Asymptotically free and safe fate of symmetry nonrestoration

Borut Bajc,[1,*] Adrián Lugo,[2,†] and Francesco Sannino[3,4,5,‡]

[1]*J. Stefan Institute, 1000 Ljubljana, Slovenia*

[2]*Instituto de Física de La Plata-CONICET, and Departamento de Física, Facultad de Ciencias Exactas, Universidad Nacional de La Plata, Argentina*

[3]*CP^3-Origins & the Danish Institute for Advanced Study, University of Southern Denmark, Campusvej 55, DK-5230 Odense, Denmark*

[4]*Dipartimento di Fisica E. Pancini, Università di Napoli Federico II & INFN sezione di Napoli, Complesso Universitario di Monte S. Angelo Edificio 6, via Cintia, 80126 Napoli, Italy*

[5]*Scuola Superiore Meridionale, Largo S. Marcellino, 10, 80138 Napoli NA, Italy*

(Received 4 March 2021; accepted 12 April 2021; published 17 May 2021)

We investigate the high temperature fate of four-dimensional gauge-Yukawa theories featuring short distance conformality of either interacting or noninteracting nature. The latter is known as complete asymptotic freedom and, as templates, we consider non-Abelian gauge theories featuring either two singlet scalars coupled to gauged fermions via Yukawa interactions or two gauged scalars with(out) fermions. For theories with interacting fixed points at short distance, known as asymptotically safe, we consider two calculable examples. Exploring the landscape of safe and free theories above we discover a class of complete asymptotically free theories for which symmetry breaks at arbitrary high temperatures. In its minimal form this class is constituted by a theory with two fundamental gauged scalars each gauged under an independent group.

DOI: 10.1103/PhysRevD.103.096014

I. INTRODUCTION

The phenomenon of symmetry nonrestoration (for a review see for example [1,2]) was first noticed by Weinberg [3] and then studied in detail by Mohapatra and Senjanović [4,5] who were the first ones to successfully apply the mechanism to phenomenology. Since then it has been employed in cosmology to address various issues like the monopole [6–8], the domain wall [9], and false vacuum problems. The phenomenon has also been invoked for other phenomena including baryogenesis [10–15] and inflation [16].

Symmetry nonrestoration at high energy can occur also due to the concomitance of other mechanisms such as the presence of large charges that can induce either Bose-Einstein condensation or superconductivity. This mechanism has been used in the literature [17–24]. For example a large charge can still be realistically related to the yet to be experimentally determined neutrino lepton number [25–30].

Symmetry nonrestoration at high temperature cannot occur in supersymmetry [31–33] unless we have flat directions [34,35] and/or nonzero fixed charge.

For nonsupersymmetric quantum field theories symmetry nonrestoration has been tested via different methods in [36–41] for global symmetries and nonrestoration for local symmetries have been investigated in [42]. The results seem to support the existence of symmetry nonrestoration although these claims have been challenged in [43–47].

Analyses including generalization to different spacetime dimensions including ϵ dimensions away from four are summarized in Refs. [48–51]. More precisely, symmetry nonrestoration at high temperature is possible also in lower [48,50,51] and noninteger dimensions [49].

A common feature of all the theories studied so far for symmetry nonrestoration at high temperature is that these can be viewed as effective theories without a well-defined ultraviolet completion. This fact implies that the arbitrary large temperature limit cannot be taken.

In this work we go one step beyond with respect to what has been done so far by analyzing Weinberg's symmetry nonrestoration hypothesis within models that are well defined at short distance. These are, according to Wilson [52,53] and Weinberg [54] classification of well-defined theories, of either asymptotically free or safe nature. Within these theories it is consistent to consider the infinite temperature limit. It is worth recalling that for these

*borut.bajc@ijs.si
†lugo@fisica.unlp.edu.ar
‡sannino@cp3.sdu.dk

theories short scale conformality guarantees the existence of a well-defined theory at high energy making them UV complete. Asymptotic safety for gauge-Yukawa theories was discovered in [55] with the corrections to the quantum potential presented in [56]. Interestingly, once asymptotic freedom is lost in the gauge-fermion sector, within perturbation theory, the fundamentality of the theory can only be reinstated via Yukawa interactions. This implies that elementary scalars are needed, for the first time, to tame the high energy behavior of the theory. The discovery of asymptotic safe quantum field theories [55] has led to an ongoing number of theoretical [57–61] and phenomenological investigations [62–69], including the recent discovery of safe nonsupersymmetric grand unified theories of [70] which naturally integrates and complements the supersymmetric story of [58].

For the issue of symmetry nonrestoration at arbitrary high temperatures we consider, at first, the landscape of complete asymptotically free non-Abelian gauge theories that feature either two singlet scalars coupled to gauged fermions via Yukawa interactions or two gauged scalars without Yukawa interaction.

The first model we encounter of complete asymptotically free theories for which symmetry breaks at arbitrary high temperatures is constituted by two gauged scalars transforming according to the fundamental representation of two distinct gauge groups with fermions also transforming in the fundamental representation and without Yukawa interactions.

To investigate the high-temperature fate of global symmetries for asymptotically safe theories we consider the Litim-Sannino model of [55] and one of its variations that has been used for perturbative safe extensions of the standard model [71]. We show that for these examples the safe quantum global symmetries are restored at high temperatures.

The paper is organized as follows: In Sec. II we study various classes of complete asymptotically free theories. We start in Sec. II A with gauge singlet scalars and show that their thermal mass is always positive in the UV. Then in Sec. II B we first show various examples of $\mathrm{SU}(N_c)$ gauge groups with scalars in the fundamental representation, for which the symmetry restores at high temperature. Then we study a theory of a product of different $SU(N)$ gauge groups in two versions: both for a pair of scalars in fundamental representation and for a pair of scalars in adjoint representations we show that for a particular choice of model parameters the symmetry can be broken at arbitrary high temperature. Then in Sec. III we study two examples of asymptotically safe theories both of which restore their global symmetries at high temperature. Three Appendixes collect all the of formulas needed for the computation. Appendix A summarizes the calculation of the one-loop renormalization group equations and the high temperature thermal masses for a generic gauge-Yukawa theory. Appendixes B and C give all of the computational details used in Secs. II A and II B.

II. COMPLETE ASYMPTOTICALLY FREE THEORIES AT HIGH TEMPERATURE

Before embarking on our main quest, which is to investigate the symmetry (non)restoration phenomenon for complete asymptotically free quantum field theories, we briefly summarize Weinberg's (nonfree) model mechanics. In its most minimal form the model features two scalars with the following quartic potential:

$$V = \frac{\lambda_1}{4}\phi_1^4 + \frac{\lambda_2}{4}\phi_2^4 - \frac{\lambda}{2}\phi_1^2\phi_2^2 \tag{2.1}$$

with discrete $Z_2 \times Z_2$ symmetry $\phi_1 \to -\phi_1$ and $\phi_2 \to -\phi_2$. For

$$\lambda_{1,2} > 0, \qquad \lambda^2 < \lambda_1\lambda_2 \tag{2.2}$$

the model is bounded from below. At high temperature the following correction arises [3]:

$$\Delta V_T = \frac{T^2}{24}((3\lambda_1 - \lambda)\phi_1^2 + (3\lambda_2 - \lambda)\phi_2^2). \tag{2.3}$$

With $\lambda > 3\lambda_2$ [but with λ_1 satisfying (2.2)] the field ϕ_2 acquires a negative thermal mass squared at high temperature which yields a nonzero vev $\langle\phi_2\rangle \neq 0$. Therefore in this case the second Z_2 breaks at sufficiently high temperatures. This theory is, however, not UV complete since the scalar couplings increase with the energy. Assuming a physical cutoff, for temperatures below this cutoff one therefore observes the phenomenon of symmetry nonrestoration.

Because the theory is limited by a physical cutoff we cannot ask the relevant question of whether the symmetry remains broken at arbitrary high temperatures. This is exactly what our work wishes to achieve, i.e., what is the ultimate fate of the symmetry in a truly UV complete theory (up to gravity) at arbitrary large temperatures.

Here we analyze complete asymptotically free theories that are natural UV completions of Weinberg's model. These require the presence of gauge fields and the gauge sector to be asymptotically free given that it is this sector that is the one responsible to drive the Yukawa and scalar couplings to be asymptotically free as well.

We divide our theories in whether they feature gauge singlets or gauged scalars.

A. Symmetry restoration with singlet scalars

To start with we consider an $SU(N_c)$ gauge group with $N_f = N_{f_1} + N_{f_2}$ Dirac fermions in the fundamental representation coupled to the scalars $\phi_{1,2}$ via the following $Z_2 \times Z_2$ symmetry[1] ($\phi_k \to -\phi_k$, $\psi_k \to i\gamma_5\psi_k$) preserving Yukawa terms:

[1]In some cases the symmetry of the theory could be larger than Z_2.

$$\mathcal{L}_Y = \phi_1 \sum_{i=1}^{N_{f_1}} y_{1i}\bar{\psi}_{1i}\psi_{1i} + \phi_2 \sum_{i=1}^{N_{f_2}} y_{2i}\bar{\psi}_{2i}\psi_{2i}. \quad (2.4)$$

Because we are searching for asymptotically free solutions we must have that $\alpha_g \propto 1/t$ for large $t = \log(\mu/\mu_0)$ with μ the renormalization scale and μ_0 a reference scale. Complete asymptotic freedom requires that all couplings (2.5) must vanish at infinity at least as fast as α_g and therefore their scaling must be proportional to $1/t^a$ with $a \geq 1$. Additionally the requirement of a negative thermal mass given in (A19), necessary for symmetry breaking, implies that at least some scalar quartic couplings cannot decrease faster than the gauge coupling, i.e., they must approach zero as $1/t^b$ with $b \leq 1$. Therefore, for the purpose of our work, it is sufficient to investigate the fixed flow solution according to which all couplings (2.5) vanish at infinity as $1/t$ [72]. This observation greatly simplifies the following analyses by transforming a set of nonlinear and coupled first order ordinary differential equations into a system of nonlinear and coupled *polynomial* equations. In practice, by defining (g is the gauge coupling)

$$\alpha_g = \frac{g^2}{(4\pi)^2}, \qquad \alpha_{y_i} = \frac{y_i^2}{(4\pi)^2}, \qquad \alpha_{\lambda_i} = \frac{\lambda_i}{(4\pi)^2}, \qquad \alpha_\lambda = \frac{\lambda}{(4\pi)^2} \quad (2.5)$$

($i = 1, 2$) we will search for solutions of the asymptotic form

$$\alpha_a = \frac{\tilde{\alpha}_a}{t}, \qquad a = g, y_1, y_2, \lambda_1, \lambda_2, \lambda, \ldots \quad (2.6)$$

with constant $\tilde{\alpha}_a$.

