CDD: 530.12

THE CENTRAL ROLE OF THE HAMILTONIAN IN QUANTUM MECHANICS: DECOHERENCE AND INTERPRETATION

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Received: 12.08.2009; Accepted: 01.03.2010

Abstract: The core of the environment-induced decoherence program relies on the interaction between the system and its environment; this interaction leads interference to vanish with respect to a definite "preferred basis". On the other hand, modal interpretations of quantum mechanics supply criteria to select the "preferred context", where observables acquire definite values. The purpose of this paper is to show the compatibility between the modal interpretative framework and the results of the decoherence program, a compatibility that comes to the light when the Hamiltonian is conceived as the main character of the quantum play.

Keywords: Quantum mechanics. Modal-Hamiltonian interpretation. Environment-induced decoherence. Measurement problem.

O PAPEL CENTRAL DO HAMILTONIANO NA MECÂNICA QUÂNTICA: DECOERÊNCIA E INTERPRETAÇÃO

Resumo: O núcleo do programa da coerência indicida pelo ambiente reside na interação entre o sistema e seu entorno; tal interação conduz ao desaparecimento da interferência com respeito a uma `base preferencial' definida. Por outra parte, as interpretações modais da mecânica quântica proveem critérios para selecionar o `contexto preferencial', onde os observáveis adquirem valores definidos. O propósito deste artigo consiste em mostrar a compatibilidade entre o marco interprettivo modal e os resultados do programa da coerência, uma compatibilidade que vem à luz quando o hamiltoniano 'e concebido como protagonista no cenário quântico.

Palavras chave: Mecânica quântica. Interpretação modal hamiltoniana. Decoerência induzida pelo ambiente. Problema da medicão.

1. INTRODUCTION

The problem of the interpretation of quantum mechanics is certainly one of the most discussed topics in the philosophy of physics. Although during many years since the birth of the theory instrumentalist interpretations prevailed, in the last decades several attempts to interpret quantum mechanics from a realist viewpoint have been presented in the literature. Any realist interpretation is committed to explain which observables acquire definite values without violating the contextuality of the theory. Many different criteria to select the "preferred" context have been proposed; however, the Hamiltonian of the system was systematically ignored in the discussions.

Since the early days of the theory, the measurement problem has been one of the most serious challenges for interpretation. Much ink has been spilled over the search of an adequate solution, and many interpretations have been deliberately designed to explain why the measuring apparatus acquires the definite properties observed in measurements. During the last decades, the idea that the physical phenomenon of decoherence supplies the answer to the measurement problem has been taken for granted in the physical community. The core of the decoherence program relies on the interaction between the measuring apparatus and its environment: the continuous "monitoring" of the environment leads interference to vanish with respect to a definite "preferred basis", which turns out to be the eigenbasis of the pointer observable of the measuring apparatus. In spite of its successful application in many areas of physics, the *environment-selection* ("*einselection*") or *environment-induced* decoherence program is still threatened by certain conceptual problems that obscure the complete understanding of the phenomenon. In particular, the criterion to select the preferred basis still remains unclear, and the identification of the system-environment "cut" is not accounted for by the theory.

The purpose of this paper is to argue for the central role played by the Hamiltonian in quantum mechanics. First, I shall recall the theoretical

elements of the environment-induced decoherence program, and I shall point out the conceptual challenges that the program still has to face. Then, I shall summarize the main features of the modal-Hamiltonian interpretation, according to which the Hamiltonian is decisive in the definition of systems and subsystems, and in the selection of the preferred context where observables acquire definite values. On this basis, I shall show that, when the total Hamiltonian involved in the process of decoherence is taken into account, one can find a general criterion to define the preferred basis and the problem of discriminating between the system and its environment vanishes. Finally, I shall stress the compatibility between the new interpretative framework and the results of the decoherence program, a compatibility that comes to the light when the Hamiltonian is conceived as the main character of the quantum play.

2. ENVIRONMENT-INDUCED DECOHERENCE

In the standard model due to von Neumann, a quantum measurement is conceived as an interaction between a system S and a measuring apparatus M. Before the interaction, M is prepared in a ready-to-measure state $|p_0\rangle$, eigenvector of the pointer observable P of M, and the state of S is a superposition of the eigenstates $|a_i\rangle$ of an observable A of S. The interaction introduces a correlation between the eigenstates $|a_i\rangle$ of A and the eigenstates $|p_i\rangle$ of P:

$$|\psi_0\rangle = \sum_i c_i |a_i\rangle \otimes |p_0\rangle \quad \rightarrow \quad |\psi\rangle = \sum_i c_i |a_i\rangle \otimes |p_i\rangle \qquad (2\text{-}1)$$

The problem consists in explaining why, being the state $|\psi\rangle$ a superposition of the $|a_i\rangle \otimes |p_i\rangle$, the pointer P acquires a definite value.

In the orthodox collapse interpretation, the pure state $|\psi\rangle$ is assumed to "collapse" to a mixture ρ^c, such that

$$\rho^c = \sum_i |c_i|^2 |a_i\rangle \otimes |p_i\rangle\langle a_i| \otimes \langle p_i| \qquad (2\text{-}2)$$

where the probabilities $|c_i|^2$ are given an ignorance interpretation. Then, in this situation it is supposed that the measuring apparatus is in one of the eigenvectors $|p_i\rangle$ of P, say $|p_k\rangle$, and therefore P acquires a definite value p_k, the eigenvalue corresponding to the eigenvector $|p_k\rangle$, with probability $|c_k|^2$. But the indeterministic and non-unitary "collapse" is introduced as an *ad hoc* assumption, completely different from the dynamical postulate according to which quantum states follow deterministic and unitary evolutions governed by the Schrödinger equation.

The key idea of the decoherence program is that macroscopic systems, like measuring apparatuses, are never isolated but always interact with their environments. When the environment E is taken into account, the initial state of the whole system $S + M + E$ becomes

$$|\psi_{SME}(0)\rangle = \left(\sum_i c_i |a_i\rangle \otimes |p_i\rangle \right) \otimes |e_0\rangle \qquad (2\text{-}3)$$

where $|e_0\rangle$ is the state of the environment before its interaction with the measuring apparatus. Zurek and his collaborators prove that, when the interaction Hamiltonian H_{ME}^{int} satisfy certain conditions (*cfr.* Zurek 1981), $|\psi_{SME}(0)\rangle$ evolves into

$$|\psi_{SME}(t)\rangle = \sum_i c_i |a_i\rangle \otimes |p_i\rangle \otimes |e_i(t)\rangle \qquad (2\text{-}4)$$

where the $|e_i(t)\rangle$ are the states of the environment associated with the different pointer states $|p_i\rangle$. According to Zurek, the state of the system $S + M$ is represented by the reduced density operator $\rho_r(t)$ resulting from tracing over the environmental degrees of freedom,

$$\rho_r(t) = Tr_{(E)}(|\Psi_{SME}(t)\rangle\langle\Psi_{SME}(t)|) =$$
$$\sum_{ij} c_i c_j^* |a_i\rangle \otimes |p_i\rangle\langle a_i| \otimes \langle p_i| \langle p_i| \langle e_i(t)|e_j(t)\rangle \qquad (2\text{-}5)$$

where the factor $\langle e_i(t)|e_j(t)\rangle$ determines the size of the off-diagonal terms at each time. Many standard models for the interaction Hamiltonian H_{ME}^{int} show that, when the environment is composed of a large number of subsystems, the states $|e_i\rangle$ of the environment rapidly approach orthogonality,

$$\langle e_i(t)|e_j(t)\rangle \longrightarrow \delta_{ij} \tag{2-6}$$

This means that the reduced density operator rapidly becomes approximately diagonal in the preferred basis $\{|a_i\rangle \otimes |p_i\rangle\}$ (compare with eq.(2-2)),

$$\rho_r(t) \longrightarrow \rho_r = \sum_i |c_i|^2 |a_i\rangle \otimes |p_i\rangle\langle a_i| \otimes \langle p_i| \tag{2-7}$$

According to Paz and Zurek, in a certain sense decoherence "explains" collapse since "*quantum entanglement will be converted into an effectively classical correlation as a result of the interaction between* M *and* E." (Paz and Zurek 2002, p. 90).

