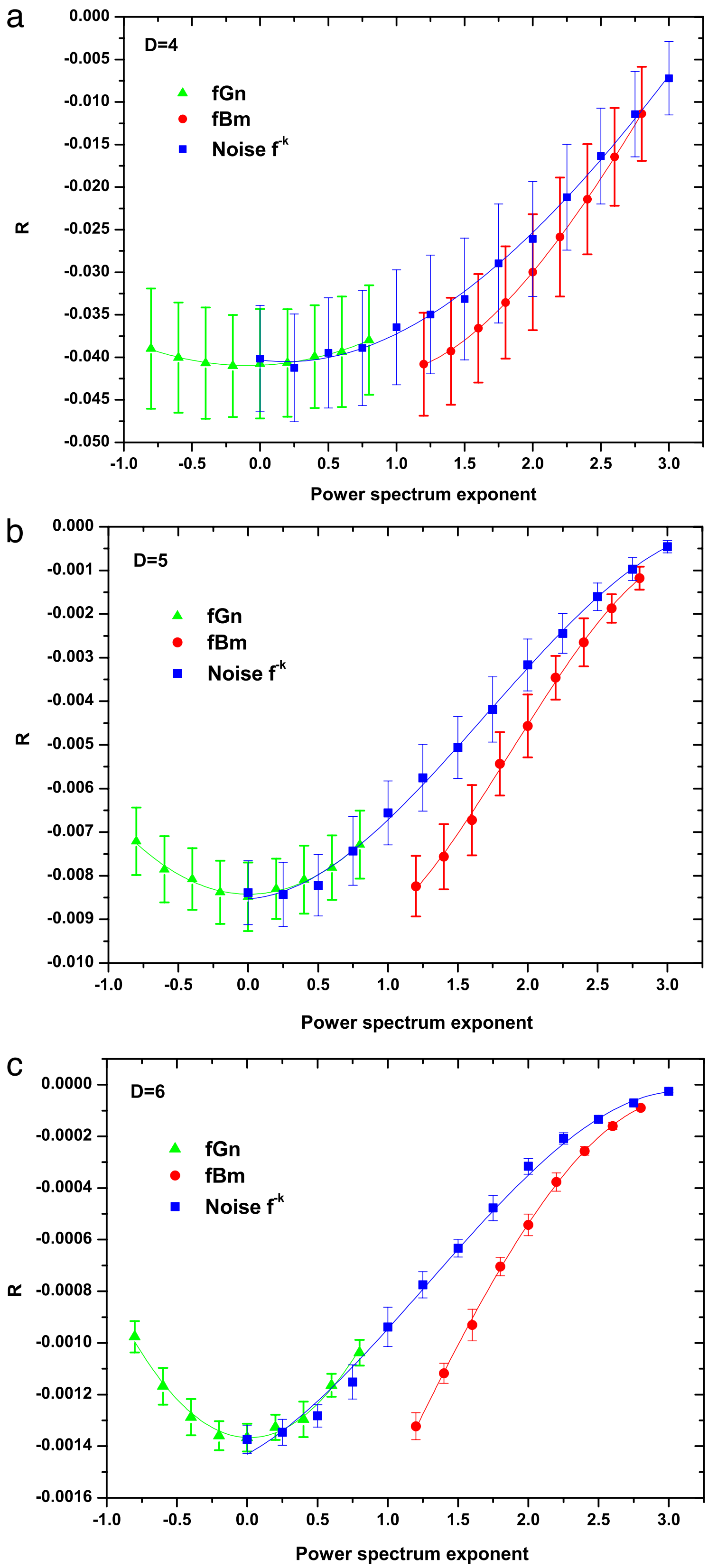

Fig. 4. Mean decay rate R with the corresponding standard deviations as a function of the power spectrum exponents (α for fBm and fGn, k for f^{-k} PS noise) for embedding dimensions (a) $D = 4$; (b) $D = 5$ and; (c) $D = 6$ for the three different stochastic process considered. The number of series M considered for embedding dimensions $D = 4$, 5 and 6 was 1000, 250 and 30 respectively. The subseries were computed using a length increment $\delta N = 5$ for $D = 4$, 5, and $\delta N = 20$ for $D = 6$.

in this direction is currently under study. In addition, we should note that the rate of decay of missing ordinal patterns (R) can be used to determine the Hurst exponent ($\mathcal{H}$) from an experimental time series. Work to implement this technique is being pursued by the authors.

