Negative parity baryon decays in the $1/N_c$ expansion

C. Jayalath,[1,2,3,*] J. L. Goity,[1,2,†] E. González de Urreta,[4,5,‡] and N. N. Scoccola[4,5,6,§]

[1]*Department of Physics, Hampton University, Hampton, Virginia 23668, USA*
[2]*Thomas Jefferson National Accelerator Facility, Newport News, Virginia 23606, USA*
[3]*Department of Physics, Peradeniya University, Peradeniya 20400, Sri Lanka*
[4]*Department of Theoretical Physics, Comisión Nacional de Energía Atómica, 1429 Buenos Aires, Argentina*
[5]*CONICET, Rivadavia 1917, 1033 Buenos Aires, Argentina*
[6]*Universidad Favaloro, Solís 453, 1078 Buenos Aires, Argentina*
(Received 12 August 2011; published 10 October 2011)

The partial decay widths of lowest lying negative parity baryons belonging to the **70**-plet of $SU(6)$ are analyzed in the framework of the $1/N_c$ expansion. The channels considered are those with single pseudoscalar meson emission. The analysis is carried out to sub-leading order in $1/N_c$ and to first order in $SU(3)$ symmetry breaking. Conclusions about the magnitude of $SU(3)$ breaking effects along with predictions for some unknown or poorly determined partial decay widths of known resonances are obtained.

DOI: 10.1103/PhysRevD.84.074012 PACS numbers: 14.20.Gk, 11.15.Pg, 12.39.Jh

I. INTRODUCTION

The extensive experimental programs at various facilities, in particular, Jefferson Lab, MAMI, ELSA, GRAAL and BES, where photo- and electro-production data as well as J/ψ decays give unprecedented access to baryon resonance parameters, will lead to significant improvements over the current knowledge of resonance masses and partial widths. With this progress, further sharpening of the theoretical approaches is needed. Although successful to a remarkable extent, the study of decays with quark models is affected by numerous choices about the mechanism of decay [1], which result in model dependencies which are difficult to quantify. An alternative approach, which gives a systematic connection to QCD is based on the $1/N_c$ expansion. As formulated for baryons, it allows one to represent quantities and observables in a systematic expansion in effective operators [2–5] where the coefficients encode the unknown dynamics. The expansion is ordered in powers of $1/N_c$, and the mentioned coefficients are determined by fitting to data. Tests of the feasibility of the expansion at each order are provided by relations which are independent of those coefficients, and by whether the magnitude of next to leading order effects are or are not of natural size.

The $1/N_c$ expansion for excited baryons is based on the classification of states and operators under the dynamical symmetry group $SU(6) \times O(3)$ [4,5]. A contracted $SU_C(6)$ spin-flavor symmetry is an emergent symmetry in the large N_c limit in baryons [2,6], which serves to organize the $1/N_c$ expansion using effective operators. The $O(3)$ symmetry is only approximate even in large N_c in the case of the **70**-plet baryons [4] (it becomes exact in the case of **56**-plets [4]), and in the large N_c limit the emergent $SU_C(6)$ is a subgroup of the $SU(6) \times O(3)$. In the real world, the breaking of $SU(6) \times O(3)$ seems to be small, even when it happens at $\mathcal{O}(N_c^0)$, as in the case of the masses of the negative parity baryon **70**-plet considered here. It is therefore natural to implement the $1/N_c$ expansion in the framework of an approximate $SU(6) \times O(3)$ symmetry[4,5,7,8].

Masses and partial decay widths are the main quantities characterizing baryon resonances. These quantities can be defined and obtained through partial wave analyses where the constraints of unitarity and analyticity of the S-matrix are fulfilled. In principle, they can be given unambiguous meaning, through pole positions in the complex energy plane. A rigorous approach in which the $1/N_c$ expansion is implemented alongside with those analyses is yet to be developed, but it has been initiated [9]. The analysis presented here aims at providing a $1/N_c$ expansion for the Breit-Wigner partial decay widths, and is therefore limited in its rigor in the sense just mentioned.

The present work extends the analysis of partial decay widths of the negative parity baryons to include the decays of the strange members of the **70**-plet as well as the decays of the non-strange members into hyperons. The original analysis with the $1/N_c$ expansion was carried out in [3], although an incomplete basis of operators was used. The improvement in the present work, which is carried out to include $1/N_c$ corrections and $SU(3)$ breaking to first order, is in including up to 2-body $SU(3)$ preserving and 1-body $SU(3)$ breaking operators. An analysis with a complete basis at the mentioned order is not possible because of the incompleteness in the input partial widths. However, it is expected that, in particular, 3-body operators are going to be dynamically suppressed. A motivation for the present study is to extend to the strangeness sector the work in the non-strange sector in the $SU(4) \times O(3)$ analysis [10], and,

*jayalath@jlab.org
†goity@jlab.org
‡emilianogdeurreta@gmail.com
§scoccola@tandar.cnea.gov.ar

in particular, to determine the importance of $SU(3)$ breaking effects. Despite the limitations due to the mentioned scarcity of information on strangeness partial widths, it is possible to conclude that $SU(3)$ breaking in partial decay widths is larger than natural size. We will be able to show this through the failure of some leading order coefficient-independent relations as well as at next to leading order where some $SU(3)$ breaking operators are shown to have contributions of unnaturally large size. Particular channels had been identified long ago [11], such as the S-wave $\Lambda(1670)\to\bar{K}N$, the D-wave $\Lambda(1690)\to\bar{K}N$, which turn out to be a problem in the different versions of the quark model. The channels into η meson were shown to be poorly described at leading order [10] as well. These channels are a most definite proof of the large $SU(3)$ breaking effects in decays. On the other hand, we will show that the $SU(3)$ preserving components of the amplitudes are well described in the $1/N_c$ expansion.

This work is organized as follows: Sec. II presents the framework, Sec. III gives the construction of the bases of operators for the partial wave decay amplitudes, Sec. IV presents the analysis of the Breit-Wigner partial decay widths as given in the Particle Data Group (PDG) [12], and Sec. V is devoted to conclusions. An Appendix presents various results needed for the calculations of matrix elements.

II. $1/N_c$ EXPANSION FRAMEWORK FOR DECAYS

In this section, we present the framework for implementing the $1/N_c$ expansion for partial decay widths of the **70**-plet baryons, briefly reviewing the bases of states and the formalism for the decays. Some additional details can be found in Refs. [13,14].

The lowest lying negative parity baryons are assumed to belong to the $[\mathrm{MS}, \ell^P=1^-]$ multiplet of $SU(6)\times O(3)$, where MS is the $SU(6)$ mixed symmetric representation $(p,q)=(N_c-1,1)$. For $N_c=3$, this is the $[\mathbf{70},1^-]$ multiplet.

The 35 generators of $SU(6)$ along with their commutation relations are

$$\begin{aligned}[][S_i,T_a]&=0,[S_i,S_j]=i\epsilon_{ijk}S_k,[T_a,T_b]=if_{abc}T_c,\\ [S_i,G_{ja}]&=i\epsilon_{ijk}G_{ka},[T_a,G_{ib}]=if_{abc}G_{ic},\\ [G_{ia},G_{jb}]&=\frac{i}{4}\delta_{ij}f_{abc}T_c+\frac{i}{6}\epsilon_{ijk}\delta_{ab}S_k+\frac{i}{2}\epsilon_{ijk}d_{abc}G_{ck},\end{aligned}\tag{1}$$

where S_i are the spin generators, T_a the flavor generators, and G_{ia} are the spin-flavor generators, and d_{abc} and f_{abc} are, respectively, the $SU(3)$ symmetric and antisymmetric invariant tensors.

Throughout the approximation of neglecting configuration mixings (i.e., mixing of $[\mathrm{MS},1^-]$ with other multiplets) will be made. This approximation is expected to be good [15], and it has to be made due to the lack of completeness and sufficient accuracy of the data on masses and partial decay widths, which are inputs to the $1/N_c$ analyses. The states in the $[\mathrm{MS},1^-]$ multiplet are constructed as follows: a fundamental $SU(6)$ representation ("excited quark") is coupled to a totally symmetric (S) representation ("core") with baryon number N_c-1. The core carries spin $S_c=S+\eta$, where S is the spin of the MS $SU(6)$ state and $\eta=\pm1/2$. The $SU(3)$ representation of the core is determined by S_c and is given by $R_c=(p_c,q_c)=(2S_c,\frac{N_c-1-2S_c}{2})$. The $[\mathrm{MS},\ell]$ states then read [7,8],

$$\begin{aligned}|JJ_3;R(Y,II_3);S\rangle_{\mathbf{MS}}=&\sum_{\eta=\pm1/2}C_{\mathrm{MS}}(R,S,\eta)\langle\ell m,SS_3|JJ_3\rangle\langle S_cS_{c3},\tfrac12\,s_3|SS_3\rangle\\ &\times\left\langle\begin{matrix}R_c & \mathbf{3}\\ Y_cI_cI_{c3} & y\,i\,i_3\end{matrix}\middle|\begin{matrix}R\\ YII_3\end{matrix}\right\rangle|S_cS_{c3};R_c(Y_c,I_cI_{c3})\rangle|\tfrac12\,s_3;\mathbf{3}(y,ii_3)\rangle|\ell m\rangle,\end{aligned}\tag{2}$$

where summation over repeated projection indices is implied, and $S_c=S+\eta$ in the sum. ℓ is the $O(3)$ quantum number of the baryon, equal to 1 here, and J is the baryon spin. R indicates the $SU(3)$ representation of the baryon. The coefficients $C_{\mathrm{MS}}(R,S,\eta)$ are given by [8]:

$$C_{\mathrm{MS}}\left(R,S,\pm\frac12\right)=\begin{cases}1 & \text{if } I_R=S\pm1\\ 0 & \text{if } I_R=S\mp1\\ \pm\sqrt{\frac{(2S+1\mp1)(N_c+1\pm(2S+1))}{2N_c(2S+1)}} & \text{if } I_R=S,\end{cases}\tag{3}$$

where I_R denotes the isospin of the zero strangeness states in the irreducible representation R of $SU(3)$. For $R=(p,q)$, $I_R=p/2$. Later on we will indicate some of the quantum numbers of the excited baryons by an upper label *.

For $N_c = 3$, one obtains the **70**-plet $\ell = 1$ states, which consist of the following $^{2S+1}R_J$ multiplets: $^2\mathbf{8}_{(1/2)}$, $^2\mathbf{8}_{(3/2)}$, $^4\mathbf{8}_{(1/2)}$, $^4\mathbf{8}_{(3/2)}$, $^4\mathbf{8}_{(5/2)}$, $^2\mathbf{10}_{(1/2)}$, $^2\mathbf{10}_{(3/2)}$, $^2\mathbf{1}_{(1/2)}$, $^2\mathbf{1}_{(3/2)}$. For generic N_c, the identification of states is given in Table XV in the Appendix. The **1** and **10** states have core states which for $N_c = 3$ are pure $S_c = 0$ and 1, respectively. The **8**'s with the same J but a different S mix to give the mass eigenstates. In the limit of $SU(3)$ symmetry, two mixing angles describe these possible mixings. Because of the breaking of $SU(3)$ symmetry by the strange quark mass, the following sets of states mix with each other: $\{N^2(\mathbf{8}_J), N(^4\mathbf{8}_J)\}$, $\{\Sigma(^2\mathbf{8}_J), \Sigma(^4\mathbf{8}_J), \Sigma(^2\mathbf{10}_J)\}$, $\{\Xi(^2\mathbf{8}_J), \Xi(^4\mathbf{8}_J), \Xi(^2\mathbf{10}_J)\}$ and $\{\Lambda(^2\mathbf{8}_J), \Lambda(^4\mathbf{8}_J), \Lambda(^2\mathbf{1}_J)\}$. The mixings, as discussed later, can be determined primarily by the decays and can be further constrained by the masses [7,8] and photocouplings, as it has been done in the non-strange sector [16]. While the two mixing angles in the $SU(2)$ symmetry limit are $\mathcal{O}(1)$, the $SU(3)$ breaking effects are $\mathcal{O}(m_s - m_{u,d})$. The dimensionless $SU(3)$ breaking expansion parameter will be denoted by ϵ, where $\epsilon \propto (m_s - m_{u,d})$. We estimate that in practice its value can be taken to be $\epsilon \sim 1/3$. Thus, the mixing angles involving the octet components should differ by $\mathcal{O}(\epsilon)$ corrections, while mixing angles involving states in different $SU(3)$ multiplets are $\mathcal{O}(\epsilon)$. Unfortunately, the available data on strong decays are not sufficient to perform an analysis that can account for all the different mixing angles. Thus, in what follows, mixing angles involving states in different $SU(3)$ multiplets are set to vanish. The $1/N_c$ analysis of the **70**-plet masses showed that this is a reasonable approximation [8]. In this case, the states $\Sigma''_J = \Sigma(^2 10_J)$, $\Xi''_J = \Xi(^2 10_J)$ and $\Lambda''_J = \Lambda(^2 1_J)$ are taken as unmixed states while the other physical states are obtained as mixture between octet states according to

$$\begin{pmatrix} B_J \\ B'_J \end{pmatrix} = \begin{pmatrix} \cos\theta_{B_{2J}} & \sin\theta_{B_{2J}} \\ -\sin\theta_{B_{2J}} & \cos\theta_{B_{2J}} \end{pmatrix} \begin{pmatrix} ^2B_J \\ ^4B_J \end{pmatrix}, \qquad (4)$$

where for each value of $J = 1/2, 3/2$ we have four different angles, i.e. $\theta_{N_{2J}}$, $\theta_{\Lambda_{2J}}$, $\theta_{\Sigma_{2J}}$ and $\theta_{\Xi_{2J}}$.

