Dynamical Casimir effect from fermions in an oscillating bag in $1+1$ dimensions

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We evaluate dissipative effects for a system consisting of a massive Dirac field confined between two walls, one of them oscillating, in $1+1$ dimensions. In the model that we consider, a dimensionless parameter characterizing each wall is tuned so that bag-boundary conditions are attained for a particular value. We present explicit results for the probability of creating a fermion pair, and relate the total probability to the imaginary part of the effective action.

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I. INTRODUCTION

Quantum field theory predicts many interesting effects in the presence of nontrivial boundary conditions. The best known example of this phenomenon is the Casimir effect [1,2] which, in the static case, manifests itself in forces due to a nontrivial dependence of the vacuum energy on the geometry of the boundary. Although, in principle, this effect is relevant for any kind of fluctuating field, the most frequently studied case corresponds to an Abelian gauge field. This is hardly surprising since, for the electromagnetic (EM) field, boundary conditions can be controlled in a rather precise and straightforward way. Nevertheless, fields other than the electromagnetic field have also been studied, like in the fermionic fields describing quarks, since their vacuum energies play an important role in the bag model of QCD [3], where part of the mass of a baryon is due to the Casimir energy of the fields which are affected by the (bag) boundary conditions. A more straightforward realization arises in the context of condensed matter physics, where Dirac fields play a preeminent role, specially in $1+1$ and $2+1$ dimensions [4]. Boundary conditions may, on the other hand, be also relevant due to the existence of impurities, domain walls, etc.

We are interested here in the dynamical Casimir effect, whereby a time dependence of the boundary may induce the creation of particles of the quantum field out of the vacuum. In [5] this has been studied, for a massless Dirac field in $1+1$ dimensions satisfying bag conditions on two moving boundaries. For massive Dirac fields, higher dimensions, and more general boundary condition, the imaginary part the effective action for a single moving boundary has been evaluated in [6]. In this paper, we consider a massive Dirac field coupled to two walls, one them moving, both imposing boundary conditions which, for a particular value of a parameter describing the coupling of the fermion to the wall, correspond to the vanishing of the component of the current which is normal to the boundary: bag conditions.

The structure of this paper is as follows: in Sec. II we introduce the concepts and define the model that we study in the rest of this work. Then, in Sec. III, we evaluate the probability of pair creation from the vacuum, assuming a small oscillation amplitude. In Sec. IV, we compare, and show the consistency of the previous result with the one that one finds from the evaluation of the imaginary part of the effective action. Finally, in Sec. V we present our conclusions.

II. THE MODEL

In the model that we consider, the (real-time) action $\mathcal{S}$, describing the fermionic field $(\psi, \bar{\psi})$ subjected to boundary conditions, is:

$$\mathcal{S}(\bar{\psi}, \psi; V) = \int d^2x \bar{\psi}(x) \mathcal{D} \psi(x) \tag{1}$$

with

$$\mathcal{D} \equiv i\partial\!\!\!/ - m - V(x), \tag{2}$$

where m is the mass of the fermion field, and $V(x)$ will be used in order to introduce the boundary conditions (see below). In our conventions, both $\hbar$ and the speed of light are equal to 1, the spacetime coordinates are denoted by x^μ,

$\mu = 0, 1$, $x^0 = t$, and the metric tensor is $g_{\mu\nu} \equiv \mathrm{diag}(1, -1)$. Dirac's γ-matrices are chosen as follows:

$$\gamma^0 \equiv \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma^1 \equiv i\sigma_3 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \tag{3}$$

and

$$\gamma^5 \equiv \gamma_5 \equiv \gamma^0\gamma^1 = \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{4}$$

with σ_i ($i = 1, 2, 3$) representing the usual Pauli's matrices.

Following the approach of [7,8], we can impose boundary conditions by a special choice of the "potential" V. Namely, V has to be proportional to a δ-function concentrated on the worldline swept by the point where the condition is imposed. For example, for a timelike curve $\mathcal{C}$, corresponding to the solution to the equation $F(x) = 0$, the potential V shall have the structure:

$$V(x) = g|N|\delta[F(x)], \tag{5}$$

where $|N| \equiv \sqrt{-N_\mu N^\mu}$, $N_\mu \equiv \pm[\partial_\mu F(x)]_{F=0}$, is defined on $\mathcal{C}$, and everywhere normal to it (therefore spacelike). There is a global sign ambiguity in N_μ, which corresponds to the two possible orientations of the normal to a curve. We will fix it by setting it to point toward the interior of the region limited by two curves.

