IET Generation, Transmission & Distribution

Research Article

The Institution of Engineering and Technology Journals

Hybrid method for power system state estimation

ISSN 1751-8687
Received on 3rd September 2014
Accepted on 5th November 2014
doi: 10.1049/iet-gtd.2014.0836
www.ietdl.org

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Abstract: State estimation in power systems is classically based on the weighted least squares method. Recently, different extensions of Kalman filters have been proposed. Among them, the 'unscented' Kalman filter (UKF) improves the results of weighted least squares methods, when there are small changes in the system, as it considers the history of the state. The novel algorithm presented in this work combines the best of both approaches. To perform this task a new index is defined to allow the algorithm to choose in real time, and for each iteration, between a static or a dynamic estimator. This combination allows overcoming the anomalies observed when the UKF faces abrupt variations of the system state and also the lack of observability that weighted least squares could present. The proposed methodology was tested with three test cases outperforming the previously mentioned algorithms.

1 Introduction

In power systems, it is crucial to maintain the operation level inside the normal condition range. This means that there is no overload, voltage levels are inside their respective ranges and the load balance is met. These levels are monitored and controlled at each operation centre, through an energy management system (EMS). The EMS provides a set of algorithms (power flow, contingency analysis and optimal power flow among other) that support the operation of the system. The input data for these algorithms may be the power, the voltage or the phase angles in the buses of the electrical network. A supervisory control and data acquisition (SCADA) system is used to obtain these values at some previously selected points since measuring at all the points is usually not feasible. The value in the non-measured points must be estimated using the redundancy given by the Ohm/Kirchhoff equations (see [1] for the original paper). This is made by the state estimator that also filters the measurement and transmission errors. Most EMS operation centres in the world base the estimation on the weighted least squares (WLS) method (see [2, 3]). WLS is a static state estimator which obtains results from the current data measured regardless recent historical information except for some pseudo-measures. See [4, 5] for a detailed survey of power system state estimation methods.

Recent literature [6] proposes using a new method called unscented Kalman filter (UKF) to estimate the system state. UKF is a dynamic state estimator which allows estimating the current state of the system and predicting the value of the state vector at the next time step.

In [6], the authors show that UKF improves the results of WLS when there are slight changes in the system load and provides a useful estimate even in the case of lack of observability of the system. However, UKF rarely outperforms WLS in every case [7] and takes some time to recover a good estimation level when the operating conditions change abruptly. Considering the advantages and disadvantages that WLS and UKF methods have, this paper presents a hybrid method that combining UKF and WLS is able to exploit the advantages of both methods. This hybrid method maintains the robustness of the WLS and incorporates state predictability through UKF.

More precisely, WLS is able to compute the most probable state vector given the observed measurements. However, if some measurements are corrupted and should be discarded, the WLS could lose its meaning due to a so called 'lack of observability', being the redundancy data not enough to compute a solution. In such case, some heuristic actions are taken, for instance, the use of historical data or heuristic rules to compute the missing measures (now called 'pseudo-measures').

On the other hand, even in the presence of bad data, UKF always gives a result, that can be far from the real state. The quality of the estimation will depend on both the current observation and the previous observations. Given a slight variation of the state, the estimation will gain in precision over the time; however, when a larger variation appears, the computed estimation will have a considerable error that will diminish in time while no other large variation appears.

In this work, we propose a combination of UKF and WLS to take profit of their advantages. Specifically, both methods are computed at each time and the best estimation is chosen. When WLS lacks observability, the UKF solution can be used to give the missing pseudo-measurements. When UKF is not accurate because of the large variations of the system, the WLS solution is provided. For the practical implementation of the hybrid method, the algorithm should decide, in an unsupervised way, which method is the best solution without the knowledge of the true state. For this purpose, an index and a threshold are defined to choose between WLS and UKF in real time. Numerical experiments where the real state of the system is known in advance show that this hybrid methodology outperforms both previously mentioned methods.

The first part of this work consists of an introduction to WLS and UKF methods. Then, the proposed hybrid method is presented showing in detail how WLS and UKF are combined. Finally, the results of two theoretical examples and a real case in a section of the 132 kV transport network in Argentina are analysed.

2 State estimation

The state of a system is defined by variables that provide a complete representation of the internal condition or status of the system at a given time [8]. In power systems, those variables are voltage magnitudes and phase angles at the buses. Solving the state estimation problem consists in acquiring the most probable state from given SCADA measurements. These measurements include voltages, power flows, power injections and/or current angles provided by conventional remote terminal units (see [9]).

Power system state estimators use the set of available measurements to estimate the system state. Considering a set of measurements and their locations, the network observability analysis will determine if a unique estimation can be found for the system state [2]. Meter failures or telecommunication errors may occasionally lead to cases where the state of the entire system cannot be estimated because of the lack of measurements. Should the system be not observable, then additional pseudo-measures may have to be placed in particular locations. In this work, an on-line observability analysis is executed after a set of measurements is received at the last SCADA scan. A detailed explanation of the observability analysis algorithm is presented in Section 3.

