

## Solving problems to learn concepts, how does it happen? A case for buoyancy

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Problem solving is a preferred activity teachers choose to help students learn concepts. At the same time, successful problem solving is widely regarded as a very good indicator of conceptual learning. Many studies have provided evidence that problem solving often improves students' chances of learning concepts. Still, the question remains relatively unexplored as to how this activity is useful to promote concept learning. In this study we explore this question in the setting of three university students solving a problem on hydrostatics, in which the concept of buoyancy is involved. We use coordination class theory to study how these students progress on their conceptual understanding. We were able to describe how this progress is related to contextual traits, as well as to students' particular epistemic stances. Finally, we discuss some implications for research and for teaching.

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### I. SOME RESULTS FROM RESEARCH IN PROBLEM SOLVING

It is safe to state that problem solving is a universally extended practice among physics teachers at all levels. It is also widely accepted that problem solving is particularly adequate to promote Physics concept learning. However, this association is based more on teachers' intuitive knowledge than on actual research results on how this learning takes place [1]. Although solving many problems does not guarantee that students will achieve the desired conceptual development, physicists do attain much of their conceptual understanding by solving a large number of problems. Thus, the question of how and when problem solving contributes to students' conceptual understanding is important for physics education research.

Although there are several studies that report progress in students' conceptual understanding after particular problem-solving tasks [2–7], they are focused on the (sometimes) successful results of these activities rather than on the learning process during them.

For example, Leonard *et al.* [5] use results from educational research to develop a framework for knowledge organization, and use it to create an instructional approach called *analysis-based problem solving*. It is designed to promote deep conceptual understanding and proficient solving abilities in students. The authors argue why such practices are potentially efficient in promoting conceptual understanding, but they do not actually explore concept learning as it occurs during those practices.

In a similar fashion, Foster [4] carries out a thorough study on the influence of a teaching strategy that explicitly includes problem-solving strategies, both on the final solving abilities of students and their understanding of physics. Results show that students taught with an explicit problem-solving strategy perform better on post-tests that assess their conceptual understanding.

Docktor *et al.* [7] describe the implementation of a framework for solving physics problems: Conceptual Problem Solving (CPS). This implementation, in operational terms, consists of engaging students in explicitly declaring physics principles, the justification for their use, and a consequent plan of action. The authors report that classroom discussions were promoted, that problem solutions, as measured in written assessments, were of higher quality, and that students also scored higher in conceptual measures.

Similarly, Gil and Martínez Torregrosa [3] describe a strategy for teaching problem solving which is based on the mechanisms of scientific inquiry, as occurs within the physics community. They report that students improve, among other operational abilities, their qualitative descriptions of physical phenomena.

These are just some examples of studies that, although different in settings and analytical frameworks, have two particular features in common. First, they show that problem solving is a potentially adequate activity to promote students' conceptual understanding. Second, they do not explore how students build that conceptual understanding *during* problem solving. Thus, the question remains as to *how* problem solving contributes to the desired conceptual learning in students.

### II. CONCEPTUAL KNOWLEDGE DURING PROBLEM SOLVING

In order to address the last question of the preceding section, we believe it is important to consider other studies

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that have focused on the conceptual knowledge used within problem-solving activities.

Almost twenty years ago, diSessa [8] made the point that understanding the *process* of conceptual change called for an account of the entities that were changing in that process. From this premise, besides developing an important theoretical contribution for the study of conceptual change (coordination class theory), he provides empirical evidence of the different ways a student coordinates the concept of force during an actual problem-solving task. The data reported come from a one-on-one interview with a student (*J*) who is addressing different questions related to objects falling, or being pushed, or slowing down when moving on real surfaces due to friction.

In 2002, Witmann showed that one can describe student reasoning in wave physics during problem solving, in terms of reasoning resources that are inappropriately linked together into an objectlike model. He used coordination class theory (developed by diSessa, [8]) to understand students' explanations consistent with a description of waves as objects rather than a description of waves as a propagating disturbance within a system.

Another example of studies addressing the details of conceptual dynamics during problem-solving activities is the work of Wagner in 2006 [9]. Although intended to address the problem of transfer, this work provides a thorough analysis of the conceptual progress that takes place throughout a number of problem-solving sessions in which the researcher holds one-on-one interviews with a student. The work details how this student, Maria (pseudonym) exhibits an evolution of her ideas on the law of large numbers.

Shortly after, in 2007, Parnafes [10] provided a detailed study of how pairs of students, working on problems that deal with oscillators, show evolving ideas of frequency and velocity. The main purpose of that work was to study the conceptual understanding of oscillation phenomena through the use of computational representations. The author focuses on the representational aspects involved in students' conceptual change, and contributes to coordination class theory by establishing how these kinds of representations constitute a support for concept learning.

In 2008, Levrini and diSessa [11] carry out a thorough and detailed analysis, describing how students' conceptual understanding of proper time changes during a single classroom episode when addressed from different definitions. In doing so, the authors show how the data analyzed are understandable from the point of view of coordination class theory, and how this theoretical approach highlights the process of comparing, contrasting, and conciliating students' different conceptual views of proper time.

In a study published in 2015, Sengupta *et al.* [12] address a similar problem to that of Parnafes [10]. These authors are interested in better understanding how a video game can be beneficial for the learning of linear momentum. They argue

Each of the following cases shows a different setting of a water container, a wooden block and a metal block on a scale. Compare the different readings on the scale.

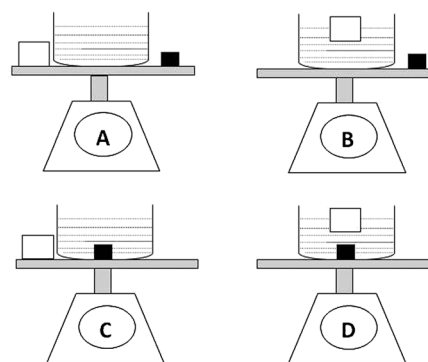


FIG. 1. The problem posed to students. Adapted from Leonard *et al.* [19].

that, in general, research has focused more on demonstrating the overall effectiveness of games than on analyzing the specific processes of conceptual change involved in students' learning. They study how conceptual understanding evolves as students play a video game that requires them to predict and produce deflections by applying different forces during different time intervals.

The reports cited in this section constitute the most immediate background for the present work. Although they vary in the concepts studied, the types of tasks students solve, and the research environments (interviews or actual classroom settings), they all share the same knowledge-in-pieces approach. This approach conceives knowledge to be made up of many fine-grained elements or pieces [13,14]. According to this ontology, learning is, in a nutshell, reorganizing those pieces in different ways when confronted with different situations. Also, it is an approach that has been successful to understand the substance of not only conceptual but also epistemic knowledge [15–18].