When $\mathcal{C}$ is the union of disconnected curves, V decomposes into a sum of terms, one for each curve. The factor g, on the other hand, is a constant.

We shall assume that there are two walls, i.e., two curves L and R (which eventually become boundaries in the bag limit). L is static and given by $x^1 = 0$, while the other, R, has the trajectory $x^1 = a + \eta(x^0)$ ($\eta(x^0) > -a$).

Applying the general structure of V discussed above to the case at hand, it will consist of two terms, namely,

$$V(x) = g_L\delta(x^1) + g_R\gamma^{-1}(\dot{\eta}(x^0))\delta(x^1 - q(x^0)), \tag{6}$$

where $\gamma(u) \equiv 1/\sqrt{1-u^2}$ is the Lorentz factor.

Here, g_L and g_R are constants which, in order to enforce bag boundary conditions, have to equal 2 (see [7]). Different values produce imperfect boundary conditions, in the sense that some current may escape the cavity. We recall that the general form of the bag boundary conditions

$$(e^{i\theta\gamma_5} + in^\mu\gamma_\mu)\psi|_{\mathcal{C}} = 0, \tag{7}$$

where θ is a real parameter which can be chosen arbitrarily, and $n^\mu \equiv \frac{N^\mu}{|N|}$. Note that, as usual, the boundary condition is assumed to be imposed on the limit of the function on which it acts, when one approaches the curve from the interior of the region delimited.

Since we are going to deal with the region limited between L and R, on L, $n^\mu = \delta^\mu_1$, while on R, $n^\mu(x^0) = -\gamma(\dot{q})(\delta^\mu_0\dot{q} + \delta^\mu_1)$.

To see the kind of boundary condition due to a singular term like the one we are considering, let us observe what happens for a singularity of strength g at $x^1 = 0$. We see from the Dirac equation, after integrating along a spatial path from $x^1 = -\epsilon$ and $x^1 = \epsilon$, that the presence of the singular term introduces a discontinuity in ψ. Therefore, following [9], we replace the integral of the δ-function times ψ by the average of the two lateral limits:

$$i\gamma^1(\psi(\epsilon) - \psi(-\epsilon)) - \frac{g}{2}(\psi(\epsilon) + \psi(-\epsilon)) = 0, \tag{8}$$

where we have omitted writing the temporal arguments, which are the same in all the terms.

Setting $g = 2$, and introducing the orthogonal projectors: $\mathcal{P}^\pm \equiv \frac{1 \pm i\gamma^1}{2}$, this is equivalent to:

$$\mathcal{P}^+\psi(-\epsilon) = -\mathcal{P}^-\psi(\epsilon), \tag{9}$$

and, therefore,

$$\mathcal{P}^+\psi(-\epsilon) = 0, \qquad \mathcal{P}^-\psi(\epsilon) = 0. \tag{10}$$

The second equation is the bag boundary condition one has on the field on L (assuming $\theta = 0$), assuming the interior of the cavity is between L and R.

This formal argument will be seen to hold true in more concrete terms, in Sec. IV, when evaluating different terms in the perturbative expansion of the effective action $\Gamma(q)$, that results by functional integrating out the Dirac field in the vacuum to vacuum transition amplitude:

$$e^{i\Gamma(q)} = \frac{\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\, e^{i\mathcal{S}(\bar{\psi},\psi;V)}}{\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\, e^{i\mathcal{S}(\bar{\psi},\psi;V_0)}}. \tag{11}$$

Here V is as defined in (6), and we have introduced V_0, the function V corresponding to $q \equiv a$, where a is a positive constant. The denominator thus incorporates the static Casimir effect, which has been evaluated for this case [7], where it has been shown that it properly reproduces the fermionic Casimir force for bag boundary conditions, when $g_L = g_R = 2$. For different values of g, the strength of the interaction is weaker.

