GENERALIZED COMPTON EFFECT

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The Compton effect equations were derived and verified experimentally in 1922 analyzing the collision of x-ray photons, with energies around several kilo electron volts (keV), and conduction electrons with energies of a few electron volts (eV). For many years this was considered as the only case of interest, that is, where the energy of the photons were greater than that of the electrons. It was during the second half of the last century that the so called "inverse Compton effect", involving the collision of relativistic electrons with laser light photons, was developed. It is interesting to regard both situations above as limiting cases of a unique equation which is derived from the relativistic equations for energy and momentum conservation in their general form. The generalized Compton effect is thus the collision of a photon and an electron (or, for that matter with any charged particle) regardless of their energy. The Compton effect occurrence in astrophysical scenarios or in the laboratory is presented here for ranges of photons and electrons energies spanning twenty two orders of magnitude, in order to illustrate the importance of this generalized effect. Examples are the generation of high energy gamma photons (around TeV's) and electrons as observed in cosmic radiation, the experiments of photonuclear reactions with gamma ray photons of hundreds MeV's of energy, or the conversion of laser photons in x-ray photons. The beams thus produced have similar properties as a laser beam, such as high intensity and collimation and high degrees of monochromaticity and polarization.

Introduction

Synchrotron radiation, or electron bremsstrahlung radiation, is produced abundantly in circular accelerators (synchrotrons) where electrons are kept accelerated to ultra-relativistic energies. These accelerators have been named "synchrotron light sources" and there are a number of them around the world in countries such as the USA, Japan, Germany, France, Brazil, Australia, etc. and access time to their use is in high demand for basic and applied research. Synchrotron radiation has outstanding properties for research in chemistry (ultra fast reactions), biology (cell structure), material science (compound's structure), medicine (diagnostic and therapy treatments), etc.

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Almost simultaneously and independently in 1922, Arthur H. Compton[1] in the USA and Peter Debye in Germany studied the collision of electrons with photons, assuming that both behaved as particles. Due to the significant experimental effort of Compton, this effect bears only his name and he was awarded the Nobel Prize in Physics, for its discovery, in 1927.

For many years the Compton effect (CE) was considered to be the transfer of energy from a photon to an electron of lower energy. In this context, in 1929, Dumond[2] developed a theory to interpret the relation between the experimental broadening of the Compton lines and the distribution of electron's momenta in atoms, thereafter named "Compton profiles". M. Cooper[3], in England, revived the interest in these studies in the sixties and numerous applications to condensed matter studies were made and are being carried out to these days[4]

In the early fifties, with the development of energy dispersive gamma ray photon detectors, the CE made it possible to explain some details of the gamma ray spectra emitted by radio nuclides. In 1948, twenty six years after the CE discovery, physicists proposed the inverse process, that is, the energy transfer from an electron to a photon (of lower energy), in order to explain the existence of photons of extremely high energies in the primary cosmic radiation flux.[5], but it was in 1965 that the term "inverse Compton effect" (ICE) was coined[6]. Another lapse of fifteen years was needed before this effect was used to produce high energy gamma rays in high energy electron accelerators[7]. It should be stressed that the gamma ray beam produced by ICE inherits the properties of high intensity, monochromaticity, collimation and polarization of the laser beam, properties which allowed performing studies of nuclear structure with high detail.

Finally, the gamma rays produced by ICE were used in nuclear physics studies, revealing valuable information about the structure of the nucleus, but now relying on the nuclear CE[4] to discover several properties, such as the existence of alpha particles as stable structures within heavy nuclei[8].

During the last several years many laboratories around the world have started to build, based on ICE and named "table-top synchrotron radiation sources"[9]. The improvement on linear accelerator technology, pushed by their applications on cancer therapy among others, provided higher currents, shorter pulses, greater stabilities and repetition rates of electron beams, which in conjunction with commercially available table-top terawatt (T^3) lasers, it is possible to obtain x-ray beams with similar, or better, properties than the synchrotron radiation produced by electron synchrotrons operating at much higher energies. Generation of intense x-ray beams with energies within 10 to 100keV are important to warrant diagnostic image quality, dose reduction and wider scope of the basic studies in applied physics. Gamma ray beams with energies around several MeV are a promise to better cancer therapy treatments. Finally, the use of these gamma beams in the treatment of radioactive wastes from nuclear power plants it is being explored, since it has been observed that they accelerate the radioactive decay[10].

Nowadays, there are three alternatives to the conventional x-ray tube, to generate x-rays with special characteristics:

- Synchrotron Radiation.
- Free electron lasers (wigglers and undulators)
- Inverse Compton effect.

All of them have significantly increased the use of x rays in all sorts of applications.

Compton Effect Generalization

Derivation of the general equation : It is a rather simple exercise of relativistic kinematics to write down the equations of energy and momentum conservation for the collision of a photon and electron in the general case. The result of solving these equations is[7]:

$$\frac{h\nu'}{h\nu} = \frac{1+\beta\cos\alpha}{1-\beta\cos\theta + \dfrac{h\nu\cdot\left[1+\cos(\alpha-\theta')\right]}{m_i c^2}} \qquad (1)$$

where hν is the energy of the incident photon , hν' is the energy of the scattered photon, β is the velocity of the electron in terms of the velocity of light, p_i and p_f are the initial and final linear momenta of the electron, θ is the scattering angle of the electron and α is the incident angle of the laser beam (see Fig. 1).