Another essential function of a state estimator is to detect measurement errors to identify and eliminate them whenever possible. Several reasons may generate errors in measurements. Random errors usually exist in measurements because of the accuracy of the meter and the data communication devices. If there exists sufficient redundancy among measurements, such errors are expected to be filtered by the state estimator, and the nature of the filtering action will depend on the specific method of estimation employed. Before state estimation, obvious bad data can be detected and eliminated. These measurements may include negative voltage magnitudes, measurements with several orders of magnitude larger or smaller than expected values, or large differences between incoming and leaving currents at a connection node within a substation. A deeper analysis of this algorithm is presented in Section 3.

2.1 Static state estimators

The standard practice in today's industry is for operators to work on an assumption of steady-state operations using an approach based on a static, or memoryless network model (see [2]) to estimate the state of the network. Most EMS operation centres in the world base the estimation on WLS (see [10]) owing to its simplicity and fast convergence properties.

2.1.1 Weighted least squares: As it was stated above, WLS is a static state estimator, which obtains results from the current measured data without considering recent historical information. Even though the accuracy of static estimators is within acceptable limits under fully observable conditions, they do not allow predicting the future operation point of the system. Let $\boldsymbol{x} \in \mathbb{R}^n$ is the state vector, whose n components represent the values to estimate and $\boldsymbol{z} \in \mathbb{R}^m$ is the observation vector, the problem is to estimate $\boldsymbol{x}$ from the measurement equation $\boldsymbol{z} = h(\boldsymbol{x}) + \boldsymbol{\varepsilon}$, where h represents the relation between state variables and measurements (through Ohm and Kirchoff laws) and $\boldsymbol{\varepsilon}$ is the measurement error vector whose distribution parameters are known, generally uncorrelated, of mean 0 and given standard deviation $(\sigma_i)_{i=1,\ \ldots,\ m}$. The WLS state estimation method requires solving the following optimisation problem

$$\min\ J(x) := \frac{1}{2} r^t \boldsymbol{W} r \tag{1}$$

$$r = \boldsymbol{z} - h(\boldsymbol{x}) \tag{2}$$

where $\boldsymbol{W}$ denotes a diagonal matrix whose components are $1/\sigma_{ii}^2$, being σ_{ii}^2 the variance of the ith measurement. The variable $\boldsymbol{r}$ is the residual between the actual measure $\boldsymbol{z}$ and the value computed by $h(\boldsymbol{x})$, where $\boldsymbol{x}$ is the state vector.

As the measurements can have important deviations from real values, it is important to include a bad data detection procedure in a state estimation method. There are several mathematical methods to detect presence of bad data [11, 12]. Considering its simplicity, in this work this task is performed by the χ^2-test [13, 14]. This method can only detect existence of bad data, and cannot identify which is/are the faulty measurement/s.

To perform the χ^2-test after the WLS computation, the measurement residuals are processed considering their statistical properties. Being the errors normally distributed with 0 mean and variance σ_i the weighted norm of the residual, $J(x)$, is distributed like χ^2 and for a confidence level α (usually 0.05) the state values are accepted if

$$J(x) > \chi^2_{m-n,\alpha} \tag{3}$$

where m–n denotes the degrees of freedom (number of measurements minus the number of state variables).

Whenever this condition is not fulfilled, it is assumed that there are wrong measurements and those are sought considering the ones with the largest normalised residual test (see [14, 15]). WLS is run again once all suspect measurements are eliminated.

The system is said unobservable when the observation equations are not enough to determine the system state. There have been several proposals to recover observability, for example, replacing erroneous data with historical data or considering other constraints involving the so called pseudo measurements [2]. This paper proposes using the predictions obtained by the UKF as is explained in Section 3.

2.2 Dynamic state estimators

Methods which use previous measurements in addition to the current measurements are referred to as dynamic state estimation, since they estimate the network state variables using models that include dynamic state variables for the network and its components.

2.2.1 Unscented Kalman filter: The Kalman filter allows estimating the state of a linear dynamic system with linear observation based on the observed measurements (see [16]). Regarding power system state estimation, the observation is not linear and even there is no equation for the dynamics. A practical way to obtain a dynamic equation for the evolution, is to assume a quasi-steady-state behaviour of the system, monitored in time steps of a few minutes. Following [6], a good approximation can be given by a linear discrete-time transition of states

$$x_{k+1} = \boldsymbol{F}_k \boldsymbol{x}_k + \boldsymbol{g}_k + q_k \tag{4}$$

where $\boldsymbol{x}_k$ is a vector of unknown state variables (voltages and angles in the bus), $\boldsymbol{F}_k$ and $\boldsymbol{g}_k$ compose the state transition matrix that define the evolution of the system. The vector $\boldsymbol{q}_k$ is the modelling error at time k. The values for F_k and g_k in (4) are obtained through a Holt method (see below).

The observation obtained through measurement of active and reactive power and voltage is represented by

$$y_k = h(\boldsymbol{x}_k) + r_k \tag{5}$$

where $\boldsymbol{y}_k$ is the observation at time k, r_k is the observation error and h is a function relating the state variables with the measured values. The explicit formula for h can be found in [2].

As noted before, (5) is not linear since it corresponds to the relation between active or reactive power with voltage and phase angle. This non-linearity precludes the utilisation of the standard Kalman filter, and the extended variant (extended Kalman filter – EKF) of the method should be used instead [17, 18]. In fact, the problem with non-linearity is that the covariance matrix of the observation is not easily deduced from the covariance of the state variable. In the linear case, if $y = H\boldsymbol{x}$ then $\mathrm{cov}(y) = H\mathrm{cov}(x)H^{\mathrm{T}}$. EKF performs a linearisation around the current point.

