Damage detection through bilinear stiffness estimation in multi-degree-of-freedom frame structures

Hernán Garrido* and Oscar Curadelli

Engineering Faculty,
National University of Cuyo,
Parque Gral. San Martín, Mendoza,
ONICET, Argentina
E-mail: carloshernangarrido@yahoo.com.ar
*Corresponding author

Abstract: During its lifetime, the integrity of structures may be affected by exposure to dynamic loads such as earthquakes. In these cases, structures might become damaged, and an early overall evaluation of their integrity is necessary. A particular pattern of damage is that produced by the existence of breathing cracks. These cracks are characterised by their opening and closure during the oscillation of the cracked structural element. In this paper, it is proposed a method for detection of breathing cracks in multi-degree-of-freedom frame structures. The method is based on bilinear stiffness direct estimation, individually for each story of the structure, by using a least-squares approach. The proposed method is demonstrated through a numerical example using the FEM to model a planar structure which presents several breathing cracks. The results suggest that this approach is more sensitive and precise than approaches based on linear stiffness models in detecting and localising damage due to breathing cracks.

Keywords: breathing cracks; damage detection; multi-degree-of-freedom; system identification; non-linear; bilinear stiffness; direct parameter estimation; least-squares; LS.

Reference to this paper should be made as follows: Garrido, H. and Curadelli, O. (2014) 'Damage detection through bilinear stiffness estimation in multi-degree-of-freedom frame structures', *Int. J. Lifecycle Performance Engineering*, Vol. 1, No. 4, pp.380–397.

Biographical notes: Hernán Garrido graduated in Electronics Engineering at the National Technological University, Argentina in 2009. At present, he is a PhD student at the National University of Cuyo, Argentina with a doctoral thesis entitled: 'Semi-Active Vibration Control in Structural Systems'. His research interests concern: vibration control of structures, non-linear control, non-linear analysis, and inverse problems.

Oscar Curadelli received his PhD in 2003 in Engineering at the Federal University of Rio Grand do Sul, Brazil, with a doctoral thesis entitled: 'Structural Vibration Control by means of Metallic Dampers'. He is currently working as a Researcher at the CONICET and an Associate Professor at the Engineering Faculty of the National University of Cuyo, Argentina. His main research interests concern vibration control.

1 Introduction

The problem of structural safety has always been crucial in the field of civil engineering (Giuliani, 2012). During its lifetime, the integrity of structures may be affected by natural degradation, aggressive environments and ambient factors, and the exposure to dynamic loads such as earthquakes, causing for instance cracks. In these cases, structures might become damaged, and an early overall evaluation of their integrity is necessary in order to take pertinent and quick decisions to avoid failures or, eventually, collapse. For example, after an earthquake, it is important to determine the current serviceability of the affected structure to ensure safe operation in the current condition, and remaining service life (Curadelli and Ambrosini, 2011).

Damage or fault detection by monitoring changes in the dynamic properties or response of the structure has received considerable attention in recent literature. The basic idea springs from the notion that spectral properties, described in terms of the so-called modal parameters (natural frequencies, mode shapes, and modal damping), are functions of the physical properties of the structure (mass, energy dissipation mechanisms and stiffness). In this context, Curadelli et al. (2008) presented a novel scheme to detect structural damage by means of instantaneous frequency and damping coefficient identification using the wavelet transform. They found that, in general, natural frequency decreases slightly whereas damping increases significantly on the presence of damage.

Since in actual structures damping mechanisms are strongly non-linear (e.g., yielding, internal friction, crack growth), damping coefficient results a function of the oscillation amplitude. For this reason, in order to assess the damage through damping change, Curadelli et al. (2008) compared instantaneous damping functions (instead of damping coefficients). On the other hand, elastic restoring mechanisms are more linear in nature. Then, instantaneous natural frequency functions are less dependent on the oscillation amplitude, as shown by Curadelli et al. (2008); so natural frequencies could be compared directly in order to assess damage. Unfortunately, as previously stated, natural frequencies are only slightly influenced by damage presence.

In practical applications an important and common type of structural damage is due to breathing cracks. These cracks are characterised by their opening and closure during oscillation of the cracked structural member. They can occur in metallic structural members, e.g., due to fatigue (Yan et al., 2013); but also in reinforced concrete structures, e.g., due to excessive loading (Law and Zhu, 2004).

A number of researchers have investigated breathing of cracks from different points of view. For example, Kisa and Brandon (2000) integrated the finite element method (FEM), the component mode synthesis method and the linear elastic fracture mechanics theory to model cracked structures. For their part, Rezaee and Hassannejad (2010) made a free vibration analysis of a simply supported beam with a breathing crack by using perturbation method. Dimarogonas (1996) gave a comprehensive survey on crack modelling approaches.

Aside all the intricate non-linear mechanisms that breathing cracks may involve, the essential non-linearity which can describe the overall dynamic behaviour of simple structural members (e.g., beams) including breathing cracks is a bilinear stiffness model. In this context, Friswell and Penny (1992) proposed and assessed a simple model of a cracked beam based on different equivalent stiffness values depending on whether the crack is open or closed. Following this simple but useful concept, Yan et al. (2013) recently proposed a method that transfers non-linear system identification into linear system identification by dividing free-vibration responses into different parts corresponding to each stiffness region according to the stiffness interface. In this way, the natural frequency of each region can be identified by using any modal identification approach applicable to linear systems.

In the present paper, it is proposed a method for breathing cracks detection in multi-degrees-of freedom (MDOF) frame structures. The method is based on bilinear stiffness direct estimation, individually for each story of the structure, assuming a shear-type reduced model. The estimation is performed by using a variation of the least-squares (LS) approach proposed by Mohammad et al. (1992).Other variations of it were reviewed by Kerschen et al. (2006).

