

SELF-INTERACTING HOLOGRAPHIC DARK ENERGY

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We investigate a spatially flat Friedmann–Robertson–Walker (FRW) universe where dark matter exchanges energy with a self-interacting holographic dark energy (SIHDE). Using the χ^2 -statistical method on the Hubble function, we obtain a critical redshift that seems to be consistent with both BAO and CMB data. We calculate the theoretical distance modulus for confronting with the observational data of SNe Ia for small redshift $z \leq 0.1$ and large redshift $0.1 \leq z \leq 1.5$. The model gets accelerated faster than the Λ CDM one and it can be a good candidate to alleviate the coincidence problem. We also examine the age crisis at high redshift associated with the old quasar APM 08279+5255.

Keywords: Interaction; holographic dark energy; dark matter.

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1. Introduction

As it is well known our universe is currently undergoing an accelerated expansion phase driven by a mysterious fuel called dark energy which exerts a negative pressure tending to drive clusters of galaxies apart. The latter fact has been corroborated by many different probes, for example the observation of type Ia supernovae,^{1–4} the data of the large scale structure from SDSS,^{5–7} and measurements of the cosmic microwave background anisotropy.^{8–10} The simplest candidate for the dark energy component is a positive cosmological constant Λ .^{11–13} Although the prediction of the cold dark matter plus cosmological constant (Λ CDM) model is mostly consistent with observational data, the cosmological constant proposal suffers from at least two puzzles.^{14–20} The first issue is known as the fine-tuning problem, that is, the theoretical prediction of the cosmological constant that is obtained as the expectation value of quantum fields differs from its cosmic observed value by 120

orders of magnitude. The measured cosmological constant in our universe is tiny but not zero, and if it were much larger, galaxies could not have formed.¹⁷ The second point of debate concerns the cosmic coincidence problem: why we observe that the fractional densities of dark matter and cosmological constant are about the same order of magnitude today.

The conflict between theoretical physics and the observational data can be alleviated by working within the framework of dynamical dark energy.^{21–23} This aforesaid idea has led to a wide variety of dark energy models such as quintessence,^{24–31} k -essence,^{32–35} quintom,^{36–45} and holography dark energy (HDE).^{46–48} In particular, the latter model was discussed extensively during the last five years.^{49–59,61–63}

The HDE model has its physical origin in the holographic principle as well as some features related with string and quantum gravity theories.^{64–68} The underlying postulate can be stated as follows⁶⁸: *the number of degrees of freedom in a bounded system should be finite and is related to the area of its boundary*. This principle also suggests that the ultraviolet (UV) cutoff scale of a system is connected to its infrared (IR) cutoff scale. In the case of a system with size L (IR length) and ultraviolet cutoff Λ without decaying into a black hole, it is required that the total energy in the region of size L should not exceed the mass of the black hole with the same size, thus, $L^3 \rho_\Lambda \leq LM_P^2$ being M_P the reduced Planck mass whereas the UV cutoff scale is defined as $\Lambda = \rho_\Lambda^{1/4}$.⁶⁷ The largest L allowed is the one which saturates the above inequality and leads to an holographic dark energy given by $\rho_\Lambda = 3c^2 M_P^2 L^{-2}$, where c is a numerical factor. Hence, this principle connects the dark energy based on the quantum zero-point energy density caused by a short distance cutoff Λ with an IR cutoff⁶⁸ that is usually taken as the large scale of the universe, for instance, Hubble horizon,^{46,47} particle horizon,⁴⁷ event horizon⁴⁷ or generalized IR cutoff.^{69–80}

A natural arena for investigating the coincidence problem is to consider a phenomenological approach where dark matter interacts with dark energy.^{81–91} From the observational point of view, an interacting dark sector is completely compatible with the current observations of standard candles and WAMP data.^{92,93} In the present paper, we show how it is possible to get a physically viable model based on a new holographic dark energy density that interacts with dark matter. More precisely, it turns to be that dark matter ρ_c feels the presence of dark energy ρ_x through the gravitational expansion of the universe plus an exchange of energy between themselves. Based on the holographic principle, we propose a dark energy model where the quantum zero point energy density ρ_Λ is equal to the dark energy density ρ_x being L an IR cutoff that will be related with a cosmological length. As a result of this, we take $\rho_x = \rho_\Lambda = 3c^2 M_P^2 L^{-2} = 3c^2 M_P^2 f(\rho, p)$ where $f(\rho, p)$ is an arbitrary positive function. This gives rise to self-interacting holographic dark energy models (SIHDE), where $\rho_x \propto f(\rho, p)$, indicating that there is a coupling to the dark matter component. The new holographic dark energy model assumes a generalized IR cutoff L that depends on the total dark sector density $\rho = \rho_c + \rho_x$ and the pressure of the mixture $p = p_c + p_x$.