III. PAIR CREATION

We evaluate there the probability of pair creation out of the vacuum, due to the motion of one of the walls, which acts as an "external source" injecting energy into the system. We will consider motions of the R wall which are parametrized by means of a function $\eta(x^0)$, which measures the departure of R from its equilibrium, time average position $a > 0$, namely,

$$q(x^0) = a + \eta(x^0). \tag{12}$$

The object we study is the S-matrix; more specifically, matrix elements of the T-matrix which describes the nontrivial part of the evolution:

$$S = 1 + iT. \tag{13}$$

For the perturbative evaluation of those matrix elements, we will make use of the interaction representation. Note, however, that bag conditions correspond to $g = 2$, thus, an expansion in powers of g is impossible. We can, however, use a reliable expansion which captures interesting physics, by taking as unperturbed system the one corresponding to two static boundaries (separated by a distance a) and the difference between the real action and the unperturbed one as perturbation. This may be justified if one assumes, as we do, that the departure η is sufficiently small. Thus, the action is split up as follows:

$$\mathcal{S} = \mathcal{S}_0 + \mathcal{S}_I \tag{14}$$

with:

$$\mathcal{S}_0 \equiv \mathcal{S}(\bar{\psi}, \psi; V_0), \quad V_0(x) = 2\delta(x^1) + 2\delta(x^1 - a), \tag{15}$$

and

$$\mathcal{S}_I \equiv -\int d^2x \bar{\psi}(x)\varphi(x)\psi(x), \qquad \varphi(x) \equiv V(x) - V_0(x). \tag{16}$$

In $\mathcal{S}_I$, the fields are in the interaction picture, so that their time evolution is dictated by the free Hamiltonian, which corresponds to the potential V_0: static walls (at a distance a).

Then, we evaluate the transition amplitudes that result by expanding T in powers of $\mathcal{S}_I$, for small departures η. Up to the second order in η, we see that $\varphi = \varphi^{(1)} + \varphi^{(2)} + \cdots$, with

$$\varphi^{(1)}(x) = -2\delta'(x^1 - a)\eta(x^0) \tag{17}$$

$$\varphi^{(2)}(x) = 2[\delta''(x^1 - a)(\eta(x^0))^2 + \delta(x^1 - a)(\dot{\eta}(x^0))^2], \tag{18}$$

where the prime denotes differentiation with respect to x^1.

Let us now evaluate, to the lowest nontrivial order in η, the transition amplitudes and transition probabilities (the latter will be of the second order in η), assuming the initial state to be the vacuum of the unperturbed system. To the first order in η, the transition amplitude from $|i\rangle$ to $|f\rangle$ is:

$$\begin{aligned} T_{fi}^{(1)} &= \langle f|\mathcal{S}_I|i\rangle = -\int d^2x \varphi^{(1)}(x)\langle f|\bar{\psi}(x)\psi(x)|i\rangle \\ &= -\int d^2x \varphi^{(1)}(x)\langle f|:\bar{\psi}(x)\psi(x):|i\rangle. \end{aligned} \tag{19}$$

The normal ordering above is justified as follows: using Wick's theorem in $\mathcal{S}_I$,

$$\begin{aligned} \mathcal{S}_I = \int d^2x \varphi^{(1)}(x)\bar{\psi}(x)\psi(x) = \int d^2x \varphi^{(1)}(x)(:\bar{\psi}(x)\psi(x): \\ - \mathrm{Tr}[S_F(x,x)]) \end{aligned} \tag{20}$$

where S_F is the fermion propagator in the presence of the static boundaries. Now, the term involving S_F vanishes. Indeed, this object is invariant under time translations: $S_F(x^0, x^1; x'^0, x'^1) = S_F(x^0 - x'^0; x^1, x^1)$. Thus,

$$\begin{aligned} &\int d^2x \varphi^{(1)}(x)\mathrm{tr}[S_F(x,x)] \\ &= \int d^2x \varphi^{(1)}(x)\mathrm{tr}[S_F(0; x^1, x^1)] \\ &= -2\left(\int dx^0 \eta(x^0)\right)\int dx^1 \delta'(x^1 - a)\mathrm{tr}[S_F(0; x^1, x^1)] \\ &= -2\int dx^0 \langle\eta\rangle \int dx^1 \delta'(x^1 - a)\mathrm{tr}[S_F(0; x^1, x^1)] = 0, \end{aligned} \tag{21}$$

where $\langle\eta\rangle$ is the time average of $\eta(x^0)$ which, by assumption, vanishes, since it is the *departure* with respect to the average position a. On the other hand, note that $\langle\eta\rangle$ is multiplied by a factor which is divergent. Indeed, the coincidence limit picks up a logarithmic divergence, so that the UV behavior of that term is

$$\int d^2x \varphi^{(1)}(x)\mathrm{tr}[S_F(x,x)] \sim -2\int dx^0 \langle\eta\rangle \frac{m}{a}\log\left(\frac{\Lambda}{m}\right), \tag{22}$$

where Λ is an UV cutoff. The physical meaning of such a term in the action, is a divergent contribution to the static energy, not to the dynamical process we want to study and therefore one could have defined the theory with the normal ordering from the very beginning without affecting transition probabilities. Also, note that, if $\langle\eta\rangle$ were a nonvanishing constant, one could still absorb that term, by a redefinition of a: $a \to a + \langle\eta\rangle$ in $\mathcal{S}_0$ and expanding to first order in $\langle\eta\rangle$, as it should be.