We are now ready to investigate the first relevant examples with singlet scalars and then we will generalize the results to a wider class of theories.

1. SU(N_c) *with two singlet scalars and fundamental fermions*

In this model, described in detail in Appendix B, we consider two singlet scalars coupled through Yukawa interactions to N_{f_1} (N_{f_2}) Dirac fermions in the fundamental representation of $SU(N_c)$. We further allow for N_{f_0} Dirac fermions in the fundamental representation of the gauge group that are inert with respect to the scalars, i.e., do not possess Yukawa couplings.

We now provide an elegant proof that at high temperature this theory, if completely asymptotically free, cannot break any symmetry. Let us start with the thermal masses for the scalars that at one loop read (B18)

$$m_i^2(T) = (4\pi)^2 \frac{T^2}{12\log T}(3\tilde{\alpha}_{\lambda_i} - \tilde{\alpha}_\lambda + 2N_c N_{f_i}\tilde{\alpha}_{y_i}), \quad (2.7)$$

written in terms of (2.6) couplings. It is sufficient to consider one of the two scalar masses to be negative. Here we choose that to be m_1^2 which requires

$$\tilde{\alpha}_\lambda - 2N_c N_{f_1}\tilde{\alpha}_{y_1} > 3\tilde{\alpha}_{\lambda_1} > 0. \quad (2.8)$$

Under the assumption that there is a completely asymptotically free solution we have (B14)

$$-\tilde{\alpha}_{\lambda_1} = 18\tilde{\alpha}_{\lambda_1}^2 + 2\tilde{\alpha}_\lambda^2 - 8N_c N_{f_1}\tilde{\alpha}_{y_1}^2 + 8N_c N_{f_1}\tilde{\alpha}_{y_1}\tilde{\alpha}_{\lambda_1} \quad (2.9)$$

for the relevant scalar coupling as a function of the other couplings. The general form of the renormalization group equations (RGE) can be found in Appendix B.

Rewriting (2.9) as

$$2(\tilde{\alpha}_\lambda^2 - 4N_c N_{f_1}\tilde{\alpha}_{y_1}^2) + \tilde{\alpha}_{\lambda_1} + 18\tilde{\alpha}_{\lambda_1}^2 + 8N_c N_{f_1}\tilde{\alpha}_{y_1}\tilde{\alpha}_{\lambda_1} = 0, \quad (2.10)$$

we notice that every term except the first one is positive. This means that to satisfy this equation, the first term must be negative for the fixed flow solution to be possible. However, since the first term can be rewritten as

$$\tilde{\alpha}_\lambda^2 - 4N_c N_{f_1}\tilde{\alpha}_{y_1}^2 = (\tilde{\alpha}_\lambda - 2N_c N_{f_1}\tilde{\alpha}_{y_1})(\tilde{\alpha}_\lambda + 2N_c N_{f_1}\tilde{\alpha}_{y_1}) + 4N_c N_{f_1}(N_c N_{f_1} - 1)\tilde{\alpha}_{y_1}^2, \quad (2.11)$$

the simultaneous requirement of the presence of a negative mass squared term implies that also the first term is positive due to (2.8).

We have therefore shown that (2.10) cannot have a solution and that the symmetry must be restored for this model at high temperature once complete asymptotic freedom is enforced.

2. *More general result for singlet scalars*

Let us consider the more general scalar potential

$$V = \frac{\lambda}{4}(\phi^T\phi)^2 - \frac{1}{2}(\phi^T\phi)\eta_{ij}\chi^i\chi^j + V(\chi) \quad (2.12)$$

where ϕ is a real vector with d_ϕ components. The global symmetry at the potential level over ϕ is $O(d_\phi)$. Under this group ϕ transforms with a $d_\phi \times d_\phi$ orthogonal matrix O of $O(d_\phi)$ as

$$\phi' = O\phi. \quad (2.13)$$

We further consider an arbitrary gauge group with Weyl fermions transforming according to an arbitrary gauge representation compatible with asymptotic freedom [73].

The Yukawa terms written directly in terms of the Weyl fermions[2] read

$$\mathcal{L}_{\mathrm{Yukawa}} = \frac{1}{2}\phi^a \psi_i Y^a_{ij} \psi_j + \mathrm{H.c.} + \mathcal{L}_{\mathrm{Yukawa}}(\chi, \psi'). \tag{2.14}$$

Under the assumption that

$$\psi' = U\psi, \qquad \tilde{O}^a_b U_{ki} Y^b_{kl} U_{lj} = Y^a_{ij} \tag{2.15}$$

with $\tilde{O}$ a rotation matrix that is part of a subgroup of $O(d_\phi)$ and U a unitary transformation with $i, j, k = 1, \ldots, N_f$ with N_f the number of Weyl matter fields, the Yukawa terms preserve the resulting subgroup of $O(d_\phi)$. The information on which fermions couple to ϕ is clearly hidden in the Yukawa matrix. The last unspecified Yukawa terms in (2.14) contain interactions of the χ scalar fields with the Weyl fermions ψ' that are not coupled to ϕ. We now show that the thermal mass of ϕ cannot be negative at high temperatures when the theory is required to be asymptotically free also in all couplings.

Let us consider the thermal mass

$$m^2_\phi(T) = (4\pi)^2 \frac{T^2}{12 \log T}((d_\phi + 2)\tilde{\lambda} - \tilde{\eta}_{kk} + \mathrm{Tr}(\tilde{Y}^\dagger_1 \tilde{Y}_1)), \tag{2.16}$$

where we used

$$\mathrm{Tr}(Y^{a\dagger} Y^b) = \delta^{ab} \mathrm{Tr}(Y^\dagger_1 Y_1). \tag{2.17}$$

and defined as usual

$$\lambda = (4\pi)^2 \frac{\tilde{\lambda}}{t}, \quad \eta_{ij} = (4\pi)^2 \frac{\tilde{\eta}_{ij}}{t}, \quad Y^a = 4\pi \frac{\tilde{Y}^a}{t^{1/2}} \tag{2.18}$$

with $\tilde{\lambda}$, $\tilde{\eta}_{ij}$, $\tilde{Y}^a$ constants.

In order not to restore the symmetry carried by the potential term and the Yukawa relative to ϕ the thermal mass (2.16) must be negative. This implies

$$\tilde{\eta}_{kk} - \mathrm{Tr}(\tilde{Y}^\dagger_1 \tilde{Y}_1) > (d_\phi + 2)\tilde{\lambda} > 0. \tag{2.19}$$

Let us now compute the RGE for $\tilde{\lambda}$ relative to achieving the fixed flow solution:

$$\begin{aligned} &2(\tilde{\eta}_{ij}\tilde{\eta}_{ij} - 2\mathrm{Tr}(\tilde{Y}^\dagger_1 \tilde{Y}_1 \tilde{Y}^\dagger_1 \tilde{Y}_1)) \\ &\quad + 2(d_\phi + 8)\tilde{\lambda}^2 + \tilde{\lambda} + 4\tilde{\lambda}\mathrm{Tr}(\tilde{Y}^\dagger_1 \tilde{Y}_1) = 0, \end{aligned} \tag{2.20}$$

where we used

$$\mathrm{Tr}(Y^{a\dagger} Y^b Y^{c\dagger} Y^d) = A\delta_{ab}\delta_{cd} + B\delta_{ac}\delta_{bd} + C\delta_{ad}\delta_{bc}, \tag{2.21}$$

which follows from the symmetry properties of the Yukawa matrices (2.15).

To obtain a solution, the first term must be negative (all the others are positive). However, we have

$$\begin{aligned} &\tilde{\eta}_{ij}\tilde{\eta}_{ij} - 2\mathrm{Tr}(\tilde{Y}^\dagger_1 \tilde{Y}_1 \tilde{Y}^\dagger_1 \tilde{Y}_1) \\ &\quad = (\tilde{\eta}_{kk} - \mathrm{Tr}(\tilde{Y}^\dagger_1 \tilde{Y}_1))(\tilde{\eta}_{kk} + \mathrm{Tr}(\tilde{Y}^\dagger_1 \tilde{Y}_1)) \\ &\qquad + 2\sum_{i<j} \tilde{\eta}_{ij}\tilde{\eta}_{ij} + ((\mathrm{Tr}(\tilde{Y}^\dagger_1 \tilde{Y}_1))^2 - 2\mathrm{Tr}(\tilde{Y}^\dagger_1 \tilde{Y}_1)^2). \end{aligned} \tag{2.22}$$

The first term on the right-hand side is positive due to the assumption of the occurrence of a negative thermal mass squared (2.19); therefore, the only possible negative term could be the last one. Since the above traces are invariant under unitary rotations of the Hermitian matrix $\tilde{Y}^\dagger_1 \tilde{Y}_1$, we are free to consider the basis with diagonal

$$(\tilde{Y}^\dagger_1 \tilde{Y}_1)_{ij} = \tilde{y}^2_{1i}\delta_{ij} \tag{2.23}$$

so that (2.22) becomes

$$\begin{aligned} &(\mathrm{Tr}(\tilde{Y}^\dagger_1 \tilde{Y}_1))^2 - 2\mathrm{Tr}(\tilde{Y}^\dagger_1 \tilde{Y}_1)^2 \\ &\quad = \sum_\mu \dim(R^1_\mu)(\dim(R^1_\mu) - 2)\tilde{y}^4_{1\mu} \\ &\qquad + 2\sum_{\mu<\mu'} \dim(R^1_\mu)\dim(R^1_{\mu'})\tilde{y}^2_{1\mu}\tilde{y}^2_{1\mu'} \end{aligned} \tag{2.24}$$

with μ and μ' running over the fermion representations. For nongauge singlet fermions we have $\dim(R_{\mu'}) \geq 2$ and therefore the right-hand side is positive. For gauge singlet fermions the only solution compatible with a UV well-defined theory is the one for which the Yukawa coupling vanishes identically and therefore the previous equation does not apply.