The present study shares this view on knowledge. We focus on understanding how contextual details that students attend to in the physical situation can orient the different coordinations of buoyancy, thus sustaining its learning. The case we present here makes a significant contribution to the previous work because (a) students make substantial and, in our opinion, impressive progress during a single problem-solving session of under 80 minutes; (b) this progress takes place in an interview setting, where no teacher is involved, and thus no instructional strategy is being carried out. However, there were certain nudges from the interviewer, which will be discussed in the data analysis.

We intend to guide the reader through the sequence that students follow in understanding buoyancy in the case of a cube sitting at the bottom of a water-filled container, which is in turn placed on a scale (see Fig. 1, case C). Students begin by assuming that buoyancy is zero, because there is no liquid below the body. Later, they decide that buoyancy

is equal to body weight because there is a thin water layer under it. After that, they understand the buoyant force is different from zero and less than body weight. Finally, they are able to arrive at the correct value for buoyancy ( $B = W - N$ ).

### III. THE STUDY

The problem-solving session in which the data for this study were collected was one of a series of interviews carried out with three university students. They were in the third semester of their career in physics. At the moment of the interview, they had already been instructed in hydrostatics. One of the first problems posed to them during those interviews, was the one in Fig. 1.

When conducting the first interview we encountered an unexpected reasoning: Students dismissed the influence of buoyancy on the body inside the fluid in situation C. The following is an excerpt from that interview when they first attempted a solution of the problem (Students will be referred to as A, M, and J):

*M: (to the interviewer) we say that the scale in A reads the same as in C, because there is no buoyancy.*

*A: we're saying that here, in C, buoyancy does not intervene, that the scale reads the weight of the three things because the body is flat against the bottom of the container... I mean, for a buoyant force to exist it has to be floating*

*J: (looking at his class notes) It says here that all bodies, totally or partially immersed in a fluid receive a buoyant force...*

*M: we never saw a case like that, a body all sunk touching the bottom*

*J: we never saw it against the bottom*

*M: if it's all flat against the bottom, then the only upward force is the normal force done by the surface, then there is no buoyancy pointing up*

*M: because there is no liquid there to... (makes upward gestures with her hands)*

A, M, and J were very confident of their ideas (so much so, that they even disregarded a piece of information that proceeds from an authoritative source: J's class notes). After that, they extended the reasoning to compare cases "B" and "D" and ultimately to state that all scales would show the same reading (a full analysis of the data can be found in Ref. [20]). At that point we realized that, although they were giving a "correct" answer, their reasoning called for a more profound analysis. Thus, in the second interview, the problem was posed to them again. We wanted to probe deeper into their conceptualization of buoyancy in that particular case. The analysis of the present work is done on the basis of the data collected at that second interview [21]. We call on coordination class theory [22], to show some critical details of how solving this particular problem impacts their conceptual understanding of buoyancy [23].

### IV. COORDINATION CLASS THEORY

Why coordination class theory? This theory presents two characteristics that render it particularly suitable for our purposes: (a) It conceives concepts as multiple elements associated in multiple possible ways, and thus, allows for multiple mechanisms for conceptual change. It is temporally and conceptually fine grained, and therefore it is suitable to study concept changes occurring within a single problem-solving session. (b) It posits the existence of elements called *read-out strategies* that account for people's perception, things they focus their attention on, and how these are linked to other pieces of conceptual knowledge. Thus, it is suitable to study the link between conceptual dynamics and the problem context.

This theory was developed to address important questions within the science education community, such as (i) what does it mean to "know" a concept? How can this vague idea be expressed in more precise terms so as to provide more useful insights for research?, and (ii) What does "conceptual change" mean?, what actually changes in students' minds and how is the process of that change? The theory attempts to provide more precise answers to these questions.

This theory considers knowledge to be a complex system consisting of multiple elements related in multiple ways. Within this perspective, learning a concept means coordinating many of those elements in many different ways. This perspective entails some nonintuitive considerations. It is not possible, for example, to set a precise limit between knowing and not knowing a concept. Learning a concept involves many different relations between multiple elements and thus, a person could exhibit a competent performance even though some of those relations could be technically incorrect. Moreover, it is reasonable to assume that people never attain a complete learning of a particular concept, or, in other words, that such a "complete" learning status is not specifiable. Nevertheless, this nonspecifiable status is not problematic, since our research interests are focused on intermediate states of learning.

A coordination class is a model for particular kinds of concepts, among which are scientific concepts. The main function of a coordination class is to allow people to read a particular kind of information out of situations in the world. This reading takes place through specific processes and strategies. Many of the difficulties students have are related to the circumstances in which they execute those particular strategies and processes. Examples of this relevant information are the existence, magnitude, and direction of a force, or the timelike distance in space-time between two events, in the case of proper time. In the remainder of this section we briefly describe the strategies and processes proposed by coordination class theory to obtain that information in any particular situation.

*The architecture of a coordination class* includes two elements: *read-out strategies* and an *inferential net*.

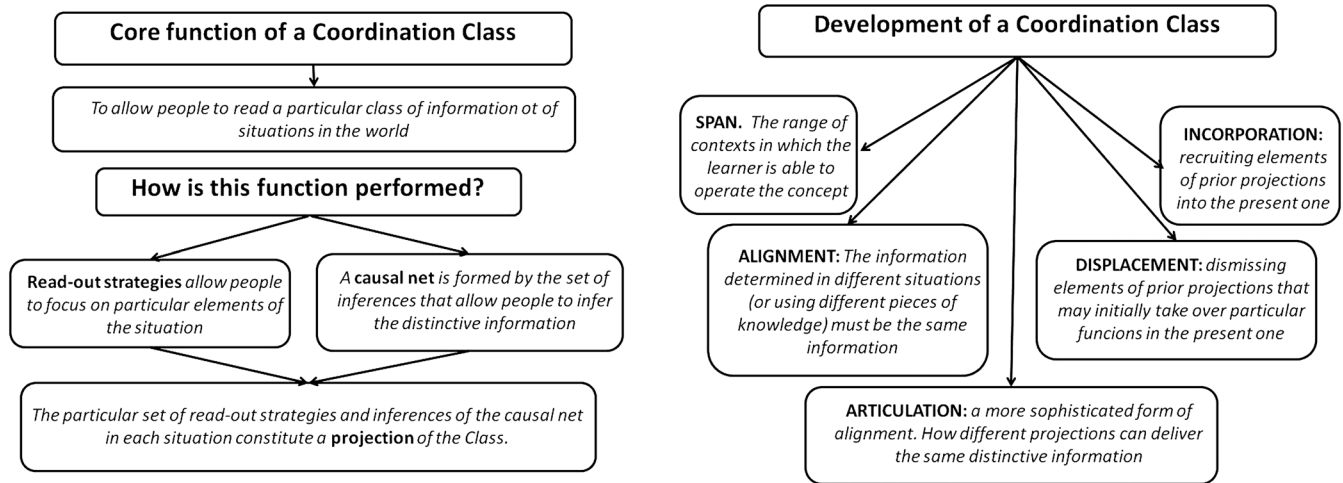


FIG. 2. Schematic representation of theoretical elements that constitute a coordination class.