We want to study particle production out of the vacuum, so that the initial state is $|i\rangle \equiv |0\rangle$; on the other hand, to this order, the only kind of final state allowed contains a fermion antifermion pair. Note that this pair will not correspond to free space particles, rather, to states contained in the bag, which are the eigenstates of the unperturbed Hamiltonian. They will be of the form

$|f\rangle \equiv b_n^\dagger d_l^\dagger |0\rangle$, with $b_n^\dagger$ and $d_l^\dagger$ being creation operators of fermions and antifermions, respectively. They are labeled by discrete indices, n and l, which correspond to spatial momenta when $a \to \infty$. Indeed, a mode-expansion of the field operator (interaction picture) may be constructed as follows:

$$\psi(x) \equiv \sum_n [b_n e^{-iE_n x^0} u_n(x^1) + d_n^\dagger e^{iE_n x^0} v_n(x^1)], \tag{23}$$

where $u_n(x^1) \equiv \psi_{n,+}(x^1)$ and $v_n(x^1) \equiv \psi_{n,-}(x^1)$, with $\psi_{n,\pm}$ are normalized solutions of Dirac equation with bag boundary conditions (7):

$$\psi_{n,\pm}(x^1) = N_n \begin{pmatrix} \pm \frac{E_n}{p_n} \sin(p_n x^1) \\ \cos(p_n x^1) + \frac{m}{p_n} \sin(p_n x^1) \end{pmatrix},$$
$$N_n \equiv \sqrt{2} p_n^2 [p_n^2 (m + 2aE_n^2) + mE_n^2 \sin^2(p_n a)]^{-1/2}, \tag{24}$$

where $E_n \equiv \sqrt{p_n^2 + m^2}$ and the values of p_n are determined by a transcendental equation. In terms of the dimensionless quantities $\rho_n \equiv p_n a$ and $\mu \equiv ma$, the energies may also be rendered in units of $1/a$, introducing dimensionless energies ϵ_n: $E_n = \frac{1}{a}\epsilon_n$, $\epsilon_n = \sqrt{\rho_n^2 + \mu^2}$, while the transcendental equation is

$$\mu \mathrm{sinc}\rho_n + \cos\rho_n = 0, \tag{25}$$

with the sinc function defined as $\mathrm{sinc} x = \frac{\sin x}{x}$. This yields a discrete spectrum [10,11]. In the massless ($\mu \to 0$) limit, this spectrum is simply $p_n = (n + \frac{1}{2})\frac{\pi}{a}$ with $n = 0, 1, \ldots$, and energies $\epsilon_n = \rho_n$. In the opposite regime, $\mu \gg 1$, the spectrum is in turn determined by the zeros of the sinc function, namely: $p_n = \frac{n\pi}{a}$, $n = 1, 2, \ldots$. Note that the lowest energy is, in this limit, the mass of the fermions.

Taking into account the mode expansion above, the transition amplitude for this kind of process becomes:

$$T_{fi}^{(1)} \equiv T_{nl} = -2\tilde{\eta}(E_n + E_l)(\bar{u}_n(x^1) v_l(x^1))'|_{x^1=a}, \tag{26}$$

where the Fourier transform of the departure is defined as: $\tilde{\eta}(\nu) \equiv \int dx^0 e^{i\nu x^0} \eta(x^0)$. Using the explicit form of the eigenstates u and v, we may write:

$$T_{nl} = -\frac{4}{a^2} \tilde{\eta}(E_n + E_l) \xi_n \xi_l (\epsilon_n - \epsilon_l), \tag{27}$$

with

$$\xi_n \equiv \frac{\epsilon_n \sin(\rho_n)}{\sqrt{2\epsilon_n + \mu(1 + \epsilon_n^2 \mathrm{sinc}^2 \rho_n)}}. \tag{28}$$

From the form of the matrix element of T it is clear that, for the transition to be possible, the energies E_n and E_l must be different.

