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Symmetry issues in mixed integer programming based Unit Commitment

J. Alemany [a,*], F. Magnago [a,b], D. Moitre [a], H. Pinto [b]

[a] Department of Electrical and Electronic Engineering, Univ. Nacional de Río Cuarto, Argentina
[b] Nexant Inc., AZ, USA

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ABSTRACT

A mixed integer linear problem is called symmetric if the variables can be permuted without changing the structure of the problem. Generally, these problems are difficult to solve due to the redundant solutions which populate the enumeration tree. In Unit Commitment problems the symmetry is present when identical generators have to be scheduled. This article presents a way to reduce the computational burden of the Branch and Cut algorithm by adding appropriate inequalities into the mixed-linear formulation of the Unit Commitment problem. In the examples considered, this approach leads to a substantial reduction in computational effort, without affecting the objective value.

1. Introduction

The Unit Commitment (UC) calculation is extensively used in daily system operation and it is an exercise of very large-scale, time-varying, non-convex, mixed-integer modeling and optimization. Security Constrained Unit Commitment (SCUC) is an extension to conventional UC with the inclusion of system network constraints. The main objective of SCUC is to ensure not only the economic operation but also the security of the system [1,2]. This extension introduces extra levels of complexity to the UC problem. This is the main reason why many methods solve the UC and the power flow-based network constraints separately as a two level optimization problem. A master level which includes the UC calculation, and the sub-problem level which checks the network security constrains. In summary, despite the complexity of SCUC problem, the master level still remains as a classic single bus UC problem. Currently, Lagrangian Relaxation (LR) and Mixed-Integer Linear Programming (MILP, branch-and-cut based algorithms) are the main UC solution contenders depending on the specific UC problems being solved.

An important factor that usually affects the performance of the algorithms is when the system has identical generating units. Unfortunately, this situation occurs frequently. Identical generators are very common in combined cycle plants, large hydraulic facilities, and thermal plants. Modeling identical generators originates an algorithmic issue known as the symmetry problem. In the case of LR algorithm will produce oscillations during the iteration process [3–5] while in branch-and-cut algorithms will produce redundant computational effort exploring equivalent search regions unnecessarily [6].

Different techniques have been applied in different areas to solve symmetry problems for the branch-and-cut methodology. They can be classified into two main methodologies: reformulation methods and removal methods. Reformulation techniques basically rewrite the problem in order to remove the symmetries [6]. They have been applied in different areas. One of the first implementation proposed a reformulation method for cutting stock problems [7]. It has also been applied to urban transit scheduling [8], airline crew scheduling [9,10], vehicle routing [11], graph coloring [12], as well as binary cutting stock problems [10]. Results proved that it is a very effective method. However, the identification of how to reformulate a specific problem is not an easy task. Furthermore, its applicability is not general and can increase the dimension of the problem to solve. On the other hand, removal methods rely on the elimination of the symmetry from the problem mainly by adding constraints. The addition of constraints can be done dynamically or statically. The first one exploits symmetry during the tree search process while the second one attempts to remove the symmetry by adding hierarchies in the selection process.

* Corresponding author.
E-mail address: jalemany@ing.unrc.edu.ar (J. Alemany).

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Different alternatives of the dynamic removal methods have been proposed [6]. A method to solve binary problems using isomorphism pruning is presented in [13]. Symmetry breaking via dominance detection used in constraint programming is described in [14–16]. As another alternative, in [17,18] the symmetry breaking was applied during the search step, ensuring that only one isomorphic solution is allowed at every tree node. These methods exploits symmetry during the tree search process, therefore, it is difficult to be implemented using off-the-shelf software.

On the other hand, the static methods try to remove the symmetry by adding hierarchies in the selection process. Several authors proposed different ways of symmetry static removal applied to different areas [19–22]. Results suggest that the static methods can be very effective. The main advantages of these methods are that they only require the addition of hierarchical constraints and no special software is needed.

The objectives of this article are: First, to identify the elements that generate symmetry, then, to evaluate the algorithm under the presence of symmetry, and finally, to use a static removal strategy to set a priority to the generator status variable, in order to reduce the computational burden of the branch-and-cut algorithm.