Several works have been devoted to obtain cosmological constraints in the case of Ricci scalar cutoff^{73–75} or generalized versions of this one.^{76,79,80} For example, the joint analysis of the 307 union sample of SNIa, together with CMB shift parameter given by WMAP5, and the BAO measurement from SDSS, suggest that the holographic Ricci dark energy exhibits a quintom-like phase, so it leads to a new model consistent with the current observation because the equation of state for the Ricci dark energy can cross the phantom line.⁷³ Using the general framework presented in Ref. 94, which suitably describes and unifies the dark sector with an exchange of energy, we will investigate a cosmological scenario where dark matter interacts with SIHDE. After that, we will confront our results with the current observational data and compare with the Λ CDM model. In the last section, we summarize our main results and conclude.

2. Evolution of the Dark Components

We consider a flat FRW universe filled with two components, dark matter and SIHDE with energy densities ρ_c and ρ_x , respectively. We also assume that the equations of states are $\omega_c = \frac{p_c}{\rho_c}$ and $\omega_x = \frac{p_x}{\rho_x}$, whereas the Einstein equations read

$$3H^2 = \rho_c + \rho_x, \quad (1)$$

$$\rho'_c + \rho'_x + (\omega_c + 1)\rho_c + (\omega_x + 1)\rho_x = 0. \quad (2)$$

Here $H = \frac{\dot{a}}{a}$ stands for the Hubble expansion rate, a is the scale factor and $'$ means derivative with respect to the variable $\eta = \ln\left(\frac{a}{a_0}\right)^3$, a_0 being the scale factor today. From Eqs. (1) and (2) the total pressure becomes, $p = -\rho' - \rho$, hence the SIHDE, $\rho_x = f(\rho, p)$ turns $\rho_x = f(\rho, \rho')$. As already mentioned in the introduction, ρ_x is related with the UV cutoff, while $L = f^{-1/2}$ is related to the IR cutoff. We now consider the simplest case of a linear SIHDE,

$$\rho_x = \frac{1}{\alpha - \beta}(\rho' + \alpha\rho), \quad (3)$$

where α and β are both free constants. Rewriting Eqs. (1) and (3) as

$$\rho = \rho_c + \rho_x, \quad (4)$$

$$\rho' = -\alpha\rho_c - \beta\rho_x \quad (5)$$

and comparing Eq. (2) with Eq. (5), we obtain a compatibility relation

$$\omega_x = (\alpha - \omega_c - 1)r + \beta - 1, \quad (6)$$

between the equation of state of both components and its ratio $r = \frac{\rho_c}{\rho_x}$. This relation allows us to use Eq. (5) with constant coefficients α and β instead of Eq. (2) with nonconstant coefficients. After solving the linear system of equations (4) and (5), we obtain the energy density of each dark component as functions of ρ and ρ'

$$\rho_c = -\frac{\beta\rho + \rho'}{\Delta}, \quad \rho_x = \frac{\alpha\rho + \rho'}{\Delta}, \quad (7)$$

where $\Delta = \alpha - \beta$ is the determinant of the linear system of equations. At this point, we introduce the interaction term, $3HQ_l$, between the dark components by splitting Eq. (5) in the following way

$$\rho'_c + \alpha\rho_c = -Q_l, \quad \rho'_x + \beta\rho_x = Q_l. \quad (8)$$

After differentiating the first equation of (7) and combining with Eq. (8), we find a second-order differential equation for the total energy density:

$$\rho'' + (\alpha + \beta)\rho' + \alpha\beta\rho = Q_l\Delta. \quad (9)$$

Once the interaction term Q_l is selected and replaced in (9), the total energy density ρ of the dark sector is determined by solving the source equation (9). Having obtained ρ , we are in a position to get ρ_c and ρ_x from Eq. (7), calculate the scale factor by integrating the Friedmann equation (1), and find the equation of state of the mixture from the relation $p = -\rho' - \rho$. In the case of pressureless dark matter ($\omega_c = 0$), the equation of state of dark energy (6) is given by

$$\omega_x = (\alpha - 1)r + (\beta - 1), \quad (10)$$

so it becomes linear in r .