The established **70**-plet states according to the Particle Data Group (PDG) [12] along with their partial decay widths are displayed in Tables I and II. Of the known states, only the $J = 5/2$ state $N(1675)$ could have a G-wave decay into for instance $\pi\Delta$, but no empirical information for such decay exists. Therefore, only S and D waves need to be considered.

The partial wave decay amplitudes via a single pseudoscalar meson can be expressed in the most general form:

$$\mathcal{M}(\ell_P Y_P I_P, B_{\rm GS}, B^*) = (-1)^{\ell_P}\sqrt{2M_{B^*}}\,\langle B_{\rm GS}|\mathcal{B}^{[\ell_P,R_P]}_{Y_P I_P}|B^*\rangle, \qquad (5)$$

TABLE I. Empirical data for the decay channels of the N's and Λ's in the $[\mathbf{70}, 1^-]$ from the PDG.

PDG Name	State	Mass [MeV]	Total Width [MeV]	Branching ratios [%] S-wave	D-wave
$N(1535)$	$N_{1/2}$	1535(10)	150(25)	πN: 45(10) ηN: 52.5(7.5)	$\pi\Delta < 1$
$N(1520)$	$N_{3/2}$	1520(5)	113(12.5)	$\pi\Delta$: 8.5(3.5)	πN: 60(5) $\pi\Delta$: 12(2) ηN: 0.23(0.04)
$N(1650)$	$N'_{1/2}$	1657(13)	165(20)	πN: 77.5(17.5) ηN: 6.5(3.5) $K\Lambda$: 7(4)	$\pi\Delta$: 4(3)
$N(1700)$	$N'_{3/2}$	1700(50)	100(50)	$\pi\Delta$: 90(5)	πN: 10(5) $K\Lambda < 3$
$N(1675)$	$N_{5/2}$	1675(5)	148(18)		πN: 40(5) $K\Lambda < 1$
$\Lambda(1670)$	$\Lambda_{1/2}$	1670(10)	37.5(12.5)	$\bar{K}N$: 25(5) $\eta\Lambda$: 17.5(7.5) $\pi\Sigma$: 40(15)	
$\Lambda(1690)$	$\Lambda_{3/2}$	1690(5)	60(10)	$\pi\Sigma^*$: 45(10)	$\bar{K}N$: 25(5) $\pi\Sigma$: 30(10)
$\Lambda(1800)$	$\Lambda'_{1/2}$	1785(65)	300(100)	$\bar{K}N$: 32.5(7.5)	
$\Lambda(1830)$	$\Lambda_{5/2}$	1820(10)	85(25)		$\bar{K}N$: 6.5(3.5) $\pi\Sigma$: 55(20) $\pi\Sigma^* > 15$
$\Lambda(1405)$	$\Lambda''_{1/2}$	1406(4)	50(2)	$\pi\Sigma$: 100	
$\Lambda(1520)$	$\Lambda''_{3/2}$	1519(1)	15.6(1)		$\bar{K}N$: 45(1) $\pi\Sigma$: 42(1)

TABLE II. Empirical data for decay channels of the Σ's and Δ's in the [**70**, 1^-] from the PDG.

PDG Name	State	Mass [MeV]	Total Width [MeV]	Branching ratios *S*-wave	Branching ratios *D*-wave
Σ(1670)	$\Sigma_{3/2}$	1675(10)	60(20)		$\bar{K}N$: 10(3) $\pi\Lambda$: 10(5) $\pi\Sigma$: 45(15)
Σ(1750)	$\Sigma'_{1/2}$	1765(35)	110(50)	$\bar{K}N$: 25(15) $\pi\Sigma < 8$ $\eta\Sigma$: 35(20)	
Σ(1775)	$\Sigma_{5/2}$	1775(5)	120(15)		$\bar{K}N$: 40(3) $\pi\Lambda$: 17(3) $\pi\Sigma$: 3.5(1.5) $\pi\Sigma^*$: 10(2)
Δ(1620)	$\Delta_{1/2}$	1630(30)	143(7.5)	πN: 25(5)	$\pi\Delta$: 45(15)
Δ(1700)	$\Delta_{3/2}$	1710(40)	300(100)	$\pi\Delta$: 37.5(12.5)	πN: 15(5) $\pi\Delta$: 4(3)

where P denotes a meson in the pseudoscalar octet ($R_P = \mathbf{8}$). B^* and $B_{\rm GS}$ are, respectively, the excited and ground state baryons, ℓ_P is the partial wave, and Y_P, I_P are the quantum numbers of the pseudoscalar meson. A factor involving the meson decay constant, $\sqrt{N_c}/F_P = \mathcal{O}(N_c^0)$, which naturally appears in the expressions of the decay amplitude [15] is absorbed into the baryon operator $\mathcal{B}^{[\ell_P R_P]}_{Y_P I_P}$. This operator represents the effective vertex $B^* B_{\rm GS} P$, and will be expanded in powers of $1/N_c$ and in $SU(3)$ breaking to first order in ϵ. The expansion is performed with a basis of effective operators and has the general form:

$$\mathcal{B}^{[\ell_P,R_P]} = \left(\frac{k_P}{\Lambda}\right)^{\ell_P} \sum_n C_n^{[\ell_P,R_P]}(k_P) \mathcal{B}_n^{[\ell_P,R_P]}, \qquad (6)$$

where k_P is the meson momentum, and for convenience, a centrifugal barrier is factored out to take into account the chief momentum dependence of the corresponding partial wave amplitude. The $\mathcal{B}_n$ represent operators in a basis, and are ordered in powers of $1/N_c$ and ϵ. $C_n(k_P)$ are effective coefficients, which encode the QCD dynamics, and will be determined by fitting to the known partial decay widths. The operators are defined and normalized such that the C_n's are all $\mathcal{O}(N_c^0)$. We arbitrarily choose the scale $\Lambda = 200$ MeV.

The basis operators are expressed in terms of spin-flavor operators $\mathcal{G}_n$:

$$\mathcal{B}_n^{[\ell_P,R_P]} = (\xi^\ell \mathcal{G}_n^{[j_n,R_P]})^{[\ell_P,R_P]}, \qquad (7)$$

with the obvious notation indicating coupling of angular momenta. ξ^ℓ is an $O(3)$ tensor operator for the transition from the $O(3)$ state with $\ell = 1$ of the excited baryon to the GS baryon where $\ell = 0$. $\mathcal{G}_n$ is a spin-flavor operator, which gives the transition from the initial $SU(6)$ MS baryon state to the GS baryon which is a symmetric state. Without any loss of generality, one can choose ξ^ℓ to satisfy $\langle 0|\xi^\ell_{m'}|\ell m\rangle = (-1)^{\ell-m}\delta_{m,-m'}$.

The partial decay width is then given in terms of the reduced matrix elements (RMEs) of the operators $\mathcal{B}_n$ as follows:

$$\Gamma_{\ell_P Q_P} = \frac{k_P}{8\pi^2}\left(\frac{k_P}{\Lambda}\right)^{2\ell_P} \frac{M_B}{M_B^*} \frac{\hat{I}^2}{(\hat{I}^*\hat{J}^*)^2} \times \left| \sum_n C_n^{[\ell_P,R_P]}(k_P) \sum_\gamma \begin{pmatrix} R^* & R_P & \| & R \\ Y^* I^* & Y_P I_P & \| & YI \end{pmatrix}_\gamma \times \mathcal{B}_n{}^\gamma(\{S,Q\},\{(\ell,S^*)J^*,Q^*\},\{\ell_P,Q_P\}) \right|^2, \qquad (8)$$

where throughout we use the customary notation for the $SU(3)$ isoscalar factors, $\hat{t} \equiv \sqrt{2t+1}$, and $Q \equiv \{R, Y, I\}$. In addition, γ labels the possible multiplicities in the coupling $R^* \otimes R_P \to R$. Since there is mixing between $S^* = 1/2$ and $3/2$ states in the case $R^* = \mathbf{8}$, this mixing is simply taken into account by replacing $\mathcal{B}_n{}^\gamma$ in the formula by the corresponding linear combination with the $S^* = 1/2$ and $S^* = 3/2$ states, and similarly for the $SU(3)$ breaking induced mixings involving states in different R^* representations, i.e., **1**, **8** and **10** if one would include those mixings.

Distinguishing the operators into $SU(3)$ preserving and $SU(3)$ breaking, the RMEs can be expressed in terms of the RMEs of the spin-flavor operators $\mathcal{G}_n$. For $SU(3)$ preserving operators, one has:

$$\mathcal{B}_n{}^\gamma(\{S,Q\},\{(\ell,S^*)J^*,Q^*\},\{\ell_P,Q_P\}) = (-1)^{j_n+J^*+\ell+S} \frac{\hat{J}^*\hat{\ell}_P}{\sqrt{\dim R}} \begin{Bmatrix} J^* & S^* & \ell \\ j_n & \ell_P & S \end{Bmatrix} \times \langle S, R \,\|\, \mathcal{G}_n^{[j_n,R_P]} \,\|\, S^*, R^*\rangle_\gamma, \qquad (9)$$

where one identifies a 6-j $SU(2)$ symbol and the RME of the spin-flavor operator. We use the $SU(2)$ conventions from Edmonds [17] and the $SU(3)$ conventions from Hecht [18].

To first order in the quark masses, the $SU(3)$ symmetry breaking can be expressed in terms of spin-flavor operators of the form:

$$(\mathcal{M}^8_f \mathcal{G}^{[j_m,R_m]}_m)^{R_P}_{\gamma_n}, \tag{10}$$

where $\mathcal{M}^8_f$ is the octet component of the quark masses (we work in the limit of exact isospin symmetry). Since in the present case $R_P = \mathbf{8}$, R_m can be $\mathbf{1}$, $\mathbf{8}$, $\mathbf{10}$, $\overline{\mathbf{10}}$, or $\mathbf{27}$. Only $R_m = \mathbf{1}, \mathbf{8}$ involve 1-body spin-flavor operators, while $\mathbf{10}$, $\overline{\mathbf{10}}$, and $\mathbf{27}$ involve 2-body operators. There is, therefore, in principle, a significant number of $SU(3)$ breaking operators. However, the reality of the matter is that the available information on partial decay widths sensitive to $SU(3)$ breaking effects is very limited, and for this reason, in the present work, only a truncated basis of such operators will be used, namely, 1-body ones.

Up to an overall constant to be absorbed by the normalization procedure discussed below, the RMEs of $SU(3)$ breaking operators can be shown to be of the most general form [19]

$$\begin{aligned}
&\mathcal{B}_n{}^{\gamma}(\{S, Q\}, \{(\ell, S^*)J^*, Q^*\}, \{\ell_P, Q_P\}) \\
&= (-1)^{j_m+J^*+\ell+S}\frac{\hat{J}^*\hat{\ell}_P}{\sqrt{\dim R}}\begin{Bmatrix} J^* & S^* & \ell \\ j_m & \ell_P & S \end{Bmatrix} \\
&\times \left(\begin{array}{cc|c} 8 & R_m & R_P \\ 00 & Y_P I_P & Y_P I_P \end{array}\right)_{\gamma_n} \langle S, R \parallel \mathcal{G}^{[j_m,R_m]}_m \parallel S^*, R^* \rangle_{\gamma}.
\end{aligned} \tag{11}$$

Note that the index m is determined by the index n.