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References

[1] G. Sugihara, R.M. May, Nature 344 (1990) 734.
[2] D.T. Kaplan, L. Glass, Phys. Rev. Lett. 68 (1992) 427.
[3] M.B. Kennel, S. Isabelle, Phys. Rev. A 46 (1992) 3111.
[4] J. Theiler, S. Eubank, A. Longtin, B. Galdrikian, J.D. Farmer, Physica D 58 (1992) 77.
[5] A.A. Tsonis, J.B. Elsner, Nature 358 (1992) 217.
[6] D.T. Kaplan, L. Glass, Physica D 64 (1993) 431.
[7] R. Wayland, D. Bromley, D. Pickett, A. Passamante, Phys. Rev. Lett. 70 (1993) 580.
[8] L.W. Salvino, R. Cawley, Phys. Rev. Lett. 73 (1994) 1091.
[9] G.J. Ortega, E. Louis, Phys. Rev. Lett. 81 (1998) 4345.
[10] J. Jeong, J.C. Gore, B.S. Peterson, Biol. Cybern. 86 (2002) 335.
[11] G.J. Ortega, E. Louis, Phys. Rev. E 62 (2000) 3419.
[12] G.J. Ortega, C. Degli Esposti Boschi, E. Louis, Phys. Rev. E 65 (2001) 016208.
[13] M. Small, C.K. Tse, IEEE Trans. Circuit and Syst. I: Fundamental theory and applications 50 (2003) 663.
[14] C. Bandt, B. Pompe, Phys. Rev. Lett. 88 (2002) 174102.
[15] M.T. Martín, A. Plastino, O.A. Rosso, Phys. Lett. A 311 (2003) 126.
[16] P.W. Lamberti, M.T. Martín, A. Plastino, O.A. Rosso, Physica A 334 (2004) 119.
[17] O.A. Rosso, H.A. Larrondo, M.T. Martín, A. Plastino, M.A. Fuentes, Phys. Rev. Lett. 99 (2007) 154102.
[18] J.M. Amigó, S. Zambrano, M.A. F Sanjuán, Europhys. Lett. 79 (2007) 50001.
[19] J.M. Amigó, S. Zambrano, M.A.F. Sanjuán, Europhys. Lett. 83 (2008) 60005.
[20] C. Bandt, G. Kellerand, B. Pompe, Nonlinearity 15 (2002) 1595.
[21] K. Keller, M. Sinn, Physica A 356 (2005) 114.
[22] A.M. Kowalski, M.T. Martín, A. Plastino, O.A. Rosso, Physica D 233 (2007) 21.
[23] O.A. Rosso, L. Zunino, D.G. Pérez, A. Figliola, H.A. Larrondo, M. Garavaglia, M.T. Martín, A. Plastino, Phys. Rev. E 76 (2007) 061114.
[24] O.A. Rosso, R. Vicente, C. Mirasso, Phys. Lett. A 372 (2008) 1018.
[25] L. De Micco, C.M. González, H.A. Larrondo, M.T. Martín, A. Plastino, O.A. Rosso, Physica A 387 (2008) 3373.
[26] L. Zunino, D.G. Pérez, M.T. Martín, M. Garavaglia, A. Plastino, O.A. Rosso, Phys. Lett. A 372 (2008) 4768.
[27] L. Zunino, D.G. Pérez, A. Kowalski, M.T. Martín, M. Garavaglia, A. Plastino, O.A. Rosso, Physica A 387 (2008) 6067.
[28] L. De Micco, H. Larrondo, A. Plastino, O.A. Rosso, Philos. Trans. R. Soc. A 367 (2009) 3281.
[29] L. Zunino, M. Zanin, B.M. Tabak, D. Pérez, O.A. Rosso, Physica A 388 (2009) 2854.
[30] O.A. Rosso, C. Masoller, Phys. Rev. E 79 (2009) 040106(R).
[31] O.A. Rosso, C. Masoller, Eur. Phys. J. B 69 (2009) 37.
[32] J.M. Amigó, L. Kocarev, J. Szczepanski, Phys. Lett. A 355 (2006) 27.
[33] H. Wold, A Study in the Analysis of Stationary Time Series, Almqvist and Wiksell, Upsala, Sweden, 1938.
[34] J. Kurths, H. Herzel, Physica D 25 (1987) 165.
[35] S. Cambanis, C.D. Hardin, A. Weron, Probab. Theory Related Fields 79 (1988) 1.
[36] G. Ouyang, X. Li, C. Dang, D.A. Richards, Phys. Rev. E 79 (2009) 041146.
[37] M. Zanin, Chaos 18 (2008) 013119.
[38] S. Sato, M. Yamamoto, T. Musha, Noise in Physical Systems and $1/f$ Fluctuation, IOS Press, Oxford, 1992.
[39] G. Bosman, Noise in Physical Systems and $1/f$ Fluctuations, World Scientific, New Jersey, 2001.
[40] F. Biagini, Y. Hu, B. Øksendar, T. Zhang, Stochastic Calculus for Fractional Brownian Motion and Applications, Springer, London, 2008.
[41] L. Zunino, D.G. Pérez, M.T. Martín, A. Plastino, M. Garavaglia, O.A. Rosso, Phys. Rev. E 75 (2007) 021115.
[42] C. Bandt, F. Shiha, J. Time Ser. Anal. 28 (2007) 646.
[43] R.B. Davies, D.S. Harte, Biometrika 74 (1987) 95.
[44] A.T.A. Wood, G. Chan, J. Comput. Graph. Statist. 3 (1994) 409.