Therefore there is no solution to the RGE for $\tilde{\lambda}$. Or, in other words, if a fixed flow solution exists, it cannot have a negative thermal mass. The previous example with a Z_2 symmetry is included here by assuming the original symmetry to be simply a Z_2 for $d_\phi = 1$.

B. Exploring symmetry nonrestoration with gauged scalars

So far we have shown that a great deal of gauge theories with scalar gauge singlets do not support symmetry nonrestoration at arbitrary high temperatures. Does this phenomenon persist when considering gauged scalar fields? This is the question we will answer in this section. To simplify the discussion we will consider theories without Yukawa terms. We will find an example with the opposite behavior, i.e., we will explicitly present a theory featuring

[2] We use the supersymmetric notation: $\psi_i\psi_j \equiv \psi^T_i(i\sigma_2)\psi_j$ $(= \psi_j\psi_i)$.

two different gauge groups displaying simultaneously complete asymptotic freedom and symmetry nonrestoration.

To motivate the introduction of a second gauge group we will first show that with a single gauge group symmetries will restore at arbitrary high temperatures with(out) fermionic matter fields.

Although the models in this section share some features with the ones investigated in [74] the main difference resides in the fact that we are interested in symmetry nonrestoration at arbitrary high temperatures. This means that we investigate theories near their UV fixed point, while in [74] the authors concentrate on symmetry nonrestoration occurring near interacting IR fixed points.

1. $SU(N_c)$ with N_s fundamental scalars

The $SU(N_c)\times SU(N_f)\times SU(N_s)$ symmetric Lagrangian is[3]

$$\mathcal{L} = -\frac{1}{2}TrF_{\mu\nu}F^{\mu\nu} + \mathrm{Tr}(\bar{Q}i\not{D}Q) + \mathrm{Tr}(D^\mu S D_\mu S^\dagger) - v(TrSS^\dagger)^2 - uTr(SS^\dagger)^2 \tag{2.25}$$

with the fields transforming as

$$Q \sim (N_c, N_f, 1), \qquad S \sim (N_c, 1, N_s). \tag{2.26}$$

The scalar thermal mass at high temperature is

$$m_S^2(T) = (4\pi)^2\frac{T^2}{24\log T}\left(4(N_sN_c+1)\tilde{\lambda}_1 + 4(N_s+N_c)\tilde{\lambda}_2 + 3\frac{N_c^2-1}{N_c}\tilde{\alpha}\right) \tag{2.27}$$

where we introduced, following [75],

$$v = (4\pi)^2\frac{\tilde{\lambda}_1}{t}, \qquad u = (4\pi)^2\frac{\tilde{\lambda}_2}{t}, \qquad g^2 = (4\pi)^2\frac{\tilde{\alpha}}{t} \tag{2.28}$$

with constant $\tilde{\lambda}_{1,2}$, $\tilde{\alpha}$.

The positivity of (2.27) follows from boundedness arguments. In fact, the $T = 0$ potential is bounded from below if and only if [75]

$$\tilde{\lambda}_2 \geq 0\colon\ N_s\tilde{\lambda}_1 + \tilde{\lambda}_2 \geq 0, \tag{2.29}$$

$$\tilde{\lambda}_2 \leq 0\colon\ \tilde{\lambda}_1 + \tilde{\lambda}_2 \leq 0. \tag{2.30}$$

Since

(1) $\tilde{\lambda}_2 \geq 0$:

$$(N_sN_c+1)\tilde{\lambda}_1 + (N_s+N_c)\tilde{\lambda}_2 = \frac{1}{N_s}(N_sN_c+1)(N_s\tilde{\lambda}_2+\tilde{\lambda}_2) + \left(N_2 - \frac{1}{N_s}\right)\tilde{\lambda}_2 \geq 0 \tag{2.31}$$

(2) $\tilde{\lambda}_2 \geq 0$:

$$(N_sN_c+1)\tilde{\lambda}_1 + (N_s+N_c)\tilde{\lambda}_2 = (N_sN_c+1)(\tilde{\lambda}_1+\tilde{\lambda}_2) + (N_s-1)(N_c-1)|\tilde{\lambda}_2| \geq 0 \tag{2.32}$$

we can now conclude that the thermal mass is always positive

$$m_S^2(T) > 0 \tag{2.33}$$

i.e., the symmetry is restored at high temperature.

2. $SU(N_c)$ with two fundamental scalars

One of the problems of the previous model was that there was too much symmetry in the scalar potential. We now take the case of two scalars, $N_s = 2$, but instead of scalar SU(2) the symmetry of the potential will be just a discrete symmetry. We can take either a single Z_2 for even $N_c = 2n$ or $Z_2\times Z_2$ for odd $N_c = 2n+1$.

In fact, we note the following:

(i) $Z_2 \subset Z_{2n}$ and for even $N_c = 2n$ the center of $SU(N_c)$ is Z_{2n} and so $Z_2 \subset SU(N_c)$. In other words, a common $\vec{\varphi}_i \to -\vec{\varphi}_i$ is already present. So in this case there is only one extra Z_2 possible, say $\vec{\varphi}_1 \to -\vec{\varphi}_1$. In other words, besides the gauge SU($2n$) symmetry there is also a Z_2 symmetry $\vec{\varphi}_1 \to -\vec{\varphi}_1$.

(ii) For odd $N_c = 2n+1$ there is no Z_2 subgroup of $SU(N_c)$. In fact using the Levi-Civita tensor the invariant out of $N_c = 2n+1$ fundamentals is possible. Here it is thus possible to have an extra $Z_2\times Z_2$ symmetry for two fundamentals.

One way or another this means that each term of the potential can have only an even number of fundamentals $\vec{\varphi}_1$ and antifundamentals $\vec{\varphi}_1^*$ and an even number of fundamentals $\vec{\varphi}_2$ and antifundamentals $\vec{\varphi}_2^*$:

$$V = \frac{\lambda_1}{2}(\vec{\varphi}_1^*\cdot\vec{\varphi}_1)^2 + \frac{\lambda_2}{2}(\vec{\varphi}_2^*\cdot\vec{\varphi}_2)^2 + \lambda_3(\vec{\varphi}_1^*\cdot\vec{\varphi}_1)(\vec{\varphi}_2^*\cdot\vec{\varphi}_2) + \lambda_4(\vec{\varphi}_1^*\cdot\vec{\varphi}_2)(\vec{\varphi}_2^*\cdot\vec{\varphi}_1) + \frac{\lambda_5}{2}(\vec{\varphi}_1^*\cdot\vec{\varphi}_2)^2 + \frac{\lambda_5^*}{2}(\vec{\varphi}_2^*\cdot\vec{\varphi}_1)^2 \tag{2.34}$$

with $\lambda_{1,2,3,4}$ real and in general λ_5 complex.

[3] If $N_c = N_s = 4$ one can add to the potential a new invariant $w\det S + w^*\det S^\dagger$.

By taking

$$g^2 = \frac{16\pi^2\tilde{\alpha}}{N_c t}, \qquad \lambda_i = \frac{16\pi^2\tilde{\lambda}_i}{N_c t} \tag{2.35}$$

with constant $\tilde{\alpha}$, $\tilde{\lambda}_i$, and

$$\tilde{\lambda}_\pm = \frac{1}{2}(\tilde{\lambda}_1 \pm \tilde{\lambda}_2) \tag{2.36}$$

the solutions to the RGE (see Appendix C 3) are

(1)

$$\tilde{\lambda}_+ = \frac{6\tilde{\alpha}-1}{4}, \qquad \tilde{\lambda}_3^2 + \tilde{\lambda}_-^2 = \frac{24\tilde{\alpha}^2 - 12\tilde{\alpha} + 1}{16} \tag{2.37}$$

(2)

$$\tilde{\lambda}_+ = \frac{6\tilde{\alpha} - 1 + a_+\sqrt{24\tilde{\alpha}^2 - 12\tilde{\alpha} + 1}}{4}, \qquad a_+^2 = 1, \qquad \tilde{\lambda}_3 = \tilde{\lambda}_- = 0 \tag{2.38}$$

acceptable only for $\tilde{\alpha} \geq (3+\sqrt{3})/12$.

The thermal potential at large N_c is

$$\Delta V_T = (4\pi)^2 \frac{T^2}{24\log T}((2(\tilde{\lambda}_1 + \tilde{\lambda}_3) + 3\tilde{\alpha})(\vec{\varphi}_1^* \cdot \vec{\varphi}_1) + (2(\tilde{\lambda}_2 + \tilde{\lambda}_3) + 3\tilde{\alpha})(\vec{\varphi}_2^* \cdot \vec{\varphi}_2)). \tag{2.39}$$

The masses are quite symmetric and the search for symmetry restoration boils down to looking for negative $\tilde{\lambda}_3 = -|\tilde{\lambda}_3|$ which leads to a negative mass square for $\vec{\varphi}_1$, i.e., a negative combination

$$\frac{12\tilde{\alpha}-1}{2} - 2\left(\sqrt{\frac{24\tilde{\alpha}^2 - 12\tilde{\alpha}+1}{16} - |\tilde{\lambda}_3|^2} + |\tilde{\lambda}_3|\right) \tag{2.40}$$

for

$$\tilde{\alpha} \geq \frac{3+\sqrt{3}}{12}, \qquad 0 \leq |\tilde{\lambda}_3| \leq \sqrt{\frac{24\tilde{\alpha}^2 - a2\tilde{\alpha} + 1}{16}}. \tag{2.41}$$

The function (2.40) is minimized for

$$|\tilde{\lambda}_3| = \frac{1}{\sqrt{2}}\sqrt{\frac{24\tilde{\alpha}^2 - 12\tilde{\alpha}+1}{16}} \tag{2.42}$$

which is however not enough for a negative mass square.