We then write the probability of creation of a specific pair in a spectral form, as follows:

$$P_{nl}^{(1)} = \int \frac{d\nu}{2\pi} \gamma_{nl}(\nu) |\tilde{\eta}(\nu)|^2. \tag{29}$$

where:

$$\gamma_{nl}(\nu) = \frac{32\pi}{a^4} \delta[\nu - (E_n + E_l)][\xi_n \xi_l (\epsilon_n - \epsilon_l)]^2. \tag{30}$$

For strictly massless fermions, this becomes:

$$\gamma_{nl}(\nu) = \frac{8\pi^5}{a^4} \delta\left[\nu - \frac{(n + l + 1)\pi}{a}\right] \left(n + \frac{1}{2}\right) \left(l + \frac{1}{2}\right) (n - l)^2. \tag{31}$$

In this case, the frequency threshold ν_0 required to produce a pair is then given by considering $n = 0$ and $l = 1$. Thus $\nu_0 = \frac{\pi}{2a} + \frac{3\pi}{2a} = \frac{2\pi}{a}$.

In the $\mu \to \infty$ limit, on the other hand, the probability is of course 0, since $\sin\rho_n$ (and therefore ξ_n) vanishes.

Finally, the *total* probability of pair creation P is obtained by summing over all values of n and l which give nonvanishing contributions.

$$P = \sum_{n,l} P_{nl}^{(1)} = \int \frac{d\nu}{2\pi} \gamma(\nu) |\tilde{\eta}(\nu)|^2, \tag{32}$$

with

$$\gamma(\nu) = \sum_{n,l} \gamma_{nl}(\nu). \tag{33}$$

In particular, for the massless case, we may write:

$$\gamma(\nu) = \frac{8\pi^5}{a^4} \sum_{k=1}^{\infty} \delta\left[\nu - \frac{(k+1)\pi}{a}\right] f(k), \tag{34}$$

with

$$f(k) = \sum_{j=0}^{k} \left(j + \frac{1}{2}\right) \left(k - j + \frac{1}{2}\right) (2j - k)^2, \tag{35}$$

where we have taken into account the fact that the minimum value of the frequency threshold is $\frac{2\pi}{a}$.

The sum over j may be explicitly evaluated, leading to the result:

$$f(k) = \frac{1}{60}(k + 1)(2k^4 + 8k^3 + 17k^2 + 18k). \tag{36}$$

IV. IMAGINARY PART OF THE EFFECTIVE ACTION

Let us here consider the (in-out) effective action Γ, in order to check the consistency of its imaginary part with the pair creation probability just derived. Γ may be written as a functional trace, in terms of the fermion propagator S_F in the presence of the static boundaries, and of $\varphi \equiv V - V_0$, as follows:

$$\Gamma(q) = -i\mathrm{Tr}\,\log\left(1 + iS_F\varphi\right). \tag{37}$$

In our conventions, S_F is determined by $[i\partial\!\!\!/ - m - V_0(x)]S_F = iI$, where I is the identity operator in both functional and spinorial spaces, also refer to its kernel. Since V_0 is time-independent, we will use its Fourier transform

$$S_F(x^0 - y^0; x^1, y^1) = \int \frac{d\omega}{2\pi} e^{-i\omega(x^0-y^0)} \tilde{S}_F(\omega; x^1, y^1). \tag{38}$$

Expanding for small departures, as in the previous section,

$$\Gamma = \Gamma^{(0)} + \Gamma^{(1)} + \Gamma^{(2)} + \ldots \tag{39}$$

where the index denotes the order of the term. It is rather straightforward to see that, since in our definition of Γ the static contribution is subtracted, then $\Gamma^{(0)} = 0$. Besides, from the assumption that the time average position of R is a, it follows that also the first order term vanishes. We thus only need to evaluate $\Gamma^{(2)}$. On the other hand, we see that in its second-order term there will be two qualitatively different contributions:

$$\Gamma^{(2)} = \Gamma^{(2,1)} + \Gamma^{(2,2)}, \tag{40}$$

with

$$\Gamma^{(2,1)} = \mathrm{Tr}(S_F\varphi^{(2)}), \qquad \Gamma^{(2,2)} = -\frac{i}{2}\mathrm{Tr}(S_F\varphi^{(1)}S_F\varphi^{(1)}). \tag{41}$$

$\Gamma^{(2,1)}$ produces a renormalization of the would be Lagrangian for the R wall, since it correspond to terms which are proportional to the square of η and of its time derivative. They are local in time, and therefore they will not contribute to any dissipative effect (which necessarily correspond to nonanalyticities in the frequency space).