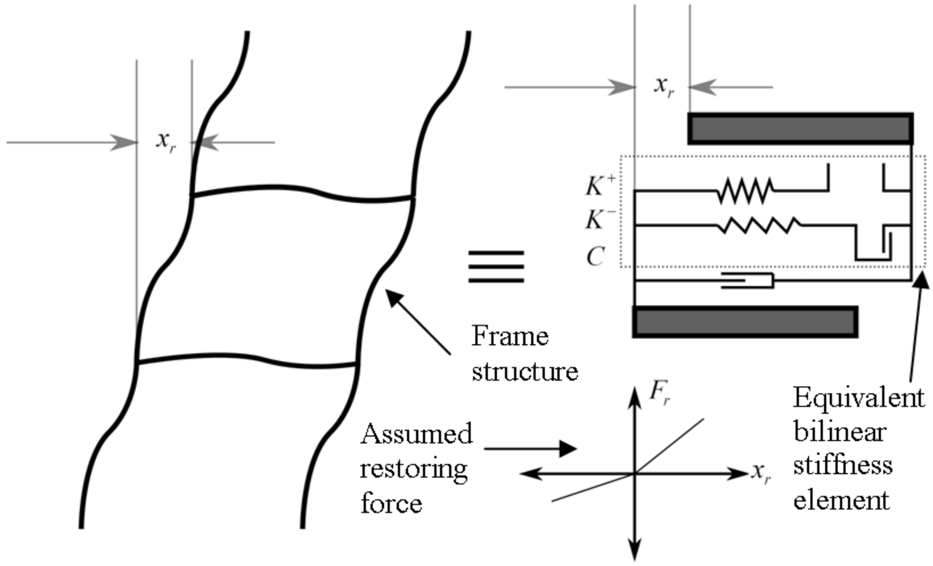
In this work, unlike the methods described in Kerschen et al. (2006), ad-hoc basis functions are proposed to be used in order to easily identify the two parameters which characterise bilinear stiffness. Another important advantage of the proposed variation of the identification method is that, instead of using one-point force excitation, it works for free-vibrations, environmental vibrations or seismically excited structures. The proposed method is demonstrated through a numerical example using the FEM to model a planar structure which presents several breathing cracks. Records of structural response obtained from FEM non-linear dynamic analysis are used as input of the proposed method (which assumes a bilinear stiffness behaviour on a shear-type reduced model). The results suggest that this approach is more sensitive and precise than linear approaches in detecting and localising this kind of damage.

2 Damage identification based on bilinear reduced model

2.1 Bilinear-stiffness element

Unlike the bilinear-stiffness oscillator considered by Friswell and Penny (1992) and Yan et al. (2013), in this approach a gap and hook macro-element with a bilinear-stiffness behaviour linking two consecutive stories of a shear-type MDOF frame structure is considered (Figure 1). Thus, the model assumes n degree-of-freedom (DOF) corresponding to the horizontal displacements of each story.

Figure 1 Sketch of frame structure, and equivalent bilinear stiffness link element between two consecutive stories



The overall shear restoring force of this macro-element is given by:

$$F_r = K^+ \operatorname{pos}(x_r) + K^- \operatorname{neg}(x_r) \tag{1}$$

in which

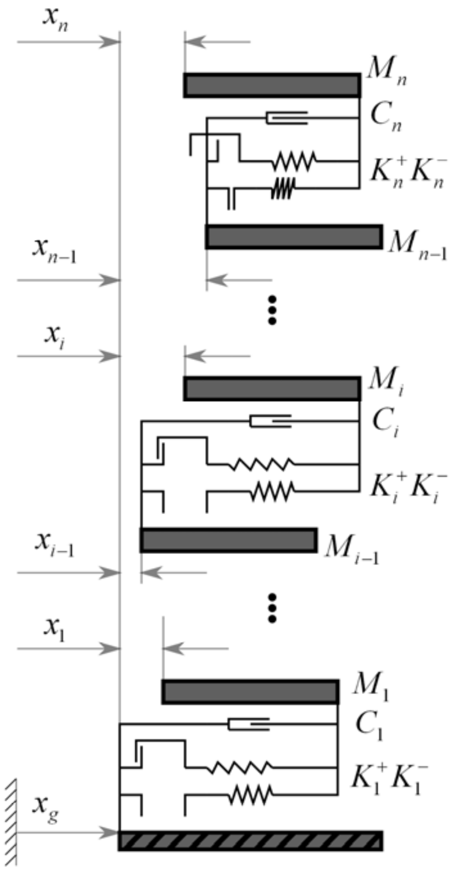
$$\begin{aligned} \operatorname{pos}(x_r) &= \begin{cases} 0 & \forall x_r \le 0 \\ x_r & \forall x_r > 0 \end{cases} \\ \operatorname{neg}(x_r) &= \begin{cases} x_r & \forall x_r < 0 \\ 0 & \forall x_r \ge 0 \end{cases}, \end{aligned} \tag{2}$$

K^+ and K^- represent the stiffness in both directions, when the crack is opened or closed, respectively, and x_r is the relative displacement between consecutive linked DOFs. Furthermore, the energy dissipative mechanisms are represented by a single viscous damper, with damping coefficient C, linking the same two DOFs.

2.2 Reduced model for LS parameter estimation method

In the parameter estimation process, a shear-type reduced model with n-DOF subjected to a seismic ground acceleration, $\ddot{x}_g$, was used (Figure 2).

Figure 2 Reduced model for the parameter estimation



On the assumption that the structural response is measured in term of absolute accelerations $(\ddot{x}_i + \ddot{x}_g)$ and relative displacement between consecutive stories $(x_i - x_{i-1})$, the equations of motion for the reduced model can be stated as:

$$
\begin{aligned}
& M_1\left(\ddot{x}_1+\ddot{x}_g\right)+C_1\dot{x}_1+K_1^+\operatorname{pos}\left(x_1\right)+K_1^-\operatorname{neg}\left(x_1\right) \\
& -K_2^+\operatorname{pos}\left(x_2-x_1\right)-K_2^-\operatorname{neg}\left(x_2-x_1\right)-C_2\left(\dot{x}_2-\dot{x}_1\right)=0 \\
& \vdots \\
& M_i\left(\ddot{x}_i+\ddot{x}_g\right)+C_i\left(\dot{x}_i-\dot{x}_{i-1}\right)+K_i^+\operatorname{pos}\left(x_i-x_{i-1}\right)+K_i^-\operatorname{neg}\left(x_i-x_{i-1}\right) \\
& -K_{i+1}^+\operatorname{pos}\left(x_{i+1}-x_i\right)-K_{i+1}^-\operatorname{neg}\left(x_{i+1}-x_i\right)-C_{i+1}\left(\dot{x}_{i+1}-\dot{x}_i\right)=0 \\
& \vdots \\
& M_n\left(\ddot{x}_n+\ddot{x}_g\right)+C_n\left(\dot{x}_n-\dot{x}_{n-1}\right)+K_n^+\operatorname{pos}\left(x_n-x_{n-1}\right)+K_n^-\operatorname{neg}\left(x_n-x_{n-1}\right)=0
\end{aligned}
\tag{3}
$$

in which M_i and C_i are the lumped mass and damping coefficient of the i^{th} DOF, K_i^+ and K_i^- are the stiffness parameters in one direction and in the opposite one, respectively, between consecutives DOF; $\ddot{x}_i$, $\dot{x}_i$, x_i are the acceleration, velocity and displacement of i^{th} DOF relative to base and $\ddot{x}_g$ is the acceleration of base.