3. $SU(N_{c_1}) \times SU(N_{c_2})$ with fundamental scalars: Symmetry breaks at high temperatures

The model we will study now is similar to the previous one, but now we have two simple groups, $SU(N_{c_1}) \times SU(N_{c_2})$, so that each φ_i is in a fundamentals representation of its $SU(N_{ci})$ and a singlet under the other one. The most general potential is

$$V = \frac{\lambda_1}{2}(\vec{\varphi}_1^* \cdot \vec{\varphi}_1)^2 + \frac{\lambda_2}{2}(\vec{\varphi}_2^* \cdot \vec{\varphi}_2)^2 - \lambda(\vec{\varphi}_1^* \cdot \vec{\varphi}_1)(\vec{\varphi}_2^* \cdot \vec{\varphi}_2). \tag{2.43}$$

Defining first

$$i = 1,2\colon\ g_i^2 = \frac{16\pi^2\tilde{\alpha}_i}{N_{ci}t}, \qquad \lambda_i = \frac{16\pi^2\tilde{\lambda}_i}{N_{ci}t}, \tag{2.44}$$

$$\lambda = \frac{16\pi^2\tilde{\lambda}}{\sqrt{N_{c1}N_{c2}}t} \tag{2.45}$$

with constant $\tilde{\alpha}_i$, $\tilde{\lambda}_i$, $\tilde{\lambda}$, the thermal effective potential becomes at large N_{ci}

$$\Delta V_T = (4\pi)^2 \frac{T^2}{24\log T}\left(\left(2\left(\tilde{\lambda}_1 - \sqrt{\frac{N_{c2}}{N_{c1}}}\tilde{\lambda}\right) + 3\tilde{\alpha}_1\right)(\vec{\varphi}_1^* \cdot \vec{\varphi}_1) + \left(2\left(\tilde{\lambda}_2 - \sqrt{\frac{N_{c1}}{N_{c2}}}\tilde{\lambda}\right) + 3\tilde{\alpha}_2\right)(\vec{\varphi}_2^* \cdot \vec{\varphi}_2)\right). \tag{2.46}$$

Introducing the new variables

$$\tilde{\lambda}_\pm = \frac{1}{2}(\tilde{\lambda}_1 \pm \tilde{\lambda}_2), \qquad \tilde{\alpha}_\pm = \frac{1}{2}(\tilde{\alpha}_1 \pm \tilde{\alpha}_2), \tag{2.47}$$

one finds the following solution[4] of the RGE (see Appendix C 4):

$$\tilde{\alpha}_- = 0, \tag{2.48}$$

$$\tilde{\lambda}_+ = \frac{6\tilde{\alpha}_+ - 1}{4}, \tag{2.49}$$

$$\tilde{\lambda}_-^2 + \tilde{\lambda}^2 = \frac{1}{16}(24\tilde{\alpha}_+^2 - 12\tilde{\alpha}_+ + 1), \tag{2.50}$$

valid for

$$\tilde{\alpha}_+ \geq (3+\sqrt{3})/12. \tag{2.51}$$

[4]The other possible solution $\tilde{\alpha}_+ = 1/4$, $\tilde{\lambda}_+ = \frac{1}{8}$, $(\tilde{\lambda}_- - \frac{3}{2}\tilde{\alpha}_-)^2 + \tilde{\lambda}^2 = \frac{1}{32}(48\tilde{\alpha}_-^2 - 1)$ describes a $T=0$ potential which is unbounded from below.

We will now prove that this solution supports symmetry nonrestoration at arbitrary high temperatures.

Denoting by μ_i^2 the coefficient in front of $(\vec{\varphi}_i^* \cdot \vec{\varphi}_i)$ in the parentheses on the right-hand side of (2.46) we have

$$\mu_1^2 = \frac{12\tilde{\alpha}_+ - 1}{2} + 2\tilde{\lambda}_- - 2\sqrt{\frac{N_{c2}}{N_{c1}}}\tilde{\lambda}, \tag{2.52}$$

$$\mu_2^2 = \frac{12\tilde{\alpha}_+ - 1}{2} - 2\tilde{\lambda}_- - 2\sqrt{\frac{N_{c1}}{N_{c2}}}\tilde{\lambda}. \tag{2.53}$$

We are searching for positive

$$\tilde{\lambda} = |\tilde{\lambda}| \tag{2.54}$$

and, up to redefinitions of what is 1 and what is 2, we can take

$$\tilde{\lambda}_- = -\sqrt{\frac{24\tilde{\alpha}_+^2 - 12\tilde{\alpha}_+ + 1}{16} - |\tilde{\lambda}|^2} \tag{2.55}$$

so that

$$\mu_1^2 = \frac{12\tilde{\alpha}_+ - 1}{2} - 2\left(\sqrt{\frac{24\tilde{\alpha}_+^2 - 12\tilde{\alpha}_+ + 1}{16} - |\tilde{\lambda}|^2} + \sqrt{\frac{N_{c2}}{N_{c1}}}|\tilde{\lambda}|\right), \tag{2.56}$$

$$\mu_2^2 = \frac{12\tilde{\alpha}_+ - 1}{2} + 2\left(\sqrt{\frac{24\tilde{\alpha}_+^2 - 12\tilde{\alpha}_+ + 1}{16} - |\tilde{\lambda}|^2} - \sqrt{\frac{N_{c1}}{N_{c2}}}|\tilde{\lambda}|\right). \tag{2.57}$$

Minimizing the expression for μ_1^2 we obtain

$$|\tilde{\lambda}|^2 = \frac{1}{1 + N_{c1}/N_{c2}} \frac{24\tilde{\alpha}_+^2 - 12\tilde{\alpha}_+ + 1}{16}. \tag{2.58}$$

The minimized mass parameter

$$\mu_1^2 = \frac{12\tilde{\alpha}_+ - 1}{2} - 2\sqrt{\frac{24\tilde{\alpha}_+^2 - 12\tilde{\alpha}_+ + 1}{16}}\sqrt{1 + \frac{N_{c2}}{N_{c1}}} \tag{2.59}$$

can now be negative by a suitable choice of number of colors.

Let us now demonstrate that the previous solution leads to a bounded potential. The latter occurs if

$$\lambda_1\lambda_2 - \lambda^2 > 0 \tag{2.60}$$

which can be rewritten first as

$$\tilde{\lambda}_+^2 - \tilde{\lambda}_-^2 - \tilde{\lambda}^2 > 0 \tag{2.61}$$

and then as

$$\frac{(6\tilde{\alpha}_+ - 1)^2}{16} - \frac{24\tilde{\alpha}_+^2 - 12\tilde{\alpha}_+ + 1}{16} = \frac{12\tilde{\alpha}_+^2}{16} \tag{2.62}$$

which is indeed positive.

Finally, requiring equal gauge couplings in the large N_{ci} limit

$$\tilde{\alpha}_1 = \tilde{\alpha}_2 \tag{2.63}$$

means that the original not rescaled couplings satisfy the relation

$$N_{c1}g_1^2 = N_{c2}g_2^2. \tag{2.64}$$

This is achieved by the following suitable choice of number of matter fermions:

$$\frac{N_{f1}}{N_{c1}} = \frac{N_{f2}}{N_{c2}}. \tag{2.65}$$

Because of (2.51), they must satisfy

$$\frac{1}{2}(2 + 3\sqrt{3}) \le \frac{N_{fi}}{N_{ci}} < \frac{11}{2}. \tag{2.66}$$

We arrive at the result, similar to Weinberg's model, that only one thermal mass is negative.

4. The IR story of $SU(N_{c_1}) \times SU(N_{c_2})$ at nonzero temperature

Interestingly the model of the previous subsection can feature also an IR Banks-Zaks fixed point, more precisely, a fixed circle [76]. This can be achieved by tuning the number of fermions to maintain both gauge couplings equality and the occurrence of a perturbative IR fixed point. Once this is achieved the remaining equations for the IR fixed point values[5] are

$$0 = 2\lambda_1^2 + 2\lambda^2 - 6\alpha_1\lambda_1 + \frac{3}{2}\alpha_1^2, \tag{2.67}$$

$$0 = 2\lambda_2^2 + 2\lambda^2 - 6\alpha_2\lambda_2 + \frac{3}{2}\alpha_2^2, \tag{2.68}$$

$$0 = 2(\lambda_1 + \lambda_2)\lambda - 3(\alpha_1 + \alpha_2)\lambda. \tag{2.69}$$

[5]N.B. These values should not be confused with the tilded $1/t$ coefficients used for the fixed flow solutions in the UV.