Another possibility is to employ the so called 'unscented Kalman filter'. Recently, in [6], it was demonstrated how UKF improves the results obtained by EKF. UKF is a combination of the Kalman filter and an unscented transformation [19]. In UKF, a set of points (called sigma points) is chosen in such a way that their mean and covariance match the current state statistics. The function h is applied to each sigma point, and the mean and covariance of the transformed sigma points are used to approximate the mean and covariance of the vector measurements $\boldsymbol{y}$. More precisely, UKF consists in:

- Compute sigma points

$$X_k = \left[\boldsymbol{x}_k \quad \boldsymbol{x}_k + \gamma\sqrt{P_k} \quad \boldsymbol{x}_k - \gamma\sqrt{P_k} \right] \tag{6}$$

- Prediction step

$$X_{k+1} = \boldsymbol{F}_k X_k + \boldsymbol{g}_k \tag{7}$$

$$x_k^- = \sum_{i=0}^{2L} W_i^{(m)} X_{k+1} \tag{8}$$

$$P_k^- = \sum_{i=0}^{2L} W_i^{(c)} \left[X_{k+1} - x_k^-\right]\left[X_{k+1} - x_k^-\right]^{\mathrm{T}} + Q \tag{9}$$

$$X_{k+1}^- = \left[x_k^- \quad x_k^- + \gamma\sqrt{P_k^-} \quad x_k^- - \gamma\sqrt{P_k^-} \right] \tag{10}$$

$$y_k = h\left(X_{k+1}^-\right) \tag{11}$$

$$y_k^- = \sum_{i=0}^{2L} W_i^{(m)} y_k \tag{12}$$

- Correction step

$$P_{y_k^- y_k^-} = \sum_{i=0}^{2L} W_i^{(c)} \left[y_k - y_k^-\right]\left[y_k - y_k^-\right]^{\mathrm{T}} + \boldsymbol{R} \tag{13}$$

$$P_{x_k^- y_k^-} = \sum_{i=0}^{2L} W_i^{(c)} \left[X_k^- - x_k^-\right]\left[y_k - y_k^-\right]^{\mathrm{T}} \tag{14}$$

$$K_k = P_{x_k^- y_k^-} P_{y_k^- y_k^-}^{-1} \tag{15}$$

$$x_{k+1} = x_k^- + K_k\left(y_k - y_k^-\right) \tag{16}$$

$$P_{k+1} = P_k^- - K_k P_{y_k^- y_k^-} K_k^{\mathrm{T}} \tag{17}$$

where L is the number of system state variables, $W_0^{(m)} = \lambda/(L+\lambda)$, $W_0^{(c)} = \lambda/(L+\lambda) + \left(1 - \alpha^2 + \beta\right)$, $W_i^{(m)} = W_i^{(c)} = 1/\{2(L+\lambda)\}$, $\lambda = \alpha^2(L+\kappa) - L$ and $\gamma = \sqrt{(L+\lambda)}$. The parameter α determines the spread of the sigma points around $\boldsymbol{x}_k$ and usually takes a value in $(10^{-4}, 1)$. The parameter β is used to incorporate the knowledge of the distribution of $\boldsymbol{x}_k$, κ is usually 0. All these parameters were obtained from [20]. In turn, $\boldsymbol{x}_k$ is defined as the vector of state variables with voltage magnitudes and angles in different buses, $\boldsymbol{Q}$ is the variance associated with the system error and $\boldsymbol{R}$ is the measurement error covariance matrix.

2.2.2 Holt method: In the case of power systems, the matrix $\boldsymbol{F}_k$ and $\boldsymbol{g}_k$ in the dynamic model (7), are not known because they represent structural changes and variable data input. Considering this fact, Valverde and Terzija [6] proposed that $\boldsymbol{F}_k$ and $\boldsymbol{g}_k$ should be predicted by the Holt method [21]. This method is a linear exponential smoothing technique which is defined as

$$\boldsymbol{F}_k = \alpha_k\left(1 + \beta_k\right)I \tag{18}$$

$$\boldsymbol{g}_k = \left(1 + \beta_k\right)\left(1 - \alpha_k\right)x_k^- - \beta_k a_{k-1} + \left(1 - \beta_k\right)b_{k-1} \tag{19}$$

where $\boldsymbol{I}$ is the identity matrix, α_k and β_k are parameters with values between 0 and 1, $\boldsymbol{x}_k^-$ is the prediction of the state vector at the previous time and the vectors $\boldsymbol{a}_k$ and $\boldsymbol{b}_k$ are obtained at time k by

$$\boldsymbol{a}_k = \alpha_k x_k + \left(1 - \alpha_k\right)\boldsymbol{x}_k^- \tag{20}$$

$$\boldsymbol{b}_k = \beta_k\left(a_k - a_{k-1}\right) + \left(1 - \beta_k\right)b_{k-1} \tag{21}$$