2. Symmetry in Mixed Integer Linear Programming

A Mixed Integer Linear Programming (MILP) problem has the following form:

$$\min\{cx + hy : Ax + Gy \geqslant b, x \in Z^n, y \in \Re^p\} \tag{1}$$

where A and G are $m \times n$ matrices, b is a m-vector, c and h are n-vectors, and x, y are the n-vector variables, where the x variables are integers and the y variables are continuous.

From the set of all feasible solutions Q, the MILP problem represented by Eq. (1) may allow multiple equivalent solutions, each of them representing a symmetry group G. Further detailed information can be found in [6]. Mathematically, the symmetry group G of MILP (1) can be defined as the set of all permutations π of the n variables mapping Q on itself and mapping each feasible solution on a feasible solution having the same objective value:

$$G = \{\pi \in \Pi^n | \forall x \in Q : \pi(x) \in Q \wedge c^T x = c^T \pi(x)\}$$

If the equivalence of these sub-problems is not identified this may lead to solve unnecessary problems, making a relatively easy problem very difficult to solve. Therefore, the main challenge in symmetry reduction is to identify a subset of a symmetry group in order to reduce the computational burden.

The static symmetry methods consists of adding constraints to the initial formulation, cutting some of the symmetric solutions, while keeping at least one optimal solution. It is a common practice to compute a subgroup G_{LP} of G [6,19], being G_{LP} the linear relaxation of Eq. (1). However, another practical option is to exploit the knowledge of the model. An alternative of this knowledge-based method is proposed in [22], where the authors identify the identical objects of their models, and they impose hierarchical decisions to mitigate the symmetry effect.

In UC problems, the identical objects that generate symmetry are the generator state variables. Given a scheduling pattern of generating units with identical production costs connected to the same system bus, it can be found several identical patterns by permuting the state variables among the identical units. Therefore, in this article, we impose hierarchical decisions based on the knowledge of power system generation, and on the ideas suggested by [22]. These hierarchies are supported by Corollary 1 [6], see Appendix A.

3. Nomenclature

T	Scheduling horizon
Gen	Number of generation plants
$C(\cdot)$	Total cost ($)
Cp_{gt}	Production cost, for unit g, at hour t ($/h)
Cs_{gt}	Startup cost ($)
p_{gt}	Active power variable (MW)
u_{gt}	Binary state variable. 1 meaning on and 0 off
D_t	Hourly system demand (MW)
$A(\cdot)$, b	Set of operational constraints
fc_g	Fixed cost ($/h)
vc_g	Variable cost ($/MWh)
K_g^τ	Cost for startup cost step τ ($/h)
E_g	Number of start up steps
B_{Gen}	Subset of identical units in a bus

4. Symmetry in MILP based Unit Commitment

The Unit Commitment (UC) problem can be formulated as a minimization problem which main objective is to determine the generation dispatch to supply the demand requirements and several other operational constraints, at minimum cost over a period of time. Mathematically can be represented as follows:

$$\min_{u,p} z = \sum_{t=1}^{T}\sum_{g=1}^{\text{Gen}} C(p_{gt}, u_{gt}) \tag{2}$$

subject to:

$$D_t - \sum_{g=1}^{\text{Gen}} p_{gt} = 0 \ \forall t \tag{3}$$

$$A(p_{gt}, u_{gt}) \leqslant b \ \forall (g,t) \tag{4}$$

where:

$$C(p_{gt}, u_{gt}) = Cp_{gt} + Cs_{gt}$$

$$Cp_{gt} = fc_g \ u_{gt} + vc_g \ p_{gt}$$

$$Cs_{gt} \geqslant K_g^\tau [u_{gt} - \sum_{n=1}^{\tau} u_{g,t-n}] \quad \forall (g,t) \quad \forall \tau = 1, \ldots, E_g$$

$$Cs_{gt} \geqslant 0 \qquad \forall (g,t)$$

The main factors that generate the existence of symmetry in UC problems are the presence of identical production (fc_g, vc_g) and start-up (K_g^τ) cost functions among generators.