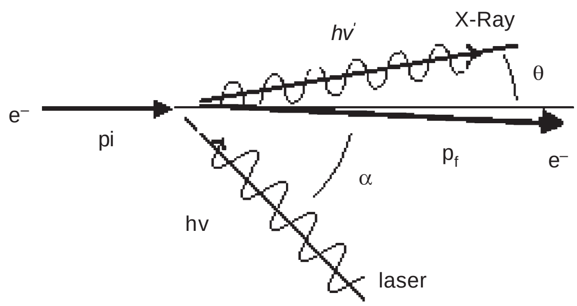

Figure 1. Schematic representation of the Generalized Compton effect.

When the incident electron is not relativistic (β→0), and instead of a laser photon it is an x-ray that collides with an electron, Eq.1 reduces to the familiar equation for the CE:

$$\frac{h\nu'}{h\nu} = \frac{1}{1+\dfrac{h\nu}{m_i c^2}(1-\cos\theta)}$$

Two simple examples illustrate the usefulness of Eq. 1 on applications of the ICE.

Example 1. The photon beam, in Fig. 1, is incident from the right and the electron beam from the left collinearly. Then α = 0 and $\theta \approx 0$ (as we shall see) in Eq. 1. After the collision, we are only concerned with the photon beam which bounces to the right. If incident electrons have large kinetic energies such that $m_i c^2 >> h\nu$, the scattered photon energy will be:

$$h\nu' \approx (1+\beta).h\nu/(1-\beta) = 4\gamma^2 h\nu \qquad (2)$$

where $\gamma = \left(1-\beta^2\right)^{-1/2}$.

Let us consider γ =41 (E_e=20MeV) and $h\nu =$ 1.9eV (red laser light), then $h\nu' \approx$ 13keV. Currently there are available violet light lasers ($h\nu$ > 5eV) which will allow to obtain x-rays of energies over 30keV. Note that in synchrotrons radiation sources, it is necessary to keep electrons accelerated to 25GeV to produce synchrotron radiation of energies of up to a few tens of keVs.

By recourse to the Klein-Nishina equation (see below) we can calculate that for = 10^7photons/s and I_e=10^{10} e^-/s, then I_x=10^7photons/s[11,12].

Example 2. The incident electron strikes the photon from its side, that is, α = 90° and θ = 0°. Again if the electron energy is large equation 1 can be written as:

$$h\nu' \approx 2\gamma^2 h\nu$$

which implies that the photon energies are a factor of two lower than in the first example (see equation 2). This arrangement is less efficient in the x-ray yield and collimation of the x-ray beam, but has the advantage over the one of the previous example in that the outgoing x-ray beam does not strike the laser apparatus.

I_ν

The Klein-Nishina differential cross section : We will mention briefly the collimation properties of the x-ray beam produced as a consequence of the collision of well collimated electron and laser photon beams. To consider the other important property of the x-ray beam, its degree of polarization, exceeds the scope of this note. The Klein-Nishina equation holds for both, relativistic and non-relativistic electrons and allows to calculate the scattering cross sections in terms of the angular arrangement of the beams in a Compton collision, the respective energies and the angular aperture of the beam[13].

If E_e>>$m_i c^2$ the Klein-Nishina equation has the form:

$$\frac{d\sigma_{KN}}{d\Omega} = \frac{1}{2} r_0^2 \frac{(\nu')^2}{\nu^2}\left(\frac{\nu'}{\nu} + \frac{\nu}{\nu'} - \sin^2\theta\right)$$

where r_0 is the "classical radius" of the electron. Figure 2 shows calculated results of the angular distribution of the x-ray photon beam produced in a head-on collision of laser light photons with high energy electron. The aperture is defined as the angle where the intensity of the x-ray beam drops to 50% of the intensity at 0°. The beam aperture is approximately given by $\theta \approx \gamma^{-2}$, as calculated by Chouffani[9]., and in this range of energies it is a few milliradians, *i.e.*, the beam has a diameter of one centimeter ten meters away from the collision region.

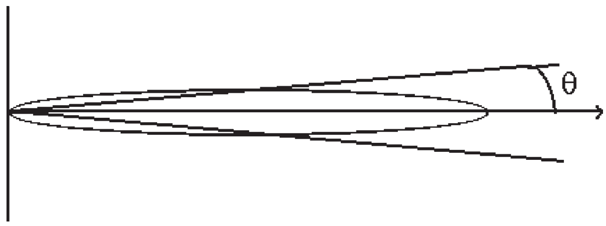

Figure 2. Calculated angular distribution of x-ray beam intensity emitted in the "forward direction", that is, along the incident electron beam propagation direction.

In Figure 3 there are depicted regions, spanning a range of 24 orders of magnitude in energy, where the Compton effect has its most striking consequences.