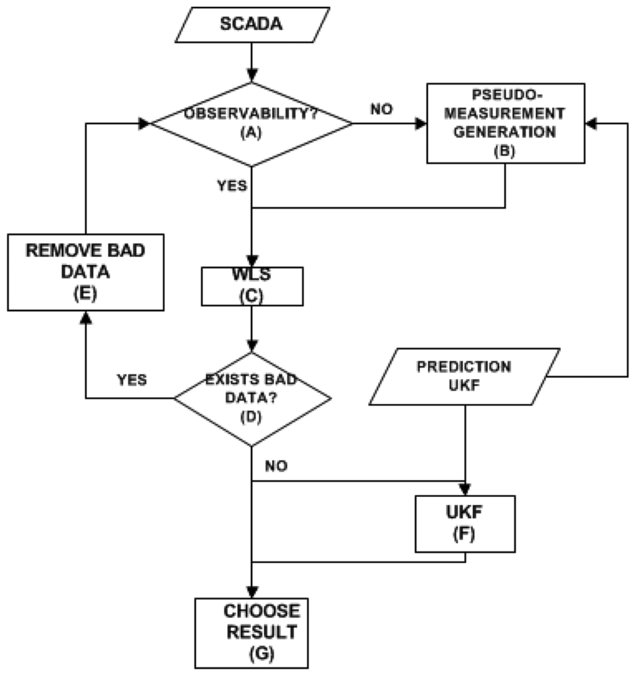

Fig. 1 *Hybrid method*

Therefore this method adapts the parameters $\boldsymbol{F}_k$ and $\boldsymbol{g}_k$ of the previous linear approximation, with each new set of measurements.

2.2.3 Bad data and anomalies: The UKF method proposed in [6] is very sensitive to the presence of bad data in the measurements and to major changes in the network (connection or disconnection of power generators and loads). These changes in the network may cause differences between the estimated result and the actual state of the system. In the last case, the results are stated as anomalous.

Distinguishing between bad data and anomalies (caused by sudden state changes) is necessary to define the action which minimises the effect of the anomaly, once detected. In [6], a method which analyses the asymmetry of the distribution is performed, and in the case of anomalies detection it is not possible

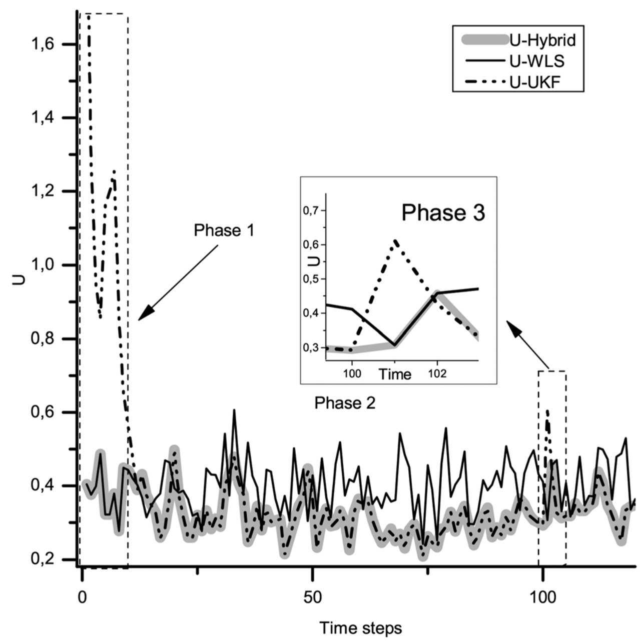

Fig. 2 *UK values for 6-bus test system*

Table 1 *U and ε average value for each 120 times*

Test case	WLS		UKF		Hybrid	
	ε	U	ε	U	ε	U
6-bus	0.0123	0.4012	0.0233	0.3841	0.0121	0.3238
14-bus (A)	0.0374	0.5646	0.0500	0.4978	0.0363	0.4242
14-bus (B)	0.0344	0.5689	0.0476	0.5381	0.0339	0.4608

to overcome. An example of this situation can be seen in Fig. 2 when at time 101 a power generator is removed.

3 Hybrid method

The method presented in this work combines the better of both, dynamic and static approaches. The objectives of this combination are to obtain the most probable state of the system, detect bad data and overcome the anomalies that appear in the UKF method and the lack of observability that WLS could present. The anomalies generated by the UKF could take place when it faces abrupt variations of the system or in the initialisation phase [6]. To perform this task a new decision index is defined to allow the algorithm to choose in real time and for each iteration between a WLS or UKF.

The hybrid method flowchart is presented in Fig. 1.

When the algorithm starts and before running the WLS, it is verified that the data read from the SCADA is enough for observability (A). Solving the observability problem is equivalent to test whether the set of equations solved for the state variables have a non-trivial solution or not. The power system is observed if the Jacobian matrix of the system (***H***) is full rank [22].

To achieve observability, a fast method for multiple measurement placement presented in [23] is used. This method performs a sequence of operations discovering in which position of the system a minimal set of pseudo measurements should be added. After generating this pseudo measurement set and placed it in the positions previously discovered the system will be rendered observable.

Pseudo measurements are created applying active and reactive power equations [24] based on system state variables (phase angle and voltage) computed in the prediction step of the UKF method (see Section 2.2.1). The process of pseudo measurements creation and placement is represented in Fig. 1 by box (B).

After achieving observability, the WLS method (C) detailed in Section 2.1.1 is executed. When WLS is finished, gross errors in the measurements must be detected and removed, in such case the observability test and the WLS method must be re-run. To detect gross errors (D) the χ^2-test mentioned in Section 2.1.1 is used and then, errors are removed (E) according to the largest normalised residual test (see [15]).