Read-out strategies allow people to focus their attention on certain information. The inferential net is the total set of inferences people make to turn information read-outs into the required relevant information. The generic processes that build up a coordination class are *incorporation* and *displacement*. Incorporating is recruiting elements from prior conceptualizations into the partial encoding [24] of the new concept. Displacement consists of dismissing elements from prior conceptualizations that may initially and inappropriately take over the function of the coordination class in certain circumstances. Typically, students exhibit two characteristic difficulties in creating a new coordination class: the problem of *span* and of *alignment*. The problem of span refers to the ability (or lack thereof) to recruit and coordinate the elements of the class in a sufficiently large set of contexts in which the concept is relevant. According to the theory, “using” a concept in different contexts may well imply retrieving different pieces of knowledge and/or articulating them in different ways. The particular knowledge and the particular way it is coordinated in specific applications of the concept is called a concept *projection*. Alignment thus refers to the possibility of obtaining the same relevant information by means of different projections of the concept.

The theory also establishes a stronger form of alignment: *articulate alignment*, or *articulation*. Articulation occurs when students are not only able to determine the relevant information in different circumstances, but can also explicitly relate those different projections, noting differences and similarities between them. This stronger form of alignment is a metaconceptual process which is a natural extension of the theory in its original form [11]. Figure 2 shows a schematic diagram of these elements.

## V. RESEARCH QUESTIONS

- (i) What are the incorporations, displacements, projections, and articulations that account for students’

conceptual progress as they solve the problem of Fig. 1?

- (ii) What is the particular role of read-outs in that progress?

## VI. METHODOLOGY

As we explained at the beginning of this paper, the data analyzed correspond to a second interview with students *A*, *M*, and *J* when solving the problem of Fig. 1. This interview is part of a series of interviews with these students solving other problems and took place 90 days after the first one.

Students voluntarily responded to a call from the research group. Although researchers were not the instructors of these students, they were recognized not only as researchers, but also as teachers of their institution. By the time of the first interview, students had been instructed in hydrostatics, as typically represented by textbooks such as Serway’s [25] or Sears and Zemansky’s [26]. All three students were in the third semester of their career and to that moment, showed an average performance, representative of their cohort; that is, they regularly attended class and had passed 80% of their courses so far.

The researcher and interviewer (first author) was in charge of posing the problem situation and interrupted students to encourage debates and to make students’ reasoning more explicit as they searched for answers. This was done using different types of interventions. These interventions varied from more neutral to more perturbative ones. Neutral interventions were, for example, asking for clarification on their points of view, nodding to their expressions, or simply expressing interest in their thoughts. More perturbative interventions included recalling prior statements or focusing on details that were not explicitly attended to. Students were, most of the time, very committed to their reasoning process and lively discussions took place. Interviews were video recorded and then transcribed.

TABLE I. Operational definitions for read-out strategies and elements of the inferential net.

Operational criteria to interpret data in students' verbalizations		
Knowledge element	Operational criteria	Some examples
Read-out strategies	They refer to specific traits in objects in the situation	<i>there is no liquid underneath the metal cube</i>
	They are directly read from the context, not questioned or mediated by any other reasoning	<i>there is liquid underneath the ball</i>
	They refer to physical traits, such as size, shape, spatial distribution, etc.	<i>both bodies (cube and sphere) displace liquid, regardless of their shape.</i>
	They do not involve abstractions, as concepts or other physical principles.	<i>the surfaces are totally smooth</i>
Elements of the inferential net	They are essential to start off an inferential net	<i>there is no water under the cube</i> <i>the "extra" layer of water at the top of the fluid</i>
	They are usually expressed in the form of if-then statements	<i>buoyancy doesn't depend on shape... so...</i>
	They involve, or are linked to, abstract elements such as concepts or physical laws.	<i>there should be a buoyant force</i>
	The inference chain is directed to the goal of producing a concept-distinctive information.	<i>because of this extra volume of water up here the pressure down here is going to be larger</i>

In order to provide a guide for the reader as to how different "pieces of knowledge" are interpreted either as a read-out strategy or as a bit of an inferential net, different criteria are summarized in Table I.

## VII. RESULTS AND ANALYSIS

Two ideas arose during the first interview that went unquestioned by the students: the cube at the bottom of the water-filled container receives no buoyant force; and all four scales show the same reading. In this second interview we revisited the same situation and asked them to consider it once again. When they tried to compare cases A and C (see Fig. 1), they stopped to understand what the scale reads in case C. This doubt generated an extended discussion (80 minutes) during which we were able to observe important changes in their conceptualization of buoyancy. Although these changes are observed throughout the complete interview; we will show four snippets ( $S_i$ ) that are representative of them in subsections A–D. They are continuous sequences of students' speech.

### A. There is no buoyant force at all

$S_1$  [3:10 min]: "Assuming there is no liquid between the small cube and the bottom of the container?"

- (1) A: *and we said like... buoyancy does not do anything to it because it is completely touching the bottom side of the beaker*
- (2) J: *there's no liquid under it*
- (3) A: *so the scale reading is the normal force that the bottom does on it, and its magnitude is the same as its weight. (J agrees)*
- (4) J: *assuming there is no liquid between the cube and the bottom*

- (5) A: *that "ideal" stuff they always tell us that happen! (laughter)*
- (6) Int.: *what if I replace that little cube for a little ball of the same material, and the same volume, just change the shape... in that case, is your answer the same as for the cube?*
- (7) J: *well, in that case it has water underneath*
- (8) Int.: *what does it mean "it has water underneath"?*
- (9) J: *because here, with the cube, I'm assuming that it is completely in contact with the base of the container, then there is no water between the cube and the container*
- (10) Int.: *ok, so, you mean, the base of the cube is dry?*
- (11) J: (smiles in a gesture that indicates that appreciation was an exaggeration) ... *yeah...*
- (12) A: *I mean, ... ideally... (ironic tone)*
- (13) M: *I don't think shape has nothing to do with it, 'cause we never cared about shape to compute the magnitude of buoyancy*
- (14) A: *yeah... I mean*
- (15) J: *sure, buoyancy doesn't depend on shape, just volume, so in this case (cube at the bottom) there should be a buoyant force...*
- (16) A: (confused)... *so it would be the same.... the same as here.... (comparing the case of the ball and of the cube)*
- (17) J: *careful, 'cause... remember when we saw why there is buoyancy, it was because the volume of water displaced was held by the water underneath... so that generates buoyancy... so it does matter if there is water underneath or not*