It is important to note that, the stiffness parameters between consecutives DOF of the reduced model take into account the beam-column jointly stiffness between consecutives stories of the actual structure.

After defining the following normalised parameters:

$$\left.\begin{aligned} m_i &= M_i / M_n \\ c_i &= C_i / M_n \\ k_i^+ &= K_i^+ / M_n \\ k_i^- &= K_i^- / M_n \end{aligned}\right\} \forall i = 1, n, \tag{4}$$

being M_n the mass of n^{th} DOF, the equations of motion (3) can be rearranged as:

$$\begin{aligned} & m_1 \left(\ddot{x}_1 + \ddot{x}_g \right) + c_1 \dot{x}_1 + k_1^+ \operatorname{pos}\left(x_1 \right) + k_1^- \operatorname{neg}\left(x_1 \right) = k_2^+ \operatorname{pos}\left(x_2 - x_1 \right) \\ & + k_2^- \operatorname{neg}\left(x_2 - x_i \right) + c_2 \left(\dot{x}_2 - \dot{x}_i \right) \\ & \vdots \\ & m_i \left(\ddot{x}_i + \ddot{x}_g \right) + c_i \left(\dot{x}_i - \dot{x}_{i-1} \right) + k_i^+ \operatorname{pos}\left(x_i - x_{i-1} \right) \\ & + k_i^- \operatorname{neg}\left(x_i - x_{i-1} \right) = k_{i+1}^+ \operatorname{pos}\left(x_{i+1} - x_i \right) \\ & + k_{i+1}^- \operatorname{neg}\left(x_{i+1} - x_i \right) + c_{i+1} \left(\dot{x}_{i+1} - \dot{x}_i \right) \\ & \vdots \\ & c_n \left(\dot{x}_n - \dot{x}_{n-1} \right) + k_n^+ \operatorname{pos}\left(x_n - x_{n-1} \right) + k_n^- \operatorname{neg}\left(x_n - x_{n-1} \right) = -(\ddot{x}_n + \ddot{x}_g), \end{aligned} \tag{5}$$

which allows the following parametric non-linear estimation procedure (LS fitting).

2.3 LS parameter estimation method

The present formulation [equation (5)] takes into account that the excitation is ground acceleration (e.g., natural minor earthquakes), with slight modifications any external force (e.g., artificial shakers) can be considered.

From records of relative displacement ($x_i - x_{i-1}$), relative velocity $(\dot{x}_{i+1} - \dot{x}_i)$ derived from relative displacement or measured, and absolute acceleration $(\ddot{x}_i + \ddot{x}_g)$ available for n measure points, at m time samples the procedure may be carried out in two steps:

2.3.1 Estimation of c_n, k_n^+ and k_n^-

From equations (5), the following over-determined system of linear equations can be stated:

$$A_n \chi_n = b_n, \tag{6}$$

in which:

$$A_n = \begin{bmatrix} \left(\dot{x}_n^{(1)} - \dot{x}_{n-1}^{(1)}\right) & \mathrm{pos}\left(x_n^{(1)} - x_{n-1}^{(1)}\right) & \mathrm{neg}\left(x_n^{(1)} - x_{n-1}^{(1)}\right) \\ \vdots & \vdots & \vdots \\ \left(\dot{x}_n^{(j)} - \dot{x}_{n-1}^{(j)}\right) & \mathrm{pos}\left(x_n^{(j)} - x_{n-1}^{(j)}\right) & \mathrm{neg}\left(x_n^{(j)} - x_{n-1}^{(j)}\right) \\ \vdots & \vdots & \vdots \\ \left(\dot{x}_n^{(m)} - \dot{x}_{n-1}^{(m)}\right) & \mathrm{pos}\left(x_n^{(m)} - x_{n-1}^{(m)}\right) & \mathrm{neg}\left(x_n^{(m)} - x_{n-1}^{(m)}\right) \end{bmatrix}, \tag{7}$$

$$\chi_n = \begin{bmatrix} c_n \\ k_n^+ \\ k_n^- \end{bmatrix}, \tag{8}$$

and

$$b_n = \begin{bmatrix} -\left(\ddot{x}_n^{(1)} + \ddot{x}_g^{(1)}\right) \\ \vdots \\ -\left(\ddot{x}_n^{(j)} + \ddot{x}_g^{(j)}\right) \\ \vdots \\ -\left(\ddot{x}_n^{(m)} + \ddot{x}_g^{(m)}\right) \end{bmatrix}, \tag{9}$$

where superscript $^{(j)}$ means measurement at the j^{th} time sample.

The over-determined system of equations (6) can be solved for χ_n in a LS sense by using Moore-Penrose (left) pseudo-inverse or singular value decomposition of A_n (Mohammad et al., 1992).

Note that pos(●) and neg(●) are basis functions of the function space of the restoring force function defined in equation (1). This ensures that the columns of A_n are linearly independent and therefore there exists the left pseudo-inverse of A_n.

