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REGULAR PAPER

Nonlinear Model Predictive Path Following Controller with Obstacle Avoidance

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Abstract

In the control systems community, path-following refers to the problem of tracking an output reference curve. This work presents a novel model predictive path-following control formulation for nonlinear systems with constraints, extended with an obstacle avoidance strategy. The method proposed in this work simultaneously provides an optimizing solution for both, path-following and obstacle avoidance tasks in a single optimization problem, using Nonlinear Model Predictive Control (NMPC). The main idea consists in extending the existing NMPC controllers by the introduction of an additional auxiliary trajectory that maintains the feasibility of the successive optimization problems even when the reference curve is unfeasible, possibly discontinuous, relaxing assumptions required in previous works. The obstacle avoidance is fulfilled by introducing additional terms in the value functional, rather than imposing state space constraints, with the aim of maintaining the convexity of the state and output spaces. Simulations results considering an autonomous vehicle subject to input and state constraints are carried out to illustrate the performance of the proposed control strategy.

Keywords Path-following · Model predictive control · Obstacle avoidance

1 Introduction

Solving path-following problems is widely required in the industry, as it finds application in the autonomous vehicles technology such as unmanned aerial vehicles or automated guided vehicles, as well as in different types of industrial robots (crane towers, milling machines), which are examples of autonomous motion systems whose correct functioning is based on the computation of a feasible collision-free trajectory. Path-following is a quite general problem in the control systems design that basically consists in tracking a predefined curve. In fact, the control problems can in general be roughly classified into setpoint stabilization, trajectory tracking and path-following and the first two can be considered particular cases of the latter -more general- problem. In the literature, this problem is also referred to by different names, such as manifold stabilization [4, 29], maneuvering problem [37], or contour control [17, 38], among others. The path-following problem can be interpreted by splitting the problem into a geometric and a dynamic task, where the geometric task consists in driving the system *to* the path and the dynamic task requires to satisfy a dynamic assignment *along* the path (for example, when there is a prescription on time, velocity, or acceleration). In recent years, optimization-based strategies for solving motion planning problems have been gaining attention [31]. In particular, Model Predictive Control (MPC) stands out as an interesting optimization-based scheme for this task. It consists of a receding horizon control

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strategy, meaning that a fixed-horizon optimal control problem is solved at every sampling time [32]. From the resulting optimal control sequence, only the first control action is applied to the plant, and the procedure is repeated. A significant advantage of the MPC framework is its ability to take into account (and to anticipate) constraints on the inputs and states, such as collision avoidance constraints. Frequently, in the context of path-following applications, the presence of obstacles must be considered, increasing the complexity of the task, which can be divided in three issues: path-following control, obstacle avoidance strategy and the coupling between the two previous goals.

1.1 Related Works

1.1.1 Model Predictive Path Following

The Path-Following problem using Nonlinear Model Predictive Control (NMPC) strategies where the reference is a parameterized curve has been extensively studied and several contributions have been recently introduced. In [39] a model predictive path-following controller is proposed which uses a fixed terminal law obtained by linearizing the error system along the state reference path and solving a Polytopic Linear Differential Inclusion Problem (PLDI). Furthermore, a unique terminal set in the error space is computed, which could be very restrictive in many relevant cases. In [1], a path-following control based on kinematics model of vehicles is proposed, by introducing a terminal stabilizing feedback law, although no considerations on dynamic tasks are discussed. For a thorough review of NMPC trajectory tracking and path-following controllers with application to non-holonomic robots the reader is referred to [26, 27]. Output path-following is deeply studied in [7], including dynamic tasks and a geometric interpretation of the problem. Similar to what is made in [3], the geometric interpretation is based on a normal-traverse form decomposition for manifold stabilization, as well as the analysis of convergence conditions. In all these works the path is required to be sufficiently often continuously differentiable. A general explanation about the relation between different control objectives, covering setpoint stabilization, trajectory tracking, path-following, and economic operation, and discussion on their approaches within the NMPC framework are included in [23].

1.1.2 Collision Avoidance

A plethora of strategies concerning the global planning of collision-free trajectories on known or partially known environments are available in the literature, with several remarkable methods such as RRT and others [15, 16, 33]. Often, hard constraints, barrier functions and artificial potential fields, among others are also used. The reader is referred to [14] for a detailed review on the subject. Real-time obstacle avoidance coupled with an accurate path-following control has been one of the major issues in the field of mobile robotics [5, 18]. Proposing adequate obstacle avoidance control strategies is a major issue in the design of reliable applications in this field, and it underlies two different issues: the obstacle detection and the computation of the system reaction [9].

1.1.3 Combined Model Predictive Path Following and Collision Avoidance

Strategies combining NMPC and Obstacle Avoidance are frequently found in works on the field of robotics research. For example, in [8] an integrated control structure combining path tracking, vehicle stabilization, and collision avoidance is presented, which uses a variable time-step and horizon, as well as so-called emergency paths -that violate the stabilization criteria- for collision-avoidance guarantees. A controller based on non-linear model predictive control for trajectory tracking and path-following is introduced in [42]. Similar to this work, the collision avoidance is achieved by introducing a penalty into the cost function. In order to guarantee the avoidance, constraints depending on the distance towards the obstacle are introduced in the optimization problem. Simulation examples on a robotic manipulator are presented. A reference tracking NMPC controller for output trajectory tracking with obstacle avoidance is presented in [34]. It is based on continuous trajectory replanning executed in real-time. The obstacles are modelled as elliptic sets, that are enlarged as means of providing a safety margin. The obstacle avoidance is imposed as soft constraints for the controller. Experimental results on 6 DoF manipulator are provided, which confirm the suitability of the approach for real time applicability. The path-following and obstacle avoidance tasks -typically when the system follows the path that leads to an obstacle- may require opposite reactions from the system. This generates a situation referred to as corner-situation, in which a local minimum on the cost is reached. In many practical situations, control objectives are locally modified with the aim to fulfill the desired task, see for example [18]. Another example of this situation can be found in [2], where trajectory tracking and path-following controllers based in model predictive control are compared and the obstacles are modeled as hard constraints. For additional optimization based collision avoidance strategies the reader is referred to [40] and references therein. The reader is also referred to recent works that implement NMPC strategies, which are focused on the setpoint tracking in unknown environments [25] and trajectory tracking tasks [30], with application in the context of quadrotor control.

1.1.4 Artificial Variables

A formulation for tracking references subject to unpredictable changes for linear systems is introduced in [19]. In that work, auxiliary variables are introduced in the optimization problem of the NMPC. This formulation provides guarantees of recursive feasibility for the optimization problem, for piecewise constant references. The auxiliary or artificial variable is constrained to belong to an invariant set which contains the equilibrium set of the dynamic system. This approach has been extended to nonlinear systems in [20]. Artificial trajectories have also been proposed in the literature; earlier works are available in the context of the control of periodic linear time-varying system [21]. A more recent formulation for trajectory tracking with obstacle avoidance, that is based on an economic NMPC strategy and makes use of auxiliary trajectories, can be seen in [36]. Following the ideas presented in [19] and further extended in [20], auxiliary variables are used as additional optimization variables. Similarly to [36], a set of variables conforming an auxiliary reference trajectory is introduced in the optimization problem as decision variables. Moreover, these variables are required to satisfy the system dynamics, meaning that the artificial trajectory is a feasible trajectory for the system.

1.2 Contributions

This paper is an extension of a work originally presented in [35], where the obstacle representation presented in [13] is merged in a new path-following framework, based on an NMPC which uses auxiliary trajectories. The work is extended in the sense that theoretical foundations for proving the stability of the control strategy are established, by introducing specific definitions and sufficient assumptions. Also, scenarios including arbitrary reference paths are discussed. The global collision-free motion planning task, which involves the planning and tracking of the optimal obstacle avoiding trajectory with respect to the complete reference path is not within the scope of this work. Rather, this work focuses in a lower-level layer, conforming a local optimizing trajectory planner and tracker which is able to handle generic reference paths, with obstacle-avoiding capabilities. The contribution of the paper is twofold. First, a formulation of model predictive path-following control is presented, which is able to optimally track reference paths that may be incompatible with the constraints and system dynamics. Also, this formulation is able to account for a-priori known obstacles or keep-out zones by introducing soft constraints into the cost functional, maintaining the convexity of the admissible state-space. This formulation is promising since it is a step towards overcoming limitations of available MPC controllers for path following in scenarios where certain conditions, such as terminal constraints or the presence of obstacles along the path, render them theoretically inapplicable. The proposed controller uses additional decision variables that provide a feasible trajectory (compatible with the system dynamics and constraints) which acts as a reference to be tracked by the system. At each iteration, a single optimization problem is solved, providing an optimal time-parameterization of the reference path, an auxiliary trajectory, and the inputs to be applied to the system. This auxiliary trajectory, which is referred to as artificial trajectory, is planned and used as reference to be tracked by the system. Besides, this trajectory fulfills certain assumptions -such as continuity, differentiability, feasibility- required in path-following formulations available in the literature. The incorporation of the artificial trajectory provides guarantees of recursive feasibility even under hard and conservative constraints, such as a terminal equality constraint. Sufficient conditions for feasibility and stability for feasible reference paths using artificial trajectories are introduced and preliminary results on stability are developed, which as far as the authors know, have not been presented in the literature. The theoretical result comprises the stability proof of the closed-loop system which applies for followable paths, considering a convex feasible state space.