As Bub (1997, p. 207) points out, during the last decades the theory of decoherence has become the "new orthodoxy" in the quantum physicists community. Many authors coming from physics and from philosophy have considered that decoherence supplies the right answer to the measurement problem. For instance, under the assumption that the only legitimate demand for a physical theory is the explanation of our perceptions (the "appearances"), d'Espagnat (2000, p. 136) says that "*for macroscopic systems, the appearances are those of a classical world* […] *decoherence explains the just mentioned appearances, and this is a most important result*." In his book on foundations of quantum mechanics, Auletta (2000, p. 289) makes a stronger claim: "*decoherence is able to solve practically all the problems of measurement which have been discussed in the previous chapters*." From a similar perspective, Anderson (2001, p. 492) asserts that "*the word «decoherence»* […] *describes the process that used to be called «collapse of the wave function»*." However,

not all are so enthusiastic: the implications of decoherence for interpretation are still far from being completely clear.

2.a. The problem of the preferred basis

With the great success of the decoherence program, the measurement problem itself has been reformulated. As Schlosshauer (2004, p. 1270) points out, at present two questions have to be distinguished:

- *The problem of definite outcomes*: why do we seem to perceive the pointer to have one definite value but not a superposition of values?
- *The problem of the preferred basis*: since the expansion of the final composite state resulting from the interaction between the system and the measuring apparatus is in general not unique, the measured observable is not univocally defined.

In the literature, the problem of definite outcomes is the traditional "measurement problem", and it has been extensively discussed in the light of the decoherence program. During the last times, many authors have advanced serious warnings about the capability of decoherence for solving this first problem (*cfr.*, *e.g.*, Healey 1995, Bacciagaluppi 2008). In particular, it has been stressed that ρ_r, being a reduced density matrix, refers to what d'Espagnat (1976) called 'improper mixture', which cannot be interpreted in terms of ignorance: in spite of decoherence, the total system $S + M + E$ is still described by a superposition $|\psi_{SME}(t)\rangle$ at any time (*cfr.* Bub 1997). For instance, Adler (2003) claims that the diagonalization of ρ_r does not allow us to say that the state of the system $S + M$ is in one of the states $|a_i\rangle \otimes |p_i\rangle$, and he concludes: "*I do not believe that either detailed theoretical calculations or recent experimental results show that decoherence has resolved the difficulties associated with quantum measurement theory*" (Adler 2003, p. 135). These and similar arguments have led even some contributors to the decoherence program to express their skepticism about the relevance of decoherence to the solution of the

definite outcomes problem; as Joos (2000, p. 14) says: "*Does decoherence solves the measurement problem? Clearly not.*"

On the contrary, the solution to the preferred basis problem supplied by the decoherence program has been considered its main contribution. For instance, Schlosshauer thinks that "*based on the progress already achieved by the decoherence program, it is reasonable to anticipate that decoherence embedded in some additional interpretive structure could lead to a complete and consistent derivation of the classical world from quantum-mechanical principles.*" (Schlosshauer 2004, p. 1287). In a similar vein, Elby (1994, p. 364) claims that "*decoherence cannot help modal, relative state, or many-world interpretations fend off general metaphysical criticisms. The value of decoherence lies in its ability to pick out a special basis.*" In fact, the theory of decoherence has been frequently used in the many-world interpretation to solve the problem of the preferred basis (Butterfield 2002, Wallace 2002, 2003), considered the main difficulty of Everett's proposal (*cfr.* Stapp 2002). Decoherence has also been integrated into the framework of modal interpretations (*cfr.* Dieks 1989), and Bacciagaluppi and Hemmo (1996) have suggested that decoherence, with its definition of the preferred basis, would allow modal interpretations to overcome the criticisms of Albert and Loewer (1990, 1993). Therefore, the identification of the preferred basis selected by the system-environment interaction is a crucial conceptual point in the understanding of the decoherence program.

In his first papers, Zurek studied physical models where the interaction between the measuring apparatus and the environment dominates the process (Zurek 1981, 1982); in those cases, the reduced density matrix ends up being diagonal in the eigenvectors of an observable P that commutes with the Hamiltonian H_{ME}^{int} describing the apparatus-environment interaction. This property is what makes P to be the pointer observable: since P is a constant of motion of H_{ME}^{int}, when the apparatus is in one of its eigenstates, the interaction with the environment will leave it unperturbed. Zurek states this result already in the abstract of his famous paper of 1981: "*The form of the interaction Hamiltonian between the apparatus and its environment is sufficient to determine which*

observable of the measured quantum system can be considered «recorded» by the apparatus. The basis that contains that record –the pointer basis of the apparatus– consists of the eigenvectors of the operator which commutes with the apparatus-environment interaction Hamiltonian" (Zurek 1981, p. 1516). Since those first works, the condition $\left[H_{ME}^{\text{int}}, P \right] = 0$ has usually be considered as the definition of the preferred –pointer– basis or of the preferred –pointer– observable P of the apparatus. For instance, Elby explains: "*Let* P' *denote an arbitrary apparatus observable that doesn't commute with the pointer reading* P. *Using 'toy' examples, along with general considerations, Zurek argues that* H_{ME}^{int} *commutes with* P, *but does not commute with any* P'. *In rough terms, the interaction between the apparatus and its environment picks out the pointer-reading basis*" (Elby 1994, p. 363). More recently, Schlosshauer claims: "*One can then find a sufficient criterion for dynamically stable pointer states that preserve the system-apparatus correlations in spite of the interaction of the apparatus with the environment by requiring all pointer state projection operators* $P_n = \left| p_n \right\rangle \left\langle p_n \right|$ *to commute with the apparatus-environment Hamiltonian* H_{ME}^{int}, *i.e.,* $\left[H_{ME}^{\text{int}}, P_n \right] = 0$, *for all* n" (Schlosshauer 2004, pp. 1278-1279).[1] This idea has given support to the usual claim that the interaction with the environment is the key feature of decoherence: as the name of the program ('*environment-induced decoherence*') suggests, it is the environment what "induces" decoherence and, as a consequence, the transition from quantum to classical.

However, the analysis of the models studied by Zurek shows the need of considering that conclusion with caution. According to the Schrödinger equation, the entangled state $\left| \psi_{SME}(t) \right\rangle$ of the whole system (see eq.(2-4)) actually evolves under the action of the total Hamiltonian H_{SME},

$$H_{SME} = H_S + H_M + H_E + H_{SM}^{\text{int}} + H_{SE}^{\text{int}} + H_{ME}^{\text{int}} \quad \text{(2-8)}$$

[1] For simplicity, in these two last quotes we have adjusted the symbolism to that used in the present paper.

Therefore, the condition $\left[H_{ME}^{\text{int}}, P\right] = 0$ as the definition of the dynamically stable pointer observable involves several assumptions. First, it is considered that the system-environment interaction and the system-apparatus interaction are zero: $H_{SE}^{\text{int}} = 0$ and $H_{SM}^{\text{int}} = 0$. This assumption is reasonable on the basis of the design of the measurement arrangement: after a short time, any interaction with the system ends and the subsystem $M + E$ follows its independent dynamical evolution; for this reason, also the self-Hamiltonian H_S of the system can be disregarded. Then, the Hamiltonian relevant for the evolution reads

$$H_{ME} = H_M + H_E + H_{ME}^{\text{int}} \tag{2-9}$$

and the stability of the pointer would strictly require that $\left[H_{ME}, P\right] = 0$. As a consequence, endowing H_{ME}^{int} with the role of preserving the time-independence of P and, thus, of defining the preferred basis requires us to justify why the self-Hamiltonians of the apparatus and the environment, H_M and H_E, can be legitimately neglected. In the philosophy of physics literature these conditions are usually taken for granted (*cfr.*, *e.g.*, Elby 1994, d'Espagnat 1976) and not sufficiently discussed.

During the years following the 1981 paper, the idea that the apparatus-environment interaction picks out the preferred basis was widely accepted; however, more than ten years later, Zurek realized that that original idea was a simplification: in more general situations, when the system's dynamics is relevant, the einselection of the preferred basis is more complicated. Zurek introduced the "*predictability sieve*" criterion (Zurek 1993, Zurek, Habib and Paz 1993) as a systematic strategy to identify the preferred basis in generic situations. The criterion is based on the fact that the preferred states are, by definition, those less affected by the interaction with the environment, in the sense that they are the ones less entangled with it. Then, the pointer basis is obtained by considering all the pure initial states of the system and computing the entropy associated with its reduced density operator after some time t: the

pointer states are those that minimize the entropy production over initial states.