2.3.2 *Estimation of c_i, k_i^+, k_i^- and m_i (from $i = n - 1$ to $i = 1$)*

From equations (5), the following over-determined system of linear equations can be written:

$$A_i \chi_i = b_i \qquad \forall i = 1, n-1, \tag{10}$$

in which:

$$A_1 = \begin{bmatrix} \left(\ddot{x}_1^{(1)} + \ddot{x}_g^{(1)}\right) & \dot{x}_1^{(1)} & \text{pos}\left(x_1^{(1)}\right) & \text{neg}\left(x_1^{(1)}\right) \\ \vdots & \vdots & \vdots & \vdots \\ \left(\ddot{x}_1^{(j)} + \ddot{x}_g^{(j)}\right) & \dot{x}_1^{(j)} & \text{pos}\left(x_1^{(j)}\right) & \text{neg}\left(x_1^{(j)}\right) \\ \vdots & \vdots & \vdots & \vdots \\ \left(\ddot{x}_1^{(m)} + \ddot{x}_g^{(m)}\right) & \dot{x}_1^{(m)} & \text{pos}\left(x_1^{(m)}\right) & \text{neg}\left(x_1^{(m)}\right) \end{bmatrix},$$

$$A_i = \begin{bmatrix} \left(\ddot{x}_i^{(1)} + \ddot{x}_g^{(1)}\right) & \left(\dot{x}_i^{(1)} - \dot{x}_{i-1}^{(1)}\right) & \text{pos}\left(x_i^{(1)} - x_{i-1}^{(1)}\right) & \text{neg}\left(x_i^{(1)} - x_{i-1}^{(1)}\right) \\ \vdots & \vdots & \vdots & \vdots \\ \left(x_i^{(1)} - x_{i-1}^{(1)}\right) & \left(\dot{x}_i^{(j)} - \dot{x}_{i-1}^{(j)}\right) & \text{pos}\left(x_i^{(j)} - x_{i-1}^{(j)}\right) & \text{neg}\left(x_i^{(j)} - x_{i-1}^{(j)}\right) \\ \vdots & \vdots & \vdots & \vdots \\ \left(\ddot{x}_i^{(m)} + \ddot{x}_g^{(m)}\right) & \left(\dot{x}_i^{(m)} - \dot{x}_{i-1}^{(m)}\right) & \text{pos}\left(x_i^{(m)} - x_{i-1}^{(m)}\right) & \text{neg}\left(x_i^{(m)} - x_{i-1}^{(m)}\right) \end{bmatrix}, \quad (11)$$

$\forall i = 2, n-1$

$$\chi_i = \begin{bmatrix} m_i \\ c_i \\ k_i^+ \\ k_i^- \end{bmatrix} \qquad \forall i = 1, n-1, \qquad (12)$$

and

$$b_i = \begin{bmatrix} k_{i+1}^+ \text{ pos}\left(x_{i+1}^{(1)} - x_i^{(1)}\right) + k_{i+1}^- \text{ neg}\left(x_{i+1}^{(1)} - x_i^{(j)}\right) + c_{i+1}\left(\dot{x}_{i+1}^{(1)} - \dot{x}_i^{(1)}\right) \\ \vdots \\ k_{i+1}^+ \text{ pos}\left(x_{i+1}^{(j)} - x_i^{(j)}\right) + k_{i+1}^- \text{ neg}\left(x_{i+1}^{(j)} - x_i^{(j)}\right) + c_{i+1}\left(\dot{x}_{i+1}^{(j)} - \dot{x}_i^{(j)}\right) \\ \vdots \\ k_{i+1}^+ \text{ pos}\left(x_{i+1}^{(m)} - x_i^{(m)}\right) + k_{i+1}^- \text{ neg}\left(x_{i+1}^{(m)} - x_i^{(m)}\right) + c_{i+1}\left(\dot{x}_{i+1}^{(m)} - \dot{x}_i^{(m)}\right) \end{bmatrix} \qquad (13)$$

$\forall i = 1, n-1,$

where superscript $^{(j)}$ means measurement at the j^{th} time sample.

Similarly, to the estimation of χ_n, the over-determined system of linear equations (10) can be solved for χ_i in a LS sense.

Note that the calculation of b_i requires a previous estimate of k_{i+1}^+, k_{i+1}^- and c_{i+1}, therefore, c_i, k_i^+, k_i^- and m_i must be estimated from $i = n - 1$ to $i = 1$.

Once having obtained k_i^+ and k_i^- for $i = 1, n$, the following pair of damage indexes (I_{D+} and I_{D-}) are proposed for each i^{th} story:

$$I_{D\pm}(i) = \frac{k_i^{d\pm} - k_i^{ud\pm}}{k_i^{d\pm} + k_i^{ud\pm}} \qquad \forall i = 1, n. \qquad (14)$$

in which superscript ^{d} means damaged state and superscript ^{ud} means undamaged state and n is the number of DOF. Because of the undamaged structure has linear behaviour, it is verified that $k_i^{ud+} = k_i^{ud-}$.

These damage indexes have the following properties: $I_{D+}(i)$ and $I_{D-}(i)$ are usually different from each other, dimensionless and can take values only in the interval [–1, 0]; being: $I_{D+}(i) = I_{D-}(i) = 0$ for the undamaged state.

3 Baseline method

In order to assess the sensitivity of the proposed method, a baseline method based on one linear stiffness k_i^d for each story is considered for comparison. In this case, k_i^d is estimated by a procedure similar to the one described above, except that the restoring force is given by:

$$F_r = K_i^d x_r. \tag{15}$$

The following damage index is defined on the baseline method:

$$I_{DBL}(i) = \frac{k_i^d - k_i^{ud}}{k_i^d + k_i^{ud}} \qquad \forall i = 1, n. \tag{16}$$

in which superscript ^{d} means damaged state and superscript ^{ud} means undamaged state respectively.

It is important to clarify that to verify the accuracy of both methods, the indexes which correspond to 'actual damage' were determined from stiffness parameters ($k_i^{ud\pm}$, $k_i^{d\pm}$, k_i^{ud} and k_i^d) obtained by FE non-linear static analyses, while the indexes which correspond to 'estimated damage' were determined from stiffness parameters estimated by the proposed methods.

4 Numerical example

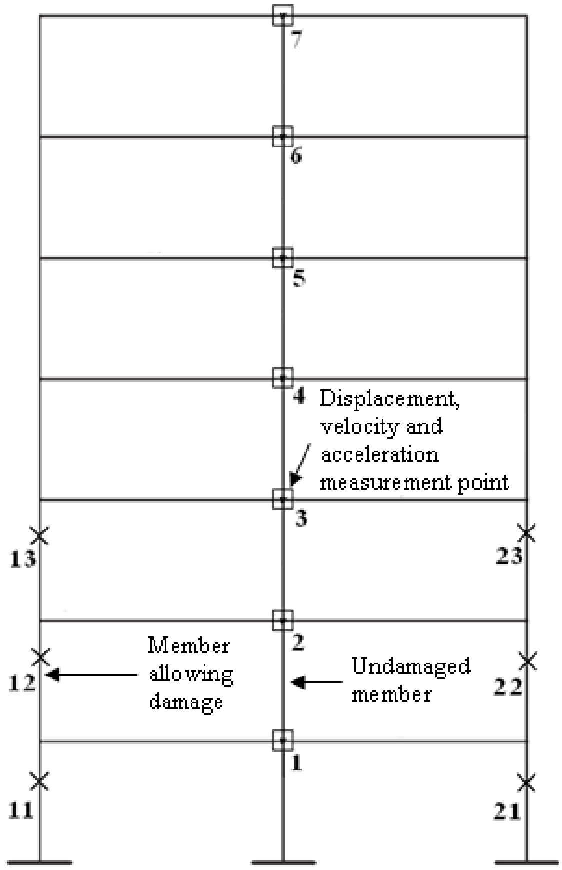
In order to demonstrate an application of the proposed method, a FEM model of a frame structure which is thoroughly detailed in SAP2000 (1997) was developed.