Complications arise due to the presence of obstacles on the path, mainly concerning constraints, such that the optimization problem might eventually become unfeasible in existing formulations. With the scheme proposed in this work, this problem is overcome thanks to the additional ingredients included in the optimization problem. Although this situation is present in the proposed controller, it can be alleviated through convenient tuning parameters that penalize the planning error (the difference between the nominal reference trajectory and the artificial variable). Lower weights on these parameters provide optimal solutions for larger deviations from the reference, as required for avoiding obstacles, at the expense of closeness of the tracking result. Another advantage resulting from the modifications introduced with respect to standard MPC controllers follows from the feasibility guarantees obtained.

1.3 Organization

The remainder of this paper is organized as follows: Section 2 describes the path-following problem, its motivation and definitions of path feasibility and followability are introduced. Next, existing MPC strategies designed to achieve path-following are discussed in Section 3. Section 4 is devoted to the presentation of the proposed controller formulation for path-following, based on extending the existing controllers by the incorporation of artificial trajectories. A stability proof for feasible reference paths is included. In Section 5, the obstacle avoidance problem is formalized and the Nonlinear Model Predictive Path-Following Control for obstacle avoidance formulation is presented. Section 6 is devoted to numerical simulations. Beginning the section, the system model and design parameters are detailed. Next, the simulation scenarios, which are used to asses the proposed control strategy, are presented, followed by the results of numerical experiments and discussion on the performance of the controller. Finally, conclusions and future works can be found in Section 7.

2 Problem Statement and Motivation

The aim of this work is to design a novel model predictive path-following control formulation for nonlinear systems with constraints, extended with an obstacle avoidance strategy. Before presenting the problem, some fundamental concepts are at first revisited. Intuitively, path-following can be understood as tracking, as close as possible, a reference that moves along a curve (in a space of lower or equal dimension than the state space). In the case of a parameterized path, the predictive control strategy also involves obtaining, at successive time instants, a time parameterization of the path parameter, which means determining how the reference should evolve or, for discrete time, which points on the reference curve are to be tracked.

Consider a continuous-time nonlinear dynamical system[1]

$$\begin{aligned} \dot{x} &= f(x,u), \quad x(0) = x_0, \\ y &= h(x), \end{aligned} \tag{1}$$

where $x \in \mathcal{X} \subseteq \mathbb{R}^n$, $y \in \mathcal{Y} \subseteq \mathbb{R}^p$, and $u \in \mathcal{U} \subseteq \mathbb{R}^m$, are the state, the output and the input at time t, respectively. The constraints sets $\mathcal{X}$, $\mathcal{Y}$ and $\mathcal{U}$ are assumed to be compact and convex. The functions $f(x,u) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ and $h(x) : \mathbb{R}^n \to \mathbb{R}^p$ are assumed to be continuous.

The output reference path is a parameterized curve $\mathcal{P}$ defined by

$$\mathcal{P} = \{r \in \mathbb{R}^p | r = y_{\text{ref}}(s)\}, \tag{2}$$

where $s \in \mathcal{S} = [s_0, s_1)$ is the path-parameter and $y_{\text{ref}} : \mathbb{R} \to \mathbb{R}^p$ is the reference function to be followed in the output space. The curve $\mathcal{P}$ can be interpreted as a set of points, r, parameterized by a scalar s. The path-parameter evolution is modeled by a single integrator dynamic system, $w = \dot{s}$, where s is handled as an internal state of the controller, and w is an *internal* input signal $w \in \mathcal{W} = [0, w_{max}]$. Formally, the path-following problem is defined as

[1]In order to avoid cluttered notation, the time dependence (t) will be dropped unless necessary.

Definition 1 (Path-following problem) Consider a constrained system described by Eq. 1 and a parameterized reference curve $\mathcal{P}$ defined by Eq. 2. Design a control law for u and w such that, as $t \to \infty$, i) $\|y - y_{\text{ref}}(s)\|$ converges to and remains inside an arbitrarily small neighborhood of zero (convergence to the path) and ii) $s(0) = s_0$, $s \to s_1$, for $t \to \infty$, and $\dot{s} \geq 0$, for all $t \geq 0$ (forward movement in the path direction).

Note that Definition 1 introduces the variable s and the input w providing an extra degree of freedom to the control system. The previous formulation accounts for the so-called *geometric task*. Now, a *dynamic task* is introduced, which requires the satisfaction of convergence to a desired speed along the path $w_{\text{ref}}(s)$. In this context, the path-following problem can be reformulated as follows:

Definition 2 (Path following with velocity assignment problem) Consider a given reference parameter velocity $w_{\text{ref}}(s)$. Solve the path-following problem given in Definition 1, with the parameter input converging to $w_{\text{ref}}(s)$, i.e.,

$$\lim_{t\to\infty} \|w - w_{\text{ref}}(s)\| = 0.$$

The path-parameter s defines a critical variable for the functionality of the controller presented in this work. This allows the spatial path to be followed more closely, at the expense of moving along the path without a strict requirement on when the system should be at each successive point on the path.
In the case the parameter velocity is enforced, $\dot{s} = w_{\text{ref}}(s)$, the path-following problem becomes a trajectory tracking one[2]. Finally, in the case $\dot{s} = 0$ (i.e., when s is a constant), it becomes a setpoint tracking problem.

Assumption 1 It is assumed that the output reference y_{ref} and the system dynamic determine a unique pair of state and input references $(x_{\text{ref}}, u_{\text{ref}})$ for a given path-parameter input value w. Moreover, there exist continuous functions Γ_x and Γ_u such that $x_{\text{ref}} = \Gamma_x(r(s), w)$ and $u_{\text{ref}} = \Gamma_u(r(s), w)$ for a prescribed path-parameter evolution s.

Remark 1 Note that Assumption 1 becomes trivial in the case of a reference expressed in the state space ($p = n$). This assumption is also fulfilled by systems that can be described with differentially flat outputs.

[2]A soft switching between path-following and trajectory tracking problem is achieved by imposing a penalty on a cost term of the form $|w(s) - w_{\text{ref}}(s)|$.

As in [39], the path-following problem can be expressed in terms of the path-following error, which is defined as

$$x^e = x - x_{\text{ref}}(s, w). \tag{3}$$

The error dynamics are given by

$$\begin{aligned} \dot{x}^e &= \dot{x} - \dot{x}_{\text{ref}}(s, w) \\ &= f(x, u) - \frac{\partial x_{\text{ref}}}{\partial s} w, \end{aligned} \tag{4}$$

where the path is assumed to be differentiable. Particularly, exact path-following means that for any initial state such that $x^e(0) = 0$, then $x^e = 0$, for $t \geq 0$. For this to be accomplished, there must exist some input w such that the reference trajectory x_{ref}, obtained by imposing it at some point on the path, is a solution to the nonlinear system equation

$$f(x_{\text{ref}}, u_{\text{ref}}) = \frac{\partial x_{\text{ref}}}{\partial s} w.$$

Also, since the path-following problem requires that $s \to s_1$ as $t \to \infty$, the condition for the equilibrium must be fulfilled considering some positive value of the input ($w > 0$) along the path. In other words, as long as the error dynamics is considered, for this kind of equilibria - referred to as *dynamic equilibria* - to exist, the path must be consistent with the system dynamics and constraints. Namely, the path must be feasible according to the following definition.

Definition 3 (Feasible path) A path $\mathcal{P}$ is feasible if there exists some positive path-parameter velocity input $w \in \mathcal{W}$, $w > 0$, such that $f(x_{\text{ref}}, u_{\text{ref}}) = \dot{x}_{\text{ref}}$ with $u_{\text{ref}} \in \mathcal{U}$, $x_{\text{ref}} \in \mathcal{X}$, for all $s \in \mathcal{S}$.

A more conservative definition of the reference path in terms of the state and inputs constraints can be the exactly followable path, expressed as follows,

Definition 4 (Exactly followable path) A path $\mathcal{P}$ is exactly followable if it is feasible and $\mathcal{P}$ is contained in the interior of the pointwise image of the state constraints $\mathcal{X}$ under the output map h, i.e $\mathcal{P} \subset int(h(\mathcal{X}))$.

The followability of a path constrains the reference state and input to belong at all times to the interior of $\mathcal{X}$ and $\mathcal{U}$ respectively, where no constraints are active, which is a more conservative requirement than *feasibility*.

Next, some technical assumptions are introduced.

Assumption 2 There exist a continuously differentiable function $g(\cdot, \cdot)$ and a control input u^e such that the error dynamic system Eq. 4 can be expressed as

$$\dot{x}^e = g(x^e, u^e). \tag{5}$$

Remark 2 Note that in the error system (5), the function g is parameter-dependent on s and w, and the input u^e is an implicit function of u, s and w.

Assumptions 1 and 2 are typical for output tracking in NMPC and are required to provide stability guarantees.

With respect to the general path-following problem, if the problem was expressed in terms of the path-following error, which is dependent on the path-parameter, an equivalent definition of the control objective would be to stabilize the system around the origin of the error dynamical system, $x^e = 0$.

Considering the error dynamics, an equilibrium on the error will be determined by

$$\dot{x}^e = g(x^e, u^e) = 0. \tag{6}$$

3 Model Predictive Path Following Control (MPPFC)

In the MPC framework, a finite horizon optimal control problem is solved successively, at each time instant t_k [6, 32]. The result of the optimization problem is the optimal input sequence -and implicitly the state evolution- predicted over horizon T. In most applications, a regular sampling interval T_s and a horizon length $N \in \mathbb{N}^+ < \infty$ is introduced such that $T := NT_s$. At each k-th time instant, an optimal control input sequence $\{u_i^*\}_k, i = 1, \cdots, N$, is obtained, from which the first control action is applied during a sampling interval T_s.

