Exploring the Relation Between Intuitive Physics Knowledge and Equations During Problem Solving

Laura Maria Buteler
Instituto de Fisica Enrique Gaviola
CONICET - Universidad Nacional de Córdoba

Enrique Andrés Coleoni
Instituto de Fisica Enrique Gaviola
CONICET - Universidad Nacional de Córdoba

Abstract

Solving many quantitative problems does not necessarily lead to an improved Physics understanding. However, physicists, who have learned physics largely through quantitative problems solving, often have a refined physical intuition. Assuming that the refinement of physical intuitions occurs, to a great extent, during problem solving, the question that guides this study is: how do equations contribute (or not) to the refinement of students' intuitions? We approach this study within a knowledge-in-pieces perspective and we describe intuitions using diSessa's (1993) phenomenological primitives. We present a study in which two cases, corresponding to two groups of students solving a problem involving buoyancy are compared. We discuss how the use of equations does or does not contribute to the refinement in students' intuitions.

Correspondence concerning this article should be addressed to: Enrique Coleoni Facultad de Matemática, Astronomía y Física. Universidad Nacional de Córdoba. Medina Allende y Haya de la Torre. Ciudad Universitaria. Córdoba, 5000. Argentina. ecoleoni@famaf.unc.edu.ar

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Introduction

The mathematical formalization of a physical situation is a fundamental process in the learning of Physics, and therefore an important goal for teachers. Thus, it is present in most curricula, either explicitly or implicitly. For teachers, it is a very valued ability, but at the same time a lot remains unknown as to how it is learned and furthermore, how it could be best taught.

There is evidence in the literature that solving quantitative problems often does not improve students' conceptual understanding in physics. Even solving many quantitative problems often does not lead to much increase in conceptual understanding (Gil Pérez, Martínez Torregrosa, & Senent Pérez, 1987; Maloney, 1994). However, advanced physics students and physics teachers, who have learned physics largely through quantitative problem solving, often have a strong conceptual understanding of physical systems.

Previous research has studied how experts and novices solve textbook-like physics problems using equations. Chi, Feltovich and Glaser (1981), Larkin (1983) and Priest and Lindsay (1992), explain the differences in expert and novice performance assuming that the equations they use during the solving process are associated to – or inferred from- *schemas*[1] of different physical principles. In these studies, subjects are assumed to have schemas that correspond to physical concepts and/or principles that, once activated, guide the solving process. These schemas allow subjects to infer which equations should be used. Although there are differences among the studies cited, they share important features. In all these studies it is assumed that: 1) the only knowledge subjects will bear is normative, i.e. physical concepts, laws, and principles, represented by schemas[2], and 2) there is an unambiguous association between these principles, laws or concepts and the equations to be used. These assumptions should be revised for at least two reasons.

There is plenty of evidence indicating that students have previous knowledge when addressing physics instruction. This knowledge does not necessarily agree with normative physics knowledge. Originated from subjects' life experiences and also from previous instruction, it constitutes the conceptual background with which they can make sense of new ideas. This knowledge has been described and conceptualized as either alternative conceptions, or misconceptions, or intuitive conceptions, intuitive theories, etc. (Caramazza & Green 1980; Clement, 1983; Ioannides & Vosniadou, 2002; McCloskey, 1983; McDermott & Redish, 1999). It would be rather naïve to consider that none of this knowledge is used by students when they solve instructional physics problems involving equations.

Also, assuming an unambiguous association between physical principles and equations does not allow to understand how and when the math used during the solving process is linked to that previous knowledge of students which is not accounted for by those schemas proposed by authors such as Larkin (1983) or Priest and Lindsay (1992). Redish and Smith (2008) challenge the assumption that mathematic manipulation can be detached from previous conceptual knowledge. They argue that equations carry a conceptual load, and their manipulation is affected by physical interpretations. In a recent report, Kuo, Hull, Gupta and Elby (2013) make the case that problem solving expertise should include opportunistically blending conceptual and formal mathematical reasoning *while* manipulating equations. Through the analysis of two cases they argue that this blending of conceptual and formal mathematical reasoning is a part of problem solving expertise. We believe it is important to address some details of how students' previous knowledge is gradually linked to equations, given that this is a relevant goal in the development of expertise. Describing this process includes understanding how this knowledge is modified in order to be coherent with normative physics knowledge. The present work attempts to increase our understanding of how previous knowledge affects the equations students build and how the manipulation of those equations impacts this knowledge.

Theoretical Background

A Definition of Physical Intuition

Students have a great deal of experience that is relevant to the study of physics. Prior to instruction, they have experimented friction, pushes, gravitational pull, cold and warm, and have observed the transformation of heat into mechanical energy or vice-versa. These experiences have added to their knowledge of the physical world. They have dealt with the physical world as part of everyday experience. It is also likely that these phenomena have been the object of previous instruction, which means that the student's ideas on those phenomena could also have been influenced by instruction. Herein we refer to these ideas as intuitive physics knowledge or *physical intuition*.

Thus, *physical intuitions* are the raw material that students use to understand situations and solve problems. They are linked to phenomenological considerations or observations (it is what happens, what is observed, what I know, what I believe). This knowledge presents certain characteristics: a) it is a combination of the student's everyday experience as well as her/his previous instruction; b) experts as well as novices have physical intuitions; experts have not rooted them out: their intuitions have been refined and can be explained in terms of formal physical knowledge; and c) it is knowledge that is necessary for future learning, that is, it is <u>useful</u> knowledge. A central issue that stems from this viewpoint is that an important component in the development of expertise can be understood as the refinement of physical intuitions. As subjects develop their expertise, they are gradually more able to decide when and why these intuitions can be successfully used to describe and explain different physical situations (diSessa, 1993).

The following example illustrates the characteristics of this approach. Many students explain the circular motion of a planet around the sun as the result of the action of two balanced forces. One of them is a centrifugal one that, if acting alone, would send the planet off its orbit. The other is a centripetal force that, if acting alone, would make the planet collapse onto the sun. This answer is based on knowledge that enables them to understand what, they believe, happens. We are calling this knowledge a physical intuition. Even though the answer that students provide, based on this intuition, is incorrect, the intuition itself has correct (there is a centripetal force) and incorrect (there is no centrifugal force) traits. An expert's refined intuition can lead him/her to state that the planet follows this orbit due to the action of one only (centripetal) force that is permanently deviating it from a straight line and causing it to "fall" towards the sun. In an expert's intuition there is simply no centrifugal force, and there is no need for one either. Throughout the paper, we will provide other examples that we hope will further illustrate the idea of *physical intuition*.