On the other hand, there are some unit constraints that can potentially help to diminish symmetry. These constraints are the minimum up/down time constraints, ramping rate constraints and power limit constraints. Nevertheless, these operational constraint differences do not entirely eliminate symmetry, mainly when identical cost units face the same dispatch conditions. Moreover, when solving SCUC problems, mainly for real-time or day-ahead planning horizons, it is a very common practice to consider a simplified model where some data and constrains are approximated or even worst not considered. Nowadays, in order to avoid symmetry in SCUC applications, it is a common practice to slightly modify the identical cost data.

Therefore, dealing with the effect of symmetry is a very important task when solving practical SCUC problems.

Table 1
Two generation units system.

Item	Symbol	Value
Fix cost	fc	8
Variable cost	vc	2
Start up cost	K_g	10
Start steps	E_g	1
Planning horizon	T	2
Fix demand	D_t	30, 50
Max power	$\overline{P}_g$	60
Min power	$\underline{P}_g$	6

4.1. Illustrative example

A small system with two identical generators is used to illustrate the symmetry issue and the application of the static symmetry method. Table 1 shows the data used for the simulations. For simplicity, from the full set of operational constraints, only power limit constraints are included into the formulation.

Applying the general formulation given by Eqs. (2)–(4), it can be formulated as follows:

$$\min_{u,p} z = \sum_{t=1}^{T=2} \sum_{g=1}^{\text{Gen}=2} [(8\ u_{gt} + 2\ p_{gt}) + Cs_{gt}]$$

$$\begin{aligned}
&\text{s.t.}\\
&Cs_{g1} \geqslant 10\ u_{g1} \quad \forall g\\
&Cs_{g2} \geqslant 10\ u_{g2} - 10\ u_{g1} \quad \forall g\\
&Cs_{gt} \geqslant 0 \quad \forall (g,t)\\
&6\ u_{gt} \leqslant p_{gt} \leqslant 60\ u_{gt} \quad \forall (g,t)\\
&30 - \sum_{g=1}^{\text{Gen}} p_{g1} = 0 \quad \forall g\\
&50 - \sum_{g=1}^{\text{Gen}} p_{g2} = 0 \quad \forall g
\end{aligned}$$

After solving this problem, a set Q containing 9 feasible solutions is obtained. The solutions for the u_{gt} variables, represented as vectors $[u_{11}, u_{21}, u_{12}, u_{22}]$, and the corresponding objective value z_i are:

$$Q = \begin{cases} \{[0,1,0,1]\} \mapsto z_1 = 186\\ \{[1,0,1,0]\} \mapsto z_2 = 186\\ \{[0,1,1,0]\} \mapsto z_3 = 196\\ \{[1,0,0,1]\} \mapsto z_4 = 196\\ \{[0,1,1,1]\} \mapsto z_5 = 204\\ \{[1,0,1,1]\} \mapsto z_6 = 204\\ \{[1,1,0,1]\} \mapsto z_7 = 204\\ \{[1,1,1,0]\} \mapsto z_8 = 204\\ \{[1,1,1,1]\} \mapsto z_9 = 212 \end{cases}$$

All the solutions with the same objective value z are equivalent. In order to see the effect of the hierarchical constraints on the set Q, the following inequalities are included into the set of operational constraints:

$$u_{1t} \geqslant u_{2t} \quad \forall t \tag{5}$$

The inclusion of these constraints modifies the set Q. Now, the set Q contains only 4 feasible solutions:

$$Q = \begin{cases} \{[1,0,1,0]\} \mapsto z_1 = 186\\ \{[1,0,1,1]\} \mapsto z_2 = 204\\ \{[1,1,1,0]\} \mapsto z_3 = 204\\ \{[1,1,1,1]\} \mapsto z_4 = 212 \end{cases}$$

Table 2
Two generation units system results.

	Num. LPs for feasible solution	Num. LPs for global optima
Base Case	4	16
Adding Eq. (5)	3	8

Hierarchy (5) has imposed an order on the binary variables that avoids recalculations on redundant solutions. Moreover, a reduced set of feasible solutions is obtained by imposing this ordering. To illustrate this fact, a simulation instance using CPLEX is performed. The following search strategies are used:

- Node selection: down branch first.
- Variable selection: minimum infeasibility variable.
- Branch selection: depth first search.