4.1 Definition of the example

Figure 3 shows a schema of the steel plane frame consisting of two-bay and seven-story with seven measurement points.

Typical story height is 3.96 m and each bay width is 9.14 m. Beams have moments of inertia of: $2.13 \cdot 10^{-3}$ m^4, for stories 1 and 2; $1.67 \cdot 10^{-3}$ m^4, for stories 3 and 4; and $1.38 \cdot 10^{-3}$ m^4, for stories 5, 6 and 7. External columns have moments of inertia of: $1.34 \cdot 10^{-3}$ m^4, for stories 1, 2 and 3; $1.11 \cdot 10^{-3}$ m^4, for stories 4 and 5; and $8.94 \cdot 10^{-4}$ m^4, for stories 6 and 7. Central columns have moments of inertia of: $1.62 \cdot 10^{-3}$ m^4, for stories 1, 2 and 3; $1.34 \cdot 10^{-3}$ m^4, for stories 4 and 5; and $1.11 \cdot 10^{-3}$ m^4, for stories 6 and 7. Typical story masses, of 85,812 kg, are distributed at the top of columns. For low amplitude vibration, the fundamental period resulted equal to 1.28 s.

Figure 3 Plane frame structure



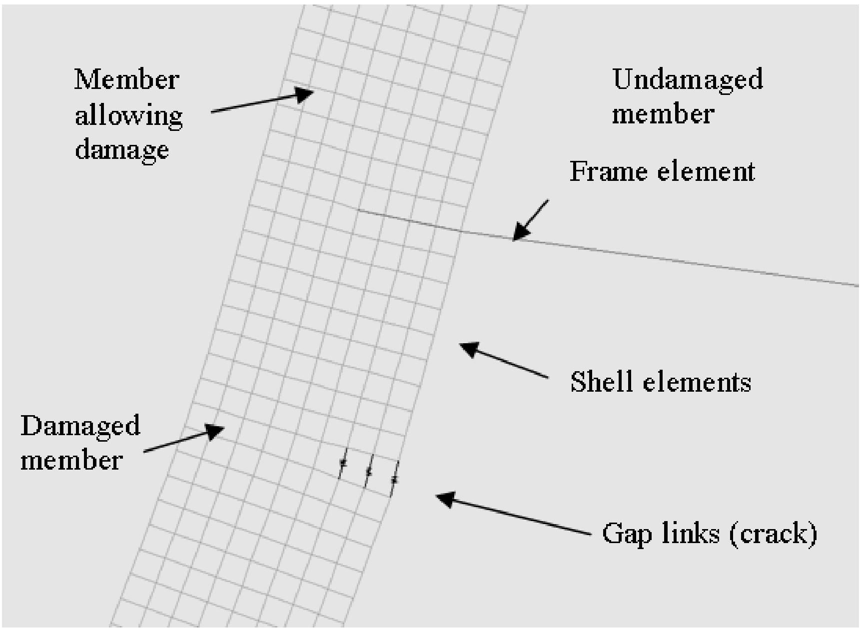
4.2 *FEM model of cracked structure*

Figure 4 shows the 2-D FEM model which uses three different element types: frame elements (2-nodes, 3 DOF per node), shell elements (4-nodes, 3 DOF per node), and gap links (2-nodes, 2 DOF per node) without tensile stiffness.

The columns which allow incorporating damage are modelled by using a set of shell elements in which a crack can be modelled by replacing some shell elements with gap links having only stiffness in compression, equivalent to that of the replaced shell elements. This model of breathing crack is a simplified variant of that proposed by Kisa and Brandon (2000). Note that the non-linearities considered in this FEM model are only concentrated in the damaged elements.

It is supposed that breathing of cracks is sufficiently small so no significant damping is added. Damping matrix was set proportional to mass and stiffness matrices assuming 5% of critical damping for first two modes (Clough and Penzien, 1995).

Figure 4 FEM model of a portion of cracked structure



4.3 Damage scenarios

Since many damage scenarios are possible on a structure, to demonstrate an application of the proposed methodology, seven scenarios with asymmetric- and symmetric-damage were considered. These cases were generated from the assumption that only the bottom section of the six columns shown in Figure 3 are susceptible to be cracked in one of the following three defined patterns:

a DP+: crack on the left side of column [Figure 5(a)]

b DP–: crack on the right side of column [Figure 5(b)]

c DP-DP+: crack on both sides of the column [Figure 5(c)].

Crack causes a behaviour which reduces the overall shear stiffness of the story when the crack opens, maintaining approximately the same stiffness when the crack closes. For illustrative purposes, the crack size was set to 3/8 of the section area and located at 0.17 m from the base of the column. Damage scenarios are detailed in Table 1.