A Model for Physical Intuitions: Phenomenological Primitives

The idea that physical intuition can play a productive role in the development of expertise is not new. It was originally suggested in the work of Smith, diSessa and Roschelle (1993). According to these authors, the shift from regarding previous knowledge as an obstacle to be overcome to considering it a necessary and useful starting point is accompanied by a shift in the ontology attributed to this knowledge. This shift, more recently also supported by other researchers (Hammer, Elby, Scherr & Redish, 2005; Redish, 2004; Sherin, 2001; Sherin, 2006), involves abandoning the idea of physics concepts, principles or laws as indivisible cognitive units, and instead to view them as a complex system formed by many smaller cognitive elements. From this *knowledge in pieces* perspective, a principle or concept is the result of a distributed encoding. It is distributed because the full meaning is distributed across many elements. Each of these elements plays a certain role in the understanding of the different facets of the principle or concept. These elemental units can be regarded as nodes in a network and the connections between nodes define their activation in relation to the activation state of their neighbors.

Different authors have provided slightly different conceptualizations for these cognitive elements. In this work we will adopt the one proposed by diSessa, who refers to those elements as *phenomenological primitives* -or *p-prims* (diSessa, 1993; diSessa & Sherin, 1998). They are phenomenological because they are often originated in nearly superficial interpretations of experienced reality. They are also phenomenological in the sense that, once established, p-prims constitute a rich vocabulary through which people remember and interpret their experience. They are primitive because they do not require any explanation; they simply reflect how things happen, and they are used as if they needed no justification. They are also primitive in the sense that they are the smallest, indivisible cognitive elements. P-prims help us develop abilities to interact with the physical world and to anticipate how a physical system will behave.

diSessa (1993) proposes a set of p-prims to understand students' intuitive ideas about physical mechanisms, which he calls *sense-of-mechanism*. The following paragraphs illustrate some of them.

A p-prim commonly used during the learning of Mechanics is *force-as-mover*. According to this p-prim, the push exerted on an object is the cause for it to move in the direction of the push. It can also be regarded as the cause of the object's slowing down, when the push is done in the direction opposite to its motion. Depending on the context in which this primitive is applied, it can lead to interpretations that are or are not in agreement with Newtonian Mechanics. Perhaps due to the predominant role of this primitive, it is so difficult for students to describe movements in which forces are not in the direction of motion, as in the case of circular motion.

Another primitive very often involved in the learning of physics is *spontaneous resistance*. This primitive refers to objects' properties to resist the effect of external agents. It is used to understand, for example, the pressure felt on one's hand when exerting a force on some object, which in physics is described in terms of an action-reaction pair. It could be also used to understand why ropes are extended when used to drag objects: objects spontaneously resist to be moved. From a Newtonian perspective, the action of an external force on an object is mediated by a feature of the object (which is not a force-exerting agent): it's mass. Thus, this primitive could play an important role in the understanding of Newton's Second Law.

Another primitive often used by children to explain certain phenomena is *supporting*. Objects naturally tend to fall to the ground unless something *supports* them and prevents them from falling. Thus, a state of rest of an object lying on a table, for instance, is explained simply in terms of the *supporting* action of the table that prevents the free fall of the object. From a Newtonian perspective, this primitive is of little use, since it does not allow for explanations in terms of forces. *Supporting* is a particular case of a more generally applicable p-prim, *blocking*, when the tendency of an object toward motion is thwarted by another object in its path.

Recalling the case of students' explanations of planetary motion, their answers can be interpreted in terms of the activation of *dynamic balance*. This primitive is probably abstracted from situations in which two opposing agents compete to provoke different results, but end up cancelling each other. It involves a conflict between two opposing dynamic actions.

This network model for physical intuitions recognizes p-prims as its elemental units, and also provides a way of describing when and how a p-prim will be cued to an active state. diSessa (1993) addresses this issue by means of two ideas: *cuing priority* and *reliability priority*. *Cuing priority* is the likelihood that a given p-prim will be activated, given some perceived characteristics in the context, which includes other previously activated p-prims. Once a p-prim is activated, *reliability priority* provides a measure of how likely it is for that p-prim to stay activated. It describes the potential feedback that can reinforce or undo the initial activation. A high reliability (with respect to a specified context) means it is unlikely that the p-prim will be turned off by subsequent processing; it is an assertion of likely reinforcement and unlikely suppression through all activation paths that return to that p-prim. These two measures work together to configure the overall activation pattern of the network. In each given context, a p-prim (or set of p-prims) with a certain *cuing priority* will be activated. This activation triggers a subsequent chain of mental events, and the *reliability priority* of this p-prim will determine whether or not it continues to stay activated. Taken together, cuing priority and reliability priority constitute what diSessa calls *structured priorities*.

Possible Changes in Phenomenological Primitives: The Refinement of Physical Intuitions

Attributing this ontology to students' knowledge allows explaining the gradual changes in this complex system in terms of small changes in its constituents. Thus, it would be possible to understand not only the differences, but also the similarities between the behavior of experts and novices. diSessa (1993) speculates that the refinement of physical intuitions could take place through different mechanisms:

1) Changes in the priorities of the individual elements. P-prims seldom used may come to be used more frequently and some p-prims can come to be less used, or not used at all. These changes would take place through incremental adjustments in the cuing priorities and reliability priorities of the p-prims occurring during instruction. Although the changes that occur during a single learning experience may be small, theses incremental changes would ultimately lead to a change in the p-prim system,
2) The generation of completely new p-prims, occurring from observation of new pieces of information in the world, and
3) P-prims taking on new functions, for instance, as heuristics to cue normative knowledge, or playing a role in "knowing" a physical law.

Goals of the Present Research

As Sherin (2006) points out, the activity of problem solving provides a suitable scenario for the refinement of intuitions. In particular, this author explores the question of how experiences with quantitative problem solving could lead to changes in intuitions. Following Sherin, we assume that the refinement of physical intuitions occurs, to a great extent, during problem solving. The question that guides this study is: how do equations contribute (or not) to this refinement during the solving process?