The solutions are

$$\lambda_+ = \frac{3}{2}\alpha_+, \tag{2.70}$$

$$\lambda_-^2 + \lambda^2 = \frac{3}{2}\alpha_+^2, \tag{2.71}$$

if and only if

$$\alpha_- = 0. \tag{2.72}$$

The thermal effective potential is

$$\Delta V_T = \frac{T^2}{24} \times \left(\left(2\left(\lambda_1 - \sqrt{\frac{N_{c2}}{N_{c1}}}\lambda \right) + 3\alpha_+ \right) (\vec{\varphi}_1^* \cdot \vec{\varphi}_1) + \left(2\left(\lambda_2 - \sqrt{\frac{N_{c1}}{N_{c2}}}\lambda \right) + 3\alpha_+ \right) (\vec{\varphi}_2^* \cdot \vec{\varphi}_2) \right) \tag{2.73}$$

so that the thermal masses are proportional to

$$\mu_1^2 = 6\alpha_+ + 2\lambda_- - 2\sqrt{\frac{N_{c2}}{N_{c1}}}\lambda, \tag{2.74}$$

$$\mu_2^2 = 6\alpha_+ - 2\lambda_- - 2\sqrt{\frac{N_{c1}}{N_{c2}}}\lambda. \tag{2.75}$$

Searching again for the branch

$$\lambda = |\lambda|, \qquad \lambda_- = -\sqrt{\frac{3}{2}\alpha_+^2 - |\lambda|^2} \tag{2.76}$$

we have first

$$\mu_1^2 = 6\alpha_+ - 2\left(\sqrt{\frac{3}{2}\alpha_+^2 - |\lambda|^2} + \sqrt{\frac{N_{c2}}{N_{c1}}}|\lambda| \right), \tag{2.77}$$

$$\mu_2^2 = 6\alpha_+ + 2\left(\sqrt{\frac{3}{2}\alpha_+^2 - |\lambda|^2} - \sqrt{\frac{N_{c1}}{N_{c2}}}|\lambda| \right). \tag{2.78}$$

μ_1^2 is minimized for

$$|\lambda|^2 = \frac{1}{1 + N_{c1}/N_{c2}} \frac{3}{2}\alpha_+^2 \tag{2.79}$$

so that the thermal mass

$$\mu_1^2 = 6\alpha_+ - 2\sqrt{\frac{3}{2}\alpha_+^2}\sqrt{1 + \frac{N_{c2}}{N_{c1}}} \tag{2.80}$$

is negative for (but still in the Veneziano limit $N_{ci} \to \infty$)

$$\frac{N_{c2}}{N_{c1}} > 5. \tag{2.81}$$

Since

$$\lambda_+^2 - \lambda_-^2 - \lambda^2 = \frac{3}{4}\alpha_+^2 > 0 \tag{2.82}$$

the parameter choice describes a $T = 0$ potential which is bounded from below.

We have therefore found an example in which symmetry nonrestoration occurs near an IR fixed point which is more minimal than the one presented in [74].

5. Another example of symmetry breaking at high T: Two adjoints in $SU(N_{c1}) \times SU(N_{c2})$

This model is similar to the previous one, except that adjoint scalars are considered instead of fundamental scalars. The details are described in Appendix C 5. The most general quartic potential is

$$V = \frac{\lambda_1'}{4} Tr\Sigma_1^4 + \frac{\lambda_2'}{4} Tr\Sigma_2^4 + \frac{\lambda_1}{4}(Tr\Sigma_1^2)^2 + \frac{\lambda_2}{4}(Tr\Sigma_2^2)^2 - \frac{\lambda}{2} Tr\Sigma_1^2 Tr\Sigma_2^2. \tag{2.83}$$

We redefine the couplings as

$$\lambda_{1,2}' = (4\pi)^2 \frac{\tilde{\lambda}_{1,2}'}{N_{c_{1,2}}} \times \frac{1}{t}, \qquad \lambda_{1,2} = (4\pi)^2 \frac{\tilde{\lambda}_{1,2}}{N_{c_{1,2}}^2} \times \frac{1}{t}, \tag{2.84}$$

$$\lambda = (4\pi)^2 \frac{\tilde{\lambda}}{N_{c_1} N_{c_2}} \times \frac{1}{t}, \qquad g_{1,2}^2 = (4\pi)^2 \frac{\tilde{\alpha}_{1,2}}{N_{c_{1,2}}} \times \frac{1}{t} \tag{2.85}$$

with all tilded quantities constants, and eventually we will take the large $N_{c_{1,2}}$ limit.

As shown in [75], the potential (2.83) is bounded from below if the parameters satisfy the following inequalities:

$$\lambda_i + \frac{\lambda_i'}{k_i} > 0 \;\; (1 \le k_i \le N_{c_i}), \qquad \left(\lambda_1 + \frac{\lambda_1'}{k_1} \right)\left(\lambda_2 + \frac{\lambda_2'}{k_2} \right) > \lambda^2. \tag{2.86}$$

If $\lambda_i' > 0$, then it is enough to check the above for $k_i = N_{c_i}$, while if $\lambda_i' < 0$, $k_i = 1$ suffices. However, for large N_{c_i}, the second case is impossible, since

$$\lambda_i + \lambda_i' > 0 \to \tilde{\lambda}_i' > 0 \tag{2.87}$$

which is in contradiction with the original assumption of $\lambda_i' < 0$.

So the only possibility is just $\lambda_1' > 0$, $\lambda_2' > 0$:

$$\tilde{\lambda}_1 + \tilde{\lambda}_1' > 0, \quad \tilde{\lambda}_2 + \tilde{\lambda}_2' > 0, \quad (\tilde{\lambda}_1 + \tilde{\lambda}_1')(\tilde{\lambda}_2 + \tilde{\lambda}_2') > \tilde{\lambda}^2. \tag{2.88}$$

The thermal mass is

$$V_T = (4\pi)^2 \frac{T^2}{48 \log T}\left(\left(\tilde{\lambda}_1 + 2\tilde{\lambda}_1' - \frac{N_{c_2}}{N_{c_1}}\tilde{\lambda} + 12\tilde{\alpha}_1\right)Tr\Sigma_1^2 + \left(\tilde{\lambda}_2 + 2\tilde{\lambda}_2' - \frac{N_{c_1}}{N_{c_2}}\tilde{\lambda} + 12\tilde{\alpha}_2\right)Tr\Sigma_2^2\right). \tag{2.89}$$

We provide here an existence proof for a negative thermal mass with parameters satisfying the boundedness of the potential constraint.

First one can show that only one sector would not work, as expected. This means that if $\tilde{\lambda} = \tilde{\alpha}_2 = \tilde{\lambda}_2 = \tilde{\lambda}_2' = 0$, there is no solution of the above fixed flow RG equations for real $\tilde{\alpha}_1, \tilde{\lambda}_1, \tilde{\lambda}_1'$ assuming $\tilde{\lambda}_1 + \tilde{\lambda}_1' > 0$ (boundedness) and $\tilde{\lambda}_1 + 2\tilde{\lambda}_1' < 0$ (negative thermal mass).

However, a solution for bounded potential with negative thermal mass squared exists for

$$\tilde{\alpha}_1 = \tilde{\alpha}_2 = \frac{2+\sqrt{2}}{2}, \tag{2.90}$$

$$\tilde{\lambda}_1' = \tilde{\lambda}_2' = 2, \tag{2.91}$$

$$\tilde{\lambda}_1 = 12(2+\sqrt{2}) - 26, \tag{2.92}$$

$$\tilde{\lambda}_2 = 16, \tag{2.93}$$

$$\tilde{\lambda} = \sqrt{120(2+\sqrt{2}) - 392}, \tag{2.94}$$

$$\frac{N_{c_2}}{N_{c_1}} = 16. \tag{2.95}$$

This is therefore another relevant example of symmetry nonrestoration at arbitrary high temperature.

III. ASYMPTOTIC SAFETY AT HIGH TEMPERATURE

Another way to achieve a UV complete theory, up to gravity, is via the presence of an interacting ultraviolet fixed point in all couplings. In fact, one can have a combination of safe and free couplings for the model to be well defined at all scales.

Due to the fact that the discovery of asymptotically safe quantum field theory is relatively recent [55] the issue of symmetry nonrestoration for this relevant class of models has never been investigated before.

We will consider here examples classified according to whether we can reuse part of the results and reasoning employed above for the complete asymptotically free theories or we need a separate in-depth analysis of the safe model.

For the first class we consider theories structurally similar to the one considered above albeit with sufficient matter fields such that asymptotic freedom is lost while assuming that perturbative asymptotic safety occurs.

To transform the previous proof valid for asymptotically free theories to the equivalent potential asymptotically safe case we need to

(i) replace all tilded quantities with untilded ones;
(ii) eliminate the $\log T$ in the denominator of the thermal mass;
(iii) replace the $16\pi^2 d\alpha_i/dt$ in the left-hand sides of the RGEs with a zero.

This means that in the theories investigated in the previous section, once asymptotic freedom is lost and potential asymptotic safety appears, symmetry restoration is a must.

A. Explicit examples of asymptotic safety

We now consider explicit constructions of asymptotically safe quantum field theories that cannot be reduced to the example above because they either have multiple gauge singlet scalar quartic terms or/and have gauged scalars with nonzero Yukawa couplings. Interestingly we anticipate that in both examples the symmetry is restored at high temperature.

1. The Litim-Sannino (LS) model

The first model we consider here is the one put forward in [55] in which asymptotically safe quantum field theories and their structure was first discovered and understood. The Lagrangian reads

$$\mathcal{L} = -\frac{1}{2}Tr(F^{\mu\nu}F_{\mu\nu}) + \mathrm{Tr}(\bar{Q}i\not{D}Q) + \mathrm{Tr}(\partial_\mu H^\dagger \partial^\mu H) + yTr(\bar{Q}_L H Q_R + \bar{Q}_R H^\dagger Q_L) - uTr(H^\dagger H)^2 - v(TrH^\dagger H)^2, \tag{3.1}$$

with symmetry

$$G = SU(N_C) \times SU(N_F) \times SU(N_F) \times U_V(1), \tag{3.2}$$

under which the fields transform as

$$Q_L \sim (N_C, N_F, 1, 1), \tag{3.3}$$

$$Q_R \sim (N_C, 1, N_F, 1), \tag{3.4}$$

$$H \sim (1, N_F, \overline{N_F}, 0). \tag{3.5}$$

We assume the Veneziano limit, needed to ensure the rigorousness of the result

$$N_F, N_C \to \infty, \qquad \frac{N_F}{N_C} = \frac{11}{2} + \epsilon, \tag{3.6}$$

with $\epsilon \ll 1$ to control the size of the UV fixed point couplings that at the relevant order in perturbation theory read

$$\alpha_g \equiv \frac{g^2 N_C}{(4\pi)^2} = \frac{26}{57}\epsilon + \mathcal{O}(\epsilon^2), \tag{3.7}$$

$$\alpha_y \equiv \frac{y^2 N_C}{(4\pi)^2} = \frac{4}{19}\epsilon + \mathcal{O}(\epsilon^2), \tag{3.8}$$

$$\alpha_h \equiv \frac{u N_F}{(4\pi)^2} = \frac{\sqrt{23}-1}{19}\epsilon + \mathcal{O}(\epsilon^2), \tag{3.9}$$

$$\alpha_v \equiv \frac{v N_F^2}{(4\pi)^2} = -\frac{1}{19}\left(2\sqrt{23} - \sqrt{20+6\sqrt{23}}\right)\epsilon + \mathcal{O}(\epsilon^2). \tag{3.10}$$

The T^2 term of the H mass squared is

$$\begin{aligned} m_T^2 &= (4\pi)^2 \frac{T^2}{24}(8\alpha_h + 4\alpha_v + 2\alpha_y) \\ &\approx 9.7\epsilon T^2 > 0, \end{aligned} \tag{3.11}$$

so that the symmetry is restored at high temperature. Therefore we arrive at the conclusion that the original model of an asymptotically safe quantum field theory is also safe with respect to global symmetries.