Considering the system (5) and a prediction horizon $T > 0$, a cost function is defined as

$$J(x_0, s_0; u(\cdot), w(\cdot)) := \int_0^T \ell(x^e(\tau), u(\tau), w(\tau))d\tau + V_f^e(x^e(T)), \tag{7}$$

where the first term, $\ell(x^e(\tau), u(\tau), w(\tau))$, is a positive definite function - referred to as *stage cost* - devoted to penalize the output and input error along the path, while the second one, $V_f^e(x^e(T))$, - denoted as *terminal cost* - penalizes the predicted terminal state.

An optimal control problem, corresponding to a stable MPC formulation, can then be written as

$$\begin{aligned} &\min_{u(\cdot), w(\cdot)} J(x_0, s_0; u(\cdot), w(\cdot)) \\ s.t. \quad &\dot{x}^e(\tau) = g(x^e(\tau), u^e(\tau)), \quad x(0) = x_0, && (8a) \\ &\dot{s}(\tau) = w(\tau), \quad s(0) = s_0, && (8b) \\ &x(\tau) \in \mathcal{X}, u(\tau) \in \mathcal{U} \quad \forall \tau \in [0, T], && (8c) \\ &s(\tau) \in \mathcal{S}, w(\tau) \in \mathcal{W} \quad \forall \tau \in [0, T], && (8d) \\ &w(\tau) \geq 0 \quad \forall \tau \in [0, T], && (8e) \\ &x^e(T) \in \mathcal{X}_f^e, && (8f) \end{aligned}$$

where constraints (8a and b) describe the error system evolution and the evolution of the path-parameter s, (8c) describes the input and state constraints, (8d and e) constitute forward movement constraints, forcing the system to move along the path in the direction of increasing values of s, and (8f) is the terminal constraint. The solution of this problem will be conformed by the optimal input sequence $\{u_i^*\}_k$ as well as the optimal path-parameter input sequence $\{w_i^*\}_k$. The terminal penalty $V_f^e(x^e(T))$ and terminal constraint $x^e(T) \in \mathcal{X}_f^e$ are the key ingredients to ensure the asymptotic convergence to the path.

Following the ideas presented in [39], it can be shown that, assuming that all assumptions are fulfilled and the reference path is followable, the previous formulation for the controller is asymptotically stable. Moreover its region of attraction is outer bounded by the controllable set to $\mathcal{X}_f^e$. An interesting case that will be considered in this work stems from the followability of the reference path, therefore it can be chosen as terminal set ($x^e(T) = 0$) for a trivial terminal control law $u^e = 0$.

Remark 3 A generic path that fulfills state constraints is not necessarily dynamically feasible to be followed. The dynamical feasibility of the path is not trivially verified for nonlinear systems. Also, simple paths obtained as concatenation of feasible paths are not necessarily feasible. This fact could hinder the applicability of this strategy, as previous feasible path generation and/or validation is required.

Path-parameter Initial Condition The selection of the initial condition for the path-parameter s_0 at the time instant t_k, $s_0 = s(t_k)$, is also a design decision. In some path-following formulations, the initial condition for the path-parameter is not fixed and is freely determined by the controller [39]. For example, it could be obtained as the closest point on the path with respect to the current system state, as

$$s(t_k) = \arg\min_s \|x - x_{\text{ref}}(s, w)\|. \tag{9}$$

Alternatively, it is often preferred to enforce a strict time evolution on the path-parameter. This can be obtained by introducing the following constraint.

$$s(t_k + T_s) = \int_{t_k}^{t_k+T_s} w_k^*(\tau)d\tau, \quad s(t_k) = s_k. \tag{10}$$

This constraint consists in propagating the path-parameter by the input w^*, which could be the result of the optimization problem (8). Note that this may lead to time-varying terminal constraints in the optimization formulation, typically required for stability proofs [7].

4 Extended Nonlinear Model Predictive Path-Following Controller (xMPPFC)

In this section, a nonlinear MPPFC that extends the previous formulation with an artificial trajectory is presented. This formulation aims to relax the requirement of feasibility of the reference path with respect to the MPPFC formulation.

The key of this formulation is the addition of an artificial trajectory reference as an extra decision variable in the optimal control problem with the aim of enlarging the region of attraction and avoiding the possible loss of feasibility which can be due to unfeasible references - e.g. discontinuities or violation of state constraints- or derived from the presence of obstacles. The obstacle can be considered as an external source that causes a loss of feasibility to a path-following problem, and consequently the existing formulations are deemed to be inapplicable.

4.1 Terminal Constraint and Equilibrium Characterization

Next, some technical definitions will be introduced.

An error equilibrium trajectory is a feasible trajectory such that for each of its points, the error with respect to their corresponding points on the reference path is constant. This is fulfilled, for example, by trajectories that follow the same evolution as the path (i.e., trajectories that have the same "shape"), although they may not be coincident (they may differ in a constant term, i.e. they are a translated copy of the path). They can be defined as follows,

Definition 5 (Path-following error equilibrium trajectory) Let $T \in (0, \infty)$ be an horizon length, let $(x(\cdot), u(\cdot))$ be a pair of state and input trajectories such that $(x(\tau), u(\tau)) \in \mathcal{X} \times \mathcal{U}$, and let $w(\cdot)$ be a path-parameter velocity input trajectory with $w(\tau) \in \mathcal{W}$, for all $\tau \in [0, T]$, that satisfy the system and path-parameter dynamics, namely $\dot{x} = f(x, u)$, $w = \dot{s}$, with initial conditions $x(0) = x_0$ and $s(0) = s_0$. For a given feasible path $\mathcal{P}$, a Path-Following Error Equilibrium Trajectory is given by both a trajectory $(x_s(\cdot), u_s(\cdot))$ and a path-parameter velocity input $w_s(\cdot)$ that satisfy (6), i.e. $\dot{x}^e(\cdot) = 0$.

In the error space, the equilibrium trajectories are fixed points (i.e., equilibria). It should be noted that a fixed and bounded feasible set for system $\dot{x} = f(x, u)$, corresponds to a time varying feasible set for the error system $\dot{x}^e = g(x^e, u^e)$. Consequently, an equilibrium trajectory may be feasible locally, but not globally.

Remark 4 In the particular case $w \equiv 0$, the reference becomes a single point and therefore, the Error Equilibrium Trajectory is also a point in both, the original state space and the error state. This can be exploited when working with systems with feasible equilibria, i.e. $f(x_s, u_s) = 0$ exists and is feasible.

Multiple path-following error equilibrium trajectories may exist for a given initial path-parameter value s, corresponding to different values of w and consequently to different trajectories. Therefore, a path-following error equilibrium set can be defined as follows.

Definition 6 (Path-following error equilibrium set) The Path-following error equilibrium set $\mathcal{X}_s^e$ is the set of points in the error space such that there exist some input trajectory $u_s(\cdot)$ and some path-parameter velocity input trajectory $w_s(\cdot)$ which make the future (predicted) error between the state $x_s(\cdot)$ and the reference path $x_{\text{ref}}(s(\cdot))$ to be constant while the predicted state trajectory remains feasible for all future time, i.e.

$$\mathcal{X}_s^e = \{x_s \in \mathcal{X} | \exists u_s(\tau) \in \mathcal{U}, w_s(\tau) \in \mathcal{W} : \quad \dot{x}^e(\tau) = g(x_s^e(\tau), u_s^e(\tau)) = 0, \\ x_s(\tau) \in \mathcal{X} \forall \tau > 0\}. \tag{11}$$

Remark 5 It is important to note that feasibility is required for increasing values of the time variable, rather than the path-parameter. This enables the selection of equilibrium trajectories that consist of a constant reference value (corresponding to a parameter input $w(\tau) = 0$ for all $\tau \geq 0$) or trajectories for which the path-parameter come to a stop after some increase interval, as well as trajectories with always increasing path-parameter evolutions.

4.2 Controller Formulation

The controller presented in the previous section is extended by the introduction of an auxiliary trajectory resulting from additional decision variables incorporated in the optimization problem. This trajectory will be referred to as artificial trajectory and may be interpreted as a reachable reference output trajectory given by $y_a(\cdot) = h(x_a(\cdot))$ [20]. The artificial state trajectory is resulting from applying the input trajectory $u_a(\cdot)$ by the dynamic system (1) to an initial state x_a. The artificial initial state and the input trajectory are decision variables in the optimization problem. In order to account for this, the stage cost will be composed of two terms, the tracking stage cost and the planning stage cost. The tracking stage cost, denoted by $\ell^a(\cdot)$, penalizes the tracking error between the predicted trajectory and the artificial trajectory. The latter, $\ell^p(\cdot)$, penalizes the error between the artificial output trajectory with respect to the reference path, and also the time-dilation w^e, namely the difference between the parameter input and the parameter reference velocity. The proposed cost functional is completed by the terminal offset penalization $V_O(\cdot)$. Summing up, the cost functional is presented:

$$J^a(x_0, s_0; u(\cdot), x_a, u_a(\cdot), w(\cdot)) = \int_0^T \ell^a(x_a^e(\tau), u_a^e(\tau)) d\tau \\ + \int_0^T \ell^p(y_O^e(\tau), w^e(\tau)) d\tau + V_O(y_O^e(T)), \tag{12}$$

with $x_a^e(\tau) = x(\tau) - x_a(\tau)$, $y_O^e(\tau) = y_a(\tau) - y_{\text{ref}}(\tau)$ and $w^e(\tau) = w(\tau) - w_{\text{ref}}(\tau)$ denote the artificial error trajectory, the artificial error output trajectory, and path-parameter input error trajectory, respectively. The feedback control is obtained by the solving at each time instant t_k, for the current state $x(t_k)$, the optimization problem $P_N^a(x_0, s_0; u(\cdot), x_a, u_a(\cdot), w(\cdot))$:

$$\min_{u(\cdot), x_a, u_a(\cdot), w(\cdot)} J^a(x_0, s_0; u(\cdot), x_a, u_a(\cdot), w(\cdot))$$