Sherin (2006) approaches the question of when and how changes in intuitive knowledge typically occur, and he speculates on the crucial experiences that could lead to its refinement. This author analyzes the case of two students (Alan and Bob) while solving the "shoved block problem"[3]. In this problem, two opposing intuitions are in conflict. One of them is supported by *force-as-mover*, and leads to predict that the heavier the block, the stronger friction force that will be exerted on it and thus, the faster it will be stopped. On the other hand, *spontaneous resistance* leads to predict that the heavier block would require a larger force to be stopped, and thus it will come to rest later. Using equations, Alan and Bob resolved this conflict in a way that was surprising to them: they found that both intuitions are applicable. They wrote an expression for the force of friction, ($F_f = mg\mu$) and they substituted this expression in *F=ma*. Cancelling the masses in the equation they obtained an expression for the acceleration of the block, $a=g\mu$, which does not depend on its mass. Comparing this case with other ones of students solving the same problem, Sherin (2006) observed that the *force-as-mover* intuition was more common and was always produced (if at all) before the *spontaneous resistance* intuition. He then speculates that this episode might have the effect of incrementally increasing the priority of the *spontaneous resistance* p-prim. Therefore, for these students, there could be a “moral” after this episode, which is that spontaneous resistance must not be overlooked.

We intend to find evidences that could support the validity of some of Sherin’s speculations, and by so doing, we attempt to better understand, in one particular case, how the use of equations leads to incremental changes in the p-prim system, particularly showing how a certain primitive takes on a new function. To do this we present and compare two cases, in which these changes are or are not seen to occur.

The Study

The present one is an exploratory study, carried out on the analysis of two cases. Case I was based on a first interview and allowed to observe some relations between the activation of certain p-prims and the equations that students used. The analysis of this first case opened new questions that were addressed through a second interview, on which Case II was based.

As in any case study, conclusions will refer to the cases analyzed, and to the reasoning of those students solving these particular problems. This methodology has, of course, strengths and weaknesses. The weakness in this type of studies is the little generalization that can be attributed to the results. Its major strength is that it enables the observation of the richness found in students’ reasoning. Although details in students' reasoning may differ between subjects, the thorough observation of these details is a very important step to understanding the learning process that occurs during problem solving.

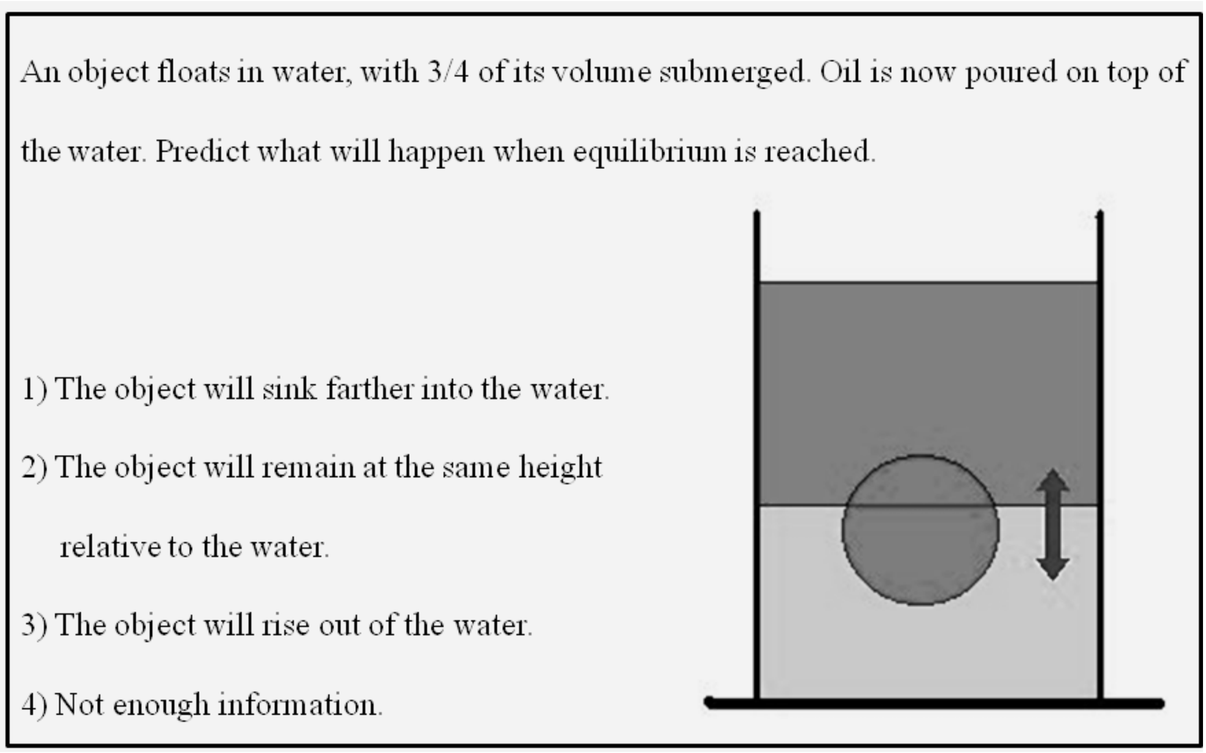
The subjects in both interviews (Cases I and II) are third-semester physics students in a public university in Argentina. When the interviews took place, students had finished a Physics course covering fluid dynamics and thermodynamics. The course was taught in a traditional way, and the total of 12 hours every week was divided into two two-hour lectures, two two-hour problem-solving sessions, and one four-hour lab. Case I and Case II were carried out with students from two cohorts corresponding to two subsequent academic years.

Each year, a call was made to all the students in the course inviting them to volunteer as participants in the research. They were aware that, among the many research groups in the institution, there is a Physics Education Research Group, and that its members (among which are the authors of the present work) are interested in improving their understanding of the mechanisms involved in learning Physics. It was also pointed out that they would be participating in a problem solving session in which they would discuss and debate on particular problems related to their studying material, and thus they were aware that besides collaborating in our research they would also make use of that time in a way that was potentially beneficial to them. From the total of approximately 15 students in each class, only two volunteered in the call of the first year (Case I): *M* (a girl) and *J* (a boy); three volunteered for the call of the second year (Case II): *A* and *T* (two girls) and *L* (a boy). Although the selection of the participants was accidental, both groups of students were average, in the sense that they were neither the highest nor poorest achieving in their class.

They are representative of most Physics students at our University. The problem all students solved is the one illustrated in Figure 1[4]. We assumed that, since this is not a typical textbook-like problem it would not invoke any rote solutions, and that it would prompt students to discuss their ideas, and thus, inform us on their intuitions. At the same time we also assumed that the problem was likely to require students to use equations. In the interviews we confirmed both these assumptions.

The gist of the problem consists in noticing that the presence of the oil poured above the water increases pressure all around the surface of the ball. These increases are larger on the points of the sphere below the oil/water interphase than above it. Pressure in *all* the points below the interphase is increased in an amount equal to the pressure of the fluid at the interphase, say P_i. Above the interphase, pressure is increased in amounts that are less than P_i. Therefore, as the resulting buoyancy force must still equal the weight of the ball, it must rise with respect to the interphase.