3. Interacting Holographic Model

In the present section, we are going to examine a proposal where the interaction term Q_l is a general linear combination of ρ_c , ρ_x , ρ and ρ' (Ref. 94)

$$Q_l = c_1 \frac{(\omega_s + 1 - \alpha)(\omega_s + 1 - \beta)}{\Delta} \rho + c_2(\omega_s + 1 - \alpha)\rho_c - c_3(\omega_s + 1 - \beta)\rho_x - c_4 \frac{(\omega_s + 1 - \alpha)(\omega_s + 1 - \beta)}{(\omega_s + 1)\Delta} \rho'. \quad (11)$$

Here ω_s is a free constant parameter and the coefficients c_i fulfill the following condition $c_1 + c_2 + c_3 + c_4 = 1$ in order to assure the existence of stable power law solution $a = t^{2/3(\omega_s+1)}$.⁹⁴ The case with $c_2 = c_3 = c_4 = 0$ was examined in Refs. 57, 85 and 92. The case $c_1 = c_2 = c_4 = 0$ was analyzed in Refs. 95–101. The linear interaction $Q_l \propto \rho'$, $c_1 = c_2 = c_3 = 0$, was introduced in Ref. 94, and now it is considered here for its study, and later considered in Ref. 102.

Using Eqs. (7) we can rewrite the interaction (11) as a linear combination of ρ and ρ' only,

$$Q_l = \frac{u\rho + (\omega_s + 1)^{-1}[u - (\omega_s - \alpha + 1)(\omega_s - \beta + 1)]\rho'}{\Delta}, \quad (12)$$

where the parameter u is defined in terms of ω_s , α and β as follows:

$$u = c_1(\omega_s - \alpha + 1)(\omega_s - \beta + 1) - c_2\beta(\omega_s - \alpha + 1) - c_3\alpha(\omega_s - \beta + 1). \quad (13)$$

Replacing the interaction term (12) into the source equation (9), we obtain a linear differential equation

$$\rho'' + (\omega_s + 1)^{-1}[(\omega_s + 1)^2 + \alpha\beta - u]\rho' + (\alpha\beta - u)\rho = 0, \quad (14)$$

whose characteristic polynomial roots are

$$\gamma^- = \omega_s + 1, \quad \gamma^+ = \frac{\beta\alpha - u}{\omega_s + 1}. \quad (15)$$

We restrict our analysis to the case with positive roots in order to avoid phantom dark energy, then we choose $0 < \omega_s + 1 < \gamma^+$. Solving Eq. (14), we obtain the total energy density in terms of the scale factor and consequently the effective pressure:

$$\rho = b_1 a^{-3\gamma^+} + b_2 a^{-3(\omega_s+1)}, \quad (16)$$

$$p = (\gamma^+ - 1)b_1 a^{-3\gamma^+} + \omega_s b_2 a^{-3(\omega_s+1)}. \quad (17)$$

From (7) and (16), we get the dark matter and dark energy densities as a function of the scale factor

$$\rho_c = \frac{(\gamma^+ - \beta)b_1 a^{-3\gamma^+} + (\omega_s - \beta + 1)b_2 a^{-3(\omega_s+1)}}{\Delta}, \quad (18)$$

$$\rho_x = \frac{(\alpha - \gamma^+)b_1 a^{-3\gamma^+} + (\alpha - \omega_s - 1)b_2 a^{-3(\omega_s+1)}}{\Delta}. \quad (19)$$

At very early times, dark matter and dark energy densities (18) and (19) behave as $a^{-3\gamma^+}$ with a constant ratio $r_e \simeq (\gamma^+ - \beta)/(\alpha - \gamma^+)$ while $p \simeq (\gamma^+ - 1)b_1 a^{-3\gamma^+}$. However, at late times the effective fluid, dark matter, and dark energy have the same behavior with the scale factor, namely, $\rho \simeq \rho_c \simeq \rho_x \simeq b_1 a^{-3(\omega_s+1)}$, leading to $r_l \simeq (\omega_s - \beta + 1)/(\alpha - 1 - \omega_s)$ and $p \simeq a^{-3(\omega_s+1)}$. The aforesaid facts indicate that the interaction term Q_l is a good candidate to represent adequately an interacting dark sector because the ratio of dark matter–dark energy r has enough parameters to adjust the cosmological observations and it also alleviates the so called coincidence problem. It is important to emphasize that the mutual exchange of energy between the dark components makes that their usual behavior with scale factor change radically; we distinguish in the dark densities two terms $a^{-3\gamma^+}$ and $a^{-3(\omega_s+1)}$. In fact, we will consider the case with $\gamma^+ = 1$ in order to get pressureless dark matter at early times.