Considering that the bad data test was successful, the WLS results are fed into the UKF method (F) detailed in Section 2.1.1. At the end of the UKF correction step, there are two possible states: one is the output of WLS and the other is the output of UKF (the WLS corrected by the UKF method).

To determine which one of those values is the best approximation is not straightforward because the true state value is not known. Generally UKF is expected to improve WLS results except when major changes affect the smooth evolution of the system. When there is a change, the accumulated dynamic history of the system degrades considerably the result of UKF, even if it is fed with WLS results. Hence, it is crucial to detect when that occurs in order to select the WLS solution as the best one. To detect such changes, two tests are performed: the first one verifies that the UKF result passes the WLS-χ^2 test. The second one is based on the following index

$$\Delta_k = \left(\delta_k - \delta_{k-1}\right)^2 \tag{22}$$

where

$$\delta_k = \left(\left[y_k^- - h\left(x_{k+1}\right)\right]^t W\left[y_k^- - h\left(x_{k+1}\right)\right]\right)/m \tag{23}$$

The value δ_k represents the difference between the predicted value y_k^- and the corrected value $h(x_{k+1})$ generated by UKF, taking into account the weight of each measurement represented by W (obtained as in 1) and divided by the measurement number m. Δ_k is the variation of δ_k with respect to the previous time.

When there is no important change in the state, it is observed that the value of δ_k stays close to 0, but when a change suddenly appears this value increases. As it is not known the possible range of the changes, Δ_k, a squared successive difference of δ_k, is considered. Obviously a change also affects the value of Δ_k and to decide when an important change occurred, μ_k, the average of the previous (successful) values of Δ_k is used.

The occurrence of an important change is declared when the WLS-χ^2 test and the following test are passed

$$\Delta_k > \gamma\mu_k \tag{24}$$

where γ is a constant greater than 1 that reflects the confidence given to the UKF method. The value of μ_k is computed by the average of the previous Δ_k, that have met the above two conditions: pass the WLS-χ^2 test and (24).

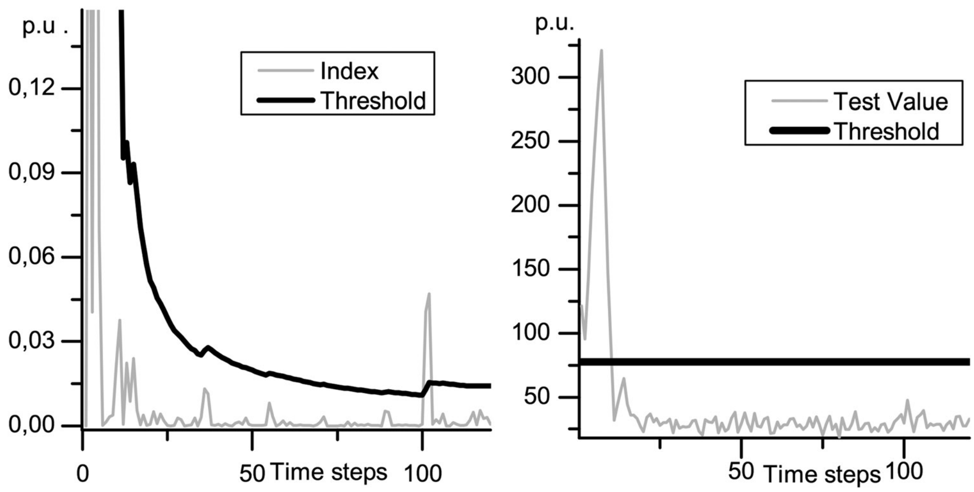

Fig. 3 *Graph of the evolution of the indices Δ_k (left) and χ^2 (right) for the 6-bus test system*