Figure 1: Problem given to students



The second author conducted the interviews, which lasted approximately 150 minutes each. Therefore, we had a total of about 5 hrs. of data on video. Registries were first analyzed and interpreted independently by each author. Both interpretations were later compared. There were minor differences between these two independent interpretations. These differences were reconciled after thorough discussions and further analysis of the registries.

Results

Case I: *M* and *J*: an incorrect, intuitive equation vs. a correct, unintuitive one

Part I-A: "the oil can push down due to the weight of this column… and it's also pushing up with a buoyancy force". The main idea in these students' discussion is that there are two opposing influences on the ball: one is a downward force that pushes on it and makes it sink farther into the water, and the other one is a buoyancy force that pushes the ball up. These two influences are competing and eventually a new balance is met with the ball located deeper in the fluid. In this new situation, the extra push from the oil column from above the ball is balanced by an increased buoyancy force. They fail to link the presence of the oil column above the ball to an increase in the water pressure (via the contact surface between the two fluids). Instead they see *two independent* influences on the ball, only one of which is caused by the oil.

***M**: there's the weight of the ball, and the buoyancy from the water… and what about the oil? What is it going to do? Because the oil can push down due to the weight of this column… and it's also pushing up with a buoyancy force… I don't know…*

***J**: I think it's going to stay in the same place, too, but I don't know why, I'm not sure*

***M**: … I think there's got to be a relation between buoyancy and the force from the oil… because I AM SURE that the oil pushes down on the ball, it has to do a force equal to the weight of this column… and that's what I don't understand, how that weight compares to buoyancy*

***J**: shouldn't we consider the pressure from the fluid on top of the ball?*

***M**: Yeah, but we don't have information…*

Part I-B: "there's more force pushing down". They come to believe that the weight of the oil column will be larger than the initial buoyancy force, and therefore the ball will sink further into the water.

***J**: look… this area, this surface up here, has to support the pressure done by the fluid* (oil) *that's on top of it… we don't know how much that pressure is, but we do know that it's going to do a downward force… so it's going to sink further down… we don't know how much further… but they don't ask that here either…*

***M**: you mean… the buoyancy that the oil could do would not be enough to push the ball up… but for sure it won't push down… because buoyancy always pushes up*

***J**: buoyancy can be, at most, equal to the weight of the displaced fluid…*

***M**: so the object will sink further down, because of the weight of the oil column on top of it… well yeah… there's more force pushing down…*

This intuition can be understood in terms of *dynamic balance* and *force-as-mover*. The first of these p-prims supports reasoning about situations in which two opposing influences try to achieve mutually exclusive results. In the present case, these influences are the weight of the oil column, and the upward buoyancy on the ball. Starting from an initially balanced situation (ball is floating on the water surface), students see this balance altered due to the downward force exerted by the oil. The ball is thus forced down into the fluid, in a new, deeper, location.

Force-as-mover states that when an impetus acts on an object, the observed result is a movement or displacement in the direction of that impetus. This p-prim accounts for a crucial aspect of these students' reasoning, namely, that the equilibrium position of the ball has been forced deeper into the fluid, because that is the direction in which the agent that has altered the balance is pushing. Therefore, students' reasoning can be understood in terms of the activation of these two p-prims. *Dynamic balance* accounts for an initial balanced situation that is altered and re-established. At the same time, *force-as-mover* allows to understand why students are confident that the new balance is achieved deeper in the fluid: the agent that has altered the initial balance has pushed its location in that direction.

Part I-C: writing the incorrect "intuitive" equation.

M*: anyway… the buoyancy from the oil is not zero; it's just smaller than the weight of the oil column on top… I mean… there IS an upward force done by the oil… I mean, there is buoyancy from the oil and from the water…. Because there's liquid* [oil] *being displaced… if that buoyancy force is too small… so…* [begins writing, see figure 2] *there is the weight of the ball, which is $\rho_p Vg$…* [and adds a minus sign before the term] *plus the buoyancy from water, which is $\rho_a 3/4Vg$, plus the buoyancy from the oil ($+E_{ac}$) minus the weight of the oil column ($-P_{col}$)*[5]

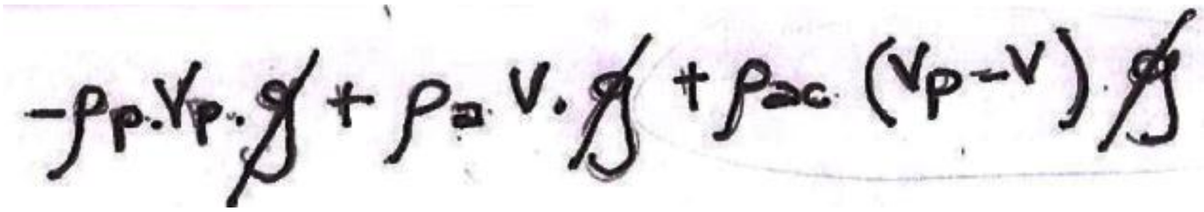
M: *oh, no, wait!* [crosses out the factor *3/4V*], *because we* <u>*don't know*</u> *if it's 3/4, we have to put it down as a variable. That's what we want to check. So we should put that volume in as a variable…*

Figure 2: First equation written by ***M*** and ***J.*** (ρ_p: density of the ball; ρ_a: water density; E_{ac}: buoyancy exerted by oil; P_{col}: weight of the oil column)

Sherin (2001) proposed that the process of generating and understanding Physics equations can be described by *symbolic forms*. They are pieces of knowledge generated by students as they learn to solve Physics problems using equations. These are intuitive schematizations of physical situations that can be directly embodied in equations. They add semantics to an equation, and enable to bypass formal principles and go straight from intuitions to equations. The equation the students generate (Figure 2) is supported by one particular symbolic form that Sherin calls *competing terms*. In this symbolic form, equations are an arrangement of terms that conflict and support or that oppose and balance. Frequently, these influences are forces, as in the present case. This form is expressed through a pattern of the kind □ ± □ ± □ ± □…Two evidences support our interpretation. First, the students explicitly match the signs of each term with the direction of each influence (minus for the downward forces such as weights, and plus for buoyancies) Second, their utterances consist of an enumeration of each influence on the ball.

Back to the analysis of how students re-address the writing of the equation, they do so using a variable, ***V***, for the unknown submerged (in water) volume. Again, as occurred while writing the first equation, terms are enumerated with signs that match the directions of each influence on the ball (see Figure 3).