Zurek and other authors applied the predictability sieve to a number of models since 1993. On the basis of the results so obtained, Paz and Zurek (1999, *cfr.* also Zurek 2003) introduced the distinction among three basically different regimes for the selection of the preferred basis: they differ in the relative strength of the system's self-Hamiltonian (the apparatus' self-Hamiltonian H_M in the measurement situation) and of the interaction Hamiltonian (H_{ME}^{int} in the measurement situation):

- The first regime is the quantum measurement situation, where the self-Hamiltonian of the system can be neglected and the evolution is completely dominated by the interaction Hamiltonian. In such a case, the preferred states are directly the eigenstates of the interaction Hamiltonian (Zurek 1981).
- The second regime is the more realistic and complex situation, where neither the self-Hamiltonian of the system nor the interaction with the environment are clearly dominant, but both induce non-trivial evolution. In this case, the preferred basis arises from the interplay between self-evolution and interaction; quantum Brownian motion belongs to this case (Paz 1994).
- The third regime corresponds to the situation where the dynamics is dominated by the system's self-Hamiltonian. In this case, the preferred states are simply the eigenstates of this self-Hamiltonian (Paz and Zurek 1999).

According to Schlosshauer (2004, p. 1280), these three regimes explain why many systems, specially in the macroscopic domain, are typically found in energy eigenstates, even if the interaction Hamiltonian depends on an observable different than energy.

When these results, obtained case by case in the study of particular models, are considered from a general perspective, the selection of the preferred basis may be viewed under a new light. If the preferred states depend on the Hamiltonian's component that dominates the whole evolution, it is reasonable to suspect that the preferred basis is defined by

the total Hamiltonian of the whole system: each regime would turn out to be a particular case of a general definition.[2]

Of course, in realistic cases the total Hamiltonian may be difficult, if not impossible to be written down. Moreover, even if we know the explicit form of the Hamiltonian, its diagonalization might be an unattainable task. However, these are practical limitations: the relevant point is to decide which is the definition of the preferred basis. In this sense, the above discussion suggests that such a definition has not to be searched in the system-environment interaction alone, as usually supposed, but in the features of the total Hamiltonian of the whole system, with the particular contributions due to its components. From this viewpoint, the interaction with the environment looses the original central role that the standard presentations assign to it: if the total Hamiltonian selects the preferred basis, the interaction is only one of the elements contributing to that selection.

2.b. The problem of defining systems

According to Zurek, "*decoherence is a process which –through the interaction of the system with external degrees of freedom often referred as the environment– singles out a preferred set of states*" (Zurek 1994, p. 176). The emergence of the preferred basis results from an environment-induced superselection which eliminates the vast majority of the "non-classical" states in the Hilbert space: einselected states are distinguished by their

[2] This conclusion would be in agreement with our "*self-induced approach*" to decoherence (Castagnino and Laura 2000, Castagnino and Lombardi 2004, 2005a, 2005b), according to which the phenomenon of decoherence does not need to "split" the whole closed system into a system of interest and an environment: for a certain subset of relevant observables, the system may decohere by destructive interference as an effect of its own dynamics, and the preferred basis is always the eigenbasis of the total Hamiltonian (for the compatibility between the environment-induced and the self-induced approaches, *cfr.* Castagnino, Laura and Lombardi 2007, Castagnino, Fortin, Laura and Lombardi 2008).

stability in spite of the monitoring environment. In Paz and Zurek's words, "*the environment distills the classical essence of a quantum system*" (Paz and Zurek 2002, p. 3).

These statements make clear that, from the einselection view, the split of the Universe into the degrees of freedom which are of direct interest to the observer –the system– and the remaining degrees of freedom –the environment– is absolutely essential for decoherence. Such a split is necessary not only to explain quantum measurement, but also to understand "*the quantum origin of the classical world*" (Paz and Zurek 2002, p. 1). In fact, Zurek and his collaborators always consider the problem of the transition from quantum to classical as the core of the discussion: "*The aim of the program of decoherence and einselection is to describe the consequences of the 'openness' of quantum systems to their environments and to study the emergence of the effective classicality of some of the quantum states and of the associated observables*" (Zurek 1998, p. 1). In this context, quantum measurement is conceived as a particular case of the general phenomenon of the emergence of classicality, that is, as an example that illustrates the quantum origin of the classical definiteness of some states in individual systems (Zurek 2003). In addition, if classicality only emerges in open quantum systems, it must always be accompanied by other manifestations of openness, such as dissipation of energy into the environment. Zurek even considers that the prejudice which seriously delayed the solution of the problem of the transition from quantum to classical is itself rooted in the fact that the role of the "openness" of a quantum system in the emergence of classicality was ignored for a very long time (Paz and Zurek 2002, Zurek 2003).

In summary, decoherence explains the emergence of classicality, but only open systems can "decohere". The question is: what about the Universe as a whole? Zurek himself admits that the Universe is, by definition, a closed quantum system, "*it is practically the only system that is effectively closed*" (Zurek 1991, p. 42); but then, the Universe cannot decohere. Zurek considers the possible criticism: "*the Universe as a whole is*

still a single entity with no 'outside' environment, and, therefore, any resolution involving its division into systems is unacceptable" (Zurek 1994, p. 181).

Nevertheless, the decoherence approach has been applied to cosmology with interesting results. In those cases, the general strategy consists in splitting the universe into some degrees of freedom which represent the "system" of interest, and the remaining degrees of freedom that are supposed to be non accessible and, therefore, play the role of an internal environment. For instance, in quantum field theory, it is usual to perform a decomposition on a scalar field ϕ, $\phi = \phi_S + \phi_E$, where ϕ_S denotes the system field and ϕ_E denotes the environment field; when it is known that the background field follows a simple classical behavior, the scalar field is decomposed according to $\phi = \phi_c + \phi_q$, where the background field ϕ_c plays the role of the system and the fluctuation field ϕ_q plays the role of the environment (*cfr.* Calzetta, Hu and Mazzitelli 2001). An analogous strategy is followed in the case of "internal" environments, such as collections of phonons or other internal excitations. The possibility of internal environments shows that the decoherence program supplies no general criterion for distinguishing between the system and its environment: the partition of the whole closed system is decided case by case, and usually depends on the previous assumption of the observables that will behave classically (for a discussion of this point, *cfr.* Castagnino and Lombardi 2004).

The absence of a general criterion to decide where to place the "cut" between system and environment is a particularly serious difficulty for an approach that insists on the essential role played by the openness of the system in the emergence of classicality. Zurek recognizes this problem as a shortcoming of his proposal: "*In particular, one issue which has been often taken for granted is looming big as a foundation of the whole decoherence program. It is the question of what are the 'systems' which play such a crucial role in all the discussions of the emergent classicality. This issue was raised earlier, but the progress to date has been slow at best*" (Zurek 1998, p. 22).

Of course, the problem of defining the systems involved in decoherence is a serious obstacle for the einselection program when the

phenomenon is described in the usual terms, that is, as a consequence of the interaction between two open systems. However, as emphasized by Omnès (2001, 2002), decoherence can be conceived as a particular case of the general phenomenon of irreversibility, where the non-unitary evolution is obtained by disregarding part of the maximal information obtainable from the system. In the quantum case, the maximal information about a system is given by the set O of all its possible observables; then, the restriction of that maximal information to a relevant part amounts to select a subset $\mathrm{O}^R \subset \mathrm{O}$ of relevant observables, with respect to which the behavior of the system will be studied. From this perspective, the identification of the system of interest S and its environment E in the decoherence program amounts to a particular selection of the relevant observables of the whole closed system. In particular, if a closed quantum system S_C is represented in the Hilbert space H and its observables O belong to the von Neumann-Liouville space $\mathrm{L} = \mathrm{H} \otimes \mathrm{H}$, the relevant observables O^R are those of the form

$$O^R = O_S \otimes I_E \in \mathrm{O}^R \subset \mathrm{L} \tag{2-10}$$

where the $O_S \in \mathrm{L}_S = \mathrm{H}_S \otimes \mathrm{H}_S$ are the observables of S, I_E is the identity operator on the von Neumann-Liouville space $\mathrm{L}_E = \mathrm{H}_E \otimes \mathrm{H}_E$ of E, and $\mathrm{L} = \mathrm{L}_S \otimes \mathrm{L}_E$. This means that, since the system S is characterized by its von Neumann-Liouville space L_S (or, equivalently, by its Hilbert space H_S), when the relevant observables O^R are selected, the system S and, with it, the environment E turn out to be precisely identified. In turn, by definition of reduced density operator, ρ_r of S is such that (*cfr.* Ballentine 1998)

$$\left\langle O^R \right\rangle_{\rho} = \left\langle O_S \right\rangle_{\rho_r} \tag{2-11}$$

Therefore, the convergence of $\rho_r(t)$ to a final ρ_r diagonal in the preferred basis (see eq.(2-7)) means that the expectation values $\left\langle O^R \right\rangle_{\rho(t)} = \left\langle O_S \right\rangle_{\rho_r(t)}$ approach final stable values:

$$\rho_r(t) \to \rho_r \quad \Rightarrow \quad \left\langle O^R \right\rangle_{\rho(t)} = \left\langle O_S \right\rangle_{\rho_r(t)} \to \left\langle O_S \right\rangle_{\rho_r} \qquad (2\text{-}12)$$

In other words, as in the general case of irreversibility, in the process of decoherence the non-unitary evolution of the reduced state expresses the convergence of the expectation values to their final values, for all the observables selected as relevant in each particular case (for a detailed discussion, *cfr.* Castagnino, Laura and Lombardi 2007).