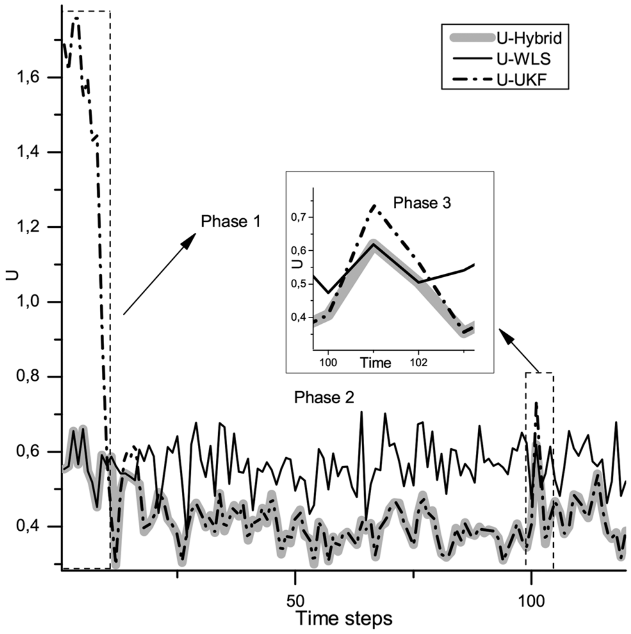

Fig. 4 *U_k values for 14-bus (A) test system*

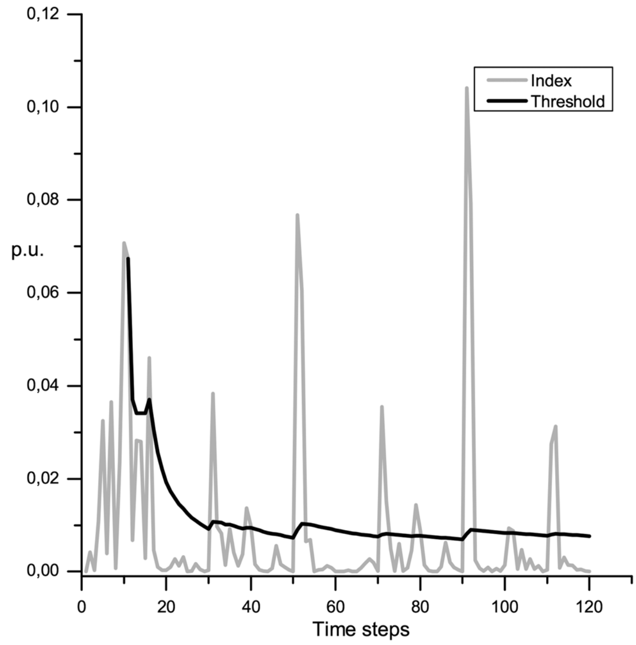

Fig. 6 *U_k values for 14-bus (B) test system*

4 Results

To analyse the performance of the proposed hybrid method, two academic test cases of different size and a real case were considered. The academic cases are the 6-buses test system presented in the book [24] and the IEEE 14-buses test system [25]. The IEEE 14-buses test case was perturbed in two different ways creating two test cases called IEEE 14-buses (A) and IEEE 14-buses (B), respectively. The real case is based on a section of the Argentinean National Interconnected System. A detailed description of the real case is presented in Section 4.1.

In the academic cases, the network simulation was performed using the PowerWorld Simulator [26]. Data exported from this commercial software are voltage magnitudes, angles and power injections for each bus, and power flows for each line. Voltage magnitudes, power injections and power flows are perturbed with an additive Gaussian error to use them as measurements.

To test the designed index, the testing scenario was complexified taking out of service some components of the system. In the 6-bus case, the generator connected to bus 3 is removed at time 101. In the 14-bus (A) case the generator connected to bus 2 is disconnected at the same time, while, in case (B), it is disconnected and connected at several times (31, 51, 71, 91 and 111). Considering the 6-bus and IEEE 14-bus (A) test cases, for the first 100 time steps an additive Gaussian error (with $\sigma^2 = 0.001$ for voltage measurements and $\sigma^2 = 0.02$ for power measurements) was added to each measurement. In 101 time step, a generator was removed and in the following 20 time steps the measures were calculated as was previously mentioned. The parameter values considered in all the test cases were: for the Holt method $\alpha_k = 0.7$, $\beta_k = 0.8$ and a confidence index $\gamma = 3$.

The obtained results were analysed using performance indices taken from [27–29], the first one is the mean absolute estimation error at each time, computed by

$$\varepsilon_k = 1/n \sum_{i=1}^{n} \left| x_{k,i} - x_{k,i}^t \right| \tag{25}$$

where $\boldsymbol{x}_{k,i}$ is the value estimated and $\boldsymbol{x}_{k,i}^t$ is the true value of state variables. The second one is a relative estimation error based on

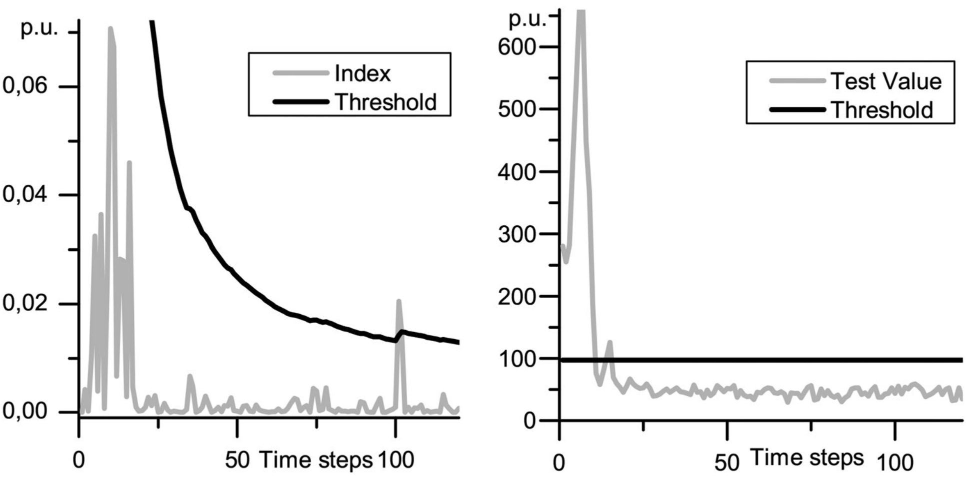

Fig. 5 *Graph of the evolution of the indices Δδ (left) and χ^2 (right) for the 14-bus (A) test system*