Figure 3: Second equation written by ***M*** and ***J***. (ρ_p: density of the ball; ρ_a: water density; ρ_{ac}: density of oil; V_P: volume of the ball; V: unknown, submerged in water, volume)



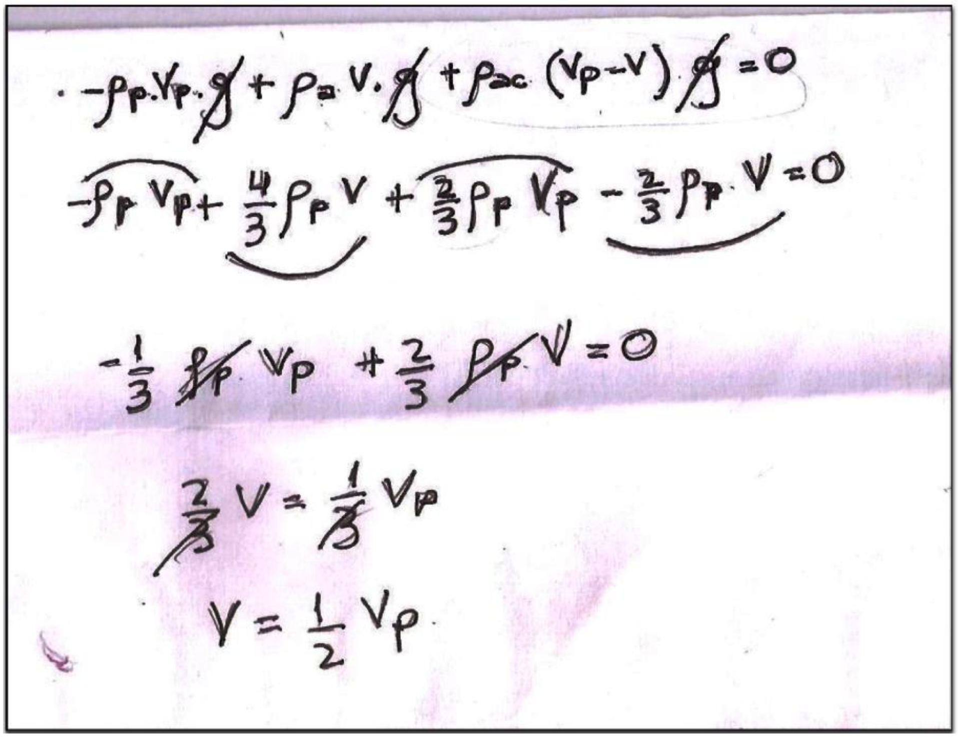
They try to include the effect of the oil column on top of the ball and they encounter a problem: the radius of the (cylindrical) column sitting right above the ball is also unknown, and they cannot relate its value to the portion of the ball submerged in water (***V***).

Part I-D: writing the correct "unintuitive" equation: confusion and distrust. At this point, the interviewer considered that an intervention could help them out of their confusion. He pointed out that the effect of the oil column is already accounted for by the buoyancy exerted by both fluids, which they have already written in two separate terms, as the weight of the displaced oil and water, respectively. This intervention clearly does not help them out. They have difficulties accepting that a downward effect (weight of the oil column), could "turn into" (at least in part) an upward buoyancy force.

Int: *Can you tell me what you've written down so far?*
M: *The weight of the ball, buoyancy from water, buoyancy from oil, and we're trying to write down the weight of this oil column.*
Int: *and why does the oil exert a force on the ball?*
M: *Because it's on top of it and pushes down on it.*
Int: *but… didn't you put that in these terms?* [points to the buoyancy terms]
M*: but, are you trying to tell me that buoyancy is cancelled by the weight of the oil column?*
Int.*: Rather than that, it accounts for the presence of the oil column*
M*: no… I don't understand… I can't do this…*
Int.*: you were already doing it, and doing it right, considering the weight of the ball and the two contributions to the buoyancy force… if you take this equation* [without the weight of the oil column] *and solve for your unknown, see what you get.*

They solve this balance equation, which does not contain the weight of the oil column (i.e. an equation that correctly describes the problem, see figure 4) and solve for the unknown submerged volume; they find that this volume is now ***½ V_P***.[6]

Figure 4: Equations in ***M*** and ***J´***s written solution, case I. (ρ_p: density of the ball; ρ_a: water density; ρ_{ac}: density of oil; V_P: volume of the ball; V: unknown, submerged in water, volume)



M: *but we don't get the same thing! Now it's only half inside the water!*
Int.: *so now considering the oil, you find that the volume submerged is different…*
M: *but before it was ¾!*
Int.: *so, what happened? Did it go up or down?*
J: (surprised) *up…*
M: (confused) *why?! There's and added push from the oil! How does that happen?*

Why are students puzzled with this (correct) result? Their intuition has been expressed in an equation through the symbolic form, *competing terms*, which has four interacting agents: weight of the ball, weight of the oil column (acting downward) and two agents pushing upward (buoyancy). *Force-as-mover*, which is a part of their intuition, dictates that the downward influence of the oil column will produce a downward effect on the ball. In students' view, this influence cannot simply be removed from the equation, let alone "be converted" into a portion of the upward effect in buoyancy, as intended by the interviewer.

It is clear that mapping *force-as-mover* only on the oil and the upper half of the ball is not, from a physical point of view, an adequate move to account for the influence of the oil column on the ball. This influence, as the interviewer states, is already accounted for by the buoyancy terms. But in students' intuition it is this p-prim, mapped in that particular way, which links the oil column to the floating body. Removing the term of the oil weight produces an equation that is no longer a symbolic form, and therefore there is no longer a link between this equation and the p-prim. Thus, the subsequent mathematical manipulation is unlikely to affect the priorities of *force-as-mover*, or its function in understanding buoyancy.

Although students cannot accept the result implied by this equation at this time, it is possible that after multiple encounters with this normative answer in the future, it will gradually start to make sense to them. However, our point is that *this particular episode* is not efficient to incorporate *force-as-mover* to understand buoyancy.

How could this efficiency be increased? We speculate this could be achieved by prompting students to estimate the resulting force acting on the body, as an integral of pressure on its surface, *including* the contribution of the oil above the ball. Although mathematically more complicated, this could lead students to refine their intuition.