Table 1 Definition of damage scenarios (only the first three stories are shown)

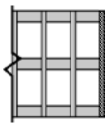
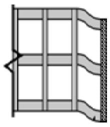
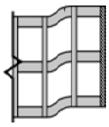
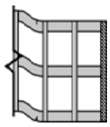
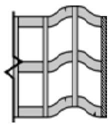
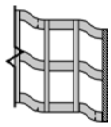
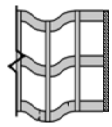
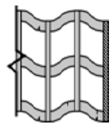
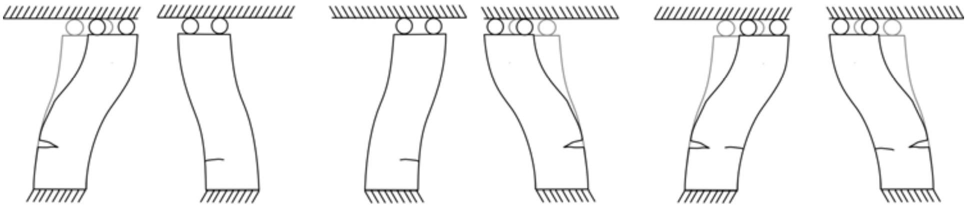
Case		*Story 1*		*Story 2*		*Story 3*		
		Col. 11	*Col. 21*	*Col. 12*	*Col. 22*	*Col. 13*	*Col. 23*	
a	CASE0 (undamaged)							
b	CASE1	DP+						
c	CASE2				DP–			
d	CASE3					DP+	DP–	
e	CASE12	DP+		DP–	DP–			
f	CASE13	DP-DP+				DP+	DP-DP+	
g	CASE23			DP-		DP-DP+		
h	CASE123	DP-DP+		DP-		DP+	DP+	

Figure 5 Schematic representation of damage patterns, (a) DP+ opening and closure of crack (b) DP– closure and opening of crack (c) DP-DP+ opening and closure of cracks on both sides



(a) (b) (c)

Note: Undamaged columns are in grey for reference.

4.4 Application of the proposed method

Non-linear dynamic analyses on the FEM model with all damage scenarios were performed using as excitation the 1/10-scaled N-S component of the 1940 El Centro earthquake, i.e., a minor earthquake.

Records of relative displacements, relative velocities and absolute accelerations were taken at the measurement points from those analyses (Figure 3), for 1,000 time samples through 20 s. Since the earthquake record decays to zero after 12 s, the response includes forced and free vibration components.

In order to consider the effect of noise on the estimation process, all records were contaminated by adding a random zero-mean white noise with a RMS (root mean square) amplitude equal to 0.5% of their RMS values.

5 Results and discussion

The damaged indexes obtained from the LS parameter estimation and baseline method are plotted in Figures 6 and 7, respectively.

Figure 6 shows that the proposed method is quite sensitive and accurate to detect ‘sign’ (DP+, DP– or DP-DP+) and location of symmetric- and asymmetric-damage. Due to spurious indications resulting from added noise, the proposed procedure is sensitive to damage sizes with indexes over 2% of their maximum values (recall that a total damage corresponds to index $I_D = -1$). In these cases [Figure 6(b), Figure 6(e), Figure 6(f), Figure 6(h)], the error in the magnitude estimation respect to the actual damage case is below 20%.

From Figure 7, it is observed that, as in the previous case, the sensitivity begins when the index value is greater than 2% of his maximum value. However, the error in the magnitude estimation respect to the actual damage case is in the order of 50% [Figure 7(e), Figure 7(f)]. Moreover, a linear identification is evidently unable to detect the ‘sign’ of damage pattern (DP+ or DP–).

Figure 6 Damage indexes estimated by means of the proposed (bilinear) method, (a) CASE0 (undamaged) (b) CASE1 (c) CASE2 (d) CASE3 (e) CASE12 (f) CASE13 (g) CASE23 (h) CASE123

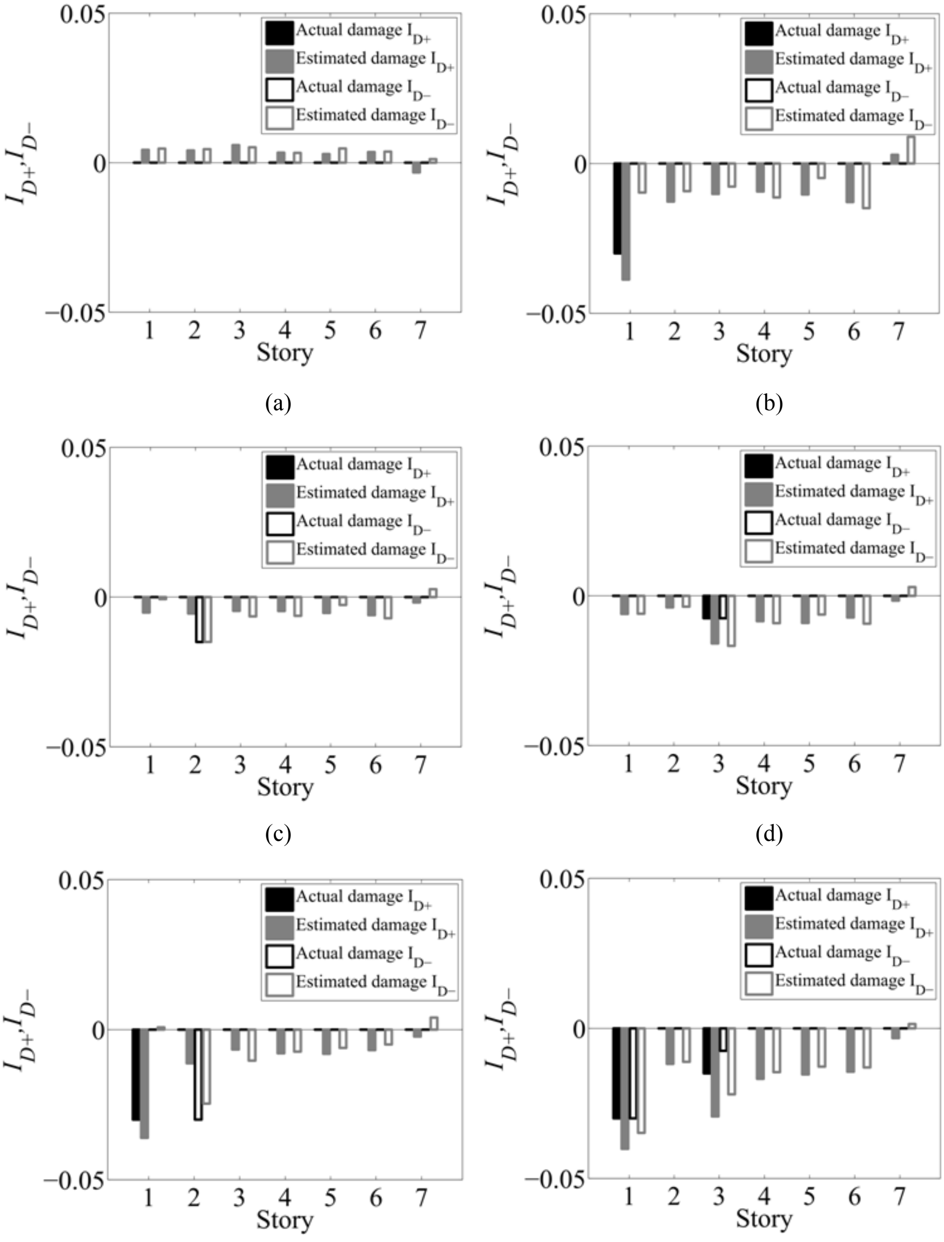
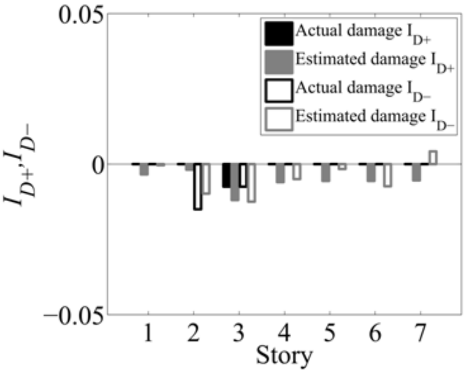
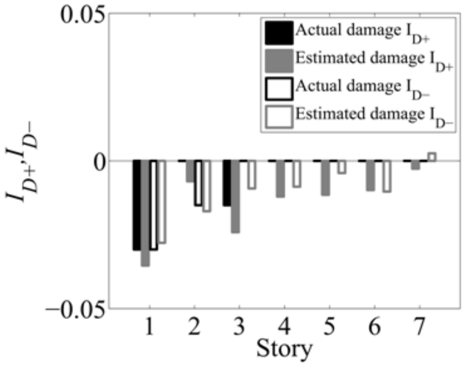