Three distinct projections can be identified in this snippet. A very particular read-out strategy seems to have triggered

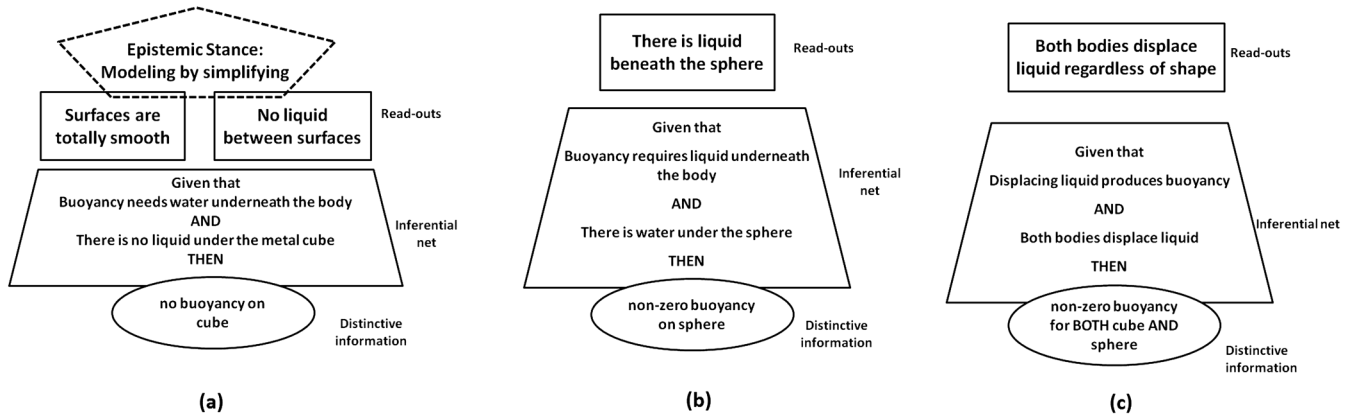


FIG. 3. Schematic diagrams of the three projections reported in  $S_1$ . (a) No buoyancy on sunk cube, (b) nonzero buoyancy for sunk sphere, and (c) nonzero buoyancy for either sunk body.

the first of these projections: *there is no liquid underneath the metal cube* and, thus, they infer that buoyancy is zero (turns 1, 2, 4, and 9). On the other hand, another read-out strategy triggers the second projection: *there is liquid underneath the ball*, and therefore, there is a nonzero buoyant force (turn 7). A third projection momentarily puts the previous two on hold. It is triggered by a read-out that indicates that *both bodies* (cube and sphere) *displace liquid, regardless of their shape*. Since the value of the buoyant force does not depend on shape, but only on displaced volume, then both bodies are affected by the same buoyant force. (turns 13, 14, 15, 16).

In turn 17, *J* is explicit in his attempt to articulate projections 1 (buoyancy is zero on the cube) and 2 (buoyancy is nonzero for the sphere). He understands, at the same time, that buoyancy is related to the volume of liquid displaced, but there is also an extra condition for this to be true: there must be liquid *underneath* the body.

Figure 3 shows a scheme of these projections. As shown in Fig. 3(a), the read-out *there is no water under the cube*, is activated together with another one: *the surfaces are totally smooth* (turns 1, 9). Both read-outs are consistent with the problem figure, in which the cube's bottom face is in complete contact with the bottom of the container. However, we also believe that the activation of these read-outs is linked to a particular epistemic stance, or idea, that is consistent with a strategy that students consider licit to solve physics problems. This can be defined as follows: "to solve physics problems it is ok to simplify aspects of the situation as needed." This idea is epistemic because it represents a stance on knowledge construction (particularly during problem solving). This piece of epistemic knowledge (turns 4, 5, 11, 12) enables students to read-out particular traits of the contacting surfaces: *the surfaces are completely smooth* and *there is no water under the cube*. These read-outs are not only triggered by the geometrical depiction of the surfaces, but are also sustained by the activation of this epistemic piece of knowledge. We will refer to this piece of epistemic knowledge as *modeling by simplifying*.

## B. Buoyancy balances weight

$S_2$  [24:37 min]: "Buoyancy is equal to weight"

After the end of  $S_1$ , and up to the beginning of  $S_2$ , students discuss for over 15 minutes

- (1) *Int.*: *ok, so, let's assume this is so, and that's because you assumed that there was a thin layer of water... so let's go back to the cube, or the ball, sitting on the bottom, whichever you prefer*
- (2) *J*: *when we saw this, remember we had done something like this?* (draws a cubic portion of the fluid within the fluid and points at the pressures above and below that imaginary cube, as well as the different depths) *and we looked at the pressures on the top and bottom faces and then computed the total force on the cube... so if now we have the cube at the bottom of the container* (draws the cube at the bottom) *down here we have the pressure of all this water column and on the top face a smaller pressure so the forces of these pressures plus the weight equals zero because the cube is at equilibrium*
- (3) *M*: *so if we take this* (buoyancy as the result of hydrostatic pressure) *then there is buoyancy... 'cause we have liquid down here, even if it's a tiny layer, pressure down there times this area, and pressure up here times this area, that difference between them yields buoyancy... otherwise the pressure would have to be the same above and below, and that can't be...*
- (4) *J*: *so if there is water underneath we have a result different from last time* (meaning that before they said  $B = 0$  and  $W = N$  for case C, and now they find a nonzero value for B)
- (5) *M*: *no, it's still the same result* (all scales read the same)
- (6) *A*: *it's still the same!*
- (7) *Int.*: *so we have weight and buoyancy, and no normal force?*

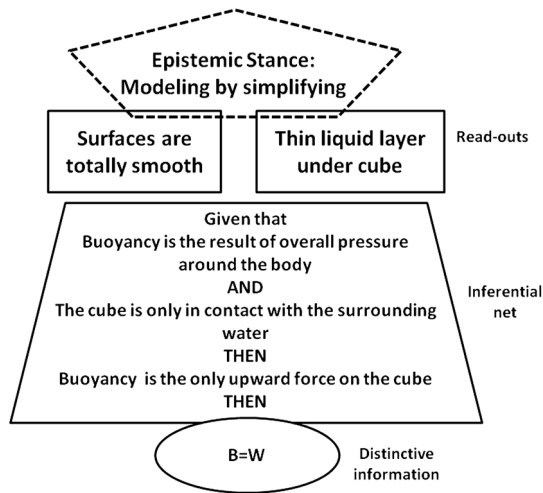


FIG. 4. Schematic diagram of the projection described in  $S_2$ .