Basis operators will be normalized such that the coefficients C_n are all $\mathcal{O}(N_c^0 \times \epsilon^0)$ and their natural size within a given partial wave be the same. In this manner, just by looking at the size of a coefficient, one can infer whether the corresponding operator is giving contributions within the expectations of its $1/N_c$ and ϵ power countings. Effects of $SU(6)$ symmetry breaking are reflected in the momentum k_P, which are then taken into account by the decay operators and are therefore encoded in the effective coefficients C_n. In the case of the $\mathbf{56}$-plet decays, where all $SU(6)$ breaking effects are $\mathcal{O}(1/N_c)$ or $\mathcal{O}(\epsilon)$, the k_P dependencies are sub-leading in the respective expansions. For this reason, such effects are simply taken into account by the expansion itself if one neglects the k_P dependence of the coefficients C_n. In the case of the $\mathbf{70}$-plet, the $\mathcal{O}(N_c^0)$ effects on mass splittings are small, and thus a similar approach of neglecting the momentum dependency of the coefficients is expected to work.

III. OPERATOR BASES

The construction of the bases of operators was already discussed in [10] for the case of $SU(4)$, and is briefly reviewed here. The spin-flavor transition operators $\mathcal{G}_n$ from the MS to S representations are built using tensor products of the $SU(6)$ generators Λ_c and λ, which operate on the "core" and "excited quark," respectively. The reduction rules established in [20] are used to reduce products of core generators, and products ($\alpha, \beta, \ldots, SU(6)$ generator indices) $\lambda_\alpha\lambda_\beta$ can be reduced to 1-body operators. In addition, using the fact that any generator of $SU(6)$ $\Lambda = \Lambda_c + \lambda$ has vanishing matrix elements between MS and S states, one obtains the additional equivalences:

$$\begin{aligned}
&\text{1-body} \quad \Lambda_{c\alpha} = -\lambda_\alpha \\
&\text{2-body} \quad \Lambda_{c\alpha}\Lambda_{c\beta} = -\lambda_\alpha\Lambda_{c\beta} - \lambda_\beta\Lambda_{c\alpha} + \text{1-body operators.}
\end{aligned} \tag{12}$$

Therefore, only the following types of operators need to be considered,

$$\begin{aligned}
&\text{1-body} \quad \lambda \\
&\text{2-body} \quad \frac{1}{N_c}\lambda_\alpha\Lambda_{c\beta} \\
&\text{3-body} \quad \frac{1}{N_c^2}\lambda_\alpha\Lambda_{c\beta}\Lambda_{c\gamma}.
\end{aligned} \tag{13}$$

There is only one 1-body operator, namely

$$\mathcal{G}^{(1)} = g^{[1,8]}, \tag{14}$$

which is identified with the axial current of the "excited quark" in quark model language. On the other hand, 2-body operators can be built with products $\lambda\Lambda_c$, which can be coupled to angular momentum $j = 0, 1, 2$. For $\ell = 1$, ℓ_P can be 0, 2, 4, therefore $j = 1, 2$. They have to be coupled to an $\mathbf{8}$ of $SU(3)$ for the decays involving pseudoscalar mesons in $R_P = \mathbf{8}$. The following possible monomials can appear in the 2-body operators:

$$\begin{aligned}
&(sT^c)^{[1,8]}, (tS^c)^{[1,8]}, (sG^c)^{[j,8]}, (gS^c)^{[j,8]}, \\
&(tG^c)^{[1,8]}, (gT^c)^{[1,8]}, (gG^c)^{[j,8]},
\end{aligned} \tag{15}$$

where $j = 1, 2$, and the coupling of the two $\mathbf{8}$s to $\mathbf{8}$ involves two possibilities, namely f-and d-type couplings. There are four such cases in this list of operators, and therefore the total number of possible monomials is equal to 14.

This set of 2-body operators can be reduced using operator identities, resulting in nine independent operators, namely:

$$\begin{aligned}
&\mathcal{G}^{(2)}_1 = \frac{1}{N_c}(sT^c)^{[1,8]} \qquad \mathcal{G}^{(2)}_2 = \frac{1}{N_c}(tS^c)^{[1,8]} \\
&\mathcal{G}^{(2)}_3 = \frac{1}{N_c}(sG^c)^{[1,8]} \qquad \mathcal{G}^{(2)}_4 = \frac{1}{N_c}(gS^c)^{[1,8]} \\
&\mathcal{G}^{(2)}_5 = \frac{1}{N_c}(gT^c)^{[1,8]}_{\gamma=1} \qquad \mathcal{G}^{(2)}_6 = \frac{1}{N_c}(gT^c)^{[1,8]}_{\gamma=2} \\
&\mathcal{G}^{(2)}_7 = \frac{1}{N_c}(sG^c)^{[2,8]} \qquad \mathcal{G}^{(2)}_8 = \frac{1}{N_c}(gS^c)^{[2,8]} \\
&\mathcal{G}^{(2)}_9 = \frac{1}{N_c}(gG^c)^{[2,8]}_{\gamma=1}.
\end{aligned} \tag{16}$$

So far, we have discussed the operators that appear in the limit of exact $SU(3)$ symmetry.

As mentioned earlier in the $SU(3)$ breaking operators, Eq. (17), $\mathcal{G}_m$ belong to **1**, **8**, **10**, $\overline{\mathbf{10}}$ or **27** of $SU(3)$. The operators of interest here are the 1-body ones, which are $\mathcal{O}(\epsilon \times N_c^0)$. The basis of 1-body $SU(3)$ breaking operators is constructed from the following three monomials:

$$\mathcal{G}_1^{(\mathrm{SB})} = (t_8 s)^{[1,8]} \qquad \mathcal{G}_2^{(\mathrm{SB})} = (t_8 g)^{[1,8]}_{\gamma=2} - \sqrt{\tfrac{2}{3}}\,\mathcal{G}^{(1)} \qquad \mathcal{G}_3^{(\mathrm{SB})} = (t_8 g)^{[1,8]}_{\gamma=1}. \tag{17}$$

Note that $\mathcal{G}_1^{(\mathrm{SB})}$ only couples to the η meson. For convenience $\mathcal{G}_2^{(\mathrm{SB})}$ has been defined in such a way that channels with emission of a π meson are not affected. $\mathcal{G}_3^{(\mathrm{SB})}$ automatically has that property. For a list of the 2-body symmetry breaking operators, see [13].

Three-body basis operators can be constructed following similar procedure. One would however expect that they will be dynamically suppressed, as it has been noted in the particular case of mass operators.

The list of basis operators used in this work is shown in Table III. Throughout, it is assumed that the number of strange quarks in the baryons is $\mathcal{O}(N_c^0)$, while of course the number of u and d quarks is assumed to be $\mathcal{O}(N_c)$. This reflects in the counting of powers of $1/N_c$ in the decay amplitudes involving strange baryons. Since the only strange baryons included as inputs in the analysis have strangeness (-1), it seems that the assumption is reasonable. We then have the following general $1/N_c$ power countings: the amplitudes of nonstrange baryons decaying via π or η meson emission are all $\mathcal{O}(N_c^0)$ up to corrections $\mathcal{O}(1/N_c)$, while their decay amplitudes via K-meson emission are $\mathcal{O}(1/\sqrt{N_c})$. Similarly, the decay amplitudes of strange baryons via π emission are $\mathcal{O}(N_c^0)$, and via K-meson emission are $\mathcal{O}(1/\sqrt{N_c})$, except for the singlet $\Lambda(1405)$ and $\Lambda(1520)$ baryons where it is reversed. These properties can be traced back to the $1/N_c$-dependence of isoscalar factors. A similar situation occurs with the $SU(3)$ breaking operators used here, except that for these the π-emission amplitudes are identically zero.

TABLE III. The effective operators for **70**-plet baryon decays. The first operator set represents the symmetric operators and the second set represents the $SU(3)$ symmetry-breaking operators. $\ell = 0$ for S-wave and $\ell = 2$ for D-wave decays.

Operator	n-bodyness	Order
$\mathcal{B}_1 = (\xi g)^{[\ell,8]}$	1	N_c^0
$\mathcal{B}_2 = \frac{1}{N_c}(\xi(sT^c)^{[1,8]})^{[\ell,8]}$	2	N_c^0
$\mathcal{B}_3 = \frac{1}{N_c}(\xi(tS^c)^{[1,8]})^{[\ell,8]}$	2	$1/N_c$
$\mathcal{B}_4 = \frac{1}{N_c}(\xi(sG^c)^{[1,8]})^{[\ell,8]}$	2	N_c^0
$\mathcal{B}_5 = \frac{1}{N_c}(\xi(gS^c)^{[1,8]})^{[\ell,8]}$	2	$1/N_c$
$\mathcal{B}_6 = \frac{1}{N_c}(\xi(gT^c)^{[1,8]}_{\gamma=1})^{[\ell,8]}$	2	N_c^0
$\mathcal{B}_7 = \frac{1}{N_c}(\xi(gT^c)^{[1,8]}_{\gamma=2})^{[\ell,8]}$	2	N_c^0
$\mathcal{B}_8 = \frac{1}{N_c}(\xi(sG^c)^{[2,8]})^{[\ell,8]}$	2	N_c^0
$\mathcal{B}_9 = \frac{1}{N_c}(\xi(gS^c)^{[2,8]})^{[\ell,8]}$	2	$1/N_c$
$\mathcal{B}_{10} = \frac{1}{N_c}(\xi(gG^c)^{[2,8]}_{\gamma=1})^{[\ell,8]}$	2	N_c^0
$\mathcal{B}_1^{\mathrm{SB}} = (\xi(t_8 s)^{[1,8]}_{\gamma=1})^{[\ell,8]}$	1	ϵ
$\mathcal{B}_2^{\mathrm{SB}} = (\xi(t_8 g)^{[1,8]}_{\gamma=2})^{[\ell,8]} - \sqrt{\frac{2}{3}}\mathcal{B}_1$	1	ϵ
$\mathcal{B}_3^{\mathrm{SB}} = (\xi(t_8 g)^{[1,8]}_{\gamma=1})^{[\ell,8]}$	1	ϵ

For arbitrary N_c, the number of $SU(3)$ preserving basis operators is equal to seven for S waves and to ten for D waves. The RMEs $\mathcal{B}_n{}^\gamma$ are shown in Tables IV and V. The tables also show the normalization factors (α_n) necessary to have each operator give natural size matrix elements. As explained earlier, these are chosen in such a way that the largest spin-isospin RME of $\mathcal{O}(N_c^0)$ operators is equal to one in magnitude when evaluated at $N_c = 3$, and $1/3$ for the case of $\mathcal{O}(1/N_c)$ $SU(3)$ preserving operators. Similarly, the latter normalization is implemented in $SU(3)$ breaking operators as well. Note that the spin-isospin RMEs are defined by the sum over γ of the product of B_n^γ with the corresponding isoscalar factor (see Eq. (8)).

The necessary isoscalar factors needed in the calculations of the actual matrix elements for the different channels are given in the Appendix.

It turns out that the sets of operators listed in Tables IV and V are linearly dependent when restricted, for arbitrary N_c, to the $SU(3)$ multiplets corresponding to the ones in the **70**-plet (according to Table XV of the Appendix). For the $SU(3)$ preserving operators, one finds the following set of bases after eliminating linear dependencies, namely, for the S-wave decays the LO basis is given by $\{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_6\}$ and the NLO basis is given by $\{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4, \mathcal{B}_6\}$ and, in addition, the $SU(3)$ breaking operators $\{\mathcal{B}_1^{\mathrm{SB}}, \mathcal{B}_2^{\mathrm{SB}}, \mathcal{B}_3^{\mathrm{SB}}\}$. For the D-wave decays, the LO basis is given by $\{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_6, \mathcal{B}_8\}$ and the NLO basis is given by $\{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_5, \mathcal{B}_6, \mathcal{B}_8, \mathcal{B}_9\}$ and the $SU(3)$ breaking operators $\{\mathcal{B}_1^{\mathrm{SB}}, \mathcal{B}_2^{\mathrm{SB}}, \mathcal{B}_3^{\mathrm{SB}}\}$. For the available data inputs to the fits, one finds that one of the D-wave $SU(3)$ breaking operators can be eliminated due to linear dependency. We will choose in this case to eliminate the third one. The normalization factors for the symmetry breaking operators are as follows: for the S-wave operators, they are $\alpha_1^{\mathrm{SB}} = \frac{\sqrt{3}}{2}$, $\alpha_2^{\mathrm{SB}} = \sqrt{\frac{2}{3}}$ and $\alpha_3^{\mathrm{SB}} = \sqrt{\frac{6}{5}}$, and for the two D-wave operators, we use in the fit, $\alpha_1^{\mathrm{SB}} = \frac{2}{3}\sqrt{\frac{10}{21}}$ and $\alpha_2^{\mathrm{SB}} = \sqrt{\frac{3}{10}}$.