4. Observational Data Analysis

In this section, we will perform some qualitative cosmological constraints for the SIHDE model interacting with dark matter through the interaction term Q_l proposed in last section. In order to do that, we start by constraining the parameter space with the Hubble data $H(z)$,^{107,109} and SNe Ia observations.¹⁰⁸ The $H(z)$ test was probably first used to constrain cosmological parameters in Ref. 111 and then in a large number of papers.^{79,80,112–123} The statistical method requires the

Table 1. Hubble data $H_{\text{obs}}(z_i)$ vs. redshift z_i .

z	$H(z)$ km s ⁻¹ Mpc ⁻¹	1 σ uncertainty	Reference
0.000	73.8	± 2.4	108
0.090	69	± 12	109
0.170	83	± 8	109
0.179	75	± 4	110
0.199	75	± 5	110
0.270	77	± 14	109
0.352	83	± 14	110
0.400	95	± 17	109
0.480	97	± 62	107
0.593	104	± 13	110
0.680	92	± 8	110
0.781	105	± 12	110
0.875	125	± 17	110
0.880	90	± 40	107
1.037	154	± 20	110
1.300	168	± 17	109
1.430	177	± 18	109
1.530	140	± 14	109
1.750	202	± 40	109

compilation of the observed value H_{obs} (Refs. 107 and 109) and the best value for the present time $z = 0$ taken from Ref. 108. Table 1 shows H_{obs} at different redshift with its corresponding 1σ uncertainty and the reference where this value was reported.

From Eqs. (1), (18) and (19), we can write the Hubble function in terms of the effective equation of state $\omega = \omega_x \Omega_x = \alpha \Omega_c + \beta \Omega_x = -2\dot{H}/3H^2$ as follows

$$H(z) = H_0 \left[\frac{(\omega_s - \omega_0)(1+z)^3 + \omega_0(1+z)^{3(\omega_s+1)}}{\omega_s} \right]^{\frac{1}{2}}, \quad (20)$$

where $\omega_0 = \alpha \Omega_{c0} + \beta \Omega_{x0} - 1$, $\Omega_{x0} = \rho_{x0}/3H_0^2$, $\Omega_{c0} = \rho_{c0}/3H_0^2$ are their present values whereas the flatness condition today reads $\Omega_{c0} + \Omega_{x0} = 1$. Taking into account the transition point z_{acc} , i.e. the moment where the universe begins to accelerate or where the deceleration parameter $q = -\ddot{a}/aH^2$ vanishes, Eq. (20) depends on $(H_0, z_{\text{acc}}, \omega_s)$ parameters:

$$H(z) = \frac{H_0(1+z_{\text{acc}})^{\frac{3}{2}}}{\sqrt{-3\omega_s - 1 + (1+z_{\text{acc}})^{-3\omega_s}}} \times \left[\frac{(1+z)^{3(\omega_s+1)}}{(1+z_{\text{acc}})^{3(\omega_s+1)}} - (1+3\omega_s) \frac{(1+z)^3}{(1+z_{\text{acc}})^3} \right]^{\frac{1}{2}}. \quad (21)$$

From Eq. (21), we see that the model has only three independent parameters $(H_0, z_{\text{acc}}, \omega_s)$ in order to be completely specified. The remaining parameters α ,

β , Ω_{c0} or Ω_{x0} are included in the transition point z_{acc} through the equation of state ω_0 . We now proceed in the following way: we perform a statistical analysis on the $(H_0, z_{\text{acc}}, \omega_s)$ parameters, confronting their best fit values with the recent available data and then, we will perform the same Hubble test using the expression (20), to obtain constraints on the others parameters, α , β and Ω_{c0} . The second approach will give us the most favored SIHDE for the Hubble's data.

The probability distribution for the θ -parameters is $P(\theta) = \mathcal{N}e^{-\chi^2(\theta)/2}$ (see e.g., Ref. 103) being \mathcal{N} a normalization constant. The parameters of the model are estimated by minimizing the χ^2 function of the Hubble data which is constructed as

$$\chi^2(\theta) = \sum_{i=1}^{N=19} \frac{[H(\theta; z_i) - H_{\text{obs}}(z_i)]^2}{\sigma^2(z_i)}, \quad (22)$$

where θ stands for the cosmological parameters, $H_{\text{obs}}(z_k)$ is the observational $H(z)$ data at the redshift z_k , $\sigma(z_k)$ is the corresponding 1σ uncertainty, and the summation is over the 19 observational $H(z)$ data. The Hubble function is not integrated over and it is directly related with the properties of the dark energy, since its value comes from the cosmological observations. Using the absolute ages of passively evolving galaxies observed at different redshifts, one obtains the differential ages $\frac{dz}{dt}$ and the function $H(z)$ can be measured through the relation $H(z) = -(1+z)^{-1}dz/dt$. The χ^2 function reaches its minimum value at the best fit value θ_c and the fit is good when $\chi_{\text{min}}^2(\theta_c)/(N-n) \leq 1$, where n is the number of parameters¹⁰³ and N counts the observational data points that in our case correspond to 19 points.