When decoherence is understood from this general perspective, the problem of defining the systems involved in decoherence is not as serious as Zurek supposes. In fact, the identification of the system of interest and its environment is just a way of selecting the relevant observables of the whole closed system. But given a closed system, there are many ways of selecting the relevant observables, and not in every case decoherence is obtained. For instance, let us consider the spin-bath model, perhaps the simplest exactly solvable model introduced by Zurek (1982): a spin-1/2 particle P immersed in a bath of N spin-1/2 particles P_i. The self-Hamiltonians of P and of the P_i are taken to be zero; P interacts with the P_i, but the P_i do not interact with each other. In the standard treatment of the model, the particle P is conceived as the system, and the bath of particles P_i as the environment. In this case, for particular system-environment interactions, numerical results show that, as N increases, the system rapidly decoheres: the terms corresponding to the interference between the two states of P become quickly and strongly suppressed. However, nothing compels this particular "cut" between system and environment: we could also select the particle P_k as the system and the rest of the particles as the environment. It is clear that, in this new partition, the system P_k does not decohere since it is almost uncoupled with its environment (for a discussion on this point, *cfr.* Castagnino, Fortin, Laura and Lombardi 2008).

More general models can be proposed. Let us consider, for instance, a collection of $M+N$ spin-1/2 particles such that (i) the particles of the group M do not interact with each other but each one of

them interacts with all the particles of the group N, and (ii) the particles of the group N do not interact with each other but each one of them interacts with all the particles of the group M. Let us call S_A the system of M particles and S_B the system of N particles. In this model, numerical results show that the behavior of S_A and S_B depends on the relationship between the numbers M and N (*cfr.* Castagnino, Fortin and Lombardi 2008):

- If $M \gg N$, the system S_A decoheres and the system S_B does not decohere. In this case, if S_A is considered the system of interest, we could say that the system decoheres as the result of the interaction with its environment S_B. But if S_B is the system of interest, decoherence is not obtained.
- If $M \ll N$, conclusions analogous to the previous case can be drawn, by interchanging S_A and S_B.
- If $M \approx N$, neither S_A nor S_B decohere.

These results clearly show that the conclusions about decoherence strongly depend on the way in which the whole closed system is partitioned into open systems. In simple models, the conclusions can be quickly inferred by the inspection of the number of degrees of freedom involved in the model and of their interactions. But in more complex situations, the model has to be described in detail in order to see, for each particular partition, whether the system of interest resulting from that partition decoheres or not.

3. THE MODAL-HAMILTONIAN INTERPRETATION

During the last decades, the research on the formal properties of the mathematical structure of quantum mechanics has shown a great advance: many results, unknown by the founding fathers of the theory, have been obtained, and this work has greatly improved the understanding of the deep obstacles that any interpretation must face. However, this interest in the features of the formalism has led to forget the physical content of quantum mechanics. In fact, in the last times,

realist interpretations usually rely on mathematical results and focus their attention mainly on the formal model of the measurement problem. But quantum mechanics is a physical theory by means of which an impressive amount of experimental evidence has been accounted for.

Recently we have proposed a new interpretation of quantum mechanics, belonging to the "modal family" (Lombardi and Castagnino 2008, Castagnino and Lombardi 2008): like previous modal interpretations, it is a realist, non-collapse approach according to which the quantum state describes the possible properties of the system but not its actual properties. However, our interpretation moves away from the present trend in the research on the subject in the sense that it places an element with a clear physical meaning, the Hamiltonian of the system, at the heart of the proposal. From our interpretational perspective, the Hamiltonian is decisive in the definition of the quantum system and in the selection of its definite-valued observables.

3.a. Systems and subsystems

In order to study the physical world, we have to identify the systems that populate it. We can cut out the physical reality in many different ways, but only when a portion of reality does not interact with others we obtain a system that obeys the dynamical postulate of quantum mechanics. For this reason, we conceive as quantum systems only those pieces of reality non-interacting with other pieces. On this basis, and by adopting an algebraic perspective, we define a *quantum system* S as a pair (O, H) such that (i) O is a space of self-adjoint operators on a Hilbert space H, representing the *observables* of the system, (ii) $H \in \mathrm{O}$ is the time-independent *Hamiltonian* of the system, and (iii) if $\rho_0 \in \mathrm{O}'$ (where O' is the dual space of O) is the *initial state* of S, ρ_0 evolves according to the Schrödinger equation in its von Neumann version.

Of course, any quantum system can be decomposed in parts in many ways; however, not any decomposition will lead to parts which are, in turn, quantum systems. This will be the case only when there is no

interaction among the components and, then, the components' behaviors are dynamically independent to each other. On this basis, we say that a quantum system $S: (\mathrm{O}, H)$ with initial state $\rho_0 \in \mathrm{O}'$ is *composite* when it can be partitioned into two quantum systems $S_1: (\mathrm{O}_1, H_1)$ and $S_2: (\mathrm{O}_2, H_2)$ such that (i) $\mathrm{O} = \mathrm{O}_1 \otimes \mathrm{O}_2$, and (ii) $H = H_1 \otimes I_2 + I_1 \otimes H_2$ (where I_1 and I_2 are the identity operators in the corresponding tensor product spaces). In this case, the initial states of S_1 and S_2 are obtained as the partial traces $\rho_{01} = Tr_{(2)}\rho_0$ and $\rho_{02} = Tr_{(1)}\rho_0$, and we say that S_1 and S_2 are *subsystems* of the *composite system*, symbolized as $S = S_1 \cup S_2$.

It has to be emphasized that, although this definition of composite quantum system is completely general, the decomposition of a quantum system into subsystems is not always possible: it may happen that there is no partition of the whole S such that the total Hamiltonian can be expressed as a sum of component Hamiltonians. In this case, the quantum system is not composite, and we call it *elemental*.

On the other hand, given any two quantum systems $S_1: (\mathrm{O}_1, H_1)$ and $S_2: (\mathrm{O}_2, H_2)$, with initial states $\rho_{01} \in \mathrm{O}_1'$ and $\rho_{02} \in \mathrm{O}_2'$ respectively, we can always define a quantum system $S: (\mathrm{O}, H)$ with initial state $\rho_0 \in \mathrm{O}'$ such that (i) $\mathrm{O} = \mathrm{O}_1 \otimes \mathrm{O}_2$, (ii) $H = H_1 \otimes I_2 + I_1 \otimes H_2 + H_{12}^{\mathrm{int}}$, where H_{12}^{int} is the *interaction Hamiltonian*, and (iii) $\rho_0 = \rho_{01} \otimes \rho_{02} \in \mathrm{O}'$. In this case, the initial state ρ_0 of S and the initial states ρ_{01} of S_1 and ρ_{02} of S_2 are still related by a partial trace. However, when the two systems S_1 and S_2 interact with each other, $H_{12}^{\mathrm{int}} \neq 0$ and, therefore, ρ_{01} and ρ_{02} do not evolve according to the Schrödinger equation. This means that, strictly speaking, S_1 and S_2 are not subsystems of S but have to be considered as mere "parts" of S; we shall symbolize this fact as $S = S_1 + S_2$. Only in the particular case that $H_{12}^{\mathrm{int}} = 0$, S_1 and S_2 will evolve unitarily, and they will properly be subsystems of S.

Summing up, the modal-Hamiltonian interpretation supplies a precise criterion for distinguishing between elemental and composite systems, and such a criterion is based on the system's Hamiltonian.

3.b. The selection of the preferred context

The subtler point in any realist interpretation of quantum mechanics is the selection of the preferred context, that is, the set of the definite-valued observables of the system. In the modal-Hamiltonian interpretation this selection is based on the *actualization rule*, which defines, among all the observables of the system, those that acquire actual, and not merely possible, values.