One comment concerning the mixing of states. In the $SU(3)$ symmetry and strict large N_c limits, the two mixing angles have definite values independent of coefficients, namely, $\theta_1 = \arccos(\frac{1}{\sqrt{3}})$ and $\theta_3 = \arccos(\frac{1}{\sqrt{6}})$ [21]. It was found [10] that to leading order in $1/N_c$ with these mixing

TABLE IV. Reduced matrix elements $\mathcal{B}_n{}^{\gamma}$ (see Eqn. (8)) for S-wave decays. The factors β_i and A_i are displayed in Table VI.

Decay chanel	$\mathcal{B}_1$	$\mathcal{B}_2$	$\mathcal{B}_3$	$\mathcal{B}_4$	$\mathcal{B}_5$	$\mathcal{B}_6$	$\mathcal{B}_7$	Overall factor
$(^2 8_{(1/2)} \to 8)_1$	$-\frac{12+N_c}{6\sqrt{2}}$	$\frac{\beta_1}{2\sqrt{2}}$	$\frac{3+N_c}{6\sqrt{2}N_c}$	$-\frac{1}{4}$	0	0	$\frac{\beta_2}{\sqrt{8}}$	A_2
$(^2 8_{(1/2)} \to 8)_2$	$\frac{2N_c-3}{6\sqrt{2}}$	$\frac{3-2N_c}{6\sqrt{2}N_c}$	$\frac{3+N_c}{6\sqrt{2}N_c}$	$\frac{N_c}{12}$	0	0	$\frac{\beta_3}{6\sqrt{2}}$	A_3
$^2 8_{(3/2)} \to 10$	$-\frac{1}{6\sqrt{2}}$	$\frac{1}{6\sqrt{2}N_c}$	$\frac{1}{6\sqrt{2}N_c}$	$\frac{3-N_c}{24N_c}$	$-\frac{1}{8N_c}$	$-\frac{1}{4\sqrt{6}N_c}$	$\frac{N_c-2}{12\sqrt{30}N_c}$	$-A_1$
$(^4 8_{(1/2)} \to 8)_1$	$\frac{1}{6}$	$\frac{2+N_c}{6N_c}$	$-\frac{1}{6N_c}$	$-\frac{1}{2\sqrt{2}N_c}$	$-\frac{1}{4\sqrt{2}N_c}$	0	$\frac{N_c+17}{12\sqrt{15}N_c}$	$-A_5$
$(^4 8_{(1/2)} \to 8)_2$	$\frac{1}{6}$	$-\frac{1}{6N_c}$	$-\frac{1}{6N_c}$	$\frac{N_c-3}{12\sqrt{2}N_c}$	$-\frac{1}{4\sqrt{2}N_c}$	0	$\frac{11-N_c}{12\sqrt{15}N_c}$	$-A_6$
$^4 8_{(3/2)} \to 10$	$\frac{1}{6}\sqrt{\frac{5}{2}}$	$-\frac{1}{6N_c}\sqrt{\frac{5}{2}}$	$\frac{1}{3N_c}\sqrt{\frac{5}{2}}$	$\frac{\sqrt{5}(N_c-3)}{24N_c}$	0	$\frac{1}{4N_c}\sqrt{\frac{5}{6}}$	$\frac{2-N_c}{12\sqrt{6}N_c}$	A_4
$^2 10_{(1/2)} \to 8$	$\frac{1}{6}$	$-\frac{1}{6N_c}$	$-\frac{2}{3N_c}$	$\frac{N_c+3}{12\sqrt{2}N_c}$	0	$-\frac{1}{4\sqrt{3}N_c}$	$\frac{2-N_c}{12\sqrt{15}N_c}$	A_9
$(^2 10_{(3/2)} \to 10)_1$	$\frac{21+N_c}{6\sqrt{2}}$	$\frac{\beta_4}{2\sqrt{2}}$	$-\frac{21+N_c}{6\sqrt{2}N_c}$	$\frac{3(N_c+1)}{4N_c}$	$\frac{21+N_c}{8N_c}$	0	$\frac{\beta_5}{\sqrt{8}}$	A_7
$(^2 10_{(3/2)} \to 10)_2$	$\frac{1}{6}\sqrt{\frac{5}{2}}$	$-\frac{1}{6N_c}\sqrt{\frac{5}{2}}$	$-\frac{1}{6N_c}\sqrt{\frac{5}{2}}$	$\frac{\sqrt{5}(N_c+3)}{24N_c}$	$\frac{\sqrt{5}}{8N_c}$	0	$-\frac{N_c+7}{12\sqrt{6}N_c}$	$-A_8$
$^2 1_{(1/2)} \to 8$	$-\frac{1}{4}$	$\frac{1}{4N_c}$	0	$\frac{1}{4\sqrt{2}N_c}$	0	$-\frac{N_c+3}{16\sqrt{3}N_c}$	$-\frac{N_c+7}{16\sqrt{15}N_c}$	A_5
α_n	$\sqrt{\frac{27}{10}}$	$\frac{9}{2}\sqrt{\frac{3}{5}}$	$\sqrt{\frac{27}{40}}$	$2\sqrt{\frac{6}{5}}$	$6\sqrt{\frac{6}{5}}$	$9\sqrt{\frac{2}{5}}$	$\frac{27}{11}\sqrt{10}$	

TABLE V. Reduced matrix elements $\mathcal{B}_n{}^{\gamma}$ (see Eq. (8)) for D-wave decays. The factors β_i and A_i are displayed in Table VI.

Decay Channel	$\mathcal{B}_1$	$\mathcal{B}_2$	$\mathcal{B}_3$	$\mathcal{B}_4$	$\mathcal{B}_5$	$\mathcal{B}_6$	$\mathcal{B}_7$	$\mathcal{B}_8$	$\mathcal{B}_9$	$\mathcal{B}_{10}$	Overall factor
$^2 8_{(1/2)} \to 10$	$\frac{1}{3}$	$-\frac{1}{3N_c}$	$-\frac{1}{3N_c}$	$\frac{N_c-3}{6\sqrt{2}N_c}$	$\frac{1}{2\sqrt{2}N_c}$	$\frac{1}{2\sqrt{3}N_c}$	$\frac{2-N_c}{6\sqrt{15}N_c}$	$\frac{1-N_c}{2\sqrt{6}N_c}$	$\frac{1}{2\sqrt{6}N_c}$	$\frac{1}{12\sqrt{2}}$	$\frac{\sqrt{5}}{4}A_1$
$(^2 8_{(3/2)} \to 8)_1$	$-\frac{N_c+12}{3}$	β_1	$\frac{N_c+3}{3N_c}$	$-\frac{1}{\sqrt{2}}$	0	0	β_2	0	0	0	$-\frac{\sqrt{10}}{4}A_2$
$(^2 8_{(3/2)} \to 8)_2$	$2N_c-3$	$\frac{3-2N_c}{N_c}$	$\frac{3+N_c}{N_c}$	$\frac{N_c}{\sqrt{2}}$	0	0	β_3	0	0	0	$-\frac{\sqrt{10}}{12}A_3$
$^2 8_{(3/2)} \to 10$	$\frac{1}{3}$	$-\frac{1}{3N_c}$	$-\frac{1}{3N_c}$	$\frac{N_c-3}{6\sqrt{2}N_c}$	$\frac{1}{2\sqrt{2}N_c}$	$\frac{1}{2\sqrt{3}N_c}$	$\frac{2-N_c}{6\sqrt{15}N_c}$	$\frac{N_c-1}{2\sqrt{6}N_c}$	$-\frac{1}{2\sqrt{6}N_c}$	$-\frac{1}{12\sqrt{2}}$	$\frac{\sqrt{5}}{4}A_1$
$^4 8_{(1/2)} \to 10$	$\frac{1}{3}$	$-\frac{1}{3N_c}$	$\frac{2}{3N_c}$	$\frac{N_c-3}{6\sqrt{2}N_c}$	0	$\frac{1}{2\sqrt{3}N_c}$	$\frac{2-N_c}{6\sqrt{15}N_c}$	$\frac{1-N_c}{2\sqrt{6}N_c}$	$\sqrt{\frac{2}{3}}\frac{1}{N_c}$	$\frac{3+N_c}{12\sqrt{2}N_c}$	$-\frac{\sqrt{10}}{8}A_4$
$(^4 8_{(3/2)} \to 8)_1$	-1	$-\frac{N_c+2}{N_c}$	$\frac{1}{N_c}$	$\frac{3}{\sqrt{2}N_c}$	$\frac{3}{2\sqrt{2}N_c}$	0	$-\frac{N_c+17}{2\sqrt{15}N_c}$	$-\frac{3\sqrt{6}}{N_c}$	$-\sqrt{\frac{3}{2}}\frac{3}{2N_c}$	$\frac{9}{4\sqrt{2}N_c}$	$\frac{1}{6\sqrt{2}}A_5$
$(^4 8_{(3/2)} \to 8)_2$	-1	$\frac{1}{N_c}$	$\frac{1}{N_c}$	$\frac{3-N_c}{2\sqrt{2}N_c}$	$\frac{3}{2\sqrt{2}N_c}$	0	$\frac{N_c-11}{2\sqrt{15}N_c}$	$\sqrt{\frac{3}{2}}\frac{3(N_c-1)}{2N_c}$	$-\sqrt{\frac{3}{2}}\frac{3}{2N_c}$	$-\frac{3}{4\sqrt{2}}$	$\frac{1}{6\sqrt{2}}A_6$
$^4 8_{(3/2)} \to 10$	-1	$\frac{1}{N_c}$	$-\frac{2}{N_c}$	$\frac{3-N_c}{2\sqrt{2}N_c}$	0	$-\frac{\sqrt{3}}{2N_c}$	$\frac{N_c-2}{2\sqrt{15}N_c}$	$\sqrt{\frac{3}{2}}\frac{N_c-1}{4N_c}$	$-\sqrt{\frac{3}{2}}\frac{1}{N_c}$	$-\frac{N_c+3}{8\sqrt{2}N_c}$	$\frac{1}{3}A_4$
$(^4 8_{(5/2)} \to 8)_1$	1	$\frac{N_c+2}{N_c}$	$-\frac{1}{N_c}$	$-\frac{3}{\sqrt{2}N_c}$	$-\frac{3}{2\sqrt{2}N_c}$	0	$\frac{N_c+17}{2\sqrt{15}N_c}$	$-\sqrt{\frac{2}{3}}\frac{1}{N_c}$	$-\frac{1}{2\sqrt{6}N_c}$	$\frac{1}{4\sqrt{2}N_c}$	$-\frac{1}{2\sqrt{2}}A_5$
$(^4 8_{(5/2)} \to 8)_2$	1	$-\frac{1}{N_c}$	$-\frac{1}{N_c}$	$\frac{N_c-3}{2\sqrt{2}N_c}$	$-\frac{3}{2\sqrt{2}N_c}$	0	$\frac{11-N_c}{2\sqrt{15}N_c}$	$\frac{N_c-1}{2\sqrt{6}N_c}$	$-\frac{1}{2\sqrt{6}N_c}$	$-\frac{1}{12\sqrt{2}}$	$-\frac{1}{2\sqrt{2}}A_6$
$^4 8_{(5/2)} \to 10$	1	$-\frac{1}{N_c}$	$\frac{2}{N_c}$	$\frac{N_c-3}{2\sqrt{2}N_c}$	0	$\frac{\sqrt{3}}{2N_c}$	$\frac{2-N_c}{2\sqrt{15}N_c}$	$\frac{N_c-1}{2\sqrt{6}N_c}$	$-\sqrt{\frac{2}{3}}\frac{1}{N_c}$	$-\frac{N_c+3}{12\sqrt{2}N_c}$	$-\frac{\sqrt{14}}{8}A_4$
$(^2 10_{(1/2)} \to 10)_1$	$\frac{N_c+21}{3}$	β_4	$-\frac{N_c+21}{3N_c}$	$\frac{3(N_c+1)}{\sqrt{2}N_c}$	$\frac{N_c+21}{2\sqrt{2}N_c}$	0	β_5	$\sqrt{\frac{2}{3}}\frac{N_c+3}{N_c}$	$\frac{N_c+21}{2\sqrt{6}N_c}$	$\frac{3-N_c}{4\sqrt{2}N_c}$	$\frac{\sqrt{5}}{4}A_7$
$(^2 10_{(1/2)} \to 10)_2$	$-\frac{1}{3}$	$\frac{1}{3N_c}$	$\frac{1}{3N_c}$	$-\frac{N_c+3}{6\sqrt{2}N_c}$	$-\frac{1}{2\sqrt{2}N_c}$	0	$\frac{N_c+7}{6\sqrt{15}N_c}$	$-\frac{N_c+5}{10\sqrt{6}N_c}$	$-\frac{1}{2\sqrt{6}N_c}$	$\frac{1}{60\sqrt{2}}$	$\frac{5}{4}A_8$
$^2 10_{(3/2)} \to 8$	1	$-\frac{1}{N_c}$	$-\frac{4}{N_c}$	$\frac{N_c+3}{2\sqrt{2}N_c}$	0	$-\frac{\sqrt{3}}{2N_c}$	$\frac{2-N_c}{2\sqrt{15}N_c}$	0	0	0	$-\frac{\sqrt{5}}{6}A_9$
$(^2 10_{(3/2)} \to 10)_1$	$\frac{N_c+21}{3}$	β_4	$-\frac{N_c+21}{3N_c}$	$\frac{3(N_c+1)}{\sqrt{2}N_c}$	$\frac{N_c+21}{2\sqrt{2}N_c}$	0	β_5	$-\sqrt{\frac{2}{3}}\frac{N_c+3}{N_c}$	$-\frac{N_c+21}{2\sqrt{6}N_c}$	$\frac{N_c-3}{4\sqrt{2}N_c}$	$\frac{\sqrt{5}}{4}A_7$
$(^2 10_{(3/2)} \to 10)_2$	-1	$\frac{1}{N_c}$	$\frac{1}{N_c}$	$-\frac{N_c+3}{2\sqrt{2}N_c}$	$-\frac{3}{2\sqrt{2}N_c}$	0	$\frac{N_c+7}{2\sqrt{15}N_c}$	$\sqrt{\frac{3}{2}}\frac{N_c+5}{10N_c}$	$\sqrt{\frac{3}{2}}\frac{1}{2N_c}$	$-\frac{1}{20\sqrt{2}}$	$\frac{5}{12}A_8$
$^2 1_{(3/2)} \to 8$	-1	$\frac{1}{N_c}$	0	$\frac{1}{\sqrt{2}N_c}$	0	$-\frac{N_c+3}{4\sqrt{3}N_c}$	$-\frac{N_c+7}{4\sqrt{15}N_c}$	0	0	0	$-\frac{\sqrt{5}}{4}A_5$
α_n	$\sqrt{\frac{6}{7}}$	$\frac{9}{5}\sqrt{\frac{3}{2}}$	$\sqrt{\frac{3}{14}}$	$\frac{12}{5}\sqrt{3}$	$\frac{4}{5}\sqrt{3}$	$6\sqrt{\frac{2}{7}}$	$\frac{18}{11}\sqrt{5}$	$\frac{12}{5}\sqrt{2}$	$\frac{3}{\sqrt{8}}$	$3\sqrt{6}$	