In order to explore the validity of our speculation, we carried out a second interview, and due to the time of the year, this had to be done with another group of students from the following cohort, solving the same problem. Students for this second interview (Case II) were chosen in the same was as those in case I. As will be seen throughout the analysis of this second case, our interventions were now oriented to foster a more conceptual understanding of buoyancy. We will discuss how these interventions were more efficient in incorporating the p-prim *force-as-mover* into a correct description of buoyancy.

The students participating in the second interview were two girls (*A* and *T*) and one boy (*L*). These students pertain to the cohort immediately following the one of *M* and *J* (Case I). They had been taught the same course, by the same teacher, and were interviewed at the same time during the following academic year. As mentioned before, academic achievement of students in Case II did not differ from that of students in Cases I.

Case II: From an alternative view of buoyancy, *A*, *T* and *L* are able to link their intuitions to equations.

Part II-A: "if you add a whole lot of oil, it feels an enormous pressure and it sinks a lot more". As in the case of *M* and *J*, these students activate the same primitives to describe what will happen to the ball: *dynamic balance* and *force-as-mover*.

T*: if we add oil, it's like doing a greater force on the ball, it is pushed down, so it will sink more.*

A*: sure, 'cause it's going to feel more pressure.*

L*: I think it sinks more, a little more, because there's an extra downward force.*

T*: Yeah, but… if you're pushing harder up here on the ball, that force is also transmitted to the water, and the water pushes back up, by means of buoyancy… so buoyancy is also going to be greater, and so it goes back to where it was*

L*: but the oil is still there, so that downward force is still there, and so it sinks further down.*

A*: it either sinks further down or stays the same, but it can't go up…*

T*: sure! There's no way it can go up!*

L*: and it also depends on how much oil you put on top… if you add a whole lot of oil, it feels an enormous pressure and it sinks a lot more*

A*: so the buoyancy on the ball is going to be equal to the weight of the ball plus the pressure I'm doing on it with that oil:* ***B=W+F*** (where ***B*** is buoyancy, ***W*** the weight of the ball, and ***F*** the force exerted by the oil column)

As occurring in case I, these students' understanding is also supported by *dynamic balance* and *force as mover*. The weight of the oil column above the ball pushes the ball down, and thus causes it to abandon its initial position where weight and buoyancy (water) were balanced. A new balance is attained at a deeper position. In this new position, a larger portion of the ball is submerged in water, and therefore the ball displaces more water than before. This increase in the magnitude of the buoyancy force of water balances the extra contribution to the downward force given by the weight of the oil column above the ball. Their understanding of this situation still does not include the effect of the oil column in increasing the pressure in all the water due to the contact of both fluids at the interface.

Part II-B: A new approach to buoyancy in a simpler context. At this point the interviewer makes a move to guide them into a more conceptual understanding of the buoyant force. This move of the interviewer is instantiated in two crucial decisions. The first is to suggest the consideration of the pressure on the points surrounding a submerged body. The second one is to consider an auxiliary situation consisting of a cube totally submerged in water. As opposed to the intervention of the interviewer in Case I (promoting the use of a "correct" equation that was not a symbolic form) this time the interviewer's "hint" was to provide them with a way of constructing a symbolic form that coincides with a "correct" equation:

***Int.**: What does buoyancy mean to you?*
***L**: It's the force water does to keep a certain volume displaced.*
***Int.**: Where does it come from?*
***A**: from the water itself*
***Int**: What's the pressure like at each point on the faces of the body?*
***L**: That depends on the height of the liquid column above that point*
***A:** And it's like perpendicular to each face of the cube…*

The interviewer proposes an estimation of the forces on each of the faces of the cube due to water pressure. Students remember a problem in which they were asked to compute the total force exerted by water on the contention wall of a dam. They arrive together at the conclusion that the lateral forces cancel out, and that the resulting force is just the difference between the forces acting on the top and bottom faces of the cube $F_2 - F_1 = \rho_A g A(h_2 - h_1)$: F_2 and F_1 are the forces on the lower and upper sides of the cube, located at depths h_2 and h_1, respectively. ρ_A is water density and, A is the area of the side of the cube).

***Int.**: So the force resulting from all those forces, where is it pointing?*
***A, T** and **L**: Up*
***Int.**: And that upward force, what do you think it is?*
***L**: (timidly) buoyancy…*
***A**: of course…*
***Int.**: So, buoyancy is the result of all the forces acting on the different points on the faces of the body, because of the pressure of the liquid on each point… you said something like that a few minutes ago, didn't you* [buoyancy comes from the water itself]*?*
***A**:*[elevating her pitch] *yeah! But now it's like, there you go! There's your buoyancy force!*

A note is in order at this point. In conversations with the teacher of these students', we had learned that the idea of buoyancy was first introduced to students through the consideration of pressures around an imaginary cubic portion of the fluid. However, after obtaining the expression ($F_2 - F_1 = \rho_A g A(h_2 - h_1)$)by themselves, these students have just built a symbolic form that is the vehicle to achieve a new, improved, and more mechanistic understanding of buoyancy. An indication of this is *A*'s tone of voice and choice of words when she says "*yeah* (we knew that buoyancy comes from the water itself) *But now it's like,* ***there you go! there's your*** *buoyancy force!"* . She *sees* the buoyancy force in the equation. She realizes something that she wasn't aware of before. They improved their understanding of buoyancy by obtaining this result after manipulating equations that made sense to them.

Part II-C: returning to the problem of the ball with the new approach to buoyancy. The interviewer takes the students back to the ball in water/oil. In this instance, they consider the pressures in the fluid all over the body's surface. To do so, they decide to consider a simpler geometry (that of a cube):

***L**: So we have oil increasing the pressure on top of the ball, making it sink further down, but oil is also increasing buoyancy… let's look at it like we did the other one… like the cube…*

***T**: I mean… if this pressure* [upper side of the ball] *were just the same as buoyancy, nothing would happen, it wouldn't go down*

***A**: yeah, but the oil is also increasing the pressure down here* [lower side of the ball], *so it's like they make up for each other… I don't know, we're guessing that's it…*

Force-as-mover not only participates in understanding the downward push of the oil column on the upper side of the body (*"we have oil increasing the pressure on top of the ball")*, but <u>also</u> in "seeing" an upward push of the underlying water on its bottom (*"but the oil is also increasing the pressure down here")*. The result of these two effects is, however, still uncertain (*"I don't know, we're guessing that's it")*.