B. A gauged scalar variant of the LS model

Here we consider an interesting example featuring a two-scalar sector with one of the scalars being gauged while the full theory remains asymptotically safe [71]. This model allows for a relevant test of symmetry (non)restoration and the Lagrangian of the model reads

$$\begin{aligned} \mathcal{L} = &-\frac{1}{2}Tr(F^{\mu\nu}F_{\mu\nu}) + \mathrm{Tr}(\bar{Q} i \not{D} Q) + \mathrm{Tr}(\partial_\mu H^\dagger \partial^\mu H) \\ &+ \mathrm{Tr}(D_\mu \tilde{S}^\dagger D^\mu \tilde{S}) + \left(\frac{y}{\sqrt{2}} Tr(\tilde{Q} H Q) + \mathrm{H.c.}\right) \\ &- u_2 Tr(H^\dagger H)^2 - u_1 (Tr H^\dagger H)^2 \\ &- w_2 Tr(\tilde{S}^\dagger \tilde{S})^2 - w_1 (Tr \tilde{S}^\dagger \tilde{S})^2, \end{aligned} \tag{3.12}$$

where the fields transform under the gauge and three global symmetries ($N_S = N_C - 2$)

$$G = SU(N_C) \times SU(N_F)_L \times SU(N_F)_R \times SU(N_S), \tag{3.13}$$

as

$$Q \sim (N_C, N_F, 1, 1), \tag{3.14}$$

$$\tilde{Q} \sim (\overline{N_C}, 1, \overline{N_F}, 1), \tag{3.15}$$

$$H \sim (1, \overline{N_F}, N_F, 1), \tag{3.16}$$

$$\tilde{S} \sim (\overline{N_C}, 1, 1, \overline{N_S}). \tag{3.17}$$

For small and positive

$$\epsilon = \frac{N_F}{N_C} - \frac{11}{2} + \frac{N_S}{4N_C} \to \frac{N_F}{N_C} - \frac{21}{4}, \tag{3.18}$$

the following relations are satisfied [71] at the UV fixed point:

$$\alpha_g \equiv \frac{N_C g^2}{(4\pi)^2} = \frac{25}{18}\epsilon, \tag{3.19}$$

$$\alpha_y \equiv \frac{N_C y^2}{(4\pi)^2} = \frac{24}{25}\alpha_g, \tag{3.20}$$

$$\alpha_{u_1} \equiv \frac{N_F^2 u_1}{(4\pi)^2} = \frac{-6\sqrt{22} + 3\sqrt{19+6\sqrt{22}}}{100}\alpha_g, \tag{3.21}$$

$$\alpha_{u_2} \equiv \frac{N_F u_2}{(4\pi)^2} = \frac{3}{25}(\sqrt{22}-1)\alpha_g, \tag{3.22}$$

$$\alpha_{w_1} \equiv \frac{N_C^2 w_1}{(4\pi)^2} = \frac{3 \pm \sqrt{3(4\sqrt{2}-5)}}{16\sqrt{2}}\alpha_g, \tag{3.23}$$

$$\alpha_{w_2} \equiv \frac{N_C w_2}{(4\pi)^2} = \frac{1}{16}(2-\sqrt{2})\alpha_g. \tag{3.24}$$

Following the analysis of the LS case but now generalized to both scalars we arrive at

$$m_T^2(H) = (4\pi)^2 \frac{T^2}{48}(2\alpha_y + 16\alpha_{u_2} + 8\alpha_{u_1}) \approx 38.4\epsilon T^2 > 0, \tag{3.25}$$

$$m_T^2(S) = (4\pi)^2 \frac{T^2}{24}(8\alpha_{w_2} + 4\alpha_{w_1} + 3\alpha_g) \approx 37.2\epsilon T^2 > 0. \tag{3.26}$$

This implies that no symmetries can be broken at high temperature.

IV. CONCLUSIONS

In this paper we analyzed Weinberg's symmetry nonrestoration idea within UV complete theories of either asymptotically free or safe nature.

The reason why these are natural models to investigate is that only for UV complete theories it is consistent to consider the arbitrary large temperature limit.

Safe and free theories share short scale conformality that insures a well-defined behavior at arbitrary high energies. Because of this, they belong to a special subset of all possible quantum field theories. The remaining field theories should be considered as effective low energy descriptions that cannot be complete without quantum gravity possibly modifying their high energy behavior. In any event, given the fact that we do not yet have a complete theory of quantum gravity, for these theories the symmetry nonrestoration test cannot be performed at arbitrary high temperatures.

As complete asymptotically free templates we commenced our investigation with $SU(N_c)$ gauge-Yukawa theories featuring N_f fundamental Dirac fermions and two singlet scalars coupled via Yukawa interactions to the fermions. We demonstrated that symmetry is restored for this class of asymptotically free theories. We then generalized the result to arbitrary (Weyl) fermion representations and to certain multiple singlet scalar theories. It was sufficient to demonstrate the incompatibility between the request of negative thermal mass squared for one of the scalars and the simultaneous need for its coupling to be asymptotically free.

We then moved to investigate the case of gauge scalars and have shown that high temperature symmetry nonrestoration appeared for the case of two gauged scalars transforming according to the fundamental representation of two independent gauge sectors. Fermions in the fundamental representation were included as well but without Yukawa couplings.

We then moved to investigate the case of asymptotically safe theories starting by noticing that the symmetry restoration results discovered for the singlet scalars discussed above could be extended to potentially safe theories.

Two more relevant examples were investigated in the asymptotically safe scenario in which either multiple quartic scalar field terms were present in the Lagrangian [55] and/or some of the scalars were gauged [71]. In these models symmetries restore at high temperature.

As an interesting class of UV complete theories featuring symmetry nonrestoration at arbitrary high temperatures we discovered the one featuring two gauged scalars, each in a fundamental representation of its own $SU(N_{ci})$ gauge group: for large enough ratios of colors, one scalar thermal mass can be negative.

So far we discussed UV complete theories before adding quantum gravity. We can imagine that a possible safe and free completion of the standard model occurs few orders of magnitude below the scale above which quantum gravity cannot be ignored. In this case our analysis still applies. It can even happen that quantum gravity is, *per se*, asymptotically free [77], and in this case we can ignore it.

The simplicity of the UV complete models discovered here featuring arbitrary high temperature symmetry nonrestoration phenomenon invites for further theoretical and phenomenological investigations. For example, it would be interesting to investigate whether UV complete grand-unified theories of the Pati-Salam type exist and that can feature the phenomenon of symmetry nonrestoration. Additionally there could be dark sectors that are gravitationally coupled to us that can be UV complete and feature early universe phase transitions from a symmetric to a broken one as the temperature increases.

ACKNOWLEDGMENTS

B. B. is very grateful to Goran Senjanović for having been introduced to the fascinating subject of symmetry nonrestoration at high temperature, as well as for illuminating interminable discussions on related subjects. B. B. acknowledges the financial support from the Slovenian Research Agency (research core funding No. P1-0035).

Note added.—While we were completing the present work, a related paper appeared [74] in which explicit examples of Banks-Zaks type CFTs were considered in which symmetry nonrestoration occurred at nonzero temperature. Differently and in a complementary manner of [74] our work investigates, rather than theories around IR fixed points, models featuring either Gaussian (completely asymptotically free) or interacting (completely asymptotically safe) UV fixed points such that we can investigate the infinite temperature limit within a given UV complete quantum field theory.

APPENDIX A: THE ONE-LOOP RG EQUATIONS

In this Appendix we summarize the relevant one-loop RG equations used in the main text starting with the normalization of the fields given by

$$\mathcal{L}_{kin} = -\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} + i\bar{\Psi}\not{D}\Psi + \frac{1}{2}D^\mu\Phi^a D_\mu\Phi^a. \tag{A1}$$

The gauge RG equation is

$$(4\pi)^2\beta_g \equiv (4\pi)^2\mu\frac{dg}{d\mu} = -b_0 g^3 \tag{A2}$$

with

$$b_0 = \frac{11}{3}T(G) - \frac{2}{3}T(F) - \frac{1}{6}T(S) \tag{A3}$$

where G, F, S stand for gauge bosons, Weyl fermions, and real scalars, respectively, and $T(R)$ is the Dynkin index of the representation R, defined as

$$Tr(T^A(R)T^B(R)) = T(R)\delta^{AB}. \quad \text{(A4)}$$

In $SU(N_c)$ we will need the following:

$$T(\text{fundamental}) = \frac{1}{2}, \qquad T(\text{adjoints}) = N_c. \quad \text{(A5)}$$

The Yukawa RG equations for Dirac fermions Ψ_i

$$\mathcal{L}_{\text{Yukawa}} = \sum_{i,j} Y^a_{ij}\bar{\Psi}_i\phi^a\Psi_j \quad \text{(A6)}$$

are [78] ($\kappa = 1$ for Dirac fermions and $\kappa = 1/2$ for Weyl fermions)

$$\begin{aligned}(4\pi)^2\beta^a_Y \equiv (4\pi)^2\mu\frac{dY^a}{d\mu} = \frac{1}{2}(Y^bY^{b\dagger}Y^a + Y^aY^{b\dagger}Y^b) \\ + 2Y^bY^{a\dagger}Y^b + \kappa Y^b Tr(Y^{b\dagger}Y^a + Y^{a\dagger}Y^b) \\ - 3g^2(C_2(F)Y^a + Y^aC_2(F))\end{aligned} \quad \text{(A7)}$$

where ϕ^a are real scalars and

$$(C_2(F))_{ij} = \sum_{kA} T^A_{ik}T^A_{kj} \quad \text{(A8)}$$

where the generators T^A are in the (in general reducible) representation of the fermions.