$$\begin{aligned}
s.t. \quad & \dot{x}^e(\tau) = g(x_a^e(\tau), u_a^e(\tau)), && x(0) = x_0, && \text{(13a)} \\
& \dot{x}_a(\tau) = f(x_a(\tau), u_a(\tau)), && && \text{(13b)} \\
& \dot{s}(\tau) = w(\tau) && s(0) = s_0, && \text{(13c)} \\
& x(\tau) \in \mathcal{X}, u(\tau) \in \mathcal{U} && \forall \tau \in [0, T], && \text{(13d)} \\
& s(\tau) \in \mathcal{S}, w(\tau) \in \mathcal{W} && \forall \tau \in [0, T], && \text{(13e)} \\
& x_O^e(T) \in \mathcal{X}_s^e, && && \text{(13f)} \\
& x_a^e(T) = 0, && && \text{(13g)} \\
& \tfrac{d}{d\tau} \ell^p(y_O^e(\tau), w^e(\tau)) \leq 0 && \forall \tau \in (0, T), && \text{(13h)}
\end{aligned}$$

where the constraint (13c) describes the evolution of the path-parameter s, $\mathcal{X}, \mathcal{U}$ are the state and the input constraint sets, respectively. The inclusion of the cost decrease constraint (13h) is required, as it conforms a fundamental tool in the stability proof. The terminal artificial error $x_O^e(T) := x^a(T) - x_{\text{ref}}(T)$ is constrained to belong to the error equilibrium set $\mathcal{X}_s^e$(13f), while the terminal equality constraint forces the predicted state to be equal to the artificial reference.

The artificial trajectories in Problem (13) induce a greater domain of attraction and provide guarantees of recursive feasibility in reference infeasibility scenarios, which may be consequence of reference paths that produce constraints violation, discontinuities on the reference, or of the presence of obstacles on the path, for instance.

As it is usual in MPC with terminal equality constraint, a controllability assumption is required to derive asymptotic stability. In this case, the following controllability condition, similar to the one proposed in [20], is stated.

Assumption 3 The system function $f(x, u)$ is differentiable at any feasible reference trajectory $(x_{\text{ref}}, u_{\text{ref}})$ and the linearized model given by the matrices $(A(x_{\text{ref}}, u_{\text{ref}}) = \frac{\partial f(x,u)}{\partial x}(x_{\text{ref}}, u_{\text{ref}}), B(x_{\text{ref}}, u_{\text{ref}}) = \frac{\partial f(x,u)}{\partial u}(x_{\text{ref}}, u_{\text{ref}}))$ is controllable. Furthermore, there exist positive constants $\varepsilon, b > 0$ and $\sigma > 1$ such that

$$\int_0^T \ell^a(x_a^e(\tau), u_a^e(\tau)) d\tau \leq b|x - x_a|^\sigma$$

holds for any feasible solution $(u(\cdot), x_{\text{ref}}(\cdot))$ of $P_N^a(x_0, s_0; u(\cdot), x_a, u_a(\cdot), w(\cdot))$ such that $|x_a^e| \leq \varepsilon$ and $|u_a^e| \leq \varepsilon$.

Assumption 3 ensures that the value functional admits at least a quadratic upper bound. This condition, similar to the one proposed in [20] for setpoint stabilization, is stronger than the weak controllability assumption proposed by [32][Ass. 2.23].

Remark 6 Differentiability of any feasible trajectory follows from the continuity of the dynamic system. This property is required only for system and artificial trajectories, which fulfill the system dynamics. Therefore, the resulting trajectories result to be differentiable. This property is also fulfilled by followable reference paths.

In the formulation (8), the domain of attraction of the controller is determined by the set of states that can be steered in the prediction horizon to the terminal set (which contains the reference path). Instead, the introduction of the artificial trajectories enlarges such domain of attraction, which for Problem (13) is the set of states that can be steered to a terminal set with respect to any artificial error trajectory that reaches $\mathcal{X}_s^e$. The terminal state of the artificial error trajectory is forced to belong to $\mathcal{X}_s^e$, which means the error can remain constant in the future, i.e. $\ell^p(y_O^e(\tau), 0) = \ell^p(y_O^e(t+T), 0)$ for all $\tau > t + T$ (see Section 4.4).

The stage cost functional and the offset cost functional must fulfill the following assumptions:

Assumption 4 1. There exists a $\mathcal{K}_\infty$ function α_ℓ such that $\ell^a(z, v) \geq \alpha_\ell(\|z\|)$ for all $(z, v) \in \mathbb{R}^{n+m}$.
2. There exists a unique minimizer of the planning cost $(\bar{y}_O^e, \bar{w}_O^e)$.
3. There exists a $\mathcal{K}_\infty$ function α such that $\ell^p(z, w) \geq \alpha(|z|)$ for all $w \in \mathcal{W}$.
4. There exists a lower bound on the offset cost $V_O(y_O^e) > \alpha_O(|y_O^e|)$, with α_O a $\mathcal{K}$-function and $V_O(0) = 0$.

These assumptions are typical in MPC for tracking formulations [20].

4.3 Stability Proof

Theorem 1 *(Asymptotic Stability) Consider that Assumptions 3 and 4 hold. Given a reference path $\mathcal{P}$ and initial parameter value s, then for any feasible initial state x_0 uch that the optimization problem (13) is feasible, the system controlled by the MPC feedback law derived from the solution of Problem (13) is stable, fulfills the constraints throughout the time and converges to*

1. *the reference path, if the path is exactly followable, or*
2. *a locally optimal (possibly non-zero) error equilibrium reference, if the path is not exactly followable.*

Proof The proof is divided into two parts. First, it is proved that the optimization problem is recursively feasible. In the second part, asymptotic stability of the optimal equilibrium error reference is proved.

i. *Recursive feasibility.*

Assume that there exists an optimal solution at time t given by $\bar{u}(\tau)$, $\bar{x}^a(\tau)$, $\bar{u}^a(\tau)$, $\bar{w}(\tau)$, with $\tau \in [0, T]$. Assuming nominal system dynamics, the state at the following sampling time $x(t+T_s)$ is coincident with the predicted state, i.e. $x(t+T_s) = \bar{x}(t+T_s)$, and the path-parameter is $s(t+T_s) = \bar{s}(T_s)$. From the error equilibrium constraint imposed on the terminal artificial error (13f) and terminal equality constraint (13g), it follows that both terminal error $x^e(T)$ and terminal artificial error $x_a^e(T)$ belong to $\mathcal{X}_s^e$. The error equilibrium state will be denoted by $x_s^e(T)$. Applying any equilibrium input trajectory $u_s(\cdot)$ and the corresponding path-parameter velocity input $w_s(\cdot)$ on the terminal equilibrium state $x_s(T) := x_s^e(T) + x_{\text{ref}}(s(T))$ results in an error equilibrium trajectory $x_s^e(\cdot)$ -following from Definition 6-. This produces an error trajectory that remains in the set $\mathcal{X}_s^e$, indefinitely. Therefore, a feasible solution at time $t + T_s$ can be obtained by extending the optimal inputs during T_s with adequate equilibrium inputs $(u_s(\cdot, w_s(\cdot))$ for a given $x_s(T))$, as follows

$$
\begin{aligned}
u^a(\tau, \bar{x}(t+T_s)) &= \begin{cases} \bar{u}^a(\tau, x(t)), & \tau \in [T_s, T), \\ u_s(\tau, \bar{x}(t+T)), & \tau \in [T, T+T_s], \end{cases} \\
u(\tau, \bar{x}(t+T_s)) &= \begin{cases} \bar{u}(\tau, x(t)), & \tau \in [T_s, T), \\ u_s(\tau, \bar{x}(t+T)), & \tau \in [T, T+T_s], \end{cases} \\
w(\tau, \bar{x}(t+T_s)) &= \begin{cases} \bar{w}(\tau, x(t)), & \tau \in [T_s, T), \\ w_s(\tau, \bar{x}(t+T)), & \tau \in [T, T+T_s], \end{cases}
\end{aligned} \tag{14}
$$

The resulting feasible state evolution is

$$
\begin{aligned}
x^a(\tau, \bar{x}(t+T_s)) &= \begin{cases} \bar{x}^a(\tau, x(t)), & \tau \in [T_s, T), \\ x_s(\tau, \bar{x}(t+T)), & \tau \in [T, T+T_s], \end{cases} \\
x(\tau, \bar{x}(t+T_s)) &= \begin{cases} \bar{x}(\tau, x(t)), & \tau \in [T_s, T), \\ x_s(\tau, \bar{x}(t+T)), & \tau \in [T, T+T_s], \end{cases}
\end{aligned} \tag{15}
$$

Note that this provides $\dot{x}_a^e(t) = 0$ for $T \leq t < T + T_s$. Since $x_a^e(T) \in \mathcal{X}_s^e$, it is able to stay in the set for all future instants, i.e. $x_a^e(T+\tau) \in \mathcal{X}_s^e$, $\tau \geq 0$. The equilibrium input renders $\mathcal{X}_s^e$ invariant, therefore the recursive feasibility is granted. □

ii. *Stability.*

Consider that the optimal solution of Eq. 13 at time t is given by

$$\bar{J}^a(x,s) = \int_t^{t+T} \ell^a(\bar{x}_a^e, \bar{u}_a^e)d\tau + \int_t^{t+T} \ell^p(\bar{y}_O^e, \bar{w}^e)d\tau + V_O(\bar{y}_O^e(t+T))$$

where $\bar{J}^a(x,s)$ denote the optimal value of the cost functional (12). In order to prove asymptotic stability, the function $W(x, x_{\text{ref}}) = \bar{J}^a(x,s)$ is used as a candidate Lyapunov function for the closed-loop system. Then, there exist $\mathcal{K}_\infty$, α_W and β_W, such that