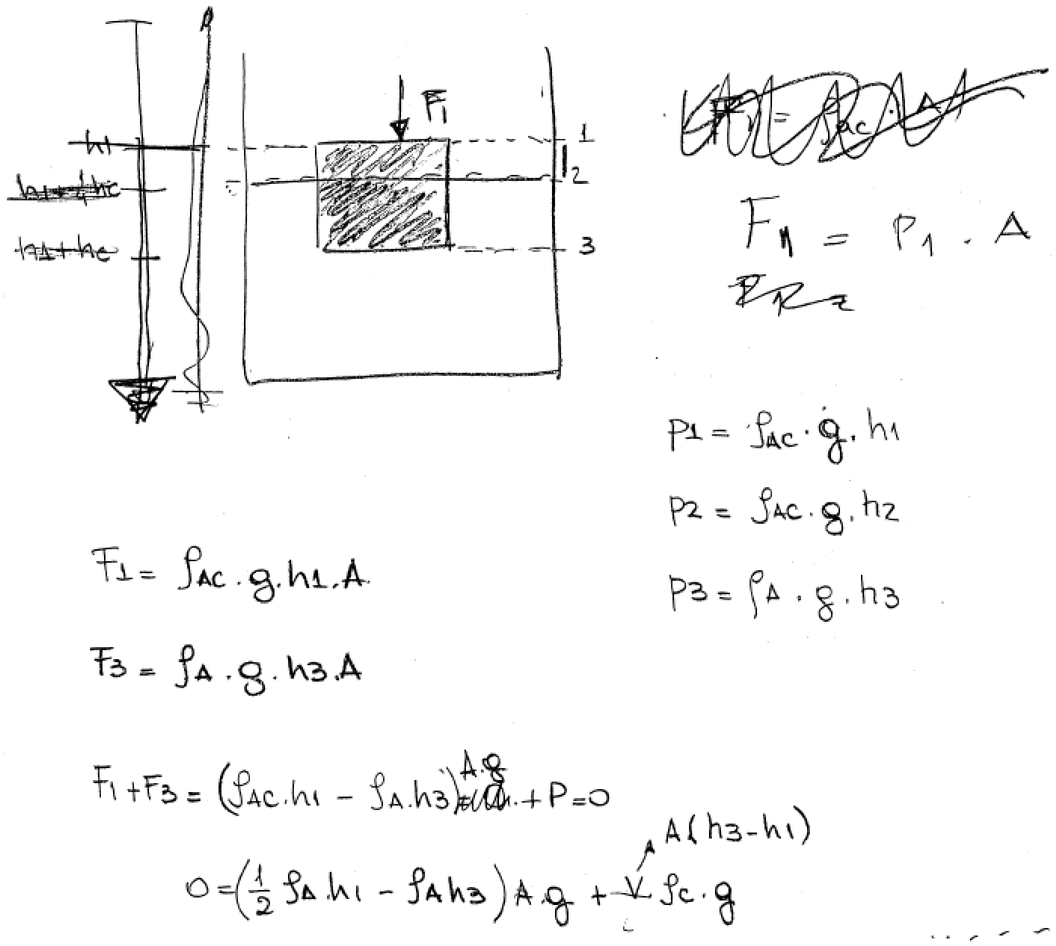
As will be seen immediately below, now that students have linked the pressures in the fluids to the resulting buoyant force, they are ready to write an equation that *is* a symbolic form (competing terms), which incorporates *dynamic balance* and *force-as-mover* into their understanding, but in a subtle, different manner. *Force-as-mover* is involved in accounting for all the influences that make up the buoyant force and thus there is no need for an independent term for the oil column above the ball. The consideration of a simpler geometry has also helped them take this approach ("*let's look at it like we did the other one… like the cube…*")

***A**: let's do it simplified…*

***L**: let's pretend it's a cube instead of a sphere!... and so we can get rid of the pressures on the lateral walls!* (laughter)

Students draw a cube submerged in water and oil. They identify a point on the upper side of the cube (see figure 5) and relate the value for its pressure (P_1) to a force (F_1), as they draw a vector pointing into the cube. At this time they write down "$F_1=P_1 A$".

Figure 5: ***A, T*** and ***L´***s written solution, case II. (***ρ_A***: water density; ***ρ_{AC}***: density of oil; ***V***: volume of the cube; ***A***: area of each side of the cube; ***h_1***, ***h_2*** and ***h_3***: depths at points ***1***, ***2*** and ***3***; ***P_1***, ***P_2*** and ***P_3***: pressures at points ***1***, ***2*** and ***3***. ***V***: unknown, submerged in water, volume)



After this, they locate two other points (2 and 3) on the same figure, and assign to each a corresponding depth, ***h***. They then write expressions for the pressures at each of these points in terms of the different depths, and densities[7]. Students are thus generating an equation in which the relevant quantities to understand the situation are pressures. These are now the quantities that account for the different influences on the body[8]. They continue the solution writing the expressions for ***F_1*** and ***F_3*** in terms of the pressures just computed. Immediately after this, they bring the cube's weight into the analysis, and balance all these forces: $(\rho_{AC}h_1 - \rho_A h_3)Ag + P = 0$ (see figure 5)

> ***A****: so we just wrote down the force up here, plus the force down here* [***F_3***] *caused by P_3* [while gesturing the upward direction of ***F_3*** on the lower face of the cube], *plus the weight, pointing down, so it will have a plus sign, and all this will equal zero.*

For A, T and L, this equation <u>is</u> a symbolic form (competing terms). It accounts for all the p-prims that students have activated. *Force-as-mover* is embodied in the first term, involved in both the downward and upward pushes. *Dynamic balance* is evidenced through the signs of each contribution and the fact that all those influences are balanced and equated to zero. Later in the interview, after correcting the expression for P_3 they solve the equation and are not at all surprised by the result that the cube actually does go up.

As opposed to case I, students in case II wrote an equation that was a symbolic form. This means that the equation worked for them as a template through which they were able to see their intuitions at play. How do we understand the fact that the same primitives (*force-as-mover* and *dynamic balance*) can account for a different intuition? More precisely, how did the use of the equation participate in this change in their intuition? We understand that the change in students' intuition has occurred because *force-as-mover* has taken a new function. This change of function has been enabled by their manipulation of the symbolic form that they had already built for the case of a cube submerged in water (F_2-F_1=$\rho_A gA(h_2$-$h_1)$. A new edition of this equation takes the form $(\rho_{AC}h_1-\rho_A h_3)Ag+P=0$ (see figure 5) In this episode, the idea of pressure has allowed them to link this p-prim to two influences at the same time. Unlike before, not only does the p-prim account for the downward influence of the oil on the body due to the fact that it is "sitting" on top of it. It now also intervenes to account for the upward influence of the increased pressure of the fluid below the body, which is a consequence of the presence of oil as well. *Force-as-mover* accounts for competing influences on the body, which together result in buoyancy. This new function of *force-as-mover* is in agreement with a mechanistic understanding of the phenomenon. Once again, a crucial point here is that this process of reorganizing the p-prim structure has been achieved through the mathematical manipulation of a symbolic form. The data show a student being quite explicit about this point. Elevating her pitch, *A* says "*...but now it's like,* ***there you go! there's your*** *buoyancy force!"*, indicating that she *sees* that buoyant force in a new way, in the equation. In this sense we conclude that in this episode, that equation, given that it was a symbolic form, served as a vehicle for her to, at least incrementally, modify her p-prim structure.