The problem is solved with and without adding the hierarchical constraint of Eq. (5). Simulation results are illustrated in Table 2, the comparison is based on the number of Linear Program solutions that each method needs.

From Table 2 can be inferred that adding hierarchical constraints leads to a significant reduction of computational burden.

4.2. Symmetry subset

Although the UC problem is a single-bus calculation, it is not always possible to break symmetry in all identical units of the power system. It is important to emphasize that UC is one part the problem in the SCUC calculation. The other issue is related to the security constraints feasibility checking. Therefore, as a consequence of the inclusion of the transmission network, only the units connected to the same bus can be considered to break the symmetry.

In order to obtain the groups of units which generate redundant solutions, Corollary 1 is used to form a subset B_{Gen} of G, restricted to N, which is the number of integer variables allowed to be permuted. The key here is to identify the generating units, candidates for permutation, and to form groups with them. These groups are identified by a specific algorithm.

Algorithm outline:

1. Obtain the subset of generation buses from the full set of system buses.
2. Form groups of units connected to the same bus, from the subset of step 1.
3. Identify units with identical production costs from the groups of step 2.
4. Form generating groups with permutation potential.
5. Form hierarchical constraints according to the groups from step 4.
6. Set priorities for the hierarchical constraints.

Once B_{Gen} is obtained, the following constraints can be included in the UC problem:

$$u_{gt} \geqslant u_{ggt}\ \forall t\ \wedge\ \pi_{(g,gg)} \in B_{\text{Gen}} \tag{6}$$

These constraints are activated according to the initial conditions of the units within the groups arranged in step 2.

5. Computational results

The test system described in [23,24] is used to illustrate the proposed method. This system has ten generation units which are all different in terms of production and start up cost. The total generating capacity is 1662 MW, with a system peak load of

1500 MW. In order to carry out different simulations, three different systems are created: a 20 generating system is built by duplicating the original system, a 50 generation system is built by five folding the 10 unit system, and a 100 generation system is built following the same pattern. This approach permits to build systems with ten symmetric groups, each of them having 2, 5 and 10 identical generation units respectively. In addition to that, a real system with 225 thermal units is also tested [25]. It has 10 groups of identical units, each of them with different number of units, the total thermal capacity is 17,031 MW with a system peak load of 10,323 MW. All the systems are modeled with the characteristics taken from references [23,24] except for operative reserve which is omitted. The model considers: piece-wise and step-wise production and start-up costs respectively; min up/down times; unit power capacities; state, start-up, and shut-down logics; initial conditions; and the energy balance equation. The model is implemented in GAMS using CPLEX as the solver, with all the parameter options set to the default values.

Each case is solved considering the original system (BC, Base Case) as well as the inclusion of static symmetry breaking constraints (SSBC). Additionally, in order to do a fair comparison between the solver and the proposed methodology, all the BC simulations were repeated changing the CPLEX symmetry-breaking parameter from moderate to aggressive. The simulation results are exhibited in Table 3.

5.1. Discussion of the results

To validate the results of this work they are compared with results given in [23,24]. Results given in [23] are used as the reference.

In [24] a Gravitational Search Algorithm (GSA) algorithm is proposed to solve a thermal UC problem. Likewise, in [23] a Genetic Algorithm (GA) is proposed to solve the same problem. In [24] the GSA total costs are compared to those of different meta-heuristics methods and no gap tolerance is reported. In [23] the GA total costs are compared to those of LR and Dynamic Programming (DP) algorithms, and a gap tolerance is reported. In addition, it is mentioned in [23] that can be expected to obtain better solutions -lower costs-.

Table 4 gives the total costs comparison:

Table 3
Examples comparison using default CPLEX options.