(8) *J: NO, there is no normal on the cube but... but buoyancy is equal to weight, and that buoyancy is what the scale reads*

The read-out strategy reported in the first snippet (*there is no water underneath the cube*) was displaced and another read-out was incorporated *there is a thin liquid layer under the cube*. In addition, *surface is totally smooth* is still activated. These two read-outs, combined, lead to infer that there is no contact between the cube’s bottom face and the surface of the container. On the other hand, buoyancy as the result of hydrostatic pressures is recovered, as shown in turns 2 and 3. These facts combined lead them to generate a still nonaligned projection (buoyancy is not equal to weight in this case).

Once again, a piece of epistemic knowledge, *modeling by simplifying*, together with a particular trait of the context (the figure of the problem depicts two completely flat contacting surfaces) have driven students’ reasoning. Read-outs seem to be activated as a result of a contextual characteristic together with a particular epistemic stance sustained by *modeling by simplifying*.

Figure 4 shows a schematic diagram of the pieces of knowledge coordinated in this projection.

### C. Buoyancy is smaller than weight

$S_3$  [49:00 min]: “buoyancy is smaller than the weight of the block”

Even though students had assumed a thin liquid layer under the cube, it was difficult for them to accept that a heavy body, one that cannot float in water, and is sinking to the bottom of the liquid, will not eventually touch the bottom of the container. This keeps them working on the problem. In this snippet, students consider an imaginary situation and conclude that buoyancy on the block is neither zero nor equal to its weight.

On the basis that all scale readings are equal (something that they are confident about all along) they break up the

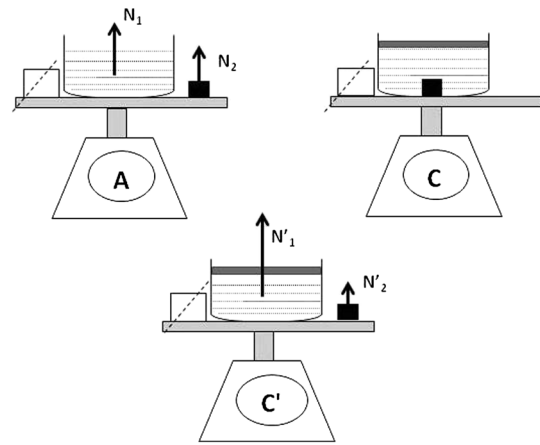


FIG. 5. Analogy proposed by students. The case  $C'$ , in which the force “supporting” the metal cube is smaller than in case A.

scale reading into its various contributions, which they refer to as “normal forces,” which correspond to each of the bodies on it. Thus, in the setting of case A, they call the force from the scale on the water  $N_1$ , and the force from the scale on the metal cube  $N_2$ . They imagine another situation, similar to case C, in which they do not need to analyze the forces on the sunk cube. This situation,  $C'$ , an analogue to case C, is depicted in Fig. 5. They imagine the same container in A, in which no cube is placed inside the liquid. Also, they consider that there is more water in the container, more specifically, an “extra” amount of water of equal volume to that of the cube. At the same time, they consider another cube lying directly on the scale, and they conclude that this cube must weigh less than the cube that was removed from within the water.  $N'_1$  is the normal force from the scale on the water in  $C'$ , and  $N'_2$  is the force from the scale on a metal cube sitting directly on it. In comparing situations A and C (or  $C'$ ), they cancel out the forces from the scale on the wooden cube and on the container. The consideration of this analogous situation,  $C'$ , helps them conclude that, if scale readings are the same for A and C (or  $C'$ ), then buoyancy on the metal cube in case C is not equal to its weight, contrary to the results they arrived at in  $S_1$  ( $B = 0$ ) or  $S_2$  ( $B = W$ ).

(1) *A: it’s like if now it (metal cube in case C) had less weight... if here you have a normal  $N_1$  (case A) that corresponds to the pressure of a water column this high, and also this normal  $N_2$  (on the metal cube, case A) that is equal to its weight. Now if you look here (case C), now the cube is inside, so all I need to look at is the new normal on the water, because of this little extra volume of water up here [the water displaced by the metal cube] and there is this new height, so the pressure down here is going to be different, larger, so there’s going to be a new normal  $N'_1$  that is larger than  $N_1$ . But we know both readings have to be the same, so if they are the same, it’s like having the metal block sitting outside,*

but with a different weight... I mean I have  $N_1$  plus  $N_2$  and it has to be equal to  $N_1$ , which is larger than  $N_1$ ... so if I want the reading to be the same and now instead of putting the cube inside I just lay it on the scale, and I assume both readings are the same, then the new normal on the cube  $N'_2$  on this cube outside the container would have to be less than  $N_2$ ... so it's like the cube had to change its weight ...

- (2) *M*: it's because now its equal to buoyancy [the new normal,  $N'_2$ , is equal to buoyancy]... so it's like you get that weight is not equal to buoyancy [in case C] but since the sum ends up being the same ... I mean buoyancy is smaller than weight, but the sum gives the same result.

*A* and *M* arrive at the same conclusion (body weight is diminished under water) through different reasonings. *M* probably continues to think that the metal cube only interacts with water, and not with the base of the beaker; thus, she (incorrectly) takes that diminished weight to be balanced by the buoyant force on the cube (in fact, buoyancy is a reasonable candidate to be the balancing force of that “new diminished weight”). On the other hand, *A* does not assume any particular interaction between the face of the cube and the bottom of the container; in fact, as she places the cube *outside* the container.

*A* and *M* are improving their conceptual understanding of buoyancy: it is different from zero but less than weight. This new projection brings together three very important features of buoyancy at the same time. It links buoyancy to the displaced liquid, since it is explicitly related to the “extra” layer of water on top of the fluid. Also, buoyancy is explicitly related to the increased liquid pressure at the bottom of the container.

Finally, buoyancy is related to the particular effect that fluids have of “diminishing” the weight of immersed bodies.

This new projection is clearly initiated by something that had not been present in their prior reasoning: the “extra” layer of water at the top of the fluid when the cube is immersed in the container. This read-out triggers a chain of inferences: increased water level → increased water pressure (bottom) →  $N'_1 > N_1$  →  $N'_2 < N_2$  → decreased *W*. *M*'s intervention in turn 2, although physically flawed, is a great contribution. It allows connecting those inferences to a value of buoyancy which is neither zero nor equal to body weight.