Figure 6 Damage indexes estimated by means of the proposed (bilinear) method, (a) CASE0 (undamaged) (b) CASE1 (c) CASE2 (d) CASE3 (e) CASE12 (f) CASE13 (g) CASE23 (h) CASE123 (continued)

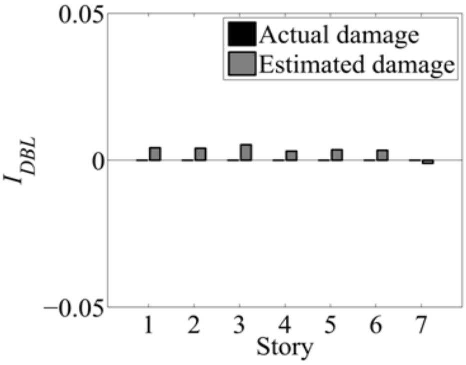


(g)

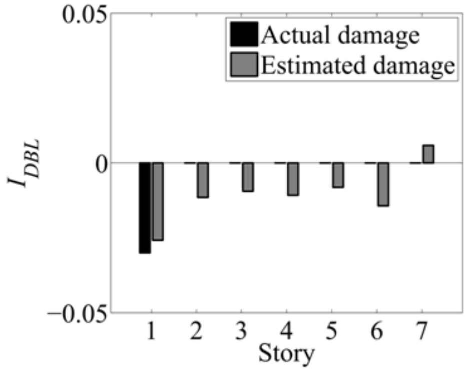


(h)

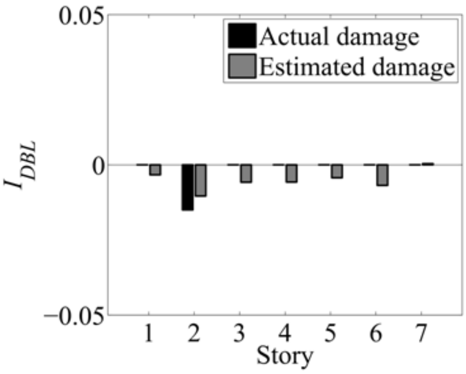
Figure 7 Damage indexes estimated by means of the baseline (linear) method, (a) CASE0 (undamaged) (b) CASE1 (c) CASE2 (d) CASE3 (e) CASE12 (f) CASE13 (g) CASE23 (h) CASE123



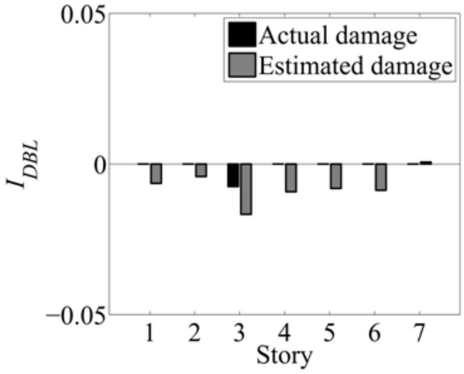
(a)



(b)