In the first approach, the parameters of the model are $\theta = (H_0, z_{\text{acc}}, \omega_s)$ therefore the 68.3% (1σ) or 95.4% (2σ) confidence levels (CL) made with the random data fulfill the inequalities $\chi^2(\theta) - \chi_{\text{min}}^2(\theta_c) \leq 3.53$ or $\chi^2(\theta) - \chi_{\text{min}}^2(\theta_c) \leq 8.02$, respectively. Figure 1 shows the CL associated with 1σ and 2σ error bars in the $H_0 - z_{\text{acc}} - \omega_s$ space; we find the best-fit values at $H_0 = 71.839 \text{ kms}^{-1} \text{ Mpc}^{-1}$, $z_{\text{acc}} = 0.7831$ and $\omega_s = -0.95$ corresponding to a $\chi_{\text{min}}^2 = 14.26$ along with $\chi_{\text{d.o.f}}^2 = \chi_{\text{min}}^2/(N-n) = 0.790$ per degree of freedom. We remark that our estimations of the actual Hubble parameter agree with the median statistics made in Ref. 104, namely, our value meets within the 1σ interval obtained with the median statistics, $H_0 = 68 \pm 5 \text{ kms}^{-1} \text{ Mpc}^{-1}$, or with the analysis performed in Ref. 105 about the impact of H_0 prior on the evidence for dark radiation. On the other hand, we obtain CL in the $z_{\text{acc}} - \omega_s$ plane obtained after having marginalized the joint probability $P(H_0, z_{\text{acc}}, \omega_s)$ over H_0 (see Fig. 2). As usual, in the case of two parameters, 68.3%, 95.4% CL are made of random data sets that satisfy the inequality $\chi^2(\theta) - \chi_{\text{min}}^2(\theta_c) \leq 2.3$, $\chi^2(\theta) - \chi_{\text{min}}^2(\theta_c) \leq 6.17$, respectively.¹⁰³ The shaded band corresponding to $\omega_s \leq -1$ is excluded in our model in order to avoid phantom dark energy. The constraint on the critical redshift is $z_{\text{acc}} = 0.78_{-0.37}^{+0.26}$, such value are in agreement with $z_t = 0.69_{-0.13}^{+0.20}$ reported in Refs. 124 and 125, and meets within the 2σ CL obtained with the supernovae (Union 2) data in Ref. 125. The critical

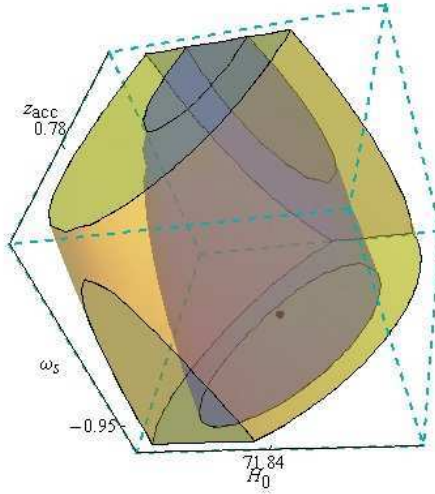


Fig. 1. Three-dimensional CL associated with 1σ , 2σ for H_0 , ω_s and z_{acc} parameters. The point indicates the best fit observational values, namely, $H_0 = 71.839 \text{ kms}^{-1} \text{ Mpc}^{-1}$, $z_{\text{acc}} = 0.7831$ and $\omega_s = -0.950$.

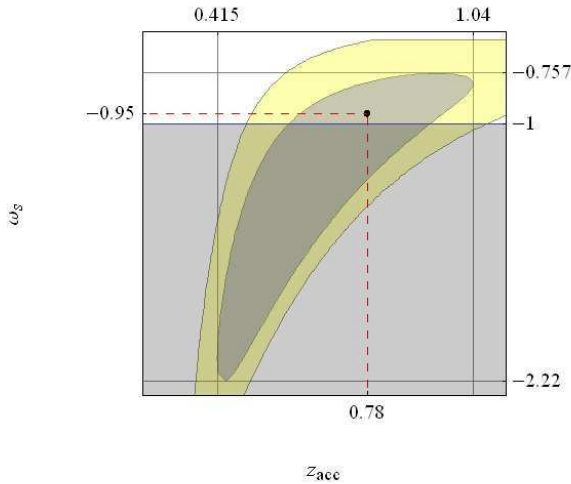


Fig. 2. Two-dimensional CL associated with 1σ , 2σ for ω_s and z_{acc} parameters, after the marginalization over the parameter H_0 was done. The point indicates the best fit observational value obtained with the $H(z)$ function.

redshift $z_{\text{acc}} = 0.78^{+0.26}_{-0.37}$ is also consistent with Union 2 + BAO + CMB data.¹⁰⁶ For the other parameter the statistical analysis leads to $\omega_s = -0.95^{+0.20}_{-1.27}$.