As it is well known, quantum mechanics is covariant under the Galilean transformations, represented by the Galilean group with its ten symmetry generators,[3] which correspond to the fundamental dynamical magnitudes of the theory (*cfr.* Ballentine 1998): the energy H (time-displacement), the momentum $P = \left(P_x, P_y, P_z\right)$ (space-displacement), the position $Q = \left(Q_x, Q_y, Q_z\right)$ (boost-transformation: boost generator $G = mQ$), the total angular momentum $J = \left(J_x, J_y, J_z\right)$ (space-rotation).[4] Since we have defined a quantum system as a closed system, its energy is constant in time and, then, the Hamiltonian H is time-independent: H is always invariant under time-displacement. Nevertheless, in a given quantum system, H may have the remaining symmetries or not. To say that the Hamiltonian is symmetric or invariant under a certain continuous transformation means that

$$e^{iKs} H e^{-iKs} = H \text{, then } \left[H, K\right] = 0 \qquad (3\text{-}1)$$

This implies that, when H is invariant under a certain continuous transformation, the generator of that transformation is a *constant of motion* of the system: each symmetry of the Hamiltonian defines a conserved quantity. For instance, the invariance of H under space-displacement in any direction implies that the momentum P is a constant of motion; the

[3] Strictly, we should speak of the central extension of the Galilean group.

[4] Strictly speaking, the generators are proportional to these observables with a factor $1/\hbar$; for instance, the time-displacement generator is $K_\tau = H/\hbar$. For simplicity, here we employ units that make $\hbar = 1$.

invariance of H under space-rotation in any direction implies that the total angular momentum J is a constant of motion. If, on the contrary, H is invariant under space-displacement only in one direction, say x, only the component P_x of P is a constant of motion.

Moreover, each symmetry of the Hamiltonian leads to an energy degeneracy. In fact, if H is invariant under a continuous transformation with generator K, we can write

$$K\,H\,|n\rangle = K\omega_n\,|n\rangle \Rightarrow H\,K\,|n\rangle = \omega_n K\,|n\rangle \qquad (3\text{-}2)$$

This means that any vector $K|n\rangle$, obtained by applying the operator K on the eigenvector $|n\rangle$, is also an eigenvector of H with the same eigenvalue (*cfr.* Meijer and Bauer 2004). As a consequence, H can be expressed as

$$H = \sum_n \omega_n P_n \qquad (3\text{-}3)$$

where P_n is the projector operator onto the subspace spanned by the degenerate eigenvectors corresponding to ω_n.

Now we have all the conceptual elements necessary to present our actualization rule. The basic idea is that the Hamiltonian of the system defines actualization; therefore, any observable that does not have the symmetries of the Hamiltonian cannot acquire an actual value, since its actualization would break the symmetry of the system in an arbitrary way. Precisely, given an elemental quantum system $S: (O, H)$, the preferred context consists of H and the observables commuting with H and having, at least, the same symmetries as H. Let us see how the rule works in different cases:

(a) The Hamiltonian H does not have symmetries (it is non-degenerate):

$$H\,|n\rangle = \omega_n\,|n\rangle \text{ with } \omega_n \neq \omega_{n'} \qquad (3\text{-}4)$$

where $\{|n\rangle\}$ is a basis of the Hilbert space H. In this case, the definite-valued observables of the system are H and all the observables commuting with H.

(b) If the Hamiltonian H has certain symmetries that lead to energy degeneracy, it can be written as

$$H|n,i_n\rangle = \omega_n|n,i_n\rangle \Rightarrow H = \sum_n \omega_n \sum_{i_n} |n,i_n\rangle\langle n,i_n| = \sum_n \omega_n P_n \quad (3\text{-}5)$$

where $\omega_n \neq \omega_{n'}$ and the index i_n expresses the degeneracy of the energy eigenvalue ω_n. Any observable of the form

$$A = \sum_{n,i_n} a_n |n,i_n\rangle\langle n,i_n| = \sum_n a_n \sum_{i_n} |n,i_n\rangle\langle n,i_n| = \sum_n a_n P_n \quad (3\text{-}6)$$

is definite-valued, since $[A,H]=0$ and A has, at least, the same degeneracy as H. On the contrary, any observable of the form

$$B = \sum_{n,i_n} b_{n,i_n} |n,i_n\rangle\langle n,i_n| \quad (3\text{-}7)$$

in spite of commuting with H, does not acquire a definite value, since the actualization of a particular eigenvalue b_{n,i_n} of B would discriminate among the degenerate eigenvectors corresponding to a single eigenvalue ω_n of H and, in this way, would introduce in the system an asymmetry not contained in the Hamiltonian.

In certain modal interpretations (*e.g.* Kochen 1985, Dieks 1988, Vermaas and Dieks 1995), the preferred context depends on the instantaneous state of the system, which continuously changes in time; this leads to the need of accounting for the dynamics of actual properties (*cfr.* Vermaas 1996). In our interpretation, on the contrary, this step is unnecessary because the dynamics of actual properties is trivial. In fact, since in any case the definite-valued observables commute with the Hamiltonian, they are constants of motion of the system: in spite of the fact that probabilities are continuously evolving, the set of definite-valued observables is time-independent and, thus, completely robust (for the

ontological picture emerging from this interpretation, *cfr.* Lombardi and Castagnino 2008, Section 8).

This actualization rule has been applied to many well-known physical situations (hydrogen atom, Zeeman effect, fine structure, etc.), leading to results consistent with experimental evidence. Moreover, it has proved to be effective for solving the measurement problem, both in its ideal and its non-ideal versions.

3.c. The quantum measurement problem

According to the modal-Hamiltonian interpretation, a quantum single measurement[5] is a three-stage process. In the first stage, the system S to be measured –represented in the Hilbert space H_S and with Hamiltonian H_S – and the measuring apparatus M –represented in the Hilbert space H_M and with Hamiltonian H_M – do not interact. During the second stage, an interaction Hamiltonian H_{SM}^{int} introduces the correlation between the eigenstates $|a_i\rangle$ of an observable A of S and the eigenstates $|p_i\rangle$ of a pointer observable P of M. In the third stage the interaction ends, and the whole system becomes a composite system $S \cup M$ with a Hamiltonian

$$H = H_S \otimes I_M + I_S \otimes H_M \tag{3-8}$$

and an initial state (see eq.(2-1))

$$|\psi_{SM}\rangle = \sum_i c_i |a_i\rangle \otimes |p_i\rangle \tag{3-9}$$

Although $|\psi_{SM}\rangle$ is an entangled state, since there is no interaction between the subsystems S and M, the actualization rule has to be applied to each one of them independently. In particular, when applied to M, the rule states that the definite-valued observables are the

[5] The difference between single measurement, frequency measurement and state measurement is explained in Lombardi and Castagnino 2008, Section 6.

Hamiltonian H_M and all the observables commuting with H_M and having, at least, the same symmetries –degeneracies– as H_M.

Of course, not any quantum process can be considered a quantum measurement. On the basis of the above description, the conditions for quantum measurements can be formulated:

(a) During a period Δt, S and M must interact through an interaction Hamiltonian $H_{SM}^{\text{int}} \neq 0$ intended to introduce a correlation between the observable A of S and the pointer P of M. The requirement of perfect correlation is not included as a defining condition of measurement, because the actualization rule explains the definite reading of the pointer P even in non-ideal measurements, that is, when the correlation is not perfect. In this case, the rule also accounts for the difference between *reliable* and *non-reliable* measurements (*cfr.* Lombardi and Castagnino 2008, Section 6).

(b) The measuring apparatus M has to be constructed in such a way that its pointer P (i) has macroscopically distinguishable eigenvalues, and (ii) commutes with the Hamiltonian H_M and has, at least, the same degeneracy as H_M. The condition $[P, H_M] = 0$ guarantees the stationarity of the eigenvectors of P, making the readings of the pointer possible.

This account of the quantum measurement has been used to explain how the initial –pure or mixed– state is reconstructed through measurement both in the ideal and in the non-ideal case, and has been successfully applied to the paradigmatic example of the Stern-Gerlach experiment, with perfect and non perfect correlation, and also in the case of an imperfect collimation of the incoming beam.