TABLE VI. Factors needed in Tables IV and V.

β_1	β_2	β_3	β_4	β_5
$\frac{3-N_c(N_c+5)}{3N_c}$	$-\frac{15+N_c(N_c+2)}{6\sqrt{15}N_c}$	$-\frac{3+N_c(N_c+1)}{\sqrt{15}N_c}$	$\frac{24+N_c(N_c+5)}{3N_c}$	$\frac{(N_c-19)(N_c+3)}{6\sqrt{15}N_c}$

A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9
$\frac{1}{N_c}\sqrt{\frac{(N_c+1)(N_c+3)(N_c+5)}{(N_c-1)}}$	$\frac{1}{N_c}\sqrt{\frac{(N_c-1)}{(N_c+3)}}$	$\frac{1}{N_c}\sqrt{\frac{(N_c+7)}{(N_c+3)}}$	$\sqrt{\frac{(N_c+1)(N_c+5)}{N_c(N_c-1)}}$	$\sqrt{\frac{(N_c-1)}{N_c}}$	$\sqrt{\frac{(N_c+7)}{N_c}}$	$\sqrt{\frac{(N_c+5)}{N_c(45+N_c(N_c+6))}}$	$\sqrt{\frac{(N_c+9)(N_c+1)(N_c-3)}{N_c(45+N_c(N_c+6))}}$	$\sqrt{\frac{(N_c-1)(N_c+7)}{N_c(N_c+5)}}$

angles, the following transitions vanish: the S-wave $N(1535) \to \pi N$, $N(1650) \to \eta N$, and the D-wave $N(1700) \to \pi N$ and $N(1520) \to \eta N$. We have checked that indeed these cancellations take place. Another channel suppressed in those limits is $\Lambda(1700) \to \pi\Sigma$. In reality, the mixing angles differ significantly from the strict large N_c limit angles, primarily because the subleading in $1/N_c$ hyperfine interaction contributes much more to the masses than the $\mathcal{O}(N_c^0)$ operators (spin-orbit type interactions), which in the strict large N_c limit give the limit angles. The deviations from those cancellations in the decay amplitudes are thus explained primarily by the mixings, but also by subleading effects in $1/N_c$.

It is useful to asses the potential predictive power of the analysis presented here. The number of possible different decay amplitudes in the isospin limit is as follows: 17 in **8→8**, 12 in **8→10** or **10 → 8**, 13 in **10 → 10**, and 4 in **1 → 8**. Thus, there are 87 S-wave and 153 D-wave amplitudes. Using the empirical baryon and meson masses together with the $1/N_c$ predictions of Ref. [8] for those in the 70-plet which are still unknown, we estimate that the number of phase space allowed decays are 52 for S waves and 94 for D waves. For S-wave transitions, there are a total of seven possible $R_{B^*} \to R_B$ transitions, which is one more than the number of $SU(3)$ preserving S-wave operators at NLO. This implies one overall relation between amplitudes in $SU(3)$ symmetric limit. In the isospin limit, we have 87 channels, and since at NLO we have included eight operators, there are in principle 79 relations. For the D waves, there are 12 $R_{B^*} \to R_B$ transitions, which in $SU(3)$ symmetry limit with the ten operators leave two relations. The number of different channels in the isospin limit is equal to 153, and thus, with the 11 operators we include, there are in principle 143 relations between amplitudes. Most of the relations would represent a test of $SU(3)$ symmetry and its breaking by the 1-body operators, as one may expect. With the available information on partial widths, we can consider a limited number of relations at leading order, which we discuss below.

IV. ANALYSIS OF THE PARTIAL WIDTHS

Only a fraction of the possible partial decay channels of the negative parity baryons has been empirically determined [12]. This relatively poor database imposes significant limitations in the accuracy and robustness of the conclusions of the present analysis, as we discuss later. Of the 87 S-wave and 153 D-wave different partial widths which would be possible in the $[\mathbf{70}, 1^-]$ decays (respectively, 52 and 94 if phase space is taken into account), only 16 S-wave and 25 D-wave partial widths are known, a few of them only as bounds.

A very important consistency check of the viability of the $1/N_c$ analysis is given in terms of coefficient independent relations, namely, relations independent of the values of the coefficients C_n. These relations exist at each order in the expansion, and can be given at the level of reduced partial decay widths defined below. These relations provide in principle predictions for unknown partial decay widths accurate to the order of the expansion. Since the widths are quadratic in the fitting coefficients, the total number of coefficient-independent relations is $N_{\rm rel} = N_{\rm ch} - N_{\rm par}(N_{\rm par} + 1)/2$, where $N_{\rm rel}$ is the number of relations, $N_{\rm ch}$ is the number of partial widths, and $N_{\rm par}$ the number of coefficients C_n. With the numbers of channels mentioned above, and the number of operators involved at LO in $1/N_c$, where the $SU(3)$ breaking effects are relegated to subleading order, one finds 81 relations for S-wave decays and 143 for D-wave decays. A smaller number results because of channels suppressed by phase space, and a much smaller number yet for the actually known decays. Various such relations in the form of ratios of reduced widths are given below. Some of those can be tested and some give LO predictions for mixing angles. We do not discuss NLO relations; in principle there are 51 for S waves and 108 for D waves, not taking into account phase space suppressed channels.

We discuss now some reduced width ratios which can be tested with the known partial decay widths. The list is not exhaustive. The reduced partial decay widths are defined by $\tilde{\Gamma} = \Gamma/f_{\rm phs}$, where $f_{\rm phs} = \frac{k_P}{8\pi^2}\frac{M_B}{M_{B^*}}(\frac{k_P}{\Lambda})^{2\ell_P}$. These are basically squares of decay amplitudes with the centrifugal factor removed. For the S-wave decays, we obtain the ratios shown in Table VII. These results show that there is a significant discrepancy in the predictions for the mixing angle θ_1 resulting from nonstrange versus strange decay channels. The mixing angles get $SU(3)$ breaking corrections which are indeed important as we will find in the NLO results. The actual ratio $\frac{\tilde{\Gamma}(N(1650)\to\pi N)}{\tilde{\Gamma}(\Sigma(1750)\to\eta\Sigma)}$ is very

TABLE VII. S-wave decays: coefficient-independent ratios of reduced widths at LO.

Ratio	LO Ratio	Empirical	Prediction
$\frac{\tilde{\Gamma}(N(1535)\to\pi N)}{\tilde{\Gamma}(N(1650)\to\pi N)}$	$\left(\frac{\sqrt{2}\cos\theta_1-\sin\theta_1}{\cos\theta_1+\sqrt{2}\sin\theta_1}\right)^2$	0.6 ± 0.2	$\theta_1=\begin{cases}0.3\pm0.1\\1.6\pm0.1\end{cases}$
$\frac{\tilde{\Gamma}(\Delta(1620)\to\pi N)}{\tilde{\Gamma}(\Delta(1700)\to\pi\Delta)}$	$2/5$	0.30 ± 0.15	
$\frac{\tilde{\Gamma}(N(1535)\to\pi N)}{\tilde{\Gamma}(\Delta(1620)\to\pi N)}$	$3+\cos2\theta_1-\sqrt{8}\sin2\theta_1$	2.1 ± 0.7	$\theta_1=\begin{cases}0.3\pm0.1\\1.6\pm0.1\end{cases}$
$\frac{\tilde{\Gamma}(N(1535)\to\eta N)}{\tilde{\Gamma}(N(1650)\to\eta N)}$	$\left(\frac{\cos\theta_1+\sqrt{2}\sin\theta_1}{\sqrt{2}\cos\theta_1-\sin\theta_1}\right)^2$	13 ± 11	$\theta_1=\begin{cases}0.7^{+0.1}_{-0.3}\\1.6^{+0.3}_{-0.1}\end{cases}$
$\frac{\tilde{\Gamma}(N(1520)\to\pi\Delta)}{\tilde{\Gamma}(\Delta(1620)\to\pi N)}$	$3-2\cos2\theta_3+\sqrt{5}\sin2\theta_3$	0.44 ± 0.20	$\theta_3=\begin{cases}2.4\pm0.1\\2.9\pm0.1\end{cases}$
$\frac{\tilde{\Gamma}(N(1650)\to\pi N)}{\tilde{\Gamma}(\Lambda(1670)\to\pi\Sigma)}$	$\left(\frac{\cos\theta_1+\sqrt{2}\sin\theta_1}{\sqrt{2}\cos\theta_1-\sin\theta_1}\right)^2$	7.6 ± 4.3	$\theta_1=\begin{cases}0.6^{+0.1}_{-0.2}\\1.3^{+0.2}_{-0.1}\end{cases}$
$\frac{\tilde{\Gamma}(N(1650)\to\pi N)}{\tilde{\Gamma}(\Sigma(1750)\to\eta\Sigma)}$	1	0.12 ± 0.12	

different from the LO prediction. This indicates large $SU(3)$ breaking. Indeed, one expects $SU(3)$ breaking to be a 30% effect at the level of amplitudes or a 60% effect at the level of the reduced widths. In this case, the deviation is significantly larger than that.

For the D-wave decays, most relations we can derive involve the mixing angles. Those not involving mixing angles and which can be tested are shown in Table VIII. We have looked at relations that can be tested when only the 1-body operator is kept. Algebraically, this corresponds to a quark model with the emission of a meson coupled to the excited quark, in the $SU(6)$ limit. For instance, the relations that can predict the mixing angles give angles very different from those obtained in the fit. In fact, we will see that a 2-body LO operator is important for the D-wave fits.

TABLE VIII. D-wave decays: coefficient-independent ratios of reduced widths at LO.