In order to get some physically relevant bounds on α , β and Ω_{c0} parameters, we now use the expression (20) and take as prior $H_0 = 71.84 \text{ kms}^{-1} \text{ Mpc}^{-1}$ which is in agreement with the median statistical constraints found in Refs. 104 and 105. Taking into account (20) for the χ^2 statistical analysis, we obtain the best-

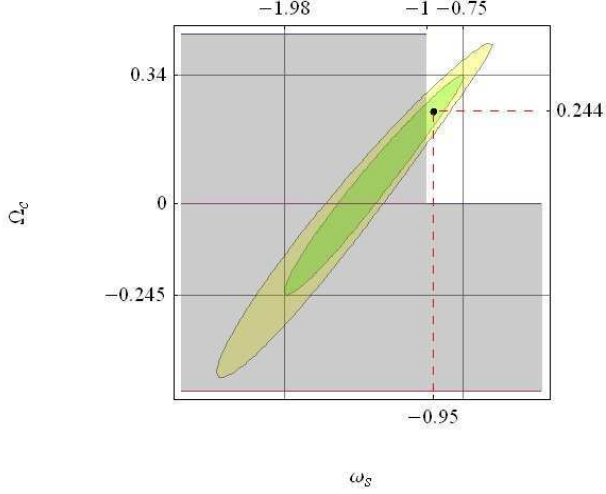


Fig. 3. Constraints in the ω_s - Ω_{c0} plane. Elliptical two-dimensional CL associated with 1σ and 2σ error bars. The dot indicates the best fit observational value obtained with the $H(z)$ function. The shaded zone is excluded because we try to avoid phantom dark energy in our model.

fit values $\alpha = 1.15$, $\beta = 0.023$ and $\Omega_{c0} = 0.243$ with $\chi_{\min}^2 = 14.2623$ along with $\chi_{\text{d.o.f}}^2 = 0.95 < 1$. Figure 3 shows two-dimensional CL in the ω_s - Ω_{c0} plane whereas the other parameters are taken as priors, namely, we fix $\alpha = 1.15$, $\beta = 0.023$ and $H_0 = 71.84 \text{ kms}^{-1} \text{ Mpc}^{-1}$. Then, the best-fit values together with their error bars are $\omega_s = -0.95^{+0.20}_{-1.03}$ and $\Omega_{c0} = 0.244^{+0.096}_{-0.489}$. We would like to use the best-fit values $\omega_s = -0.95$ and $\Omega_{c0} = 0.244$ to calculate the magnitude redshift relation for standard candles and contrast with the supernova data. As it is well known the observations of SNe Ia have predicted and confirmed that our universe is currently passing through an accelerated phase of expansion. Since then, the observational data coming from these standard candles have been taken very seriously. It is commonly believed that by measuring both their redshifts and apparent peak flux gives a direct measurement of their luminosity distances and thus SNe Ia provide the strongest constraint on the cosmological parameters. The theoretical distance modulus is defined as

$$\mu(z) = 5 \log_{10} \mathcal{D}_L + \mu_0, \quad (23)$$

where $\mu_0 = 43.028$ and \mathcal{D}_L is the Hubble-free luminosity distance, which for a spatially flat universe can be recast as

$$\mathcal{D}_L(z) = (1+z)H_0 \int_0^z \frac{dz'}{H(z')}. \quad (24)$$

Replacing the best-fit values of H_0 , ω_s and Ω_{c0} in Eqs. (20)–(24) we get the theoretical distance modulus $\mu(z)$ for our model (see Fig. 4) whereas the observational data with their error bars, $\mu_{\text{obs}}(z_i)$, are taken from Ref. 108. As we can see from

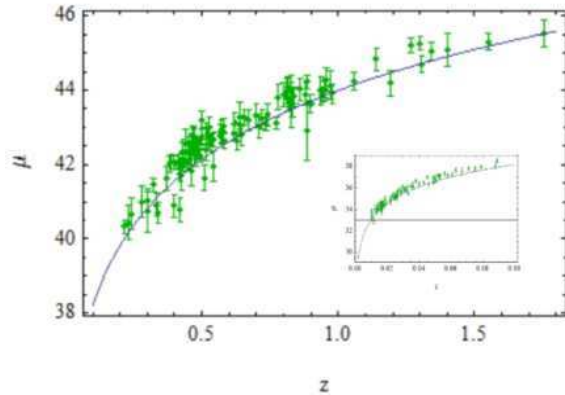


Fig. 4. The plot of the theoretical distance modulus (solid line) versus the redshift. The observational data (point) was taken from Riess¹⁰⁸ and covers two different regions of redshift, $z \leq 0.1$ and $0.1 \leq z \leq 1.5$. We obtain that the best-fit values, obtained for the interacting model, are in agreement with the supernovae data.