Conclusions

How did the use of equations contribute to the refinement of students' intuitions? In Case I, the interviewer's suggestion of a "correct" equation, and students subsequent manipulation did not impact their understanding of buoyancy. If anything, obtaining a "physically correct" result only left them with puzzlement. Our analysis indicates that this occurred because the equation they were hinted to work with was not a symbolic form and thus was disconnected from their intuitions. Changes in students intuitions are unlikely under these conditions.

Case II was carried out to address the issues arisen in Case I. More precisely, how could we be more efficient in helping students refine their intuitions regarding the concept of buoyancy? Prompting students to consider pressures around a submerged body in a simplified geometry helped them build an equation that was a symbolic form to them. These equations were ($F_2 - F_1=\rho_A\, g\, A\, (h_2 - h_1)$ and $(\rho_{AC}\, h_1 - \rho_A\, h_3)Ag + P = 0$, and they enabled students to modify the function of *force-as-mover* in their understanding of buoyancy. This p-prim now intervenes to account for the competing influences on the body (F_2 and F_1 ; $\rho_{AC}\, h_1$ and $\rho_A\, h_3$), which together result in a buoyant force. This new function of *force-as-mover* is in agreement with a mechanistic understanding of buoyancy. Thus, Case II shows how the manipulation of this symbolic form allowed students to refine their intuition by changing the function of a particular p-prim (*force-as-mover*) in the understanding of buoyancy. Prior to this episode, *force-as-mover* did not participate in the understanding of buoyancy. Buoyancy was regarded as a force related only to the amount of fluid displaced. Still, *force-as-mover* was called for to account for the influence of the oil sitting on top of the body, and students explicitly included it in their equations but separately from buoyancy. Recalling the case reported by Sherin (2006), there was a "moral" after that episode which was that *spontaneous resistance* should not be overlooked. Similarly, for *A, T* and *L*, after this episode there is a moral: *force-as-mover* is involved in the buoyant force, not only for the action of the fluid below the submerged body, but also for the fluid above it. There is a key observation on this regard. The fact that *A* raises her tone and exclaims "*...but now it's like,* ***there you go! there's your*** *buoyancy force!"* as she understands how this force is made up from the contribution of pressures around the body. Obtaining this symbolic form has been the vehicle for her to achieve this understanding.

We speculate that this micro-learning episode of buoyancy observed in Case II could be a part of a larger "virtuous circle". When approaching a new problem-solving situation, students can write equations that either are o are not symbolic forms. If they can start out from a symbolic form, its manipulation has better chances of refine their intuitions (for example, by changing the function of a particular p-prim, as observed in this case). Those refined intuitions will be the new starting point for the next loop of this virtuous circle. The modification of students' intuitions, of course, is not a process that occurs once and forever, and it involves many different micro-episodes of this kind. Students in Case I also started off from their intuitions. They wrote an equation that was to them a symbolic form. They were "forced" to abandon their symbolic form, and thus to work with an equation detached from their intuitions. The subsequent manipulation of this equation had very little chance of refining their intuitions.

Implications for Instruction

Despite the fact that interviews were carried out outside the classroom, certain aspects are worthy of mention in regard to teaching practices. In both cases, the interviewer intervened with the purpose of helping students out. However, this purpose was attained in Case II and not in Case I. How could this be taken into account in regards to teaching practices? We believe it is important for instructors to be aware of the gap there may be between correct and useful equations and equations that are only correct. Prompting students to solve a correct equation clearly does not guarantee that they will improve their understanding of the concepts involved in the equation.

In Case I, *M* and *J* started off from an incorrect equation, and in order to help them, the interviewer prompted them to solve a modified, correct version of that equation. This clearly did not promote any refinement in their understanding neither of buoyancy nor of the particular situation. In the second case, *A*, *T* and *L* started off from the same incorrect description of the situation, but the interviewer made a different move: he provided an auxiliary situation (in a geometrically simpler context) and prompted them to consider pressures on all the surfaces of the submerged body. This was done on the assumption that by considering pressures, they could be able to connect their intuitions (force-as-mover) to the mathematical description of the situation in a different way. As a result, they also seemed to have changed the function of force-as-mover in their understanding of buoyancy ("*there you go! there's your buoyant force!"*)

In this sense, we use the term “useful” to refer to equations that are symbolic forms, and as such have the potential of refining students’ intuitions through mathematical manipulation.

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Footnotes

[1] The term *schema* stems from Cognitive Psychology. A review on the meaning of this term can be found in Pozo (1989).

[2] Larkin considers a first naïve representation in the solving process, but this representation is linked to a physical representation (schemas) through formal physical knowledge. Thus, what guides the solving process are normative physical principles.

[3] A person gives a block a shove so that it slides across a table and them comes to rest. Talk about the forces and what’s happening. How does the situation differ if the block is heavier? (Assume that the heavier block starts with the same initial speed)

[4] From Leonard, W., Dufresne, R., Gerace, W. & Mestre, J. (2001).

[5] The subindex “p” corresponds to the word “pelota” (Spanish for ball), the subindex “a” to “agua” (Spanish for water), and the subindex “ac” to “aceite” (Spanish for oil); “ρ” is used for fluid density, “*E*” for “Empuje” (Spanish for buoyancy), and “P_{col}” for “Peso” (Spanish for weight) of the oil column.

[6] Implicit in this equation is the fact that $\rho_a = 4/3\ \rho_p$, i.e. that water density is 4/3 of the ball’s density. This relation had been previously obtained in the first part of the problem, in which the ball floated in water, with only ¾ of its volume submerged.

[7] The subindex “AC” corresponds to the word “Aceite” (Spanish for oil), and “AG” to “Agua” (Spanish for water); “ρ” is used for fluid density and “*A*” for the area of each side of the cube.

[8] Despite the fact that the expression for P_3 is incorrect: it corresponds to the pressure in point 3 if there was only water above it.