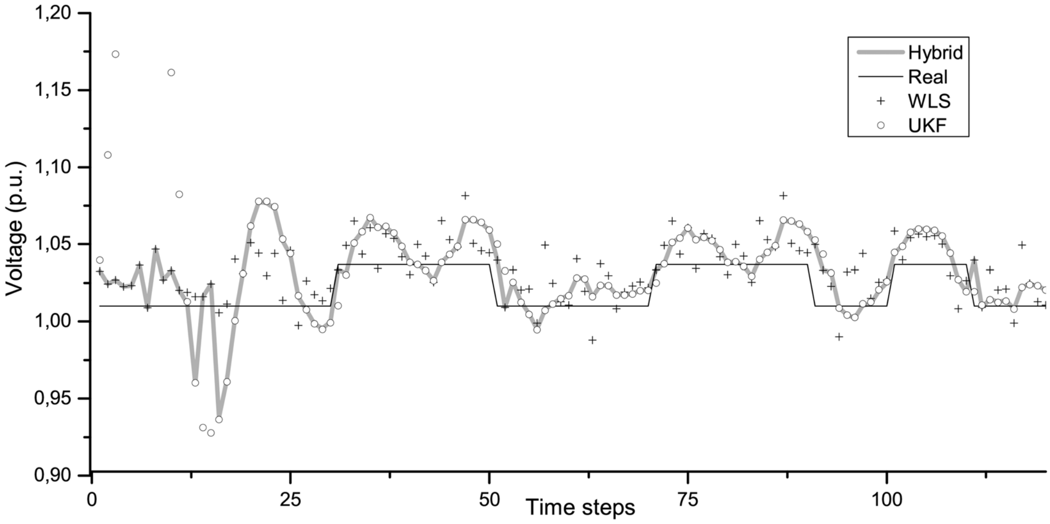

Fig. 7 *Voltage magnitudes for bus 3 of the 14-bus (B) test system*

the current and the estimated observations, computed by

$$U_k = \frac{\sum_{i=1}^{n} \left| h\left(x_{k,i}\right) - y_{k,i}^t \right|}{\sum_{i=1}^{n} \left| y_{k,i} - y_{k,i}^t \right|} \tag{26}$$

this last index compares the estimated measured values $h(x_{k,i})$ against true values $\boldsymbol{y}_{k,i}^t$ and measured values $\boldsymbol{y}_{k,i}$.

The average value for ε_k and U_k are also introduced as $\boldsymbol{\varepsilon}$ and U, respectively, and these values are shown in the following table for 120 time steps.

Table 1 shows that the hybrid method improves the results obtained for UKF and WLS methods. Indeed, the values of $\boldsymbol{\varepsilon}$ and U are lowest.

In Fig. 2, a comparison of the U_k values obtained with UKF, WLS and the hybrid method for the 6-buses test system is presented. To show the hybrid algorithm behaviour, three different phases were defined in this picture. In each of those phases, the hybrid method considers different criteria to choose between WLS results (see Fig. 1 – box C) and WLS + UKF results (see Fig. 1 – box F).

In phase 1, the algorithm chooses the WLS result because, as it is shown in Fig. 3, the UKF-χ^2 test value is above its threshold. This situation occurs because the Holt method (U-UKF) does not have enough previous data to predict the dynamic behaviour of the model. Therefore, the UKF method obtains worse results than the WLS, and consequently the hybrid method uses the WLS results generated during these steps.

As is shown in Fig. 2, in phase 2 the hybrid method chooses UKF result, because the UKF-χ^2 test value (see Fig. 3) and the index Δ_k 22 values (see Fig. 3) are below their threshold. This phase represents the steps when the Holt method has accumulated sufficient historical data and the UKF begins to perform better than the WLS.

Phase 3 represents the time when the generator was taken out of service (just after step 101). This system change makes the UKF method give an anomalous result, which is detected by the hybrid method that decides to select the result of the WLS. In order to detect this anomalous behaviour the proposed method analyses the value of the index Δ_k presented in Fig. 3. As it was shown, during this period (time step 101) $\Delta_k > 3\mu$, that is, it surpasses for more than $\gamma = 3$ times the threshold μ.

Fig. 4 shows the results of the application of the hybrid method for the 14-bus (A) test system. The index used by the algorithm to select among the UKF and WLS results is shown in Fig. 5.

Fig. 6 depicts the index and threshold values for the 14-bus (B) test case considering that the power generator was continuously connected and disconnected from the corresponding bus. As it was shown the algorithm continuously detect the UKF anomalies and switch to WLS results whenever is necessary.