Fig. 4, our model exhibit an excellent agreement with the observational data, at least in the zones corresponding to small redshifts [$z \leq 0.1$] and large redshifts [$0.1 \leq z \leq 1.5$].

4.1. *The age problem*

We now turn our attention to the age problem, namely, the universe cannot be younger than its constituents (see Ref. 126). For example, the matter-dominated FRW universe can be ruled out because its age is smaller than the ages inferred from old globular clusters. The age problem becomes even more serious when we consider the age of the universe at high redshift. Now, there are some old high redshift objects (OHROs) discovered, for instance, the 3.5 Gyr old galaxy LBDS 53W091 at redshift $z = 1.55$,^{127,128} the 4.0 Gyr old galaxy LBDS 53W069 at redshift $z = 1.43$,¹²⁹ the 4.0 Gyr old radio galaxy 3C 65 at $z = 1.175$,¹³⁰ and the high redshift quasar B1422+231 at $z = 3.62$ whose best-fit age is 1.5 Gyr with a lower bound of 1.3 Gyr.¹³¹ Also the old quasar APM 08279+5255 at $z = 3.91$, whose age is estimated to be 2.0–3.0 Gyr,^{132,133} is used extensively. To assure the robustness of our analysis, we use the most conservative lower age estimate 2.0 Gyr for the old quasar APM 08279+5255 at $z = 3.91$,^{132,133} and the lower age estimate 1.3 Gyr for the high redshift quasar B1422+231 at $z = 3.62$.¹³¹ Many authors have examined the age problem within the framework of the dark energy models, see e.g., Refs. 126, 134–141 and references therein. The age problem within the context of holographic dark energy model was explored in Refs. 139 and 142–144. In this section, we would like to consider the age problem for the SIHDE model with linear interaction.

The age of our universe at redshift z can be obtained from the dimensionless age parameter^{126,135}

$$T_z(z) = H_0 t(z) = H_0 \int_z^\infty \frac{dz'}{(z'+1)H[z']}. \quad (25)$$

At any redshift, the age of our universe should be larger or equal than the age of the old high redshift objects

$$T_z(z) \geq T_{\text{obj}} = H_0 t_{\text{obj}}, \quad \text{or} \quad S(z) = \frac{T_z(z)}{T_{\text{obj}}} \geq 1, \quad (26)$$

where t_{obj} is the age of the OHRO. It is worth noting that from Eq. (25), $T_z(z)$ is independent of the Hubble constant H_0 . On the other hand, from Eq. (26), T_{obj} is proportional to the Hubble constant H_0 that we consider as $H_0 = 71.84 \text{ kms}^{-1} \text{ Mpc}^{-1}$.

In Table 2, we show the ratio $S(z) = \frac{T_z(z)}{T_{\text{obj}}}$ at $z = 3.91, 3.62, 1.55, 1.43, 1.175$ taking into account the best-fit values obtained in the last section. We obtain that $T_z(z) > T_{\text{obj}}(z)$ at $z = 3.62, 1.55, 1.43, 1.175$ but $T_z(z) < T_{\text{obj}}(z)$ at $z = 3.91$, so the old quasar APM 08279+5255 cannot be accommodated as the others old objects. Perhaps, the age crisis at high redshift in the case of dark energy holographic models^{139,142} could be alleviated by taking into account another type of interaction. This fact will be explored in a future research.

Table 2. It shows the ratio $S(z) = \frac{T_z(z)}{T_{\text{obj}}}$ at $z = 3.91, 3.62, 1.55, 1.43, 1.175$ for the best-fit values obtained with the Hubble data.

$S(3.91)$	$S(3.62)$	$S(1.55)$	$S(1.43)$	$S(1.175)$
0.854555	1.19781	1.20467	1.12826	1.31645

4.2. Kinematic analysis

Figure 5 shows the behavior of the deceleration parameter with redshifts. Using the values $\Omega_{c0} = 0.244$ and $\Omega_{x0} = 0.75$, we obtain that the deceleration parameter vanishes at $z_{\text{acc}} = 0.78$, so the universe enters the accelerated phase earlier than