1. $W(x, x_{\text{ref}}) \geq \alpha_W(|x^e|)$.
   Considering Assumption 4, then we infer that

$$\begin{aligned} W(x, x_{\text{ref}}) &\geq \int_t^{t+T} \ell^a(\bar{x}_a^e, \bar{u}_a^e)d\tau + \int_t^{t+T} \ell^p(\bar{y}_O^e, \bar{w}^e)d\tau + V_O(\bar{y}_O^e) \\ &\geq \alpha_\ell(|x-\bar{x}_a|) + \alpha_p(|\bar{x}_a - x_{\text{ref}}|) + \alpha_O(|\bar{x}_a - x_{\text{ref}}|) \\ &\geq \alpha_W(|x^e|). \end{aligned}$$

2. $W(x, x_{\text{ref}}) \leq \beta_W(|x^e|)$.
   Assume that there exists an ϵ such that Assumption 3 holds and $P_N^a(\cdot)$ is feasible for $x_a = x_{\text{ref}}$ and for all x such that $|x - x_{\text{ref}}| \leq \epsilon$. Let (u, x_{ref}) be a feasible solution of Problem 13, then

$$\bar{J}^a(x,s) \leq \int_0^T \ell^a(x - x_{\text{ref}}, u - u_{\text{ref}})d\tau$$

   Due the controllability of x_{ref} (Assumption 3) there exists a $\mathcal{K}_\infty$ function β_W such that

$$\begin{aligned} W(x, x_{\text{ref}}) &= \bar{J}^a(x,s) \\ &\leq \int_0^T \ell^a(x - x_{\text{ref}}, u - u_{\text{ref}})d\tau \\ &\leq \beta_W(|x^e|) \end{aligned}$$

3. $W(x^+, x_{\text{ref}}^+) - W(x, x_{\text{ref}}) \leq -\alpha_W(|x^e|)$

Continuing with the proof, we first prove that the cost is decreasing if $x \neq x_a$. Since the artificial error trajectory is contained in $\mathcal{X}_s^e$, then the application of input u_s and the parameter velocity w_s provides a system evolution that is feasible and fulfills the constraints of Problem 13, such that, at time $t + T_s$, produces the feasible cost is given by

$$\begin{aligned} J^a(x^+, s^+) = &\int_{t+T_s}^{t+T} \ell^a(\bar{x}_a^e, \bar{u}_a^e)d\tau + \int_{t+T}^{t+T+T_s} \ell^a(x_a^e, u_a^e)d\tau \\ &+ \int_{t+T_s}^{t+T} \ell^p(\bar{y}_O^e, \bar{w}^e)d\tau + \int_{t+T}^{t+T+T_s} \ell^p(\bar{y}_O^e, w_s^e)d\tau \\ &+ V_O(y_O^e(t+T+T_s)). \end{aligned}$$

The state and path parameter immediately after the sampling interval are denoted by x^+ and s^+.

Then, considering that the term $\int_{t+T}^{t+T+T_s} \ell^a(x_a^e, u_a^e)d\tau$ generated by the feasible extension (14) and (15) is zero due to the terminal equality constraint, the optimal value of the cost functional $\bar{J}^a(x^+, s^+)$ fulfills

$$\begin{aligned} \bar{J}^a(x^+, s^+) - \bar{J}^a(x,s) \leq &J^a(x^+, s^+) - \bar{J}^a(x,s) \\ \leq &- \int_t^{t+T_s} \ell^a(\bar{x}_a^e, \bar{u}_a^e)d\tau \\ &- \int_t^{t+T_s} \ell^p(\bar{y}_O^e, \bar{w}^e)d\tau + \int_{t+T}^{t+T+T_s} \ell^p(\bar{y}_O^e, w_s^e)d\tau. \end{aligned} \tag{16}$$

Since the terminal state of the artificial trajectory is constrained to belong to the error equilibrium set, there is a feasible solution that $V_O(y_O^e(t+T+T_s)) = V_O(y_O^e(t+T))$. Since $\ell^a(\cdot)$ is positive definite and from constraint (13h) the latter two terms add up to a non-positive value, it follows that the feasible value and, consequently, the optimal value is monotonically decreasing, i.e.

$$\bar{J}^a(x^+, s^+) - \bar{J}^a(x,s) \leq - \int_t^{t+T_s} \ell^a(\bar{x}_a^e, \bar{u}_a^e)d\tau. \tag{17}$$

Then, in order to prove asymptotic stability, it suffices to demonstrate convergence to $(x_{\text{ref}}, u_{\text{ref}})$. To this aim, using the same arguments as in [20], see that it has been proved that $W(x, x_{\text{ref}})$ is a positive function that satisfies the following inequality

$$W(x^+, x_{\text{ref}}^+) - W(x, x_{\text{ref}}) \leq -\alpha_\ell(|x - \bar{x}_a|).$$

Then, it can be derived that

$$\lim_{t\to\infty} |x - \bar{x}_a| = 0.$$

Given that $W(x, x_{\text{ref}}) \geq 0$, then

$$\lim_{t\to\infty} W(x, x_{\text{ref}}) = W_\infty.$$

From Lemma 1, included in the Appendix, we have that if $|x - \bar{x}_a| = 0$, then $W(x, x_{\text{ref}}) = 0$. Therefore, we have that

$$\lim_{t\to\infty} W(x, x_{\text{ref}}) = W_\infty = 0.$$

Taking into account the bounds of function $W(\cdot)$, this is equivalent to

$$\lim_{t\to\infty} \alpha_W(|x - x_{\text{ref}}|) \leq \lim_{t\to\infty} W(x, x_{\text{ref}}) = 0$$

and then

$$\lim_{t\to\infty} |x - x_{\text{ref}}| = 0.$$

Note that for $x^e = 0$, since the path is assumed feasible at the reference velocity w_{ref}, the optimality of the cost implies $w^e = 0$. Thus, the proof is complete.

4.4 Enlarged Region of Attraction:

One of the main advantages of the proposed formulation is the enlarged region of attraction obtained by the introduction of artificial variables. For illustrative purposes, the following scenario is depicted. Consider a perfectly followable path and a convex state space. For simplicity, we define terminal equality constraint for MPPFC and xMPPFC controllers, i.e. a) $x(T) = x_{\text{ref}}(s(T))$ and b) $x(T) = x_a(T)$, respectively. For the MPPFC, condition a) implies that the system reaches the reference after an interval T. In contrast, condition b) forces the system to reach the artificial trajectory $x_a(T)$, which is a decision variable of the problem and, in particular, is to be chosen as an error equilibrium state. Note that the reference path at $x_{\text{ref}}(\cdot)$ is an error equilibrium, and therefore is contained in the largest error equilibrium set. Consequently, constraint b) is less restrictive and a larger region of convergence is obtained in xMPPFC. For example, consider an initial system state which is outside the controllable set in a T interval to the reference. This state is outside the region of convergence of the MPPFC. Nevertheless, if the state lies within the controllable set to *any* error equilibrium trajectory, i) there exists a reachable error equilibrium, ii) the state is contained in the region of convergence of the xMPPFC, and iii) the reference is asymptotically stable.

5 Obstacle Avoidance

As described above, the aim of this work is attaining a model predictive formulation for obstacle avoiding path-following control. To begin with, the current problem definition is stated:

Definition 7 (Obstacle avoiding path-following problem) Consider a constrained system described by Eq. 1 and let $r(s(t))$ be a parameterized output trajectory reference, being the path-parameter $s(t) \in [s_0, s_1)$. Design a control law for u and $w(t) = \dot{s}(t)$ such that, as $t \to \infty$, a collision free trajectory is locally planned and tracked, which is as close as possible to the trajectory reference. In other words, the control law is such that the path-following is performed while avoiding fixed obstacles.

In the recent work [13], the authors propose to consider the obstacles as equality constraints and their shapes are determined by the intersection set of n_i inequalities of the form

$$\mathcal{O} = \{y \in \mathbb{R}^p : h_i(y) > 0, \quad i = 1, \cdots, n_i\},$$

where $h_i(y) : \mathbb{R}^p \to \mathbb{R}$ are continuously differentiable functions with Lipschitz continuous gradients. Then, the obstacle-avoidance constraint ($y \notin \mathcal{O}$) can be written as follows:

$$\exists i \in 1, \ldots, n_i : h_i(y) \leq 0,$$

which means that to avoid the obstacle, at least one of the inequalities defined by $\mathcal{O}$ must be violated.

A very general class of obstacles can be described using the above formulation. For example, any polyhedral set can be cast as a set of affine constraints

$$\mathcal{O} = \{y \in \mathbb{R}^p : b_i - A_i^T y > 0, i = 1, \ldots, n_i\}, \tag{18}$$

which is a particular case of the general one, for $h_i(y) = b_i - A_i^T y$. Furthermore, any other complex obstacle can be approximated by the smallest polyhedron containing it.

The avoidance condition can also be rewritten as the following equality (nonlinear) constraint

$$\Psi(y) := \prod_{i=1}^{n_i} [h_i(y)]_+ = 0, \tag{19}$$

where the operator $[h_i(y)]_+$ is defined as $[h_i(y)]_+ = \max(h_i(y), 0)$.