There is also a strategic move here. At some point during the sequence of inferences reported above, an epistemic idea is activated which can be stated as follows: “when you get stuck solving a problem, it is ok to replace the given situation by an analogous one, make conclusions in it, and then translate those conclusions back to the original situation.” In this particular case, the strength in this move is enabling students to approach the question of how buoyancy and normal forces are acting on the body from

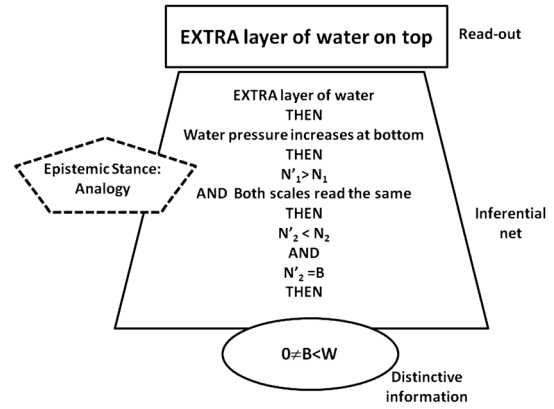


FIG. 6. Schematic diagram of the projection described in  $S_3$ .

a perspective that avoids the problematic contact surface at the cube's bottom face. We will refer to this epistemic stance simply as *analogy*. Figure 6 gathers the pieces coordinated in this projection in a schematic way.

**D. Changing the model for the contact surface**

$S_4$  [65:03 min]: “just imagine a surface like this one...”

For several minutes, students engage in a discussion in which *A* and *M* try to show *J* how the “made up” situation, *C'*, leads them to a new, different result ( $B < W$ ). Eventually, they get back to the issue of whether the cube at the bottom of the container is or is not touching the surface. The interviewer comments that being in contact does not necessarily mean that there is no water at all between the surfaces of the cube and the container. From this idea, they decide to consider a rough surface and they identify water pressure on the top and bottom faces of the cube, they express the relation between those pressures and buoyancy (except for a missing area), and they finally write down all the forces on the submerged cube, as shown in Fig. 7. They immediately arrive at the

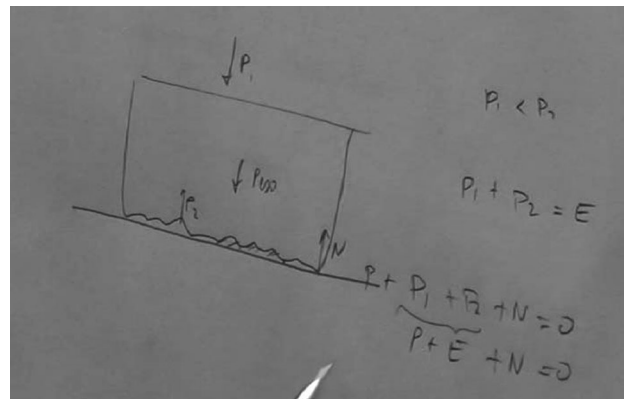


FIG. 7. Model proposed by the students from a “microscopic” point of view. ( $P_1$  and  $P_2$  are pressures,  $P$  is weight,  $N$  is normal force, and  $E$  stands for buoyancy).



vector equation:  $W + B + N = 0$  (In the figure, this equation reads  $P + E + N = 0$ )

As before, they break down the scale reading, for case C, into the partial contributions that correspond to each of the bodies on it. Thus, they arrive at the conclusion that the scale reading will be equal to the sum of four contributions: one corresponding to the wooden block, one corresponding to the container, one corresponding to the liquid, considering the “extra amount of water”, and one corresponding to the submerged metal cube (which they now know is  $W-B$ )

- (1) *Int.: contact does not necessarily mean that there is no water, right?*
- (2) *A: ok, but, imagine that you have a surface like this one... on these points it does touch the bottom [Fig. 7]*
- (3) *J: Oh! Right! I assumed the surface was completely smooth!*
- (4) *A: so then, since there's a little bit of water in these places, there you have the water pressure at the bottom ( $P_2$  in the figure), but also, you've got these contact points and they do an upward normal ( $N$  in the figure)... so you have buoyancy that is the result of these two pressures, plus the normal, plus weight ... (writes the vector equation in Figure 6,  $P + E + N = 0$ , which stands for  $W + B + N = 0$  in its English version)*
- (5) *M: But if you take directions into account, weight is buoyancy plus normal.*
- (6) *Int.: ok... so this that you just did, does it solve the problem you had before?*
- (7) *A: Yes... because weight is not equal to buoyancy, but it isn't equal to the normal force, either... and before we said that weight was either equal to the normal force, or equal to buoyancy... we never considered all three altogether...*
- (8) *J: we never considered the cube to have a rough surface... we always thought that it was completely smooth ...*
- (9) *A. so, if the scale reading is made up of normal forces... we have three normal forces: the one on the wooden cube, the one on the “increased” water (because the metal cube is inside), and the one on the metal cube... oh, but wait!... this new normal on the water should be related to buoyancy, that's what I'm not sure about...*
- (10) *M: the normal force on the metal cube is its weight minus buoyancy ...*

During this fourth snippet students modeled the surface as a macroscopically smooth and microscopically rough one. This model constituted a fundamental piece that allowed students to produce an aligned projection of buoyancy for this situation.

One read-out is displaced (*surfaces are completely smooth*) and another one is incorporated (*surface is microscopically rough*), in a very quick and natural way.

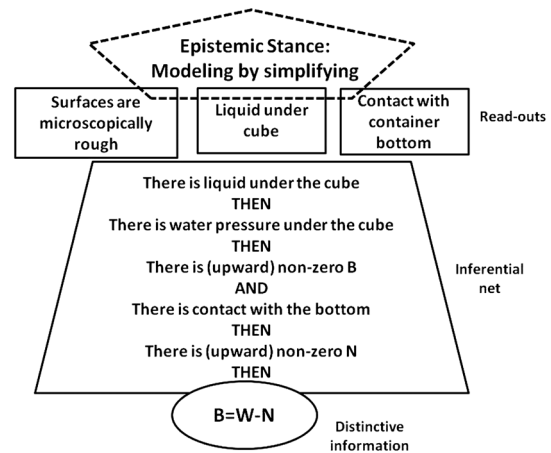


FIG. 8. Schematic diagram of projection described in  $S_4$ .

How can this move occur so easily and in an unproblematic way? We believe that this move is supported by the same epistemic piece of knowledge already reported in  $S_1$  and  $S_2$ : *modeling by simplifying* (“to solve physics problems it is ok to simplify aspects of the situation as needed”). Given this epistemic stance, it is only natural to assume different microscopic characteristics for that surface, so that water pressure and contact with the container can coexist. This new model for the contacting surface still contains simplifications: water pressure is considered uniform underneath the cube, and contact points do not seem to alter the overall value of the contact area.