4. PUTTING THE PIECES TOGETHER

Harshman and Wickramasekara (2007a, 2007b) use the expression '*tensor product structure*' (TPS) to call any partition of a closed system S, represented in the Hilbert space $\mathrm{H} = \mathrm{H}_A \otimes \mathrm{H}_B$, into parts S_A and S_B represented in H_A and H_B respectively. They point out that any

quantum system admits a variety of TPSs, each one leading to a different entanglement between its parts. In the case of the TPS $\mathrm{H} = \mathrm{H}_A \otimes \mathrm{H}_B$, if the eigenvectors $\{|j\rangle\}$ of the observable $O_A \in \mathrm{H}_A \otimes \mathrm{H}_A$ span the space H_A and the eigenvectors $\{|k\rangle\}$ of the observable $O_B \in \mathrm{H}_B \otimes \mathrm{H}_B$ span the space H_B, then any state $|\varphi\rangle \in \mathrm{H}$ can be expressed as a linear combination of the $|j\rangle \otimes |k\rangle$,

$$|\varphi\rangle = \sum_{j,k} a_j b_k |j\rangle \otimes |k\rangle = \sum_i c_i |i\rangle \tag{4-1}$$

where $\{|i\rangle\}$ is a basis of H, eigenbasis of the complete set of commuting observables (CSCO) $\{O_A \otimes I_B, I_A \otimes O_B\}$, and $\sum |c_i|^2 = 1$. When the system S is in the state $|\varphi\rangle$, the entanglement between S_A and S_B is measured by the entropy of the entanglement $E_{AB}(\varphi)$, which is a function of the coefficients $|c_i|^2$,

$$E_{AB}(\varphi) = -\sum_i |c_i|^2 \, log_2 |c_i|^2 \tag{4-2}$$

If we choose a different tensor product structure TPS', $\mathrm{H} = \mathrm{H}_{A'} \otimes \mathrm{H}_{B'}$, the argument is analogous, and the state $|\varphi\rangle$ will be expressed as

$$|\varphi\rangle = \sum_{\beta,\chi} a'_\beta b'_\chi |\beta\rangle \otimes |\chi\rangle = \sum_\alpha c'_\alpha |\alpha\rangle \tag{4-3}$$

with its entropy of the entanglement

$$E_{A'B'}(\varphi) = -\sum_\alpha |c'_\alpha|^2 \, log_2 |c'_\alpha|^2 \tag{4-4}$$

where the basis $\{|\alpha\rangle\}$ of H is the eigenbasis of the CSCO $\{O_{A'} \otimes I_{B'}, I_{A'} \otimes O_{B'}\}$. According to the authors, "*the moral of the story is that the observables determine the TPS, which in turn determine the notions of separability and entanglement*" (Harshman and Wickramasekara 2007a, p. 3). In other words, separability and entanglement are TPS-dependent, and each TPS is defined by the observables chosen to identify the component

parts. This moral agrees with the conclusion drawn in Subsection 2.b: there are many ways of splitting the whole closed system into two open systems, and such a splitting is just the selection of a subset of relevant observables.

Since a CSCO of the form $\{O_A \otimes I_B, I_A \otimes O_B\}$ defines a TPS, a symmetry transformation of the TPS in general leads to another TPS. Given a symmetry transformation, a particular TPS may be invariant under the symmetry group of those transformations. Since entanglement depends on the TPS, if a TPS is symmetry-invariant with respect to a certain group of transformations, then the entanglement is also invariant with respect to that symmetry. As we have seen, a particular symmetry transformation is time-displacement, whose generator is the Hamiltonian of the system. Then, when a particular TPS is invariant under time-displacement, it is *dynamically invariant*, and the corresponding entanglement is also dynamically invariant. This is the case when the total Hamiltonian $H \in \mathrm{H} \otimes \mathrm{H}$ can be decomposed as

$$H = H_A \otimes I_B + I_A \otimes H_B \tag{4-5}$$

where $H_A \in \mathrm{H}_A \otimes \mathrm{H}_A$ is the Hamiltonian of S_A and $H_B \in \mathrm{H}_B \otimes \mathrm{H}_B$ is the Hamiltonian of S_B (*cfr.* Harshman and Wickramasekara 2007a, p. 5). In other words, the dynamically invariant TPS is the particular TPS for which the interaction Hamiltonian H_{AB}^{int} between S_A and S_B is zero.

It is not difficult to see that the concept of dynamically invariant TPS corresponds to the definition of composite system in the context of the modal-Hamiltonian interpretation. In fact, from this interpretative perspective, when there is no interaction between S_A and S_B, they are strictly subsystems of the composite system $S = S_A \cup S_B$. In this case, we have a robust notion of entanglement: although ρ_0 evolves in time, its entanglement is dynamically invariant. Moreover, this definition of composite system does not imply that the initial state ρ_0 of S is the tensor product $\rho_{0A} \otimes \rho_{0B}$: this factored or uncorrelated state is a very

special kind of state, used in practice to describe independently prepared systems (*cfr*. Ballentine 1998). On the contrary, in general the initial state is a correlated or entangled state $\rho_0 \in O'$; nevertheless, since there is no interaction between the subsystems S_A and S_B, $[H_A \otimes I_B, I_A \otimes H_B] = 0$ and, then,

$$exp[-iHt] = exp[-iH_A t]exp[-iH_B t] \tag{4-6}$$

Therefore,

$$\begin{aligned} &\rho_A(t) = Tr_{(B)}\rho(t) = Tr_{(B)}\left[e^{-iHt}\rho_0 e^{iHt}\right] = \\ &e^{iH_A t}\left[Tr_{(B)}\rho_0\right]e^{-iH_A t} = e^{iH_A t}\rho_{0A}e^{-iH_A t} \end{aligned} \tag{4-7}$$

$$\begin{aligned} &\rho_B(t) = Tr_{(A)}\rho(t) = Tr_{(A)}\left[e^{-iHt}\rho_0 e^{iHt}\right] = \\ &e^{iH_B t}\left[Tr_{(A)}\rho_0\right]e^{-iH_B t} = e^{iH_B t}\rho_{0B}e^{-iH_B t} \end{aligned} \tag{4-8}$$

This means that, in spite of the correlations, the subsystems S_A and S_B are *dynamically independent*: each one of them evolves under the action of its own Hamiltonian.

On the other hand, in a generic TPS, S_A and S_B interact ($H_{\text{int}} \neq 0$). In this case, since the time t_0 when the interaction begins, S_A and S_B are mere parts of the system S because the initial states $\rho_{0A} = Tr_{(B)}\rho_0$ and $\rho_{0B} = Tr_{(A)}\rho_0$ do not evolve unitarily according to the Schrödinger equation. This means that, strictly speaking, when $H_{AB}^{\text{int}} \neq 0$, S_A and S_B are not quantum systems.

In the light of these considerations, the conceptual difficulties of the decoherence program acquire a new reading, and the compatibility between the modal-Hamiltonian interpretation and the results of the decoherence program can be argued for.

4.a. A new reading of the preferred basis problem

In Subsection 2.a, I have addressed the problem of the preferred basis, and I have argued that the results obtained case by case by means of the predictability sieve might be explained by assuming that the preferred basis is defined by the total Hamiltonian of the whole closed system. Now we have the elements to rethink the problem from a perspective based on the central role played by the Hamiltonian.

Let us consider a closed system S_C where we distinguish two parts: the open "system" of interest S and its environment E, in such a way that $S_C = S + E$. According to Zurek, when S decoheres in interaction with E, the preferred basis, identified by the predictability sieve, turns out to depend on the relative strength of the system's self-Hamiltonian H_S and the interaction Hamiltonian H_{SE}^{int}. But now let us forget Zurek's criterion for a moment, and recall the defining requirement for the preferred basis, that is, for the set of the eigenvectors of a pointer observable P_S of S: P_S has to remain unperturbed during the time evolution in spite of the interaction between S and its environment. If we call H_S the Hilbert space of S and H_E the Hilbert space of E, then $P_S \in \mathrm{H}_S \otimes \mathrm{H}_S$; but the same property can be represented in the total von Neumann-Liouville $\mathrm{L} = \mathrm{H} \otimes \mathrm{H}$ space of S_C, where $\mathrm{H} = \mathrm{H}_S \otimes \mathrm{H}_E$, as

$$P = P_S \otimes I_E \in \mathrm{H} \otimes \mathrm{H} \tag{4-9}$$

On the other hand, the total Hamiltonian H of the –closed– system S_C reads

$$H = H_S \otimes I_E + I_S \otimes H_E + H_{SE}^{\text{int}} \in \mathrm{H} \otimes \mathrm{H} \tag{4-10}$$

According to plain quantum mechanics, the requirement that the pointer remains unperturbed during the time evolution amounts to the requirement that P be a constant of motion of the system, that is,

$$[P, H] = \left[P_S \otimes I_E, H_S \otimes I_E + I_S \otimes H_E + H_{SE}^{\text{int}}\right] = 0 \quad (4\text{-}11)$$

But since always $\left[P_S \otimes I_E, I_S \otimes H_E\right] = 0$, then the stability requirement for the pointer observable becomes that it commute with the Hamiltonian $H_S \otimes I_E + H_{SE}^{\text{int}}$, where the self-Hamiltonian of the environment is not included:

$$\left[P, H_S \otimes I_E + H_{SE}^{\text{int}}\right] = 0 \quad (4\text{-}12)$$

This argument clearly shows that the condition introduced in the first papers on decoherence, that is, that the pointer commutes with the interaction Hamiltonian, is a particular case which holds only when the self-Hamiltonian of S can be disregarded. And it is also clear that the three regimes distinguished by Zurek as the result of the application of the predictability sieve to a number of models (see Subsection 2.a) turn out to be the three particular cases of condition (4-12), and can be redescribed in terms of that condition:

- When $H_S \otimes I_E \gg H_{SE}^{\text{int}}$, the self-Hamiltonian of S can be neglected, and then $\left[P, H_{SE}^{\text{int}}\right] = 0$. Therefore, the preferred basis is defined by the interaction Hamiltonian H_{SE}^{int}.
- When $H_S \otimes I_E \approx H_{SE}^{\text{int}}$, neither the self-Hamiltonian of S nor the interaction with the environment are clearly dominant. In this case, the preferred basis is defined by condition (4-12).
- When $H_S \otimes I_E \ll H_{SE}^{\text{int}}$, the dynamics is dominated by self-Hamiltonian of S and, then, $\left[P, H_S \otimes I_E\right] = \left[P_S, H_S\right] = 0$. Therefore, the preferred states are simply the eigenstates of H_S.