This latter form of the obstacle avoidance condition permits to cast the hard constraints as soft constraints, by mean of the addition of a penalty function to the MPC cost functional. In fact, it is straightforward to construct a quadratic penalty function of the form

$$\nu = \frac{1}{2}\mu\Psi(y)^2, \tag{20}$$

with a penalty factor $\mu > 0$, in such a way that fulfilling the constraint (19) is indicated by obtaining zero from the evaluation of the function at y. Furthermore, the obstacle avoidance penalty function has the advantage of being continuously differentiable, in contrast to an exact penalty formulation of the obstacle avoidance constraints. In order to account for obstacles in the formulation, the following term is added to the cost functional

$$J^{obs}(y(\cdot), y_a(\cdot), \mathcal{O}) = \sum_{o=1}^{N_{obs}} \int_0^T (\nu_o(y_a(\tau)) + \nu_o(y(\tau)))d\tau, \tag{21}$$

where $\{\mathcal{O}\}$ denotes a set of obstacles with N_{obs} elements in it, and ν_o is the penalty function corresponding to the $o-$th obstacle. The obstacle-aware cost is given by

$$\begin{aligned} J^{oa}((x_0, s_0, \{\mathcal{O}\}; u(\cdot), x_a, u_a(\cdot), w(\cdot))) &= J^a(x_0, s_0; u, x_a, u_a, w) \\ &\quad + J^{obs}(y(\cdot), y_a(\cdot), \mathcal{O}). \end{aligned} \tag{22}$$

It is important to note that the artificial trajectory is a feasible trajectory for the dynamical system (13a). For trajectories running over the obstacle, the term $\nu(x_a)$ will increase, therefore, trajectories avoiding the obstacle are convenient in terms of the cost. This way, the artificial trajectory provides an alternative path to the system, which simultaneously fulfills the dynamics of the system and, by optimality of the cost, penetrating into of the obstacles is discouraged. Note that this provides no strict guarantees of obstacle avoidance -for that, hard constraint or barrier functions should be imposed-, but it is a effective strategy for practical applications, as it will be shown in the simulation examples. A requirement for terminal prediction of the artificial trajectory is to belong to the Path-Following Error Equilibrium set, so recursive feasibility is achievable.

6 Simulations Results

Two scenarios are presented in order to demonstrate properties of the proposed controller. First, a piecewise-followable track consisting of the discontinuous concatenation of two sinusoidal paths is presented. The feasibility is compromised due the discontinuity and cannot be guaranteed in the traditional formulations. It will be evident that the artificial trajectories provide intermediate references which maintain the feasibility of the constrained optimization problem. The second scenario consists of an eight-shaped path which is not feasible, as it violates the inclusion in the feasible set $\mathcal{X}$, and also obstacles are located on the path. The proposed controller can handle this situation, as the artificial variables provide feasible trajectories that enable following the path while avoiding the obstacles.

6.1 Sampled-data Implementation

The optimal control formulations (13) with costs definitions (12) and (22) were introduced to formally present the main ideas in the continuous-time domain. In order to numerically solve the constrained optimization problem, a reformulation in discrete-time is required. The controller is implemented in the sampled-data MPC fashion [12]. In this type of implementation, the input signal u is considered to be sampled and applied to a zero-order hold, which keeps a constant value throughout each sampling interval, and consequently u is a piecewise constant signal. A discretization of the nonlinear continuous-time model is required for the prediction, and will be conducted using Runge-Kutta methods, although several alternatives are available [11]. The discrete-time model is denoted as

$$\begin{aligned} x_{k+1} &= f_d(x_k, u_k), \\ y_k &= h(x_k). \end{aligned} \tag{23}$$

It is convenient to adequate the value functional after the discretization. This is done by incorporating additional terms that penalize the inputs. In case no input reference u_{ref} is available, it is typical to consider $u_{\text{ref}} \equiv 0$, and the corresponding penalization term is referred to as *regularization term*. Under such conditions, for a given time instant t_k, the MPPFC cost is given by

$$\begin{aligned} \tilde{J}^a_{oa} = &\sum_{j=0}^{N-1} (l^a(x^e_{a,j}, u^e_{a,j}) + l^O(y^e_{O,j}, u^e_{O,j})) + l^w(w^e_j) \\ &+ V_f(x^e_a(N)) + V_O(y^e_O(N)) + J^{obs}(y(\cdot), y_a(\cdot), \mathcal{O}), \end{aligned} \tag{24}$$

where the cost $\ell^a(\cdot)$ has been split into two terms, $\ell^O(\cdot)$ and $\ell^w(\cdot)$, that penalize the planning error on state and time dilation, respectively. Often in the literature, stage and terminal cost terms are chosen to be quadratic. In the remainder of this work it will be assumed so. Then, the penalty terms can be expressed as

$$\begin{aligned} l^a &= \|x^e_a\|^2_{\mathcal{Q}} + \|u^e_a\|^2_{\mathcal{R}}, \\ l^O &= \|y^e_O\|^2_{\mathcal{K}} + \|u^e_O\|^2_{\mathcal{S}}, \\ l^w &= \|w^e\|^2_{\mathcal{T}}, \\ V_O(y^e_O(N)) &= \|y^e_O(N)\|^2_{\mathcal{K}_f}, \\ J_{obs}(y(\cdot), y_a(\cdot), \mathcal{O}) &= \sum_{o=1}^{N_{obs}} \textstyle\sum_{j=0}^{N} (\nu_o(y_{a,j}) + \nu_o(y_j)), \end{aligned}$$

where $\mathcal{Q}$, $\mathcal{R}$, $\mathcal{S}$, and $\mathcal{T}$ are positive definite penalization matrices of adequate dimensions, while the penalization of the o-th obstacle $\nu_o(y_k)$ follows the definition given in Eq. 20.[3]

In order to implement the cost-decrease constraint in the sampled-data framework, the constraint is approximated in a discrete form,

$$l^p(y^e_{O,j+1}, w^e_{j+1}) - l^p(y^e_{O,j}, w^e_j) \leq 0,\ j \in [0, N-1],$$

where $l^p(y^e_{O,k}, w^e_k) = l^O(y^e_O(k), u^e_O(k)) + l^w(w^e(k))$.

6.2 Underactuated Vehicle Model

Several robotic systems and vehicles (wheeled mobile robots, waterborne vehicles or fixed wing aerial vehicles moving at constant altitude) can be modeled with sufficient accuracy for different applications by the two dimensional nonlinear system classically known as Dubin's car. This is a first order kinematic model which has been used in the design of control systems for unmanned vessels [41], unicycle-like robots [24], and even -with a slight adaptation on the input ω- for unmanned aerial vehicles [22]. The system is modeled as

$$\begin{bmatrix} \dot{r}_x \\ \dot{r}_y \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos(\psi) & 0 \\ \sin(\psi) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{25}$$

[3]Notation: second subindex j indicates the j-th sample of the discrete time variable.

where x and y give the vehicle's location, and ψ its heading. The system's inputs v and ω are the translation speed and the angular velocity, respectively. By inspection of Eq. 25, its is evident that states r_x and r_y, do not affect its dynamics. In other words, the dynamics are invariant to translations on r_x and r_y. The stabilizing terminal set, penalization and control law are computed on the error space in terms of $x^e(T)$ -the error of the predicted terminal state with respect to the reference path-. These sets could also be used for $x_a^e(T)$ -the error between the predicted states with respect to the terminal state of the artificial trajectory- as long as the reference and artificial variable have the same heading ψ. For the numerical simulations, box constraints in the inputs and the position are considered, such that $0 \leq v \leq 1[m/seg]$, $-1 \leq \omega \leq 1[rad/seg]$, $-5.5 \leq r_x \leq 6.5[m]$ and $-2.5 \leq r_y \leq 3.5[m]$. The parameter input for the adjoint system (8b) is constrained by $0 \leq w \leq 1$.

6.3 Terminal Set

Exploiting the fact that the selected unicycle model has feasible equilibrium points at the entire feasible space, i.e. $f(x, 0) = 0, \forall x \in \mathcal{X}$, then $\mathcal{X}$ can be used as terminal set with a feasible path-parameter input $w = 0$, such that Eq. 13f can be replaced by $x^a(T) \in \mathcal{X}$. For a more general computation of terminal error equilibrium sets, the reader is referred to [20] where LTV partitions of the system are used for the calculation of the terminal ingredients.

6.4 Heuristic Tuning

In order to provide a basic tuning strategy, we propose a heuristic approach. We start by ordering the path-following objectives in a hierarchy: preventing the path-parameter to stall (due to infeasibilities or obstacles) is of highest priority. Reducing the tracking errors are at a secondary level. Finally, low levels of the input are desired, which is of interest in many applications. The obstacle avoidance is achieved by introducing relatively large obstacle penalizations ν_o, which are competing against the tracking costs. Note that this cost is only effective while in the vicinity of the obstacles. High values of the time-dilation penalty $\mathcal{T}$ are initially chosen, and the rest of the tuning values are relatively low. Iteratively, the time-dilation penalization is reduced until the path-parameter input is not saturating, specially when the system state is close to the obstacles or infeasible regions. This means the path-following control is producing some effects. At that point, slightly increasing the tracking penalties $\mathcal{Q}$ and $\mathcal{K}$ provides a better path tracking while maintaining the time-dilation functionality. Finally, the input penalization $\mathcal{R}$ and regularization $\mathcal{S}$ can be adjusted in order to obtain softer input curves. The development tuning strategies for specific applications is left for future work, the interested reader is referred to [10] and [28] for related works.

6.5 Numerical Results

In all simulations, the system evolution simulation (continuous time) is implemented with a timestep of $0.0250[seg]$, and the Runge-Kutta integration method is applied. The NMPC is implemented using a sampling interval of $1[s]$ and a horizon length of $N = 6$. The discrete time predictions were also obtained by the Runge-Kutta method of fourth order. Terminal equality constraints of the form $x(T) = x_a(T)$ are imposed, this means the terminal predicted state and terminal artificial state are coincident or, equivalently, $x_a^e(T) = 0$.