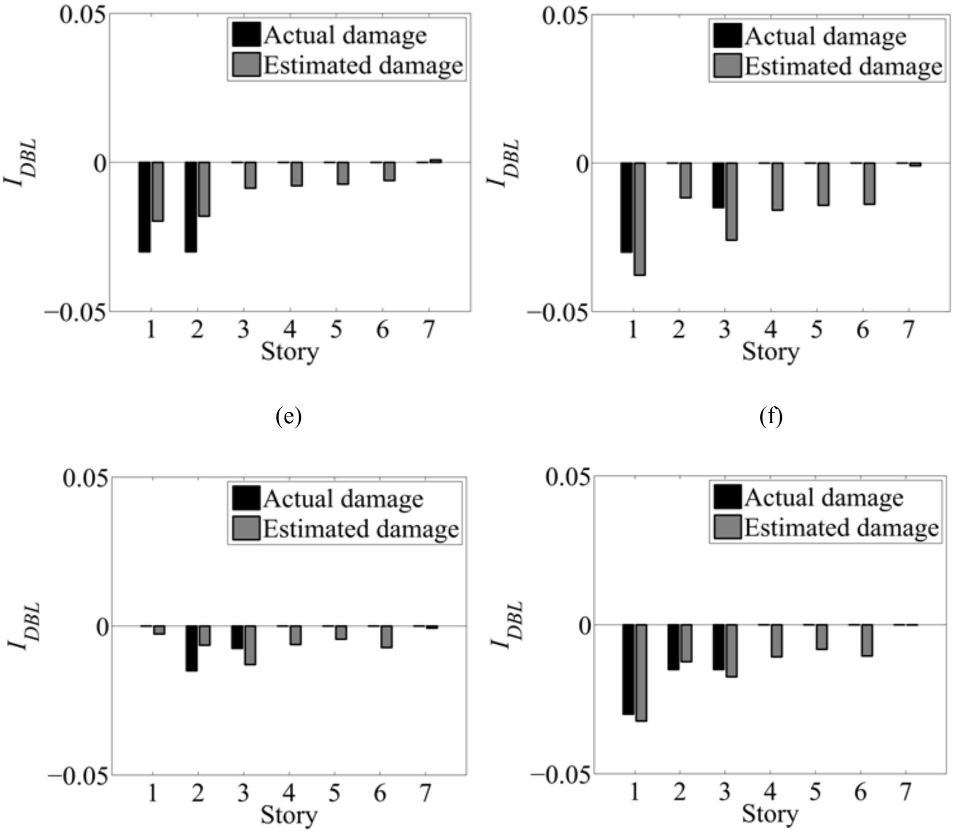


(c)



(d)

Figure 7 Damage indexes estimated by means of the baseline (linear) method, (a) CASE0 (undamaged) (b) CASE1 (c) CASE2 (d) CASE3 (e) CASE12 (f) CASE13 (g) CASE23 (h) CASE123 (continued)



(e) (f)

(g) (h)

From Figure 7(b), Figure 7(c) and Figure 7(e) it can be seen that, when the damage changes the stiffness in only one direction (asymmetric breathing cracks), the linear baseline method always underestimates the stiffness reduction (that has been confirmed by Yan et al. (2013) in cantilever cracked beams), suggesting that for this pattern of damage a bilinear stiffness model leads to better assessment of stiffness reduction than the linear stiffness model. A general overview of Figures 6 and 7 shows that the Signal-to-Noise Ratio, which measures the capability of the method to distinguish between 'healthy' and 'damaged' states, is greater than 2 for the proposed (bilinear) method and less than 2 for the baseline (linear) method. In the cases of symmetric or almost symmetric damage patterns, both methods give roughly similar estimates [Figure 6(f) and Figure 7(f)].

6 Conclusions

This work is intended to develop a method for detection of breathing cracks in MDOF frame structures based on a LS approach using a bilinear stiffness model. A damage index was defined on the basis of two stiffness parameters (a stiffness value for each direction) for each story of the frame. The following conclusions can be drawn from the results:

1 while the sensitivity depends on the noise level, for the assumed types of damage the proposed method is significantly more sensitive than a similar method based on a linear stiffness model

2 the proposed method allows identifying: the location, 'sign' (DP+ or DP–) and magnitude of damage from both stiffness parameters

3 the proposed method is a non-destructive dynamic method using the structural response from excitations such as natural minor earthquakes or artificial shakers.

Without loss of generality, the method proposed in this work can be applied to other damage patterns causing bilinear stiffness, as for instance clearance in joints or missing bolts in frame or truss structures.

Acknowledgements

The authors would like to express their thanks to CONICET and National University of Cuyo for the financial support and to Eng. Tomás Schnetzer for his helpful suggestions on FEM modelling.

References

Clough, R.W. and Penzien, J. (1995) *Dynamics of Structures*, 3rd ed., Computers & Structures, Inc., Berkeley, CA USA [online] http://trid.trb.org/view.aspx?id=50306 (accessed 28 June 2012).

Curadelli, O. and Ambrosini, D. (2011) 'Damage detection in elevated spherical containers partially filled with liquid', *Engineering Structures*, Vol. 33, No. 9, pp.2708–2715 [online] http://linkinghub.elsevier.com/retrieve/pii/S0141029611002331 (accessed 14 August 2013).

Curadelli, R.O. et al. (2008) 'Damage detection by means of structural damping identification', *Engineering Structures*, Vol. 30, No. 12, pp.3497–3504 [online] http://linkinghub.elsevier.com/retrieve/pii/S014102960800206X (accessed 14 August 2013).

Dimarogonas, A.D. (1996) 'Vibration of cracked structures: a state of the art review', *Engineering Fracture Mechanics*, Vol. 55, No. 5, pp.831–857.

Friswell, M.I. and Penny, J.E.T. (1992) 'A simple nonlinear model of a cracked beam', *10th International Modal Analysis Conference*, San Diego, CA, pp.516–521.

Giuliani, L. (2012) 'Structural safety in case of extreme actions', *Int. J. Lifecycle Performance Engineering*, Vol. 1, No. 1, pp.22–40.

Kerschen, G. et al. (2006) 'Past, present and future of nonlinear system identification in structural dynamics', *Mechanical Systems and Signal Processing*, Vol. 20, No. 3, pp.505–592 [online] http://linkinghub.elsevier.com/retrieve/pii/S0888327005000828 (accessed 15 August 2013).

Kisa, M. and Brandon, J. (2000) 'The effects of closure of cracks on the dynamics of a cracked cantilever beam', *Journal of Sound and Vibration*, Vol. 238, No. 1, pp.1–18 [online] http://linkinghub.elsevier.com/retrieve/pii/S0022460X00930992 (accessed 2 September 2013).

Law, S.S. and Zhu, X.Q. (2004) 'Dynamic behavior of damaged concrete bridge structures under moving vehicular loads', *Engineering Structures*, Vol. 26, No. 9, pp.1279–1293 [online] http://linkinghub.elsevier.com/retrieve/pii/S0141029604001257 (accessed 5 September 2013).

Mohammad, K.S., Worden, K. and Tomlinson, G.R. (1992) 'Direct parameter estimation for linear and non-linear structures', *Journal of Sound and Vibration*, Vol. 152, No. 3, pp.471–499.

Rezaee, M. and Hassannejad, R. (2010) 'Free vibration analysis of simply supported beam with breathing crack using perturbation method', *Acta Mechanica Solida Sinica*, Vol. 23, No. 5, pp.459–470.

SAP2000 (1997) *SAP2000 Integrated Finite Element Analysis and Design of Structures Verification Manual*, Computers and Structures, Inc., Berkeley, California, USA.

Yan, G. et al. (2013) 'A novel approach to detecting breathing-fatigue cracks based on dynamic characteristics', *Journal of Sound and Vibration*, Vol. 332, No. 2, pp.407–422 [online] http://linkinghub.elsevier.com/retrieve/pii/S0022460X12006979 (accessed 11 September 2013).