Ratio	LO Ratio	Empirical
$\frac{\tilde{\Gamma}(\Lambda(1830)\to\pi\Sigma)}{\tilde{\Gamma}(\Sigma(1775)\to\pi\Lambda)}$	3	2.7 ± 1.4
$\frac{\tilde{\Gamma}(N(1520)\to\pi N)}{\tilde{\Gamma}(\Lambda(1690)\to\pi\Sigma)}$	1	0.5 ± 0.2
$\frac{\tilde{\Gamma}(N(1520)\to\pi N)}{\tilde{\Gamma}(\Sigma(1670)\to\pi\Lambda)}$	3	2.2 ± 1.4
$\frac{\tilde{\Gamma}(\Sigma(1670)\to\pi\Lambda)}{\tilde{\Gamma}(\Sigma(1670)\to\pi\Sigma)}$	1/2	0.12 ± 0.10
$\frac{\tilde{\Gamma}(N(1675)\to\pi N)}{\tilde{\Gamma}(\Lambda(1830)\to\pi\Sigma)}$	1	0.9 ± 0.4
$\frac{\tilde{\Gamma}(N(1675)\to\pi N)}{\tilde{\Gamma}(\Sigma(1775)\to\pi\Lambda)}$	3	2.3 ± 0.6
$\frac{\tilde{\Gamma}(N(1675)\to\pi N)}{\tilde{\Gamma}(\Sigma(1775)\to\pi\Sigma)}$	3/2	7 ± 4
$\frac{\tilde{\Gamma}(\Sigma(1775)\to\pi\Sigma)}{\tilde{\Gamma}(\Sigma(1775)\to\pi\Sigma^*)}$	8/7	0.06 ± 0.03
$\frac{\tilde{\Gamma}(\Lambda(1690)\to\pi\Sigma)}{\tilde{\Gamma}(\Sigma(1670)\to\pi\Lambda)}$	3	4.6 ± 3.2

Now we proceed to discuss the fits. The coefficients and mixing angles obtained in the fits are depicted in Table IX. Tables X, XI, XII, XIII, and XIV show the fitted widths as well as the predictions. The LO fit is performed using the operator bases $\{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_6\}$ for the S-wave amplitudes and $\{\mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_6, \mathcal{B}_8\}$ for the D-wave amplitudes. No $SU(3)$ breaking effects are therefore included. The matrix elements are not expanded in $1/N_c$. Both S- and D-wave partial widths are fitted simultaneously since both depend on the mixing angles θ_1 and θ_3, which are fitted. As a check, we performed a fit of the nonstrange sector of decays, with similar results to those obtained in previous work [10] (we note that the input partial widths have slightly changed from that analysis to the present one). In particular, the main problem with that fit is the discrepancy in the $N(1535)\to\eta N$. When all available channels are included, the $\chi^2_{\rm dof}$ is quite large, predominantly due to the difficulty in describing the η channels as well as several of the excited hyperon decays. For instance, if one would like to have the mixing angles $\theta_{1,3}$ similar to the ones from the nonstrange analysis, one has to remove the channels $\Lambda(1670)\to\bar{K}N$ and $\Lambda(1690)\to\bar{K}N$. In fact, in quark models these channels had been found long ago to be problematic to describe [11]. The issue of large $SU(3)$ breaking effects is clearly manifested by the shortcomings of the LO fit, as well as the LO ratios we discussed earlier. In particular, we find that the result for the S-wave partial width $\Delta(1700)\to\pi\Delta$ is rather sensitive to the removal of the mentioned Λ channels, and others are also significantly affected. After analyzing the NLO fit, one realizes that indeed the mentioned Λ channels and the η channels cannot be described consistently at LO. This is based on the stability of the coefficients of the dominant operators in

TABLE IX. Parameters from LO and NLO fits. Note ambiguities in some angles; the rest of the parameters differ within the errors, therefore parameters have been given for the first quoted angle in the ambiguous cases.

S-wave coefficient	LO	NLO	*D*-wave coefficient	LO	NLO
c_{S1}	20.6(1.0)	19.6(1.6)	c_{D1}	2.79(0.12)	2.59(0.20)
c_{S2}	0.92(1.2)	−3.1(1.2)	c_{D2}	0.03(0.08)	−0.28(0.09)
c_{S3}	0	5.2(5.8)	c_{D3}	0	0.92(1.0)
c_{S4}	0	9.2(4.0)	c_{D5}	0	1.54(0.39)
c_{S6}	−6.36(1.2)	−8.2(1.8)	c_{D6}	0.03(0.17)	−0.26(0.26)
b_{S1}	0	40.3(14)	c_{D8}	1.18(0.12)	1.28(0.15)
b_{S2}	0	−0.25(14)	c_{D9}	0	0.09(0.85)
b_{S3}	0	−3.0(14)	b_{D1}	0	1.0(1.0)
			b_{D2}	0	5.6(0.5)
Angle	LO	NLO		LO	NLO
θ_{N_1}	0.58(0.06)	0.42(0.07)	$\chi^2_{\rm dof}$	4.8	1.52
θ_{N_3}	{2.82(0.05), 2.36(0.06)}	{2.74(0.09), 2.36(0.10)}	dof	30	18
θ_{Λ_1}	θ_{N_1}	0.99(0.09)			
θ_{Λ_3}	θ_{N_3}	1.56(0.07)			
θ_{Σ_1}	θ_{N_1}	{0.28(0.16), 3.03(0.17)}			
θ_{Σ_3}	θ_{N_3}	2.19(0.47)			

the LO fit and of the mixing angles. One learns from the LO fit that the dominant operators are for the *S*-wave decays the 1-body $\mathcal{B}_1$ and important but with smaller contributions the 2-body operator $\mathcal{B}_6$, and for the *D*-wave decays the $\mathcal{B}_1$ and the 2-body $\mathcal{B}_8$ operators.

The NLO fit is complicated by the large number of coefficients and angles, a total of 23 parameters. Since the number of inputs is at most 41, there are multiple minima of the χ^2 function which have similar values. The criterion is imposed that the coefficients of dominant

TABLE X. The decay widths of *N* states in **70**-plet whose mass is currently experimentally known. Values are in MeV.

	$N(1535)$ πN	ηN	$\pi\Delta$	$N(1520)$ $\pi\Delta$		πN	ηN
PW	*S*	*S*	*D*	*S*	*D*	*D*	*D*
LO	57(17)	33(6)	0.3(0.2)	8.9(4.3)	8.1(1.0)	77(7)	0.09(0.01)
NLO	57(19)	73(44)	0.9(0.7)	9(11)	10(2)	72(11)	0.26(0.07)
Exp	68(19)	79(17)	0.8(0.8)	9.6(4.1)	13.6(2.7)	69(10)	0.26(0.05)

	$N(1650)$ πN	ηN	$K\Lambda$	$\pi\Delta$	$N(1700)$ $\pi\Delta$		πN	ηN	$K\Lambda$	$K\Sigma$
PW	*S*	*S*	*S*	*D*	*S*	*D*	*D*	*D*	*D*	*D*
LO	143(26)	2.5(1.6)	9.8(2.9)	4.8(2.6)	215(57)	2.9(2.4)	11.4(8.5)	0.52(0.25)	0.13(0.08)	∼0
NLO	133(33)	12.5(11.0)	11.5(6.4)	5.1(5.8)	297(111)	0.3(2.0)	12(13)	≤ 0.15	≤ 0.03	∼0
Exp	128(33)	10.7(5.9)	11.5(6.7)	6.6(5)			10(7)		1.5(1.5)	

	$N(1675)$ πN	ηN	$K\Lambda$	$\pi\Delta$
PW	*D*	*D*	*D*	*D*
LO	52(8)	2.6(0.4)	0.02(0.01)	72(9)
NLO	51(12)	6.3(2.5)	≤ 0.1	75(24)
Exp	59(10)		0.75(0.75)	

TABLE XI. The decay widths of Λ states in **70**-plet whose mass is currently experimentally known. Values are in MeV.

	$\Lambda(1670)$				$\Lambda(1690)$				
	$\bar{K}N$	$\eta\Lambda$	$\pi\Sigma$	$\pi\Sigma^*$	$\pi\Sigma^*$		$\bar{K}N$	$\eta\Lambda$	$\pi\Sigma$
PW	*S*	*S*	*S*	*D*	*S*	*D*	*D*	*D*	*D*
LO	113(24)	0.11(0.12)	1.8(2.0)	0.16(0.09)	7.3(3.5)	9(1)	60(6)	~0	9.0(0.9)
NLO	9(15)	6.1(4.3)	15(11)	0.04(0.10)	114(49)	2.1(1.5)	16(5)	~0	5.3(2.9)
Exp	9.4(3.6)	6.6(3.6)	15(7.5)				15(4)		18(6.7)

	$\Lambda(1800)$				$\Lambda(1830)$				
	$\bar{K}N$	$\eta\Lambda$	$\pi\Sigma$	$\pi\Sigma^*$	$\bar{K}N$	$\eta\Lambda$	$\pi\Sigma$	$K\Xi$	$\pi\Sigma^*$
PW	*S*	*S*	*S*	*D*	*D*	*D*	*D*	*D*	*D*
LO	43(13)	30(4)	150(20)	3.0(1.6)	3.0(1.6)	3.5(0.3)	69(6)	~0	54(7)
NLO	100(73)	94(47)	109(25)	5.9(5.2)	12(4)	9.6(2.5)	38(11)	~0	57(18)
Exp	98(40)				5.5(3.4)		46.7(22)		

	$\Lambda(1405)$	$\Lambda(1520)$	
	$\pi\Sigma$	$\bar{K}N$	$\pi\Sigma$
PW	*S*	*D*	*D*
LO	50(19)	2.7(0.4)	8.2(1.3)
NLO	50(9)	6.7(1.1)	6.9(1.8)
Exp	50(5)	7(0.5)	6.5(0.5)

operators at LO and the mixing angles which are not affected by $SU(3)$ breaking effects, namely, θ_{N_1} and θ_{N_3} deviate from the LO results as expected by NLO corrections. A fit satisfying that criterion is obtained. This fit shows that the coefficients of the $SU(3)$ preserving operators are of natural size, and most of them are actually smaller than natural size. On the other hand, the $SU(3)$ breaking *S*-wave operator $\mathcal{B}_1^{\rm SB}$ and the *D*-wave operator $\mathcal{B}_2^{\rm SB}$ have coefficients roughly a factor two larger than natural size. $\mathcal{B}_1^{\rm SB}$ only contributes to η channels, and eliminating it increases the $\chi^2_{\rm dof}$ from 1.5 to 1.9, and at the same time, the coefficients of the *S*-wave operators

TABLE XII. The decay widths of Σ states in **70**-plet whose mass is currently experimentally known. Values are in MeV.

	$\Sigma(1670)$				
	$\pi\Sigma^*$		$\bar{K}N$	$\pi\Lambda$	$\pi\Sigma$
PW	*S*	*D*	*D*	*D*	*D*
LO	1.5(0.7)	1.5(0.2)	2.1(0.5)	4.8(0.5)	46(5)
NLO	4(11)	1.5(0.9)	2.5(1.4)	7.0(2.9)	28(11)
Exp			6(2.7)	6(3.6)	27(12.7)

	$\Sigma(1750)$					
	$\bar{K}N$	$\pi\Lambda$	$\pi\Sigma$	$\eta\Sigma$	$\bar{K}\Delta$	$\pi\Sigma^*$
PW	*S*	*S*	*S*	*S*	*D*	*D*
LO	45(8)	51(7)	6.2(5.3)	14(2)	0.07(0.04)	0.5(0.3)
NLO	30(34)	38(12)	4.2(7.6)	53(28)	0.4(0.2)	0.4(0.5)
Exp	27.5(21)		4.4(4.4)	38.5(28)		

	$\Sigma(1775)$					
	$\bar{K}N$	$\pi\Lambda$	$\pi\Sigma$	$\eta\Sigma$	$\bar{K}\Delta$	$\pi\Sigma^*$
PW	*D*	*D*	*D*	*D*	*D*	*D*
LO	39(3)	27(3)	3.0(1.2)	0.08(0.01)	1.6(0.2)	7(1)
NLO	55(12)	14(4)	0.6(0.8)	0.22(0.06)	3.9(0.8)	7.4(2.3)
Exp	48(7)	20.4(4.4)	4.2(2)			12(2.8)

TABLE XIII. The decay widths of Ξ states in **70**-plet whose mass is currently experimentally known. Values are in MeV.

	$\Xi(1820)$				
	$\pi\Xi^*$		$\bar{K}\Lambda$	$\bar{K}\Sigma$	$\pi\Xi$
PW	*S*	*D*	*D*	*D*	*D*
LO	2.3(0.6)	2.6(0.3)	10(1)	14(1)	4.2(0.9)
NLO	2.4(2.2)	3.2(0.6)	18(3)	29(4)	0.3(0.6)
Exp					

TABLE XIV. The decay widths of Δ states in **70**-plet whose mass is currently experimentally known. Values are in MeV.