Here and in the following a repeated index gets summed [a, b over real scalars, α over $SU(N_c)$ generators, i, j, k over (bi)spinors] even when the explicit sum is not written.

Notice that the Yukawa matrices in (A6) are Hermitian by definition.

The scalar sector is defined by

$$V = \frac{1}{4!}\lambda_{abcd}\phi_a\phi_b\phi_c\phi_d \quad \text{(A9)}$$

Following [79] we introduce the completely symmetric tensors

$$\Lambda^2_{abcd} = \frac{1}{8}\sum_{\text{perm}}\lambda_{abef}\lambda_{efcd}, \quad \text{(A10)}$$

$$\Lambda^Y_{abcd} = \frac{1}{12}\sum_{\text{perm}} Tr(Y^{a\dagger}Y^e + Y^{e\dagger}Y^a)\lambda_{ebcd}, \quad \text{(A11)}$$

$$H_{abcd} = \frac{1}{4}\sum_{\text{perm}} Tr(Y^{a\dagger}Y^bY^{c\dagger}Y^d), \quad \text{(A12)}$$

$$\Lambda^S_{abcd} = \frac{1}{6}\sum_{\text{perm}}\sum_{A=1}^{N_c^2-1}(T^A(S)T^A(S))_{ae}\lambda_{ebcd}, \quad \text{(A13)}$$

$$A_{abcd} = \frac{1}{8}\sum_{\text{perm}}\sum_{A,B=1}^{N_c^2-1}\{T^A(S),T^B(S)\}_{ab}\{T^A(S),T^B(S)\}_{cd} \quad \text{(A14)}$$

where the sum over "perm" means that we sum over all 4! permutations of the indices a, b, c, and d so to make the left-hand sides completely symmetric in all indices. The matrices $T^A(S)$ are the Hermitian $SU(N_c)$ generators in the representation of the scalars. Since ϕ^a are taken real, these generators are imaginary and antisymmetric. For real representations of $SU(N_c)$ this is automatic, while for complex representations one has to work out the form of these matrices. More precisely, they are found in the covariant derivative:

$$D_\mu\phi^a = \partial_\mu\phi^a - igW^A_\mu(T^A(S))^a{}_b\phi^b. \quad \text{(A15)}$$

For the case of more gauge couplings g_α of gauge groups with generators T^A_α, one should remember that

$$A_{abcd}\frac{\phi^a\phi^b\phi^c\phi^d}{4!} = (M^2_W)^{AB}(M^2_W)^{AB} \quad \text{(A16)}$$

with the W mass

$$(M^2_W)^{AB} = \frac{1}{2}\phi^a g_\alpha g_\beta\{T^A_\alpha(S),T^B_\beta(S)\}_{ab}\phi^b. \quad \text{(A17)}$$

The one-loop RG equations then read [79]

$$\begin{aligned}16\pi^2\frac{d\lambda_{abcd}}{dt} = \Lambda^2_{abcd} + 2\kappa\Lambda^Y_{abcd} - 8\kappa H_{abcd} \\ - 3g^2\Lambda^S_{abcd} + 3g^4A_{abcd}.\end{aligned} \quad \text{(A18)}$$

Finally, at high temperature the thermal mass matrix is given by [3] (see also [80])

$$\begin{aligned}m^2_{ab}(T) = \frac{T^2}{24}(\lambda_{abcc} + 2\kappa Tr(Y^{a\dagger}Y^b + Y^{b\dagger}Y^a) \\ + 6g^2(T^A(S)T^A(S))_{ab}).\end{aligned} \quad \text{(A19)}$$

It is useful to rewrite the above formulas by multiplying the various quantities by constant $\phi^a\phi^b\phi^c\phi^d/4!$ and summing over the indices a, b, c, d. We thus define

$$V_{\Lambda^2} \equiv \Lambda^2_{abcd}\frac{\phi^a\phi^b\phi^c\phi^d}{4!} = \frac{1}{2}\frac{\partial^2 V}{\partial\phi^a\partial\phi^b}\frac{\partial^2 V}{\partial\phi^a\partial\phi^b}, \quad \text{(A20)}$$

$$V_{\Lambda^Y} \equiv 2\kappa\Lambda^Y_{abcd}\frac{\phi^a\phi^b\phi^c\phi^d}{4!} = \kappa\phi^a Tr(Y^{a\dagger}Y^e + Y^{e\dagger}Y^a)\frac{\partial V}{\partial\phi^e}, \quad \text{(A21)}$$

$$V_H \equiv -8\kappa H_{abcd}\frac{\phi^a\phi^b\phi^c\phi^d}{4!} = -2\kappa Tr(Y^{a\dagger}Y^bY^{c\dagger}Y^d)\phi^a\phi^b\phi^c\phi^d, \tag{A22}$$

$$V_{\Lambda^S} \equiv -3g^2\Lambda^S_{abcd}\frac{\phi^a\phi^b\phi^c\phi^d}{4!} = -3g^2\phi^a(T^A(S)T^A(S))_{ae}\frac{\partial V}{\partial\phi^e}, \tag{A23}$$

$$V_A \equiv 3g^4A_{abcd}\frac{\phi^a\phi^b\phi^c\phi^d}{4!} = \frac{3}{8}g^4(\phi^a\{T^A(S),T^B(S)\}_{ab}\phi^b)(\phi^c\{T^A(S),T^B(S)\}_{cd}\phi^d). \tag{A24}$$

Equation (A18) can thus be written as

$$16\pi^2\frac{\phi^a\phi^b\phi^c\phi^d}{4!}\frac{d\lambda_{abcd}}{dt} = V_{\Lambda^2} + V_{\Lambda^Y} + V_H + V_{\Lambda^S} + V_A \tag{A25}$$

while the equivalent of (A19) is

$$\Delta V(T) \equiv \frac{1}{2}m^2_{ab}(T)\phi^a\phi^b = \frac{T^2}{48}\left(2\frac{\partial^2 V}{\partial\phi^a\partial\phi^a} + 6g^2\phi^a(T^A(S)T^A(S))_{ab}\phi^b + 2\kappa\phi^a Tr(Y^{a\dagger}Y^b + Y^{b\dagger}Y^a)\phi^b\right). \tag{A26}$$

APPENDIX B: SU(N_c) WITH TWO SINGLET SCALARS AND FUNDAMENTAL FERMIONS

In this model the two singlet scalars with the potential (2.1) couple through Yukawa couplings to N_{f_1} (N_{f_2}) Dirac fermions in the fundamental representation of $SU(N_c)$. We further allow for N_{f_0} Dirac fermions in the fundamental representation of the gauge group that are inert with respect to the scalars, i.e., do not possess Yukawa couplings.

The gauge coupling one-loop RGE is

$$16\pi^2\frac{dg}{dt} = -b_0g^3, \tag{B1}$$

with

$$b_0 = \frac{11}{3}N_c - \frac{2}{3}(N_{f_0} + N_{f_1} + N_{f_2}). \tag{B2}$$

The solution is

$$\alpha_g = \frac{g^2}{(4\pi)^2} = \frac{\tilde{\alpha}_g}{t}, \tag{B3}$$

with

$$\tilde{\alpha}_g = \frac{1}{2b_0}. \tag{B4}$$

The Yukawa RGE are

$$16\pi^2\frac{dy_i}{dt} = (3 + 2N_cN_{f_i})y_i^3 - 3g^2\frac{N_c^2-1}{N_c}y_i, \quad i = 1, 2. \tag{B5}$$

Assuming the ansatz

$$\alpha_{y_i} = \frac{y_i^2}{(4\pi)^2} = \frac{\tilde{\alpha}_{y_i}}{t}, \tag{B6}$$

the fixed flow solution is given by

$$\tilde{\alpha}_{y_i} = \frac{6\frac{N_c^2-1}{N_c}\tilde{\alpha}_g - 1}{2(3 + 2N_cN_{f_i})}, \quad i = 1, 2 \tag{B7}$$

and has positive solutions only if the gauge coupling is big enough

$$6\tilde{\alpha}_g\frac{N_c^2-1}{N_c} - 1 > 0, \tag{B8}$$

which reduces to a constraint on the number of Dirac fermion fundamentals:

$$\frac{22}{4}N_c - \frac{9}{2}\left(N_c - \frac{1}{N_c}\right) < N_{f_0} + N_{f_1} + N_{f_2} < \frac{22}{4}N_c. \tag{B9}$$

The RG equations for the scalar couplings are

$$16\pi^2\frac{d\lambda_1}{dt} = 18\lambda_1^2 + 2\lambda^2 - 8N_cN_{f_1}y_1^4 + 8N_cN_{f_1}y_1^2\lambda_1, \tag{B10}$$

$$16\pi^2\frac{d\lambda_2}{dt} = 18\lambda_2^2 + 2\lambda^2 - 8N_cN_{f_2}y_2^4 + 8N_cN_{f_2}y_2^2\lambda_2, \tag{B11}$$

$$16\pi^2\frac{d\lambda}{dt} = -8\lambda^2 + 6\lambda(\lambda_1 + \lambda_2) + 4N_c(N_{f_1}y_1^2 + N_{f_2}y_2^2)\lambda. \tag{B12}$$