Figure 8 shows the different elements coordinated in this projection.

### E. The role of the interviewer

How much is the interviewer involved in students’ progress? It could be argued that it was the interviewer’s intervention that originated some of the reasonings that were later reported as students’ conceptual progress. This is particularly so in  $S_1$  and  $S_4$ . The interviewer did indeed affect students’ thoughts. However, we will show they did not abandon their original ideas.

In  $S_1$ , we reported three different projections (no water under the cube leads to no buoyancy; liquid under the sphere leads to non-zero buoyancy; and both bodies displace liquid means buoyancy is non-zero in both cases). These last two were most probably prompted by the interviewer in turn 6. She asks them to compare the case of the cube with the one of a sphere. This is a perturbative intervention because it introduces an element not present in students’ reasoning. However, their first projection is still there, and two more projections are being considered. In fact, students are attempting to articulate those projections. Again, this articulation could be attributed to the interviewer’s prompt in turn 10. She focuses on an otherwise unnoticed fact: no water under the cube means the cube’s bottom face is dry, as if it had never been introduced in the

container. Students, however, do not back up from their original idea that the cube is not affected by buoyancy. On the contrary, their reasoning is refined as they view the underlying liquid as an “extra” condition for buoyancy to occur; and this condition is indeed a very reasonable one, in physical terms.

$S_4$  illustrates how students changed the original simplification of two smooth surfaces, totally in contact with each other, for another one in which macroscopically flat surfaces present roughness at a microscopic level. One cannot help noticing that it was the interviewer’s comment in turn 1 that prompted this change (contact does not necessarily mean there is no water). The critical point is that it just introduced the possibility that there could be contact with both water and container at the same time. Students are the ones who actually decided to revise the model for the surfaces as they did and they did so spontaneously.

In both snippets ( $S_1$  and  $S_4$ ) the interviewer’s participations did not override students’ reasoning. On the contrary, students’ spontaneous reasoning was enriched.

## VIII. DISCUSSION

The analysis of this case was originated by the fact that, although giving a correct answer (all scales do read the same), students were “incorrectly” using the concept of buoyancy. Interpreting this as a robust misconception on buoyancy, would only have called for its report as such and not for any further study. Instead, a knowledge-in-pieces perspective lead us to pursue the present study and to witness students’ conceptual progress at a very small scale. We were able to understand students’ progress even when they momentarily held views that are inconsistent with normative physics.

### A. What are the evidences of students’ conceptual progress?

In  $S_1$ , they still have not been able to decide what the value of buoyancy is, however they are making conceptual progress. They are committed to the expansion of the class to the particular context of a cube sitting flat against the bottom of the container. This involves three different projections and an articulation between them. This process leads them to explicitly consider the importance of the liquid underneath the body, which is very relevant to a physical description of buoyancy.

A particular element is seen to play an important role in the process described in the preceding paragraph. An epistemic stance is seen to participate: *modeling by simplifying*. This stance can be regarded as an epistemic resource in agreement with previous research that has focused on epistemic knowledge from a knowledge-in-pieces approach [11,22–24]. In this view, people’s attitude in relation to knowledge and knowing is the context-sensitive result of activating different epistemic resources.

*Modeling by simplifying* allows students to make assumptions that are not explicitly suggested in the problem and these assumptions are critical for the read-out *surfaces are totally smooth*. The idea that epistemic resources can either foster or hinder the activation of productive knowledge that students have available is neither new nor surprising [27]. What constitutes an interesting aspect of the present discussion is that in this study we are able to describe how this epistemic piece of knowledge is related to the activation of a particular read-out.

*Modeling by simplifying* is also involved in the displacement of a particular read-out strategy (*there is no liquid between the surfaces*) and the incorporation of another one (*there is a thin liquid layer under the cube*) which occur in  $S_2$ . This change implies that students continue to make efforts to expand the class for buoyancy.

The conceptual progress observed in  $S_3$  exhibits some striking features. Students generate, of themselves, a new context in which they can avoid the issues that are keeping them from (confidently) obtaining the actual value for buoyancy. In doing so, they start off with a new read-out strategy: *the extra liquid up there*. Besides, a very useful element is incorporated into the inferential net: the epistemic resource of *analogy*. This epistemic stance not only validates a particular type of reasoning, it also helps them make inferences that connect elements of the actual situation with elements of the analogous case. Once more, we are able to observe how this epistemic piece of knowledge operates in relation to the if-then sequence in the inferential net. As a result, buoyancy is greater than zero and less than weight, in closer agreement with its actual normative value.

Finally, in  $S_4$ , students displace the read-out *surfaces are totally smooth* and incorporate *surface is microscopically rough*. Thus, they are able to assume a new simplification that allows for buoyancy and a contact force at the same time. As in the case of the read-outs displaced and incorporated in  $S_1$  and  $S_2$  (*there is no water under the cube* and *there is a thin liquid layer*), this displacement or incorporation is linked not only to particular geometrical characteristics, but also to assumptions at a microscopic level that are enabled by *modeling by simplifying*.

Focusing on the role of read-outs in the overall conceptual progress one is confronted to a particular finding. Read-outs seem to be activated by salient features of the problem, that students can directly perceive, but also by a particular epistemic stance: *modeling by simplifying*. There is nothing in the problem depiction or in the problem statement that could lead students to see a thin layer of liquid completely separating both surfaces, or to see the assumed roughness of the cube’s bottom face at a microscopic level. These things are read-out because of a particular epistemic positioning: *to solve physics problems it is ok to simplify aspects of the situation as needed*.

The role of epistemic resources described in the preceding paragraph constitutes a natural expansion of coordination class theory that includes epistemic stances within its ontology.

### B. Implications

The present work has strong implications for researchers addressing the question of how and when conceptual learning takes place. It shows how a knowledge-in-pieces perspective, as the one adopted here, allows understanding otherwise inaccessible processes. Viewing students' ideas as robust misconceptions, for example, calls for detecting, recognizing, and eventually guiding students to replace those misconceptions for "correct" physical ideas. On the contrary, from the present view, research can help us understand conceptual progress independently of the "correctness" of students' conceptual understanding at any particular time. This is much more in agreement with actual experiences as students and as teachers: learning involves changing ideas, and even ideas that are not completely correct may well represent progress in one's understanding.