	$\Delta(1620)$		$\Delta(1700)$			
	πN	$\pi\Delta$	$\pi\Delta$		πN	$K\Sigma$
PW	*S*	*D*	*S*	*D*	*D*	*D*
LO	34(5)	62(7)	215(39)	20(4)	22(4)	~ 0
NLO	34(12)	64(14)	157(52)	18(8)	18(11)	≤ 0.04
Exp	35.7(7.4)	64.3(21.7)	112(53)	12(10)	45(21)	

$\mathcal{B}_{2,3}^{\rm SB}$ become unnaturally large. This exercise indicates that there is significant correlation among the coefficients of the S-wave $SU(3)$ breaking operators. Evidently, the $SU(3)$ breaking operators are crucial for describing the η channels. One also finds that $SU(3)$ breaking is significant in the mixing angles; in particular, it is unnaturally large for the Λ baryons.

The analysis leads to the following observations:

(1) The 1-body operators $\mathcal{B}_1$ are dominant in S and D waves. This supports the quark model picture in which the meson is predominantly emitted from the excited quark.

(2) The 2-body operators are less important but necessary to obtain good fits. These operators will, in particular, encode the longer range dynamics of the decays. At LO, they have smaller or much smaller coefficients than the 1-body operator $\mathcal{B}_1$, and at NLO their coefficients are in general smaller than the natural size. Note that this conclusion is under the criterion of selecting the fits which have LO coefficients stable as one moves to the NLO fit. Thus, one can conclude that NLO fits consistent with a $1/N_c$ power counting are possible. We also note that such fits are indeed the ones with lowest χ^2 we have found.

(3) As Table IX shows, several operators carry coefficients consistent with zero within error. One can eliminate those operators and perform a NLO fit where the coefficients of the relevant operators do not change significantly, and $\chi^2_{\rm dof} \sim 1.2$.

(4) A lower $\chi^2_{\rm dof} \sim 1$ can be obtained by reducing by a factor ~ 0.6 the exponent of the centrifugal barrier. This however does not give any significant change to the outcome of the analysis.

(5) For the nucleon's mixing angles, previous analysis of the nonstrange sector gave $\theta_{N_1} = 0.39 \pm 0.11$ and $\theta_{N_3} = 2.82$ or 2.38 ± 0.11, to be compared, respectively, with 0.42 ± 0.07 and 2.74 or 2.36 ± 0.09 obtained in the present analysis. We note that the ambiguity in θ_{N_3} found in the previous analysis persists in the present analysis at both LO and NLO level. A similar ambiguity is found at NLO for the mixing angle $\theta_{\Sigma 1}$. For the nucleons, the ambiguity can be sorted out by analyzing the photo-couplings [16,22].

(6) The $SU(3)$ breaking effects are of unnaturally large magnitude by roughly a factor two. In particular, they manifest themselves in the η channels, where the LO fit gives very poor description. This problem had been noticed when those channels were included in the analysis of the nonstrange decays [10]. The very important S-wave channel $N(1535) \to \eta N$ is too small at LO by a factor two, while the small S-wave $N(1650) \to \eta N$ is also under-predicted at LO by a factor four. On the other hand, the πN channels are well described at LO for both resonances.

(7) The S-wave decay $\Lambda(1405) \to \pi\Sigma$ is well described in all fits. It is sensitive to the presence of the 2-body operator $\mathcal{B}_6$. On the other hand, the D-wave decay $\Lambda(1520) \to \pi\Sigma$ is well described while the $\Lambda(1520) \to \bar{K}N$ is poorly described at LO. A clear example of $SU(3)$ breaking effects.

(8) The decays $\Lambda(1670) \to \bar{K}N$ (S-wave) and $\Lambda(1690) \to \bar{K}N$ (D-wave) are poorly described at LO if one requires that the mixing angles $\theta_{N_{1,3}}$ are similar to the values obtained in fits of the non-strange decays only or the NLO fits. At NLO, these

decays are improved because of the $SU(3)$ breaking in the mixing angles, in particular.

(9) The S-wave decay $\Delta(1700) \to \pi\Delta$ is particularly sensitive to the 2-body operator $\mathcal{B}_2$, and found to be very sensitive to the inclusion or not of the latter Λ channels in the LO fit.

(10) The NLO results show that the mixing angles are strongly affected by $SU(3)$ breaking effects. To obtain a more accurate picture, it will be necessary to carry out an analysis of masses and photocouplings along with the decays [23].

(11) We provide predictions for the unknown channels of known **70**-plet states, and here we discuss some of them. Little can be concluded from predictions of small partial widths, except that the corresponding channels will be most likely experimentally inaccessible. On the other hand, several large partial widths are predicted, which require some discussion.

(i) The $N(1700)$ is given in the PDG with three stars, but its existence is challenged by several recent analyses [24–26], while other analyses confirm it [27]. Since the photocouplings and the electrocouplings of the $p(1700)$ and $n(1700)$ are small [12,22,28], the access to $N(1700)$ by these means is limited. In our fits, we included the PDG estimate for the πN channel, but disregarding this input gives no significant change to the fits. The S-wave partial width of $N(1700) \to \pi\Delta$ is predicted to be the largest one while the D-wave channel has a small width. This contradicts partial wave analyses [27], where it is claimed that the D-wave is significantly larger than the S-wave. According to the PDG [12], the total width of the $N(1700)$ is rather uncertain, and an estimate of the $N(1700) \to \pi\Delta$ partial width from the πNN width is viable provided one assumes it to proceed mostly through the $\pi\Delta$ S-wave channel if one is to obtain a reasonable fit at NLO. This however affects the fit significantly. We have chosen not to include as input the estimate of the $N(1700) \to \pi\Delta$ S-wave partial width, and this remains therefore an open problem requiring further empirical progress.

(ii) A similar problem to that with the $N(1700)$ is found for the $\Lambda(1690) \to \pi\Sigma^*$, where the empirical estimate for this partial width, assumed to be S-wave and estimated from the dominance of this channel in the decays $\pi\pi\Lambda$ and $\pi\pi\Sigma$, is around 20 MeV, while the fits predict it to be larger than 100 MeV. Giving that estimate as input has similar effects as the input of $N(1700) \to \pi\Delta$. We do not include this input either.

(iii) Briefly, the inclusion of these S-wave inputs leads to the following: the inclusion of either estimate leads to the same values of the coefficients within errors, the main changes with respect to not including either of them being the values of the S-wave coefficients, which in an indirect way alter significantly the angle θ_{N_1}.

(iv) The prediction for the D-wave partial width $N(1675) \to \pi\Delta$ of 75 ± 22 MeV is in good agreement with the estimate based on the total width and the other significant partial width into πN.

(v) The S-wave partial widths of the $\Lambda(1800)$ to $\eta\Lambda$ and to $\pi\Sigma$ are predicted to be approximately equal to the empirical one for the channel $\bar{K}N$, and in agreement with the total width.

(vi) The D-wave partial width $\Lambda(1830) \to \pi\Sigma^*$ is predicted to be the largest one, and it agrees with empirical lower bound of 15% for the branching ratio. The total predicted width is 116 ± 20 MeV, to be compared with the PDG [12] estimate of 60 to 110 MeV.

(vii) For the D-wave $\Xi(1820)$, the PDG [12] gives a total width $\sim 25 \pm 15$ MeV [12]. The predictions given in Table XIII are dependent on the choice of the mixing angle $\theta_{\Xi 3}$ which cannot be determined by the current analysis. The predictions are given for the choice $\theta_{\Xi 3} = \theta_{N_3} = 2.74$. The dominant channels are the $\bar{K}\Sigma$ and $\bar{K}\Lambda$. The total width predicted is about a factor two larger than the one from the PDG. If one requires that the total width be reproduced, the mixing angle becomes $\theta_{\Xi 3} \sim 1.9$.

(12) Several channels where there can be $SU(3)$ symmetry-breaking effects are predicted to have significant partial widths, and could therefore become empirically accessible. Such channels are the S-wave $\Lambda(1690) \to \pi\Sigma^*$, $\Lambda(1800) \to \eta\Lambda$, $\Lambda(1800) \to \pi\Sigma$, $\Sigma(1670) \to \pi\Sigma^*$, and $\Sigma(1750) \to \pi\Lambda$, and the D-wave $N(1675) \to \eta N$, $\Lambda(1830) \to \eta\Lambda$, $\Lambda(1830) \to \pi\Sigma^*$, $\Sigma(1670) \to \pi\Sigma^*$ and $\Sigma(1775) \to \bar{K}\Delta$. All these channels involving the π meson are not affected by the 1-body $SU(3)$-breaking effects, and would give additional information on breaking at the 2-body level.

V. SUMMARY AND CONCLUSIONS

This work has extended previous analyses based on the $1/N_c$ expansion of the low-lying negative-parity baryon partial-decay widths. The extension includes all known decay channels, and was carried out to first subleading order in the $1/N_c$ expansion and first order in $SU(3)$ symmetry breaking. The approximations involved were the following ones: for $SU(3)$ preserving amplitudes, only up to 2-body operators and for $SU(3)$ breaking amplitudes, only 1-body operators were included. Mixings between states in different $SU(3)$ multiplets as well as $SU(6) \times O(3)$ configuration mixings were neglected.

These approximations are necessary due to the limitations in the available empirical inputs for the partial decay widths. One important focus of the analysis was on $SU(3)$ symmetry breaking, whose significance is noticed first by violations in LO coefficient-independent relations. In fact, the symmetry-breaking effects are of unnaturally large magnitude as the NLO analysis shows. On the other hand, the $1/N_c$ expansion seems to work rather well, as the coefficients of NLO order symmetry-preserving operators are in general of natural or smaller than natural magnitude. This agrees with the conclusions drawn in previous work where the nonstrange sector had been analyzed. The existence of other solutions to the NLO fit where the χ^2 is not significantly larger than the one presented in this work is an issue. Such solutions do have, however, unnaturally large NLO coefficients and lead to an inconsistent $1/N_c$ expansion. Obviously, additional and more accurate partial-decay widths should be available for settling this issue. An interesting open problem left by the analysis are the two S-wave decays $N(1700) \to \pi\Delta$ and $\Lambda(1690) \to \pi\Sigma^*$, which are predicted to be rather large, and are well correlated in the analysis. More accurate information on these widths has the potential to modify the results for the S-wave coefficients and the mixing angle θ_{N_1} in turn.

From the present work, one can draw some conclusions that seem quite robust, namely, that the 1-body operator plays a key role in both S- and D-wave decays, as one would have expected from the quark model picture, and that 2-body operators are crucial for obtaining an overall consistent description, although they have smaller strength than the 1-body ones. Some uncertainties concerning mixing angles, in particular, those affected by large $SU(3)$ breaking corrections seem to remain. Further refinement of the present analysis, with the aim at improving, in particular, the determination of mixing angles, can be carried out by simultaneously analyzing masses, decays, and photocouplings of the **70**-plet baryons [23]. In addition, present progress in the analysis of recent and new data will very likely lead to an improvement on the inputs, allowing for a refinement of the present analysis and conclusions.

ACKNOWLEDGMENTS

We thank Michael Döring, Hiroyuki Kamano, Victor Mokeev and Igor Strakovsky for informative discussions. J. L. G. and N. N. S. thank the Grupo de Particulas y Campos, Centro Atómico Bariloche, and, in particular, Professor Roberto Trinchero, for the hospitality extended to them during completion of part of this work. This work was supported by DOE Contract No. DE-AC05-06OR23177 under which JSA operates the Thomas Jefferson National Accelerator Facility, by the National Science Foundation (USA) through grant PHY-0555559 and PHY-0855789 (J. L. G. and C J.), by CONICET (Argentina) Grant No. PIP 00682 and by ANPCyT (Argentina) Grant. No. PICT 07-03-00818 (N. N. S.).

APPENDIX: $SU(3)$ ISOSCALAR FACTORS

This appendix gives the isoscalar factors needed for the calculations carried out in this work. They correspond to the emission of mesons belonging to an **8** of $SU(3)$, and are given for the irreducible representations of $SU(3)$ of interest for generic N_c. We denote the isoscalar factors by:

$$\left(\begin{matrix} (p,q) & (1,1) \\ YI & yi \end{matrix}\middle\| \begin{matrix} (p',q') \\ Y'I' \end{matrix}\right)_{\gamma}, \tag{A1}$$

where the $SU(3)$ representations are identified in terms of the two labels defining the Young tableu, namely, (p, q), where $p + 2q = N_c$.

For baryons, the correspondences between multiplets for generic odd N_c and $N_c = 3$ are as follows: $(p = 0, q = \frac{N_c-3}{2}) \to \mathbf{1}$, $(p = 1, q = \frac{N_c-1}{2}) \to \mathbf{8}$, and $(p = 3, q = \frac{N_c-3}{2}) \to \mathbf{10}$. Table XV displays these correspondences more explicitly.

Tables XVI, XVII, XVIII, XIX, and XX give the isoscalar factors of interest. The first row of the tables depicts the excited baryon state, and the second row the final baryon and meson. The isoscalar factor arguments are in the order $(B^*, P\|B)$, where B^*, P and B are the quantum numbers of the excited baryon, the meson and the final baryon, respectively.