Let us then extract from the second order term its imaginary part, which is related to the total probability of pair creation P. Indeed, the vacuum persistence probability is related to Γ by:

$$|\langle 0_{\rm out}|0_{\rm in}\rangle|^2 = e^{-2\mathrm{Im}\Gamma} \simeq 1 - P, \tag{42}$$

where the last equality is valid for $\mathrm{Im}\Gamma \ll 1$. This is essentially the equation for probability conservation, where $P = 2\mathrm{Im}\Gamma$ is the probability of the transition of the vacuum to a state with a nonvanishing particle content. Because the first nontrivial process is the creation of a particle and antiparticle pair, by computing $\mathrm{Im}\Gamma$ we should obtain the pair-production probability.

In $\Gamma^{(2,2)}$, for bag boundary conditions, and evaluating the trace over the spatial coordinates

$$\Gamma^{(2,2)} = -2i\int_{x^0,y^0} \eta(x^0)\eta(y^0)\partial_{x_1}\partial_{y_1} \times \mathrm{tr}[S_F(x,y)S_F(y,x)]|_{x_1=y_1=a}. \tag{43}$$

To evaluate the integrals in the last expression, rather than using the time Fourier transforms, and evaluate the convolution of the propagators, we take into account that we are interested in a process whereby real particles are created. Therefore, the flux of energy will have a definite sense in the diagram and, in the spirit of the "largest time equation" [12], we have found it convenient to use the following decomposition of the propagator in terms of positive- and negative-energy projectors:

$$S_F(x,y) = \sum_n [\theta(x_0 - y_0)e^{-E_n(x_0-y_0)}\mathcal{P}_n^+(x_1,y_1) - \theta(y_0 - x_0)e^{-E_n(y_0-x_0)}\mathcal{P}_n^-(x_1,y_1)]. \tag{44}$$

The energy projectors are written in terms of the solutions of the Dirac equation with bag boundary conditions:

$$\mathcal{P}_n^+(x_1,y_1) = u_n(x^1)\bar{u}_n(y^1), \quad \mathcal{P}_n^-(x_1,y_1) = v_n(x^1)\bar{v}_n(y^1). \tag{45}$$

Evaluating the effective action with the previous representation for the propagator, we obtain the expression:

$$\Gamma^{(2,2)} = 2i\sum_{n,l}|(\bar{u}_n(x^1)v_l(x^1))'|^2|_{x^1=a}\int_{x^0,y^0}\eta(x^0)\eta(y^0) \times [\theta(x^0 - y^0)e^{-i(E_n+E_l)(x^0-y^0)} + \theta(y^0 - x^0)e^{-i(E_n+E_l)(y^0-x^0)}]. \tag{46}$$

Finally, using the integral representation of Heaviside's step function, and expressing the function η in terms of its Fourier transform we get:

$$\Gamma^{(2,2)} = -4\sum_{n,l}|(\bar{u}_n(x^1)v_l(x^1))'|^2|_{x^1=a} \times \int \frac{d\nu}{2\pi}\frac{|\tilde{\eta}(\nu)|^2}{\nu - (E_n + E_l) + i\epsilon}. \tag{47}$$

The imaginary part of the last result may be taken in a rather straightforward way, leading to the result:

$$P = 2\mathrm{Im}\Gamma^{(2,2)} = 4\sum_{n,l} |\tilde{\eta}(E_n + E_l)|^2 |(\bar{u}_n(x^1) v_l(x^1))'|^2|_{x^1=a}. \quad (48)$$

Namely,

$$P = \sum_{n,l} |T_{nl}|^2, \quad (49)$$

with T_{nl} as given in (26); therefore in total agreement with the results previously obtained.

V. CONCLUSIONS

In this work, using an S-matrix approach, we have evaluated the fermion pair creation propability for a trembling cavity which enforces bag boundary conditions on the Dirac field, in $1 + 1$ dimensions. The results may be expressed in a rather general form in terms of the eigenenergies of the static cavity, which in turn correspond to the roots of a transcendental equation. In the massless case, results may be written more explicitly.

We have shown the consistency of those results with the ones stemming from the imaginary part of the effective action, for the evaluation of which we have used a shortcut approach.

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