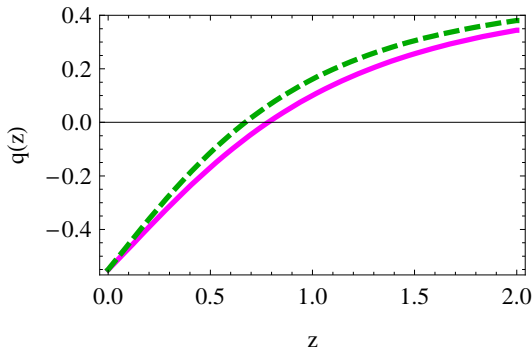


Fig. 5. (color online) Plot of deceleration parameter $q(z)$ taking into account the best-fit values $\Omega_x = 0.75$, $\alpha = 1.15$, $\beta = 0.024$ and $\omega_s = -0.95$ (magenta, solid line). It also shows the deceleration parameter for the Λ CDM model (green, dashed line).

the Λ CDM model. Regarding the effective equation of state, it stays in the range $-1 < \omega(z) < 0$ for $z \geq 0$, more precisely, $\omega(z)$ starts as nonrelativistic cold matter, decreases rapidly around $z = 2$ and then ends with the asymptotic value $\omega_s = -0.95$. The dark energy equation of state ω_x stays in the range $-1 < \omega_x(z) < 0$ also. The ratio of dark matter-dark energy,

$$r = \frac{\omega_0(\beta - \omega_s - 1) + (1 - \beta)(\omega_0 - \omega_s)(1 + z)^{3\omega_s}}{\omega_0(\omega_s + 1 - \alpha) + (\alpha - 1)(\omega_0 - \omega_s)(1 + z)^{3\omega_s}}, \quad (27)$$

evaluated at the best fit values $\alpha = 1.15$, $\beta = 0.024$, $\omega_s = -0.95$ and $\omega_0 = 0.78$ indicates that interaction Q_l helped to alleviate the coincidence problem.

5. Summary and Conclusions

In this paper, we have considered a flat FRW universe composed of an interacting dark matter and SIHDE. We have shown that the compatibility between SIHDE and the conservation equation gives a constraint between the equations of state of the dark components. We have selected linear SIHDE and a linear interaction in the dark sector and find that this model describes properly the evolution of both dark components. We have also shown that a general linear interaction, Q_l , is a good candidate for alleviating the cosmic coincidence problem.

Taking into account the Hubble data (see Table 1) and using the χ^2 statistical method, we have obtained the best-fit values at $H_0 = 71.839 \text{ kms}^{-1} \text{ Mpc}^{-1}$, $z_{\text{acc}} = 0.7831$ and $\omega_s = -0.95$ along with $\chi^2_{\text{d.o.f}} = 0.790 < 1$ per degree of freedom (see Fig. 1). The value of H_0 is in agreement with the one reported in the literature¹⁰⁸ or with the median statistical constraints found in Refs. 104 and 105. Having marginalized the joint probability $P(H_0, z_{\text{acc}}, \omega_s)$ over H_0 (see Fig. 2) we build two-dimensional CL and obtained the best-fit values with their 1σ error bars, namely, $z_{\text{acc}} = 0.78^{+0.26}_{-0.37}$ and $\omega_s = -0.95^{+0.20}_{-1.27}$. The critical redshift is in agreement with $z_t = 0.69^{+0.20}_{-0.13}$ reported in Refs. 124 and 125, and meets within the 2σ CL obtained with the supernovae (Union 2) data in Ref. 125. It is also consistent with Union 2 + BAO + CMB data.¹⁰⁶ Using as priors $\alpha = 1.15$, $\beta = 0.023$, and $H_0 = 71.84 \text{ kms}^{-1} \text{ Mpc}^{-1}$, we build two-dimensional CL in the ω_s - Ω_{c0} plane and estimated the best-fit values $\omega_s = -0.95^{+0.20}_{-1.03}$ and $\Omega_{c0} = 0.244^{+0.096}_{-0.489}$ (see Fig. 3).

Replacing the best-fit values of H_0 , ω_s and Ω_{c0} , we obtained the theoretical distance modulus $\mu(z)$ for our model (see Fig. 4) and confronted with supernovae data $\mu_{\text{obs}}(z_i)$ taken from Ref. 108. Figure 4 shows that our model exhibits an excellent agreement with the observational data. Besides, we have found that the age crisis at high redshift cannot be alleviated because the old quasar APM 08279+5255 at $z = 3.91$ (Refs. 132 and 133) seems to be older than the universe; so it will be needed to consider other kind of interaction (cf. Table 2) or perhaps to propose a nonlinear SIHDE for exploring this issue. Finally, we have found that our model enters the accelerated phase faster than the Λ CDM model (see Fig. 5). Concerning the effective equation of state and the dark energy equation of state, we found that both do not cross the phantom divide line.⁴¹ In a future research, we are going

to explore the linear SIHDE proposal where the dark sector is also coupled to a radiation or baryonic term; we will examine the changes introduced in the behavior of dark energy at early times.¹⁴⁵

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