Remark 7 In this application, the xMPPFC controller is thought of as a high-level kinematic controller which generates the reference velocity for a low-level dynamics controller. This enables the application for robotics with fast dynamics using longer sampling intervals. It is not in the scope of this work the discussion on the computational efficiency of the numerical solutions or the computation time demanded by the optimization algorithm. Nevertheless, simulations with available high efficiency solvers have been conducted and results are obtained within reasonable time intervals. Therefore, real-time applicability appears achievable.

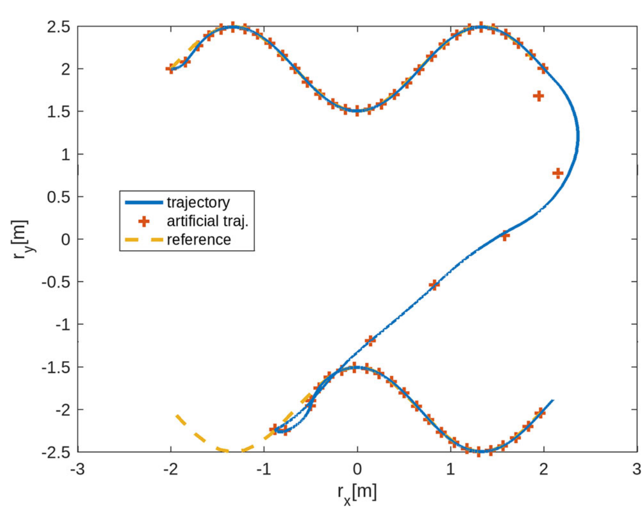

Fig. 1 Resulting trajectories for path-following problem using xMPPFC applied to a discontinuous trajectory. The actual trajectory of the vehicle (blue) converges to the reference path (yellow, dashed). The artificial variables provide locally feasible references that enable the continuation of the path-following problem around the discontinuity

6.5.1 Piecewise Followable Path

A piecewise-continuous output path defined by

$$p_1(s) = \left\{ \begin{bmatrix} r_x \\ r_y \end{bmatrix} = \begin{bmatrix} -2 + 4\frac{s}{(T_t/2)} \\ 2 + \frac{1}{2}\sin(\frac{6\pi}{T_t}s) \end{bmatrix} \right.,$$

$$p_2(s) = \left\{ \begin{bmatrix} r_x \\ r_y \end{bmatrix} = \begin{bmatrix} -2 + 4\frac{s-(T_t/2)}{(T_t/2)} \\ -2 + \frac{1}{2}\sin(\frac{6\pi}{T_t}s) \end{bmatrix} \right.,$$

with $T_t = 60$ is tracked, the reference is changed from p_1 to p_2 when s reaches $T_t/2$. The vehicle initial configuration is set to $x_0 = [-2 \;\; 2 \;\; 0]^T$. The weighting matrices are chosen $\mathcal{Q} = diag(10, 10, 10)$, $\mathcal{R} = diag(10, 10)$, $\mathcal{T} = 3$, $\mathcal{K} = diag(10, 10)$, and $\mathcal{S} = diag(0.01, 0.01)$. This illustrative example presents one of the strongest benefits of the formulation with artificial trajectories. It can be observed that the system correctly tracks the feasible parts of the reference. Note that due to discontinuity of the path, perfect path tracking is rendered unfeasible. In fact, traditional MPPFC with terminal equality constraints would stall before the discontinuity as the posterior section of the path is unreachable, leaving the objectives

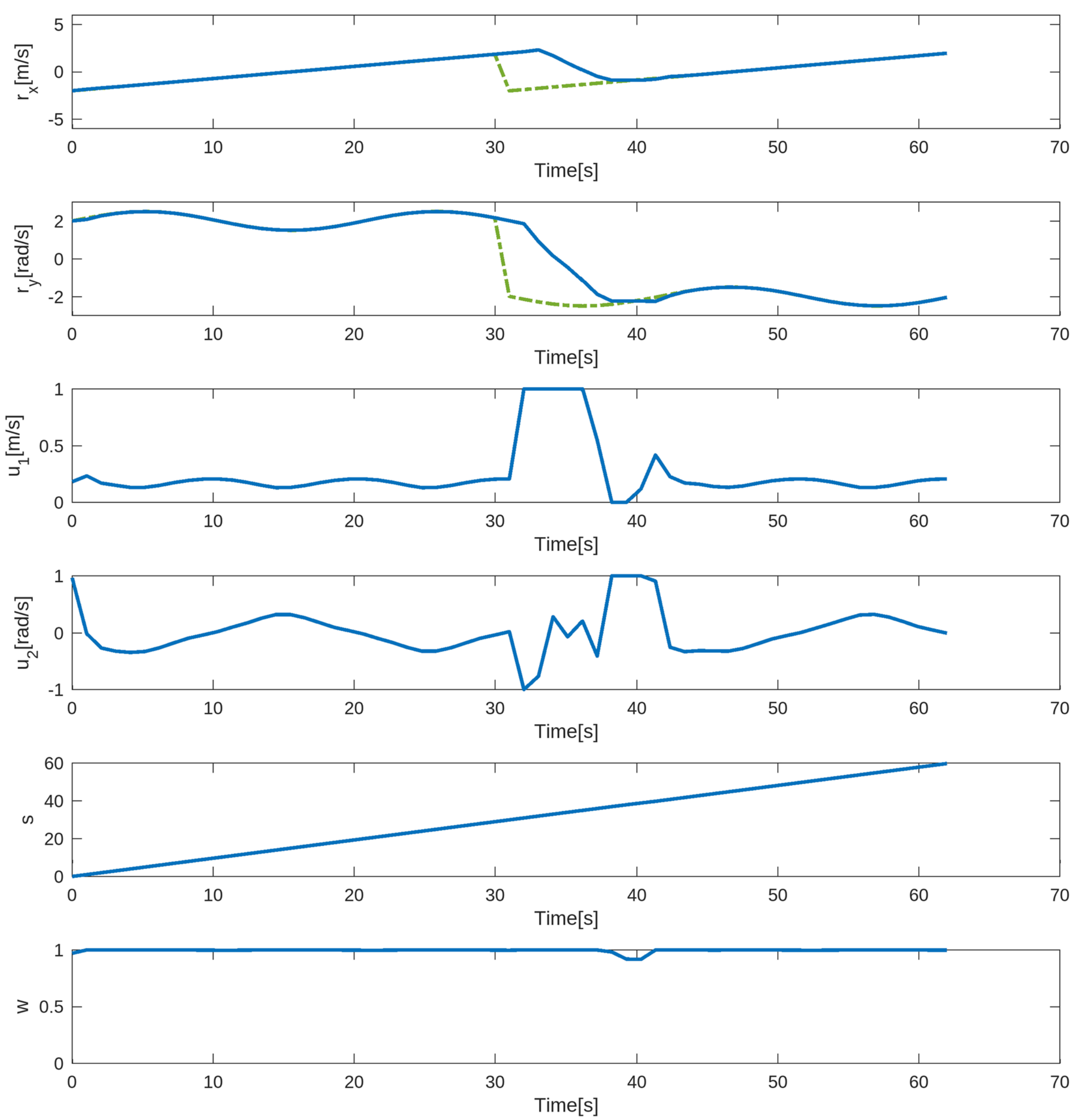

Fig. 2 Outputs and Inputs (blue,solid) plot for the piecewise followable discontinuous path. The system tracks the reference while feasible and maintains feasibility regardless of the discontinuity. The inputs and states are clearly within bounds, and it can be seen that the state and inputs constraints are enforced. There reference curves are also plotted (green, dot-dashed)

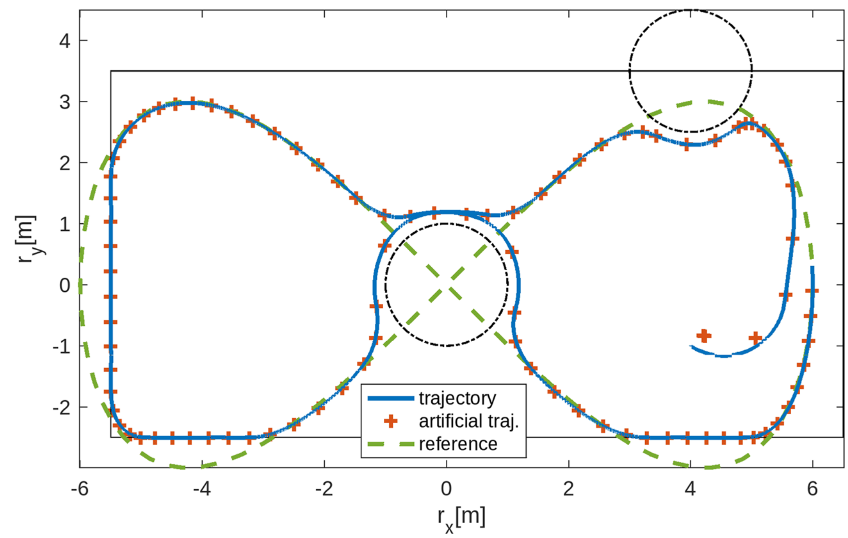

Fig. 3 Resulting trajectories for obstacle avoidance with xMPPFC. The actual trajectory of the vehicle (blue) converges to the reference path (green, dashed) while feasible and avoids the obstacle. Artificial variables show the alternative path used as effective reference

of the path-following problem unaccomplished. Here the artificial trajectories provide reachable, feasible references for the system, even with the restrictive terminal equality constraint. The controller is then able to continue driving the system towards the following section of the path. The effective system and artificial trajectories can be seen in Fig. 1, and in Fig. 2 the corresponding inputs and outputs plots are presented.