	Case	Units 20	50	100	225
Equations	BC	5821	14,476	26,401	74,302
	SSBC	6071	14,976	28,501	75,452
B Variables	BC & SSBC	1500	3750	7500	16,817
$\mathfrak{R}$ Variables	BC & SSBC	4026	10,026	17,526	33,768
Non zeros	BC	22,724	56,735	103,420	248,640
	SSBC	23,224	57,735	107,620	250,940
Optimal Cost	BC & SSBC	1,101,377	2,748,513	5,494,339	424,787
Nodes	BC	463	19,513	11,802	3627
	SSBC	139	1854	1076	1738
Relative gap	BC	0.0	0.0	0.02%	0.0
	SSBC	0.0	0.0	0.0%	0.0
Solution time	BC	8.17	163.72	261.73	163.80
	SSBC	3.43	66.81	115.64	120.00
	SYM 1	8.05	159.07	276.11	168.12
	SYM 2	8.04	165.96	262.65	176.30
	SYM 3	7.18	222.42	205.76	127.25
	SYM 4	6.88	228.64	206.71	error
	SYM 5	7.20	386.51	200.17	153.35

BC: Base Case. SSBC: With Static Symmetry Breaking Constraints. SYM #: CPLEX symmetry-breaking parameter.

Table 4
Total cost comparison with respect to the benchmark system [23].

		20 Units	100 Units	Relative difference with [23]	
[23]		1.130.660	5.627.437	20 Units	100 Units
MILP proposed approach		1.101.377	5.494.339	−2.59%	−2.37%
[24]	With ramps	1.124.475	5.606.413	−0.55%	−0.37%
	WO ramps	1.123.216	5.600.883	−0.66%	−0.47%

The differences described in Table 4 are less than 3%, being our optimal costs lower. The differences can be justified by two main reasons:

- The algorithms used. It is important to mention that our MILP implementation allowed us to get global optimal solutions, except for the Base Case, 100 units instance, which gives a gap of 0.02%.
- The system operative reserve. These constraints are omitted in this article.

6. Conclusion

Classic MILP UC models have inherent disadvantages when they have to deal with systems with identical generating units connected to the same bus. In this paper, a static strategy of symmetry breaking is applied to solve MILP based UC problems with identical generating units. Numerical results for different systems show that the proposed methodology leads to a considerable reduction of the search on the branch-and-cut enumeration tree. As a consequence, a significant time computation reduction is also obtained.

Appendix A. Hierarchical constraints

A fundamental region F for G can be defined as [6]:

- $\forall g \in G, g \neq I$,
- $g(int(F)) \cap int(F) = \emptyset$,
- $\cup_{g \in G}\ g(F) = \mathfrak{R}^n$,

where $int(F)$ represents the interior of region F and I the identity permutation. The last equality implies that F includes at least one optimal solution of the MILP problem.

Theorem 1 [6]. *Let G be the symmetry group for MILP problem (1) and let F be a fundamental region for G. Then, an optimal solution of the MILP problem (1) can be found by optimizing over the intersection of the feasible set of Eq. (1) with F.*

Theorem 2 [26]. *Let G be the symmetry group for MILP problem (1) and let $\bar{x} \in \mathfrak{R}^n$ such that $g(\bar{x}) \neq \bar{x}$ for all $g \in G$, $g \neq I$. Then,*

$$F = \{x \in \mathfrak{R}^n | \forall g \in G,\ g \neq I : (g(\bar{x}) - \bar{x}) \bullet x \leqslant 0\} \quad \text{(A.1)}$$

is a fundamental region for G.

In practice, simple sets of static symmetry breaking inequalities are used, and most of them can be derived from the following corollary:

Corollary 1 [6]. *Theorem 1 remains true when the fundamental region F is replaced by the region obtained from Theorem 2 by relaxing its statement in the following ways:*

- Inequalities (A.1) are written only for a subset of permutations in G.

- The condition $g(\bar{x}) \neq \bar{x} \quad \forall g \in G,\ g \neq I$ is removed.

Consequently, if the MILP problem has n integer variables $0 \leqslant x_i \leqslant k$ for $i = 1, \ldots, n$. G is restricted on these variables and the following inequalities can be added:

$$x_1 \geqslant x_2 \geqslant \cdots \geqslant x_n \tag{2}$$

These inequalities are called *Hierarchical constraints* and they can be applied to MILP based UC problems.

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