Comment on "Exploring chaos in the Dicke model using ground-state fidelity and Loschmidt echo"

Ignacio García-Mata,[1,2,*] Augusto J. Roncaglia,[3] and Diego A. Wisniacki[3]

[1]*Instituto de Investigaciones Físicas de Mar del Plata (IFIMAR, CONICET), Universidad Nacional de Mar del Plata,
Mar del Plata, Argentina*

[2]*Consejo Nacional de Investigaciones Científicas y Tecnológicas (CONICET), Argentina*

[3]*Departamento de Física "J. J. Giambiagi," and IFIBA, FCEyN, Universidad de Buenos Aires, 1428 Buenos Aires, Argentina*

In a recent paper [Phys. Rev. E **90**, 022920 (2014)] a study of the ground-state fidelity of the Dicke model as a function of the coupling parameter is presented. Abrupt jumps of the fidelity in the superradiant phase are observed and are assumed to be related to the transition to chaos. We show that this conclusion results from a misinterpretation of the numerics. In fact, if the parity symmetry is taken into account, the unexpected jumps disappear.

In Ref. [1] a study of the ground-state fidelity of the Dicke model (DM) is presented. The ground-state fidelity is defined as

$$F = |\langle \psi(\lambda_0)|\psi(\lambda_0 + \delta\lambda)\rangle|^2, \tag{1}$$

which is the overlap between ground states with slightly different interaction strengths. The DM describes the interaction of the global spin of N spin $1/2$ particles with an external field. The Hamiltonian that describes this system is

$$H = \omega_0 J_z + \omega a^\dagger a + \frac{\lambda}{\sqrt{2j}}(a^\dagger + a)(J_+ + J_-). \tag{2}$$

For parameter values $\omega = \omega_0 = 1$ there is a superradiant quantum phase transition at $\lambda_c = 0.5$. At approximately the same value the level spacing distribution changes from Poisson to Wigner-Dyson, which can be identified with an integrability-chaos transition [2].

In the main numerical result shown in Fig. 3 of Ref. [1], abrupt jumps (so called "aperiodic oscillations") can be observed and since they happen for values of λ_0 beyond the superradiant and integrability transitions, the authors attribute these jumps to chaos. In what follows we provide a clear proof that this is not the case.

The DM has a well defined parity symmetry, and the Hilbert space can be split into two noninteracting subspaces. When the ground-state fidelity is computed without taking into account the parity symmetry abrupt jumps can be observed in Fig. 1 (red line). These jumps are completely analogous to those presented in Fig. 3 of Ref. [1]. On the other hand, if we now take into account the parity symmetry, the jumps disappear completely (black curve of Fig. 1).

The curves presented in Fig. 1 clearly show that the jumps attributed to chaos in Ref. [1] are due to nothing but a numerical issue: For coupling values beyond the superradiant transition the system becomes gapless, i.e., the smallest energies corresponding to different parity states become machine-precision degenerate (see Figs. 1 and 2). Therefore, the diagonalization routine cannot resolve between them, and from λ_0 to $\lambda_0 + \delta\lambda$ it randomly yields ground states with different parities which
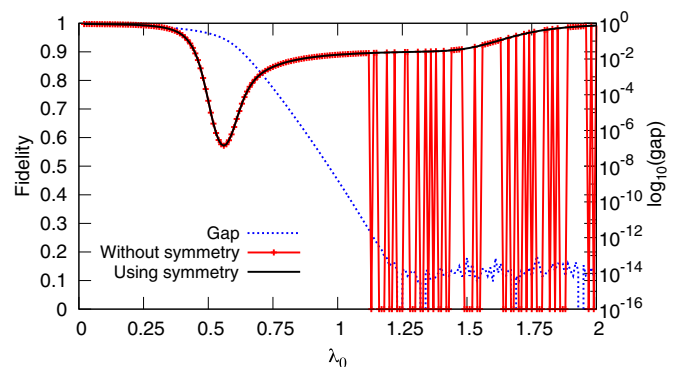
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*i.garcia-mata@conicet.gov.ar

FIG. 1. (Color online) Ground-state fidelity for the DM with $J = 5$ and $n_{cut} = 40$. Red line (and + signs): without looking at the parity of the ground state. Black solid line: taking into account parity flips. Dotted blue line (scale on the right y axis): (logarithm of the) gap $|E_0^{(even)} - E_0^{(odd)}|$ as a function of the coupling λ_0.

of course are orthogonal (thus $F = 0$). In Fig. 2 we show the lowest energy for both even and odd spaces as a function of λ_0. It is clearly observed that after λ_c both lower energies $E_0^{(even)}$ and $E_0^{(odd)}$ become increasingly close. In Fig. 1 (blue curve) we plot the gap $|E_0^{(even)} - E_0^{(odd)}|$, and we see that it is precisely when this difference reaches the order of machine precision ($\sim 10^{-14}$) is when the oscillations start to appear.
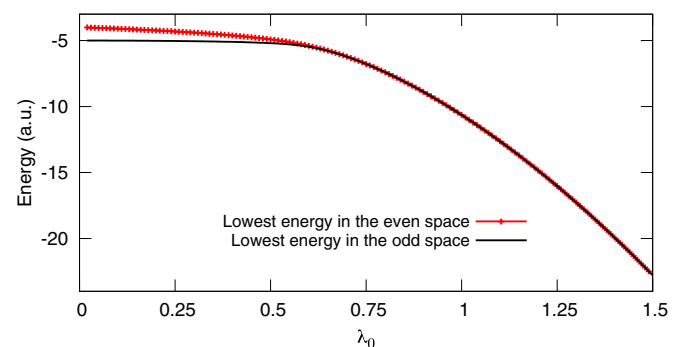

FIG. 2. (Color online) Energy of the ground state for both odd and even spaces of the DM for $J = 5$ and $n_{cut} = 40$.

[1] U. Bhattacharya, S. Dasgupta, and A. Dutta, Phys. Rev. E **90**, 022920 (2014).

[2] C. Emary and T. Brandes, Phys. Rev. Lett. **90**, 044101 (2003).