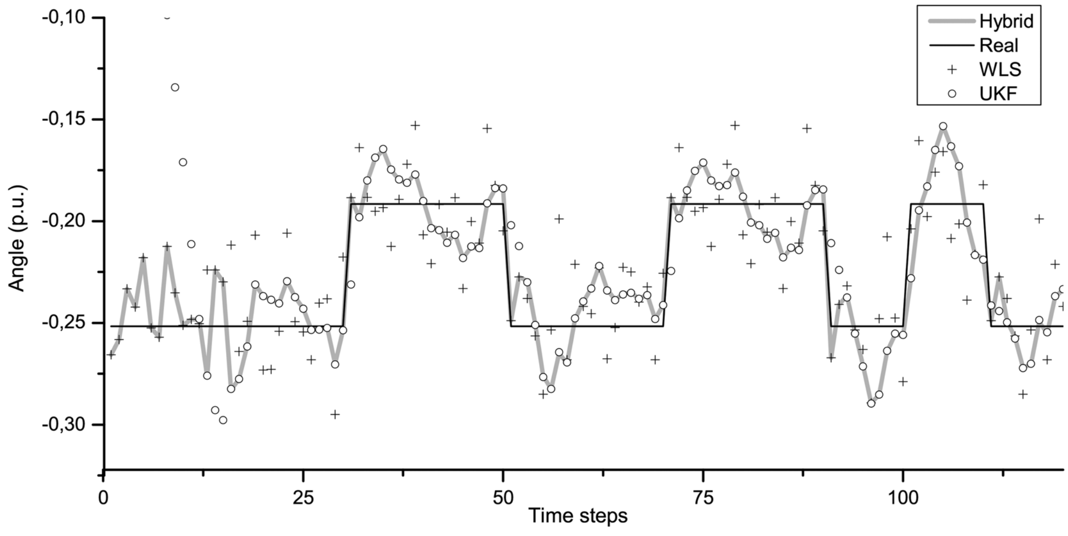

Fig. 8 *Angle values for bus 6 of the 14-bus (B) test system*

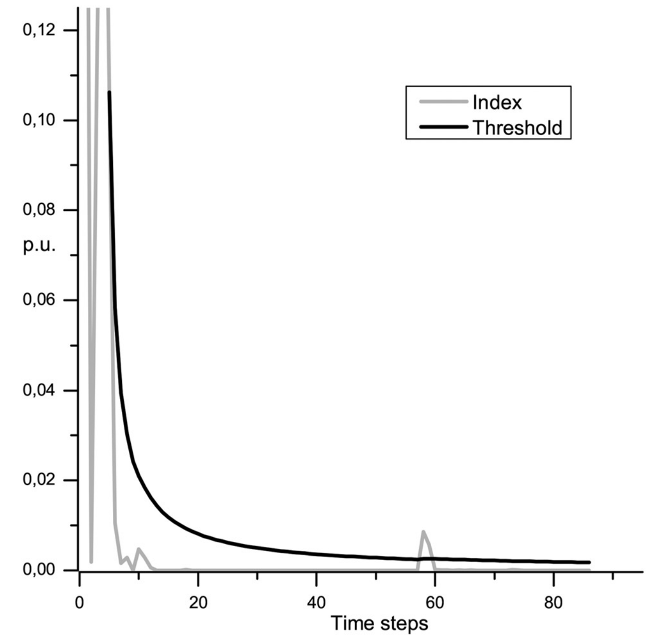

Fig. 9 *Index against threshold comparison for the real case*

Figs. 7 and 8 show the results of the three methods for state variables estimation and the real value. For the sake of simplicity, two state variables were selected, the voltage magnitude for bus 3 in Fig. 7 and the angle for bus 6 in Fig. 8. As it is depicted in both figures, during 120 time steps the hybrid method outperforms WLS and UKF.

4.1 Real case

The proposed algorithm was also applied to a section of the Argentinean National Interconnected System. This network is located in the Upper Valley zone and includes the provinces of Neuquén and Río Negro. This medium-sized power network has thermal and hydraulic generation units. The network modelled considers the 132 kV voltage level areas and lower voltage buses and lines, that reach the generators and demands. This system has 87 buses, 23 thermal and 6 hydraulic units. The one-line diagram is presented in [30].

The SCADA implemented in this system reports 52 voltage measurements and 64 power flow measurements (32 active and 32 reactive). σ^2 was defined as 0.0004 for voltage measurements and 0.008 for power measurements. Also a confidence index $\gamma = 3$ was considered for this example. The method was applied during 24 h considering 15 min intervals. In Fig. 9, a comparison between Δ_k index 22 and the threshold μ is presented.

It is noted that in the first 15 observed steps (3 h and 45 min) the calculated index Δ_k is larger than $\gamma\mu$. Considering this fact, the proposed method selects result values from the WLS method during these steps. Since the fourth hour (step 16) the index Δ_k reduces its values to acceptable levels Fig. 9 shows a detailed view of the index Δ_k where the anomalous behaviour because of the start of a generator (close to step 75) should be observed.

5 Conclusions

In the power system industry, the problem of state estimation is solved using WLS and more recently UKF method. When there are slight changes in the system, UKF improves the results of WLS and provides a useful estimate even in the case of unobservability. However, when load/generation levels change abruptly, UKF needs some time to give a good estimation.

The hybrid method proposed here is able to exploit the advantages of both methods, maintaining the robustness of WLS and incorporates state predictability through UKF. An index and a threshold that evolves dynamically allows the method to decide for the best result among UKF and WLS methods.

Specifically, both methods are computed at each time and the best estimation is chosen. When WLS lacks observability, the UKF solution can be used to give the missing pseudo-measurements. When UKF is not accurate because of large variations of the system, the WLS solution is provided. For the practical implementation of the hybrid method, the algorithm should decide, in an unsupervised way, which method is the best solution without the knowledge of the true state. For this purpose, an index and a threshold that evolve dynamically, are defined to choose between WLS and UKF in real time. Numerical experiments where the real state of the system is known in advance show that this hybrid methodology outperforms both previously mentioned methods.

Tests were performed on three academic systems where the knowledge of the true value of the state variables allows to compute the real error and to analyse the performance and behaviour of the method. The method was also applied to a real case which corresponds to a section of the Argentinean National Interconnected System to show its performance in a medium-sized real network. These numerical results confirm the improvement of the proposed method over the use of UKF or WLS alone.

6 References

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