TABLE XV. Representation correspondences for arbitrary odd N_c. Displayed are the $SU(3)$ generic N_c multiplets corresponding to the ones at $N_c = 3$, namely, **1**, **8**, and **10**.

1 Baryon $(p,q)=(0,\frac{N_c-3}{2})$		**8** Baryons $(p,q)=(1,\frac{N_c-1}{2})$		**10** Baryons $(p,q)=(3,\frac{N_c-3}{2})$		Mesons $(p,q)=(1,1)$	
State	(Y,I)	State	(Y,I)	State	(Y,I)	State	(Y,I)
Λ	$(\frac{N_c-3}{3},0)$	N	$(\frac{N_c}{3},\frac{1}{2})$	Δ	$(\frac{N_c}{3},\frac{3}{2})$	π	(0,1)
		Σ	$(\frac{N_c-3}{3},1)$	Σ^*	$(\frac{N_c-3}{3},1)$	η	(0,0)
		Λ	$(\frac{N_c-3}{3},0)$	Ξ^*	$(\frac{N_c-6}{3},\frac{1}{2})$	K	$(1,\frac{1}{2})$
		Ξ	$(\frac{N_c-6}{3},\frac{1}{2})$	Ω	$(\frac{N_c-9}{3},0)$	$\bar{K}$	$(-1,\frac{1}{2})$

TABLE XVI. Isoscalar factors for $\mathbf{8} \to \mathbf{8}$ decays. The listed values should be multiplied by $f_1 = \frac{1}{(N_c+3)}$ and $f_2 = \frac{1}{(N_c+3)}\sqrt{\frac{(N_c-1)}{(N_c+7)}}$ to obtain the actual isoscalar factors for $\gamma = 1$ and $\gamma = 2$, respectively.

	N				Λ			
	ηN	πN	$K\Sigma$	$K\Lambda$	$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	$K\Xi$
$\gamma = 1$	N_c	3	$\sqrt{3(N_c-1)}$	$\sqrt{3(N_c+3)}$	$-\sqrt{\frac{3(N_c+3)}{2}}$	0	$N_c - 3$	$3\sqrt{\frac{N_c-1}{2}}$
$\gamma = 2$	3	$-(N_c+6)$	$\frac{N_c+15}{\sqrt{3(N_c-1)}}$	$-\sqrt{3(N_c+3)}$	$\sqrt{\frac{3(N_c+3)}{2}}$	$-\sqrt{\frac{(N_c+3)^3}{3(N_c-1)}}$	6	$\frac{9-N_c}{\sqrt{2(N_c-1)}}$

	Σ				
	$\bar{K}N$	$\eta\Sigma$	$\pi\Sigma$	$\pi\Lambda$	$K\Xi$
$\gamma = 1$	$3\sqrt{\frac{N_c-1}{2}}$	$N_c - 3$	$2\sqrt{6}$	0	$\sqrt{\frac{3(N_c+3)}{2}}$
$\gamma = 2$	$\frac{N_c+15}{\sqrt{2(N_c-1)}}$	$\frac{2(N_c-9)}{N_c-1}$	$-\sqrt{\frac{2}{3}}\frac{(N_c-3)(N_c+7)}{N_c-1}$	$\sqrt{\frac{(N_c+3)^3}{N_c-1}}$	$-\frac{5N_c+3}{N_c-1}\sqrt{\frac{N_c+3}{6}}$

	Ξ			
	$\bar{K}\Sigma$	$\bar{K}\Lambda$	$\eta\Xi$	$\pi\Xi$
$\gamma = 1$	$-\sqrt{N_c+3}$	$3\sqrt{N_c-1}$	$N_c - 6$	3
$\gamma = 2$	$\frac{5N_c+3}{3(N_c-1)}\sqrt{N_c+3}$	$\frac{9-N_c}{\sqrt{N_c-1}}$	$\frac{7N_c-15}{N_c-1}$	$\frac{N_c^2+3N_c+36}{3(N_c-1)}$

TABLE XVII. Isoscalar factors for $\mathbf{10} \to \mathbf{10}$ decays. The listed values should be multiplied by $f_1 = \frac{1}{\sqrt{45+N_c(N_c+6)}}$ and $f_2 = \sqrt{\frac{5(N_c-3)(N_c+5)}{(N_c+1)(N_c+9)(45+N_c(N_c+6))}}$ to obtain the actual isoscalar factors for $\gamma = 1$ and $\gamma = 2$, respectively. Note that f_2 vanishes at $N_c = 3$, as in that case there is a unique recoupling $\mathbf{10} \otimes \mathbf{8} \to \mathbf{10}$.

	Δ			Σ^*			
	$\eta\Delta$	$\pi\Delta$	$K\Sigma^*$	$\bar{K}\Delta$	$\eta\Sigma^*$	$\pi\Sigma^*$	$K\Xi^*$
$\gamma = 1$	N_c	$3\sqrt{5}$	$\sqrt{3(N_c+5)}$	$-\frac{3\sqrt{N_c+5}}{2}$	$N_c - 3$	$2\sqrt{6}$	$\sqrt{6(N_c+3)}$
$\gamma = 2$	3	$-\frac{N_c+6}{\sqrt{5}}$	$\frac{3-N_c}{\sqrt{3(N_c+5)}}$	$\frac{N_c-3}{2\sqrt{N_c+5}}$	$\frac{4(N_c+3)}{N_c+5}$	$-\frac{N_c^2+10N_c+33}{\sqrt{6}(N_c+5)}$	$\frac{3-N_c}{N_c+5}\sqrt{\frac{2(N_c+3)}{3}}$

	Ξ^*				Ω	
	$\bar{K}\Sigma^*$	$\eta\Xi^*$	$\pi\Xi^*$	$K\Omega$	$\bar{K}\Xi^*$	$\eta\Omega$
$\gamma = 1$	$-2\sqrt{N_c+3}$	$N_c - 6$	3	$3\sqrt{N_c+1}$	$-3\sqrt{\frac{N_c+1}{2}}$	$N_c - 9$
$\gamma = 2$	$\frac{2(N_c-3)\sqrt{N_c+3}}{3(N_c+5)}$	$\frac{5N_c+9}{N_c+5}$	$-\frac{N_c^2+9N_c+36}{3(N_c+5)}$	$\frac{(3-N_c)\sqrt{N_c+1}}{N_c+5}$	$\frac{N_c-3}{N_c+5}\sqrt{\frac{N_c+1}{2}}$	$\frac{6(N_c+1)}{N_c+5}$

TABLE XVIII. Isoscalar factors for $\mathbf{8} \to \mathbf{10}$ decays. The listed values should be multiplied by $f = \frac{\sqrt{2}}{\sqrt{(N_c+1)(N_c+5)}}$ to obtain the actual isoscalar factors.

N		Λ	
$\pi\Delta$	$K\Sigma^*$	$\pi\Sigma^*$	$K\Xi^*$
$-\sqrt{\frac{(N_c-1)(N_c+5)}{2}}$	$-2\sqrt{\frac{N_c-1}{3}}$	$-\sqrt{\frac{(N_c+3)(N_c-1)}{3}}$	$-\sqrt{2(N_c-1)}$

Σ				Ξ			
$\bar{K}\Delta$	$\eta\Sigma^*$	$\pi\Sigma^*$	$K\Xi^*$	$\eta\Xi^*$	$\pi\Xi^*$	$\bar{K}\Sigma^*$	$K\Omega$
$\sqrt{N_c+5}$	2	$\frac{N_c+1}{\sqrt{6}}$	$\sqrt{\frac{2(N_c+3)}{3}}$	2	$\frac{2N_c}{3}$	$\frac{2\sqrt{N_c+3}}{3}$	$2\sqrt{N_c+1}$

TABLE XIX. Isoscalar factors for **10** → **8** decays. The listed values should be multiplied by $f = ((N_c + 7)(N_c - 1))^{-1/2}$ to obtain the actual isoscalar factors.

Δ		Ξ^*			
πN	$K\Sigma$	$\bar{K}\Sigma$	$\bar{K}\Lambda$	$\eta\Xi$	$\pi\Xi$
$-\sqrt{(N_c - 1)(N_c + 5)}$	$2\sqrt{\frac{N_c+5}{3}}$	$\frac{2\sqrt{N_c+3}}{3}$	$2\sqrt{N_c - 1}$	-2	$-\frac{2N_c}{3}$

		Σ^*			Ω
$\eta\Sigma$	$\bar{K}N$	$\pi\Sigma$	$\pi\Lambda$	$K\Xi$	$\bar{K}\Xi$
-2	$\sqrt{2(N_c - 1)}$	$-\frac{N_c+1}{\sqrt{6}}$	$-\sqrt{(N_c + 3)(N_c - 1)}$	$\sqrt{\frac{2(N_c+3)}{3}}$	$\sqrt{2(N_c + 1)}$

TABLE XX. Isoscalar factors for **1** → **8** decays.

	Λ		
$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$	$K\Xi$
1	$\sqrt{\frac{2}{N_c-1}}$	$\sqrt{\frac{6}{N_c+3}}$	$\sqrt{\frac{12}{(N_c+3)(N_c-1)}}$

[1] S. Capstick and W. Roberts, Prog. Part. Nucl. Phys. **45**, S241 (2000), and references therein.
[2] R. Dashen and A. V. Manohar, Phys. Lett. B **315**, 425 (1993); **315**, 438 (1993).
[3] Ch. D. Carone, H. Georgi, L. Kaplan, and D. Morin, Phys. Rev. D **50**, 5793 (1994).
[4] J. L. Goity, Phys. Lett. B **414**, 140 (1997).
[5] D. Pirjol and T.-M. Yan, Phys. Rev. D **57**, 5434 (1998); **57**, 1449 (1998).
[6] J. L. Gervais and B. Sakita, Phys. Rev. Lett. **52**, 87 (1984); Phys. Rev. D **30**, 1795 (1984).
[7] C. E. Carlson, Ch. D. Carone, J. L. Goity, and R. F. Lebed, Phys. Lett. B **438**, 327 (1998); Phys. Rev. D **59**, 114008 (1999).
[8] C. L. Schat, J. L. Goity, and N. N. Scoccola, Phys. Rev. Lett. **88**, 102002 (2002); J. L. Goity, C. L. Schat, and N. N. Scoccola, Phys. Rev. D **66**, 114014 (2002).
[9] T. D. Cohen and R. F. Lebed, Phys. Rev. D **67**, 096008 (2003); **68**, 056003 (2003); T. D. Cohen, D. C. Dakin, A. Nellore, and R. F. Lebed, Phys. Rev. D **70**, 056004 (2004).
[10] J. L. Goity, C. L. Schat, and N. N. Scoccola, Phys. Rev. D **71**, 034016 (2005).
[11] R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D **3**, 2706 (1971).
[12] C. Amsler *et al.* (Particle Data Group), J. Phys. G **37**, 075021 (2010).
[13] C. Jayalath, Ph.D. thesis, Hampton University, 2010 (unpublished).
[14] T. D. Cohen and R. F. Lebed, Phys. Rev. D **70**, 096015 (2004).
[15] J. L. Goity, Phys. At. Nucl. **68**, 624 (2005).
[16] E. Gonzalez de Urreta and N. N. Scoccola (unpublished).
[17] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, 1974).
[18] K. T. Hecht, Nucl. Phys. **62**, 1 (1965).
[19] J. L. Goity, C. Jayalath, and N. N. Scoccola, Phys. Rev. D **80**, 074027 (2009).
[20] R. F. Dashen, E. E. Jenkins, and A. V. Manohar, Phys. Rev. D **51**, 3697 (1995).
[21] D. Pirjol and C. Schat, Phys. Rev. D **67**, 096009 (2003); T. D. Cohen and R. F. Lebed, Phys. Rev. Lett. **91**, 012001 (2003); Phys. Rev. D **72**, 056001 (2005).
[22] N. N. Scoccola, J. L. Goity, and N. Matagne, Phys. Lett. B **663**, 222 (2008).
[23] J. L. Goity, E. Gonzalez de Urreta, C. Jayalath, and N. N. Scoccola (unpublished).
[24] R. A. Arndt, W. J. Briscoe, I. I. Strakovsky, and R. L. Workman, Phys. Rev. C **74**, 045205 (2006).
[25] M. Döring *et al.*, Nucl. Phys. A **829**, 170 (2009).
[26] N. Suzuki *et al.*, Phys. Rev. Lett. **104**, 042302 (2010).
[27] D. M. Manley and E. M. Saleski, Phys. Rev. D **45**, 4002 (1992); U. Thoma *et al.*, Phys. Lett. B **659**, 087 (2008).
[28] V. Mokeev (private communication).