Therefore, the fact, noted by Schlosshauer (2004, p. 1280), that many systems are typically found in energy eigenstates although the interaction Hamiltonian depends on an observable different than energy, far from being surprising, necessarily results from the requirement of stability for the preferred basis and from the central role that the total Hamiltonian and its components play in meeting this requirement.

4.b. Dissolving the defining systems problem

In Subsection 2.b, I have claimed that the problem of defining the systems involved in decoherence is not as serious as Zurek himself supposed: the identification of the system of interest and its environment depends on the subset of the observables considered relevant in each particular situation. Now we have better conceptual elements to argue for this conclusion.

As we have seen, any quantum system S_C can be partitioned according to different TPSs, each one leading to a different entanglement between the component parts. From a dynamical viewpoint, the only privileged TPSs are those dynamically invariant: each component unitarily evolves independently governed by its own Hamiltonian, and entanglement is constant with time. The characterization of the dynamically invariant TPS, which depends on the particular form of the total Hamiltonian, corresponds to the definition of subsystems in the modal-Hamiltonian interpretation. Therefore, in this conceptual framework we can say that the subsystems of a quantum system, resulting from a dynamically invariant TPS, never decohere to the extent that they evolve unitarily according to the Schrödinger equation.

On the other hand, the non-dynamically invariant TPSs correspond to different partitions of the whole quantum system. As Harshman and Wickramasekara (2007a, 2007b) stress, each particular TPS $\mathrm{H} = \mathrm{H}_A \otimes \mathrm{H}_B$ is determined by the observables that define the bases of the Hilbert spaces H_A and H_B. In the language of the modal-Hamiltonian interpretation, each partition expresses the split of the whole space of observables O into spaces O_A and O_B such that $\mathrm{O} = \mathrm{O}_A \otimes \mathrm{O}_B$. It is clear that, in this non-dynamically invariant case, the parts S_A and S_B evolve non-unitarily as a consequence of the interaction and, eventually, they may decohere. But the point to stress here is that there is no privileged non-dynamically invariant splitting: each partition into S_A and S_B is just a way of selecting the spaces of observables O_A and O_B. And in each partition, there is no essential

criterion for identifying the "open system" and its "environment": we are free of considering S_A as "the system" and S_B as "the environment", or vice versa. Therefore, the identification of the system of interest S and the environment E amounts to the selection of the observables considered relevant in each situation.

This argument shows that the splitting of the whole system into an open "system" and its environment is just the adoption of a descriptive perspective. There are many ways of performing that splitting: each decomposition represents a decision about which degrees of freedom are relevant and which can be disregarded in any case. Since there is no privileged or essential decomposition, there is no need of an unequivocal criterion for deciding where to place the cut between "the" system and "the" environment. Decoherence is not a yes-or-not process, but a phenomenon *relative* to the chosen partition of the whole quantum system, and in each case the particular form of the resulting Hamiltonians determine whether the so defined "system of interest" decoheres or not.

Summing up, quantum mechanics is a theory whose dynamical postulate refers to closed systems: the time-behavior of the parts resulting from different partitions of the closed system has to be inferred from that postulate. This means that the total Hamiltonian rules the dynamical evolution of the closed system, and the time-behavior of its parts depends on the form in which the Hamiltonian is decomposed in each particular partition. If all these elements are taken into account, Zurek's "looming big problem" does not constitute a real threat to the decoherence program: the supposed challenge dissolves once the relative nature of decoherence is admitted.

The irony of this story is that the problem of defining the systems involved in decoherence is the consequence of what has been considered to be the main advantage of the decoherence program: its "open-system" perspective. This perspective is what deprives the program of a precise definition of systems and subsystems, and of a clear distinction between subsystems and parts of a quantum system. As I have shown, those

precise definitions can be given when the Hamiltonian of the system is seriously taken into account.

4.c. Two compatible accounts of quantum measurement

As we have seen in Subsection 3.c, the actualization rule of the modal-Hamiltonian interpretation explains the definite reading of the pointer P of the measuring apparatus M by considering that P commutes with the Hamiltonian H_M of M and does not break the degeneracies of such a Hamiltonian. This account of the quantum measurement seems to be at odds with the explanation given by the decoherence program, according to which the decoherence of the measuring apparatus in interaction with its environment is what causes the apparent "collapse" that suppresses superpositions. In fact, in the modal-Hamiltonian interpretation, the environment is absent: M is a closed quantum system unitarily evolving with its own Hamiltonian H_M. Moreover, this seems to flagrantly contradict the fact that real measuring apparatuses are never isolated, but they interact significantly with their environments. However, this apparent conflict vanishes when the physical situation is considered in detail from a "closed-system" perspective.

If measurement is described in terms of the quantum –and, therefore, closed– systems involved in the process, the measuring apparatus M has not to be considered as an open macroscopic device surrounded by a "bath" of particles in interaction with it. In the third stage of the measurement process, the measuring apparatus is the entire quantum system that interacted with the system S in the second stage: it is this system what has to have an observable pointer commuting with its Hamiltonian H_M. On this basis, we can now analyze the elements participating in the process as described in the framework of the modal-Hamiltonian interpretation.

- The system M –*e.g.*, the device *and* the bath of particles– is certainly a macroscopic system, whose Hamiltonian is the result of the interaction

among a huge number of degrees of freedom. Since, in general, symmetries are broken by interactions, the symmetry of a Hamiltonian decreases with the complexity of the system. Then, a macroscopic system having a Hamiltonian with symmetries is a highly exceptional situation: in the generic case, the energy is the only constant of motion of the macroscopic system. As a consequence, in realistic measurement situations, H_M is non-degenerate,

$$H_M \left|\omega_n\right\rangle = \omega_n \left|\omega_n\right\rangle \text{ with } \omega_n \neq \omega_{n'} \tag{4-13}$$

and, therefore, $\left\{\left|\omega_n\right\rangle\right\}$ is a basis of the Hilbert space H_M of M. This means that, when $\left[P, H_M\right] = 0$, we can guarantee that P has, at least, the same degeneracies as H_M because H_M is non-degenerate.

- The pointer P cannot have such a huge number of different eigenvalues as H_M, because the experimental physicist must be able to discriminate among them (for instance, in the Stern-Gerlach experiment the pointer has three eigenvalues). This means that P is a "collective" observable of M (*cfr.* Omnés 1994, 1999), that is, a highly degenerate observable that does not "see" the vast majority of the degrees of freedom of M:

$$P = \sum_n p_n P_n \tag{4-14}$$

where the set $\left\{P_n\right\}$ of the eigenprojectors of P spans the Hilbert space H_M of M. In other words, the eigenprojectors of P introduce a sort of "coarse-graining" into the Hilbert space H_M. Therefore, if the Hamiltonian H_M is non-degenerate (see eq.(4-13)), the condition $\left[P, H_M\right] = 0$ implies that P can be expressed in terms of the energy eigenbasis $\left\{\left|\omega_n\right\rangle\right\}$ as

$$P = \sum_n p_n P_n = \sum_n p_n \sum_{i_n} \left|\omega_{i_n}\right\rangle\left\langle\omega_{i_n}\right| \tag{4-15}$$

This expression shows that, since $p_n \neq p_{n'}$, P has more degeneracies than H_M.