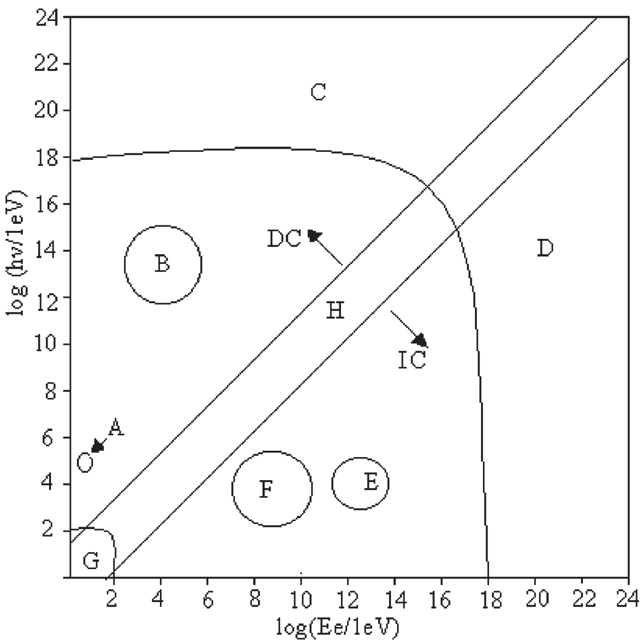

Figure 3. Main regions of interest of the generalized Compton effect. Vertical axis depicts the photon's energy. The horizontal axis the electron's energy.

It would be necessary to draw this figure in 4-D, initial and final energies of the electron and the photons, but we will only consider the initial energies of the electron and the photon, just for illustration purposes and in each different region we will refer to the characteristics of the outgoing particles.

Region "A" contains the range of electron and photon energies where A.H. Compton carried out his groundbreaking experiments.

Region "B" is where "nuclear Compton effects" have been and are currently performed, including Compton scattering with quarks inside nucleons.

Regions "C" and "D", is where astrophysical processes everywhere in the universe produce electrons and photons of the highest energies and the Compton effect is responsible for producing the highest energy of the observed electrons and photons in cosmic radiation arriving to the earth.

In regions "E" and "F" inverse Compton experiments of high and low energies take place. In zone "G" Compton collision effects are unobservable because $\lambda << \lambda_C$ and along the diagonal "H" photons and electrons have roughly the same energy and energy transfer among them is negligible. The top arrow signals the region of the direct Compton (DC) and the lower arrow the region where inverse Compton effect (ICE) is dominant.

The Double Inverse Compton Effect

A few MeV energy electron accelerators[16,17], pulsed at around 100 Hz, offer the possibility to yield x-ray source beams of characteristics similar to the so called synchrotron light sources[9]. Nowadays, new high intensity pulsed lasers are available at moderate prices and low energy accelerators are also slashing their prices in a highly competitive market. It is thus possible to build a double inverse Compton experiment, with an arrangement such that the outgoing x-ray beam does not impinge on the laser. Its proof of principle is described below.

In the arrangement of Fig 4, a pulsed electron beam is injected along the lower right corner horizontal path. The splitter wave guide, currently under design, turns one pulse upwards and lets the next to continue along the horizontal path. Bending magnets, labelled with the letter M, deflect the electron pulses at angles of 90°. If the timing of the laser and electron beams is such that they collide in the region A-B, in the upper part of the arrangement, a low energy x-ray beam pulse is produced by a first ICE and directed towards the left. By a suitable spacing of magnets, it is possible to make the second electron pulse to arrive at point B, right in time to collide with the left moving x-ray beam pulse and yield a much higher energy x-ray beam pulse moving to the right.

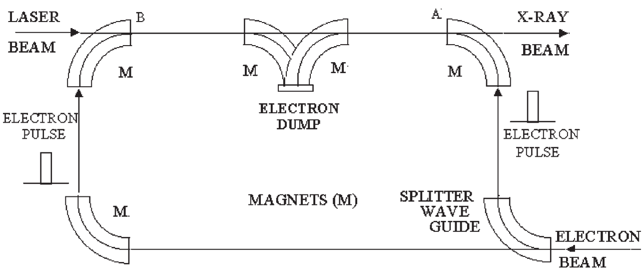

Figure 4. Schematic arrangement of a double inverse Compton scattering experiment.

The beam intensity from the second ICE is several orders of magnitude lower, but preliminary calculations show that pulses of intensities comparable to commercial x-ray tubes, are easily achieved with the advantages of output energy selection, collimation and monochromaticity, thus many applications of this arrangement are possible.

Conclusions

Several textbooks and countless research articles treat the "direct" and "inverse Compton effects" as two separate effects, as if the physical phenomena involved were different. As equation 1 shows, it is one and only one effect with different outcomes depending on the relative energies of the particles. It must be noted that in Eq. 1 the photon is considered a particle and that electrons need to be treated with the kinematical relativistic expressions for conservation of energy and momentum. It is seen that a joint treatment has a greater teaching value and exposes the consequences and applications beyond the usual ones so frequent in radiation detection and measurement. ❒

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