6.5.2 Obstacle Avoiding Path-following

An ∞-sign shaped path is used, given by $p(s) = [6\cos(\frac{2\pi}{T_t}s) \quad 3\sin(\frac{4\pi}{T_t}s)]$, with $T_t = 90$. Two circular exclusion zones with radius of one meter, representing a convex bound around an obstacle, centered at (0,0) and (4,3) are introduced. The reference path has been designed such that it violates also the state constraints. Due to the fact that the obstacle related cost term is not a barrier function, due to optimality of the local solution, the resulting trajectory may overlap some parts of the obstacle when reference becomes unfeasible as it is moving through the obstacle. In consequence, a $0.2[m]$ safety margin was introduced around the obstacle. The weight μ is set to $5 \cdot 10^5$. The initial configuration of the vehicle is $x(0) = [4 \; -1 \; -\pi/2]^T$. The weighting matrices are chosen as $\mathcal{Q} = diag(10, 10, 10)$, $\mathcal{R} = diag(10, 10)$, $\mathcal{T} = 10$, $\mathcal{K} = diag(0.5, 0.5)$, and $\mathcal{S} = diag(0.01, 0.01)$. By inspecting Fig. 3, it can be seen that both obstacles are avoided, constraints are enforced, and the artificial variables are effective in providing a feasible reference for the obstacle avoidance task. Consequently, the path-following with obstacle avoidance problem is solved. The states and inputs plots can be seen in Fig. 4, where the coincidence between the reference and the solution when feasible is evident. It is also remarkable that the inputs are smooth while following the path. Besides, strong inputs are applied when avoiding the obstacle or returning to the feasible path. The path-following extra degree of freedom can be noted in the w input, as it is low valued for $0 < t < 2$, which means that the reference is not advancing, as long as the system state prediction is not getting closer to the reference evolution.

7 Conclusion

This work presented a new formulation of MPC for path-following with obstacle avoidance, that solves both path-following and obstacle avoidance in a unified control law. The solution approach is proposed as an extension of the MPC for path-following by the introduction of auxiliary trajectories. The controller solves the local trajectory planning and tracking tasks in a combined manner. These variables not only enlarge the region of attraction of the controller, enabling the recursive feasibility under arbitrary reference paths, but also provide the necessary flexibility for obstacle avoidance in practice, as shown in the simulation scenarios. Two simulation examples are introduced to show the versatility and main benefits of the proposed approach, handling changes of reference paths and managing the presence of obstacles on the path, with application as kinematic control for a constrained nonlinear vehicle system. Future works include implementation and evaluation using efficient numerical solvers and the generic characterization of terminal sets. The development of a more general stability proof, based on a specifically designed cost function which might result in a Lyapunov

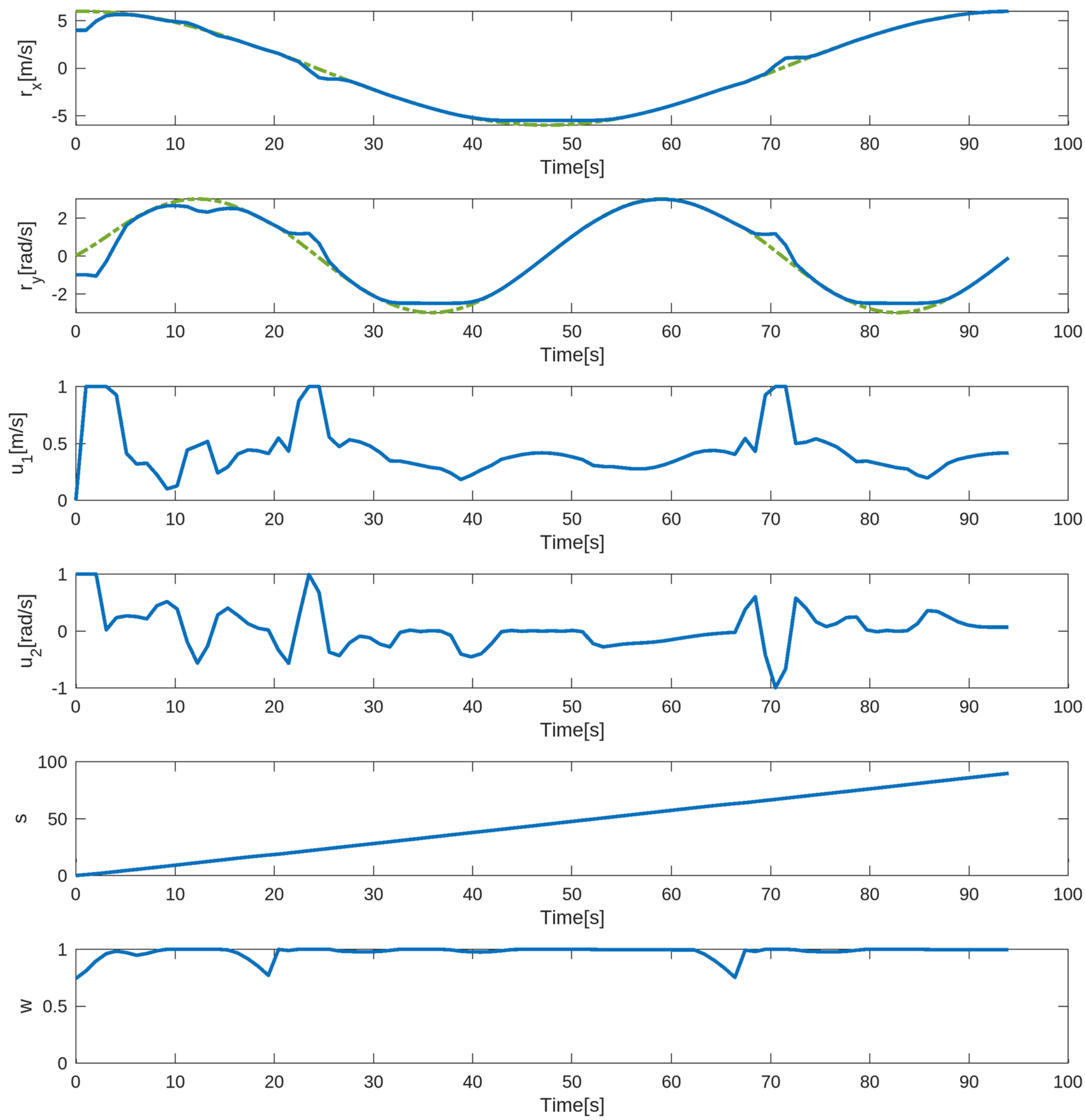

Fig. 4 Outputs and Inputs plots (solid, blue) for obstacle avoidance with xMPPFC. The system tracks the reference while feasible and it makes a detour for the avoidance task. The inputs and states are clearly within bounds and it can be seen that the state and inputs constraints are enforced. It can be noted that strong inputs are applied when avoiding the obstacle, as the system quickly returns to the reference path. The low value of the path-following input w at the beginning and at point where the reference reaches the obstacles indicates that the reference advances at slower pace that desired. The references are also plotted (green, dotted-dashed)

function for the closed-loop system, is being conducted. Finally, simulation and experimentation on different vehicle models and other systems will be conducted.

Appendix

Using the same arguments as in [20] the following lemma is introduced.

Lemma 1 *Consider that Assumption 3 and 4 hold. Consider also a reference path x_{ref} and assume that for a given state x the optimal solution of $P_N^a(\cdot)$ is such that $x = \bar{x}_a(x, x_{\text{ref}})$. Then the function $W(x, x_{\text{ref}}) = 0$.*

Sketch of Proof. The key idea of the proof is to show that if the system converges to an equilibrium trajectory $\bar{x}_a$, then this trajectory is already equal to the reference path x_{ref}. In other words, the convergence of the system trajectory to the artificial one, $\bar{x}_a$, and the convergence of the artificial trajectory to the reference, x_{ref}, are not consecutive, but simultaneous, and the convergence rate of the former is an upper bound for the latter. The formal proof can be done by contradiction, by assuming that the system state and input, (x, u), converge to an artificial trajectory that is an equilibrium different from the one corresponding to the reference trajectory, i.e., $(x, u) = (\bar{x}_a, \bar{u}_a) \neq (x_{\text{ref}}, u_{\text{ref}})$. Then, given that both trajectories $(\bar{x}_a, \bar{u}_a)$ and $(x_{\text{ref}}, u_{\text{ref}})$ belong to a convex set, there exists a feasible trajectory $(\hat{x}, \hat{u}) = \beta(\bar{x}_a, \bar{u}_a) + (1 - \beta)(x_{\text{ref}}, u_{\text{ref}})$, with $\beta \in (0, 1)$, which produces a cost function smaller than one obtained at $(\bar{x}_a, \bar{u}_a)$. This proves that there exists a $\beta \in (0, 1)$ such that the cost of moving the system from $\bar{x}_a$ to $\hat{x}$ is smaller than the cost of remaining in the error equilibrium solution $\bar{x}_a$. This contradicts the optimality of the solution to problem $P_N^a(\cdot)$, and hence $x = \bar{x}_a = x_{\text{ref}}$.

For more details of the complete proof of this Lemma the reader can refer to [20].

Remark 8 Lemma 1 covers the case of a feasible trajectory reference x_{ref}. However, the analysis made in the proof still holds true for infeasible references. Indeed, by a simple convex analysis, if x_{ref} is not reachable the system will converge to the optimal feasible equilibrium trajectory with respect to x_{ref}. This means that the controller finds by itself an optimal trajectory (in the sense of the proximity to x_{ref}) even in the case x_{ref} is infeasible/unreachable.

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Availability of data and materials All data necessary to reproduce the results are given in the paper.

Declarations

Ethical Approval Not applicable.

Consent to Participate Not applicable.

Consent to Publish Not applicable.

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