- The requirement $[P, H_M] = 0$, far from being an *ad hoc* condition necessary to apply the actualization rule, has a clear physical meaning: it is essential to preserve the stationary behavior of P during the third stage of the measurement process. If this requirement did not hold because of the uncontrollable interaction among the microscopic degrees of freedom of the macroscopic device or between the macroscopic device and an external "bath", the reading of P would constantly change and measurement would be impossible. It is precisely at this point that the skills of the experimental physicist play a central role: he has to be capable of designing a complete experimental arrangement such that the uncontrollable degrees of freedom of M –internal or external to the macroscopic device– do not affect significantly the stationary character of the pointer. This goal may be achieved by many different technological means; but, in any case, measurement has to be a controlled situation where the reading of a stable pointer can be obtained.

Now let us analyze the third stage of the measurement process from the viewpoint of the decoherence program. In this context, during the third stage the "measuring apparatus" does no longer interact with the measured system S but interacts with the "environment". If we call, as before, M the whole system that interacted with S in the second stage but remains closed during the third stage, the question is how to identify the open parts of M to be conceived as "measuring apparatus" and as "environment". This is a legitimate question because, as we have seen, a whole closed system may be partitioned in many different ways, none of them more "essential" than the others.

The usual assumption is to consider the macroscopic, material device D built for measurement as "the apparatus", and the bath B of the particles scattering off D as "the environment"; then, $M = D + B$ is the closed system resulting from the interaction between D and B. From this usual position, it is supposed that D is the open system that

decoheres: the reduced density operator $\rho_r^D(t)$ of D should converge to a final time-independent ρ_r^D, diagonal in the preferred basis of D, that is, of its Hilbert space H_D, and the pointer P should define such a basis. However, even if apparently "natural", this is not the best choice for the splitting of M, since it does not take into account the environment *internal* to the device D. In fact, being a macroscopic body, D also has a huge number of degrees of freedom, which have to be "coarse-grained" by P if it is to play the role of the pointer. In other words, since the pointer P must have a small number of different eigenvalues to allow the experimenter to discriminate among them, P is a highly degenerate observable on the Hilbert space H_D of D and, as a consequence, it does not define a *basis* of H_D.

When we recall that the only univocally definable entity is the –closed– quantum system, and that a quantum system can be partitioned in many, equally legitimate manners, the system M can be split in a theoretically best founded way in the measurement case. Let us recall that the pointer P is the observable whose eigenvectors became correlated with the eigenvectors of an observable of the measured system during the second stage of the process, and that the interaction in that stage was deliberately designed to introduce such a correlation. So, if we want that during the third stage P really defines a *basis*, the open "measuring apparatus" A must be the part of M corresponding to the Hilbert space H_A where the pointer is non-degenerate; if we call P_A the pointer belonging to $\mathrm{H}_A \otimes \mathrm{H}_A$, it reads

$$P_A = \sum_n p_n \left| p_n \right\rangle \left\langle p_n \right| \tag{4-16}$$

where $\left\{ \left| p_n \right\rangle \right\}$ is a basis of H_A. Then, the relevant partition (the relevant TPS) is $\mathrm{H}_M \equiv \mathrm{H}_A \otimes \mathrm{H}_E$, where H_E is the Hilbert space of the "environment" E. If $\left\{ \left| e_i \right\rangle \right\}$ is a basis of H_E, the pointer acting on H_M = can be expressed as a highly degenerate observable:

$$P = P_A \otimes I_E = \left(\sum_n p_n \left| p_n \right\rangle \left\langle p_n \right| \right) \otimes \left(\sum_m \left| e_m \right\rangle \left\langle e_m \right| \right) = \sum_n p_n \sum_m \left| p_n \right\rangle \otimes \left| e_m \right\rangle \left\langle p_n \right| \otimes \left\langle e_m \right| = \sum_n p_n P_n \tag{4-17}$$

This agrees with the features of P in the modal-Hamiltonian framework: P introduces a sort of "coarse-graining" into the Hilbert space H_M (compare eq.(4-17) with eq.(4-14)). The many degrees of freedom corresponding to the degeneracies of P in H_M play the role of the "environment", composed by the microscopic degrees of freedom of the device D –internal environment– and the degrees of freedom of the bath B –external environment–.

Now we can introduce the condition for the stability of the pointer. As we have discussed in Subsection 4.a, if the total Hamiltonian of $M = A + E$ is

$$H_M = H_A \otimes I_E + I_A \otimes H_E + H_{AE}^{\text{int}} \tag{4-18}$$

then the self-Hamiltonian H_E of the environment does not affect the behavior of $P = P_A \otimes I_E$, since $\left[P_A \otimes I_E , I_A \otimes H_E \right] = 0$. Therefore, the stability condition results (compare with eq.(4-12)):

$$\left[P , H_A \otimes I_E + H_{AE}^{\text{int}} \right] = 0 \tag{4-19}$$

When decoherence is viewed from this perspective, Zurek's "first regime" for the selection of the preferred basis (see Subsection 2.a) can be justified on general grounds. According to Zurek, the first regime is the quantum measurement situation, where the self-Hamiltonian of the apparatus system can be neglected and the evolution is completely dominated by the interaction Hamiltonian: this means that $H_A \otimes I_E \ll H_{AE}^{\text{int}}$ (see Subsection 4.a). If the apparatus is now conceived as the part of the system M "viewed" by the pointer P, and the environment carries over

almost all the degrees of freedom of M, it seems reasonable to suppose that, in general, the Hamiltonian corresponding to the interaction with that huge number of degrees of freedom is much greater than the self-Hamiltonian of the "small" part defined by the pointer. Therefore, the condition $H_A \otimes I_E \square H_{AE}^{\text{int}}$ leading to the "first regime" turns out to have a physical justification.

On the basis of the above considerations, it is clear that both accounts of quantum measurement, the one supplied by the modal-Hamiltonian interpretation and the one given by the decoherence program, are not in conflict with each other. The stability condition for the pointer is the same in both cases: for the modal-Hamiltonian interpretation, it reads

$$\left[P, H_M\right] = 0 \tag{4-20}$$

which, for a pointer P highly degenerate in the –internal and external– degrees of freedom of the environment, is equivalent to the condition in the decoherence context,

$$\left[P, H_A \otimes I_E + H_{AE}^{\text{int}}\right] = 0 \tag{4-21}$$

and almost equivalent to the condition for the measurement regime,

$$\left[P, H_{AE}^{\text{int}}\right] = 0 \tag{4-22}$$

Moreover, the observables selected by decoherence and by the actualization rule also agree:

- The classical-like states einselected by the interaction with the environment, according to the decoherence program, are the eigenvectors of the pointer (the elements of the preferred basis).
- According to the modal-Hamiltonian interpretation, the pointer is a definite-valued observable belonging to the preferred context.

As a consequence, the seeming conflict between both approaches only reflects the different viewpoints adopted by each for describing the same physical situation: a closed-system perspective in the modal-Hamiltonian approach, and an open-system perspective in the decoherence approach.

5. CONCLUSIONS

The crucial role played by the Hamiltonian in mechanics, both classical and quantum, is well-known: as a two-faced Janus, it represents the conserved magnitude of the system and, simultaneously, it governs the time-evolution. So, it should not be surprising that, when the Hamiltonian is seriously taken into account, it also proves to be relevant in the interpretation of quantum mechanics and in the understanding of the phenomenon of decoherence.

In this paper, conceptual difficulties of the decoherence program and matters of the interpretation of quantum mechanics were brought under a comprehensive framework. From this perspective, I have argued that (i) the challenges that still threaten the decoherence program can be overcome on the basis of an interpretation that defines with precision the concepts of quantum system and of quantum subsystem, and (ii) the modal-Hamiltonian interpretation supplies that definition and gives an explanation of quantum measurement compatible with the account in terms of decoherence.

The agreement between the modal-Hamiltonian interpretation and the environment-induced decoherence approach contributes to our understanding of the phenomenon of decoherence and, at the same time, may count for the adequacy of the interpretation. The purpose of this work has been to point out that those conclusions can only be drawn when the Hamiltonian is given a leading role both in interpretation and in decoherence.

Acknowledgements: I want to thank all the members of the Buenos Aires Group on Philosophy and Foundations of Physics and Chemistry,

to which I belong, for supplying me the necessary intellectual context to develop these ideas. In particular, I am grateful to Mario Castagnino, Sebastian Fortin and Juan Sebastian Ardenghi for their patient reading of the manuscript and their stimulating comments. I am also specially indebted to Roberto Torretti: Section 4.c was written as the direct result of his clever observations. Nevertheless, any weakness of the paper is my own responsibility. This work has been supported by grants of CONICET, ANPCyT, UBA and SADAF, Argentina.

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