The results of the present analysis could also be useful for teachers and curriculum developers to inform their work. The first thing that this study highlights is that the situations we offer students may have certain details that can go unnoticed to us, but which can have a great impact on their reasoning. Of course, we cannot say that cubes are either a better or a worse option than spheres to teach buoyancy. Nevertheless, we can affirm that the comparison of both cases is the most sensible choice, as it offers the best chances of expanding students' conceptual span.

The second thing that this study alerts us to is the use of (implicit) simplifications often present in problem statements. Do they actually facilitate learning? Or can they constitute an extra obstacle for students? The discussions analyzed are evidence that students had to work hard to overcome that implicit simplification in order to understand how different forces were acting in this case. The use of problems with implicit simplifications may end up in conceptual difficulties for students that can go unnoticed to us. This is clear evidence of the extent to which modeling should be regarded as an instructional goal in itself and not something that students will always learn on their own.

Finally, the ways in which interviewer's prompts affect students' reasoning can also be an important input for teachers. As such, we are constantly making micro on-the-spot decisions, while conducting classroom or small group discussions. What kinds of interventions are most helpful for students' knowledge construction? Data show that it is possible to keep them working on their own ideas and, at the same time, to nudge them into more productive reasoning.

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- [1] T. Byun and G. Lee, Why students still can't solve physics problems after solving over 2000 problems, *Am. J. Phys.* **82**, 906 (2014).
- [2] C. B. Labra, A. Gras-Martí, and J. Martínez-Torregrosa, Análisis de la resolución de problemas de física en secundaria y primer curso universitario en Chile, *Enseñanza de las Ciencias* **22**, 275 (2004).
- [3] D. Gil Pérez and J. Martínez Torregrosa, A model for problem solving in accordance with scientific methodology, *Eur. J. Sci. Educ.* **5**, 447 (1983).
- [4] T. Foster, Ph.D. thesis, University of Minnesota, Minnesota, 2000, <http://groups.physics.umn.edu/physed/People/Tom's%20Thesis/tom.html>.
- [5] W. Leonard, W. Gerace, and R. Dufresne, Resolución de problemas basada en el análisis: Hacer del análisis y del razonamiento el foco de la enseñanza de la Física, *Enseñanza de las Ciencias* **20**, 387 (2002).
- [6] J. Mestre, R. Dufresne, W. Gerace, P. Hardiman, and J. Tonger, Promoting skilled problem-solving behavior among beginning physics students, *J. Res. Sci. Teach.* **30**, 303 (1993).
- [7] J. Docktor, N. Strand, J. Mestre, and B. Ross, Conceptual problem solving in high school physics, *Phys. Rev. ST Phys. Educ. Res.* **11**, 020106 (2015).
- [8] A. diSessa and B. Sherin, What changes in conceptual change?, *Int. J. Sci. Educ.* **20**, 1155 (1998).
- [9] J. Wagner, Transfer in pieces, *Cognit. Instr.* **24**, 1 (2006).
- [10] O. Parnafes, What does "Fast" mean? Understanding the physical world through computational representations, *J. Learn. Sci.* **16**, 415 (2007).
- [11] O. Levrini and A. diSessa, How students learn from multiple contexts and definitions: Proper time as a coordination class, *Phys. Rev. ST Phys. Educ. Res.* **4**, 010107 (2008).
- [12] ] P. Sengupta, K. D. Krinks, and D. B. Clark, Learning to deflect: Conceptual change in physics during digital game play, *J. Learn. Sci.* **24**, 638 (2015).

- [13] A. diSessa, B. Sherin, and M. Levin, Knowledge Analysis: An Introduction, in *Knowledge and Interaction*, edited by A. diSessa, M. Levin, and N. Brown (Routledge, New York and London, 2016), pp. 30–71.
- [14] A. diSessa, A Bird’s-Eye View of the “Pieces” vs. “Coherence” Controversy, in *International Handbook of Research on Conceptual Change*, edited by S. Vosniadou (Routledge, New York and London, 2008), pp. 35–60.
- [15] D. Hammer and A. Elby, On the form of a personal epistemology, in *Personal Epistemology: The Psychology of Beliefs about Knowledge and Knowing*, edited by B. K. Hofer and P. R. Pintrich (Lawrence Erlbaum, Mahwah, NJ, 2002), pp. 169–190.
- [16] D. Hammer, A. Elby, R. E. Scherr, and E. F. Redish, Resources, framing, and transfer, in *Transfer of Learning from a Modern Multidisciplinary Perspective*, edited by J. P. Mestre (IAP, Greenwich, CT, 2005), pp. 89–120.
- [17] E. F. Redish, in A Theoretical Framework for Physics Education Research: Modeling Student Thinking, *Proceedings of the International School of Physics “Enrico Fermi” Course CLVI*, (Varenna, Italy, Jul 15–25, 2003), <http://www.eric.ed.gov/ERICWebPortal/contentdelivery/servlet/ERICServlet?accno=ED493138>.
- [18] S. Rosenberg, D. Hammer, and J. Phelan, Multiple epistemological coherences in an eighth-grade discussion of the rock cycle, *J. Learn. Sci.* **15**, 261 (2006).
- [19] W. Leonard, R. Dufresne, W. Gerace, and J. Mestre, *Minds on Physics—Complex Systems* (Kendall/Hunt Publishing Company, Iowa, 2001).
- [20] L. Buteler, E. A. Coleoni, and M. A. Perea, Aprendiendo empuje durante la resolución de problemas: un análisis desde la Teoría de Clases de Coordinación, *Enseñanza de las Ciencias* **32**, 511 (2014).
- [21] Preliminary results were reported in SIEF XII: L. Buteler and E. Coleoni, Sobre el aprendizaje (y la enseñanza) del concepto de empuje: un análisis de la dimensión cognitiva y comunicativa de lo que ocurre durante una entrevista grupal, *Revista de Enseñanza de la Física* **26**, 17 (2014).
- [22] A. diSessa and J. Wagner, What Coordination Has to Say About Transfer, In *Transfer of Learning: From a Modern Multidisciplinary Perspective*, edited by J. Mestre (Information Age Publishing, Greenwich, 2005), p. 121.
- [23] The problem could be solved without considering buoyant forces at all. However, these students approached the solution including this force as a central component of their reasoning.
- [24] Partial encoding means that they will be used in some circumstances, but not in others.
- [25] R. Serway, *Física, Tomo I* (Interamericana Editores, Distrito Federal, 1997).
- [26] F. Sears and M. Zemansky, *Física General* (Aguilar, S. A. de Ediciones, Madrid, 1966).
- [27] A. Gupta and A. Elby, Beyond Epistemological Deficits: Dynamic explanations of engineering students’ difficulties with mathematical sense-making, *Int. J. Sci. Educ.* **33**, 2463 (2011).