The ansatz

$$\alpha_{\lambda_i} = \frac{\lambda_i}{(4\pi)^2} = \frac{\tilde{\alpha}_{\lambda_i}}{t}, \qquad \alpha_\lambda = \frac{\lambda}{(4\pi)^2} = \frac{\tilde{\alpha}_\lambda}{t} \tag{B13}$$

reduces the system of ODEs (B10)–(B12) to a system of algebraic equations

$$-\tilde{\alpha}_{\lambda_1} = 18\tilde{\alpha}_{\lambda_1}^2 + 2\tilde{\alpha}_\lambda^2 - 8N_c N_{f_1}\tilde{\alpha}_{y_1}^2 + 8N_c N_{f_1}\tilde{\alpha}_{y_1}\tilde{\alpha}_{\lambda_1}, \tag{B14}$$

$$-\tilde{\alpha}_{\lambda_2} = 18\tilde{\alpha}_{\lambda_2}^2 + 2\tilde{\alpha}_\lambda^2 - 8N_c N_{f_2}\tilde{\alpha}_{y_2}^2 + 8N_c N_{f_2}\tilde{\alpha}_{y_2}\tilde{\alpha}_{\lambda_2}, \tag{B15}$$

$$-\tilde{\alpha}_\lambda = -8\tilde{\alpha}_\lambda^2 + 6\tilde{\alpha}_\lambda(\tilde{\alpha}_{\lambda_1} + \tilde{\alpha}_{\lambda_2}) + 4N_c(N_{f_1}\tilde{\alpha}_{y_1} + N_{f_2}\tilde{\alpha}_{y_2})\tilde{\alpha}_\lambda. \tag{B16}$$

To this we add (B4) and (B7). We look for strictly positive solutions for all 6 couplings $\tilde{\alpha}_{g,y_1,y_2,\lambda_1,\lambda_2,\lambda}$, with

$$N_c > 1, \qquad N_{f_{0,1,2}} \geq 0, \qquad N_{f_1} > 0 \quad \text{or} \quad N_{f_2} > 0 \tag{B17}$$

and $N_{f_0} + N_{f_1} + N_{f_2}$ in the interval (B9).

Once this is obtained one can compute the thermal mass for the scalars:

$$m_i^2(T) = (4\pi)^2 \frac{T^2}{12\log T}(3\tilde{\alpha}_{\lambda_i} - \tilde{\alpha}_\lambda + 2N_c N_{f_i}\tilde{\alpha}_{y_i}). \tag{B18}$$

It turns out that there are 1784 inequivalent (we do not count those obtained by $N_{f_1} \leftrightarrow N_{f_2}$) choices of colors and flavors which satisfy (B17) and (B9). However we are not only looking for fixed flow solutions, what we also need is that they lead to a negative thermal mass.

We will now prove in general that there are no solutions with symmetry nonrestoration.

Let it be $m_1^2(T) < 0$. To be so one needs

$$\tilde{\alpha}_\lambda - 2N_c N_{f_1}\tilde{\alpha}_{y_1} > 3\tilde{\alpha}_{\lambda_1} > 0. \tag{B19}$$

We can now rewrite (B14) as

$$2(\tilde{\alpha}_\lambda^2 - 4N_c N_{f_1}\tilde{\alpha}_{y_1}^2) + \tilde{\alpha}_{\lambda_1} + 18\tilde{\alpha}_{\lambda_1}^2 + 8N_c N_{f_1}\tilde{\alpha}_{y_1}\tilde{\alpha}_{\lambda_1} = 0. \tag{B20}$$

All the terms except the first one are manifestly positive, so to satisfy the equation, the first term should be negative. However, the first term can be rewritten as

$$\tilde{\alpha}_\lambda^2 - 4N_c N_{f_1}\tilde{\alpha}_{y_1}^2 = (\tilde{\alpha}_\lambda - 2N_c N_{f_1}\tilde{\alpha}_{y_1})(\tilde{\alpha}_\lambda + 2N_c N_{f_1}\tilde{\alpha}_{y_1}) + 4N_c N_{f_1}(N_c N_{f_1} - 1)\tilde{\alpha}_{y_1}^2. \tag{B21}$$

This is positive, since the last term is non-negative, while the first product is positive due to (B19). Equation (B20) thus cannot have a solution.

We conclude this Appendix summarizing the result for the model presented: *there is no fixed flow solution once a negative thermal mass is assumed.*

APPENDIX C: GAUGED SCALARS

We consider in this Appendix various examples of scalars in nontrivial representations of the gauge group.

1. $SU(2)$ with two scalar triplets

First we take the two scalar fields as gauge SU(2) triplets, coupled each to one fermion SU(2) doublet ($N_{f_1} = N_{f_2} = 1$). To use almost all of the old results we still keep the $Z_2 \times Z_2$ discrete symmetry. There is now an extra quartic term:

$$V = \frac{\lambda_1}{4}(\vec{\varphi}_1 \cdot \vec{\varphi}_1)^2 + \frac{\lambda_2}{4}(\vec{\varphi}_2 \cdot \vec{\varphi}_2)^2 - \frac{\lambda_{11}}{2}(\vec{\varphi}_1 \cdot \vec{\varphi}_1)(\vec{\varphi}_2 \cdot \vec{\varphi}_2) - \frac{\lambda_{12}}{2}(\vec{\varphi}_1 \cdot \vec{\varphi}_2)^2. \tag{C1}$$

Denoting

$$\phi = (\vec{\varphi}_1, \vec{\varphi}_2) \tag{C2}$$

we compute the quartic couplings directly from the definition

$$V = \frac{\lambda_{abcd}}{4!}\phi^a\phi^b\phi^c\phi^d \tag{C3}$$

i.e.,

$$\lambda_{abcd} = \frac{\partial^4 V}{\partial\phi^a \partial\phi^b \partial\phi^c \partial\phi^d}, \qquad a, b, c, d = 1, \ldots, 6. \tag{C4}$$

For the Yukawa term we take

$$\mathcal{L}_{\text{Yukawa}} = \sum_{i=1}^{2} y_i \bar{\psi}_i \left(\frac{\vec{\tau}}{2} \cdot \vec{\varphi}_i\right)\psi_i \tag{C5}$$

with τ^A, $A = 1, 2, 3$ the Pauli matrices.

The (reducible) generators for the fermions [two fundamental representations of SU(2)]

$$\Psi = (\psi_1, \psi_2) \tag{C6}$$

are

$$T^A = \frac{1}{2}\begin{pmatrix} \tau^A & 0 \\ 0 & \tau^A \end{pmatrix}, \qquad A = 1,2,3. \tag{C7}$$

The fixed flow RGE are

$$\tilde{\alpha}_g = 2b_0\tilde{\alpha}_g^2, \tag{C8}$$

$$-\tilde{\alpha}_{y_i} = \frac{5}{2}\tilde{\alpha}_{y_i}^2 - 9\tilde{\alpha}_g\tilde{\alpha}_{y_i}, \tag{C9}$$

$$-\tilde{\alpha}_{\lambda_i} = +22\tilde{\alpha}_{\lambda_i}^2 + 6\tilde{\alpha}_{\lambda_{11}}^2 + 4\tilde{\alpha}_{\lambda_{11}}\tilde{\alpha}_{\lambda_{12}} + 2\tilde{\alpha}_{\lambda_{12}}^2 - \tilde{\alpha}_{y_i}^2 + 4\tilde{\alpha}_{\lambda_i}\tilde{\alpha}_{y_i} + 12\tilde{\alpha}_g^2 - 24\tilde{\alpha}_g\tilde{\alpha}_{\lambda_i}, \tag{C10}$$

$$\tilde{\alpha}_{\lambda_{11}} = 6\tilde{\alpha}_g^2 + 24\tilde{\alpha}_g\tilde{\alpha}_{\lambda_{11}} + 8\tilde{\alpha}_{\lambda_{11}}^2 - 10\tilde{\alpha}_{\lambda_{11}}(\tilde{\alpha}_{\lambda_1} + \tilde{\alpha}_{\lambda_2}) + 2\tilde{\alpha}_{\lambda_{12}}^2 - 2\tilde{\alpha}_{\lambda_{12}}(\tilde{\alpha}_{\lambda_1} + \tilde{\alpha}_{\lambda_2}) - 2\tilde{\alpha}_{\lambda_{11}}(\tilde{\alpha}_{y_1} + \tilde{\alpha}_{y_2}), \tag{C11}$$

$$\tilde{\alpha}_{\lambda_{12}} = 6\tilde{\alpha}_g^2 + 24\tilde{\alpha}_g\tilde{\alpha}_{\lambda_{12}} + 16\tilde{\alpha}_{\lambda_{11}}\tilde{\alpha}_{\lambda_{12}} + 10\tilde{\alpha}_{\lambda_{12}}^2 - 4\tilde{\alpha}_{\lambda_{12}}(\tilde{\alpha}_{\lambda_1} + \tilde{\alpha}_{\lambda_2}) - 2\tilde{\alpha}_{\lambda_{12}}(\tilde{\alpha}_{y_1} + \tilde{\alpha}_{y_2}). \tag{C12}$$

The thermal mass squared results are

$$m_i^2(T) = (4\pi)^2\frac{T^2}{12\log T}(\tilde{\alpha}_{y_i} + 5\tilde{\alpha}_{\lambda_i} + 6\tilde{\alpha}_g - 3\tilde{\alpha}_{\lambda_{11}} - \tilde{\alpha}_{\lambda_{12}}). \tag{C13}$$

The gauge beta function is known as

$$b_0 = \frac{22}{3} - \frac{2}{3}(N_{f_0} + 2) - \frac{2}{3} = \frac{16 - 2N_{f_0}}{3} \to \tilde{\alpha}_g = \frac{3}{4(8 - N_{f_0})} \tag{C14}$$

from where, to get $\tilde{\alpha}_g > 1/9$, see (C8), we need

$$2 \leq N_{f_0} < 8. \tag{C15}$$

By explicit search one can find that there are no solutions of the fixed flow RGE for positive $\tilde{\alpha}_g$, $\tilde{\alpha}_{y_{1,2}}$, $\tilde{\alpha}_{\lambda_{1,2}}$ and real $\tilde{\alpha}_{\lambda_{11,12}}$.

2. $SU(2)$ with one scalar singlet and one scalar triplet

We take now one adjoint scalar and one singlet scalar that couple to the fermions (again in the fundamental representation, $N_{f_1} = N_{f_2} = 1$) with the following Yukawa term:

$$\mathcal{L}_{Yuk} = y_1\bar{\psi}_1\phi_1\psi_1 + y_2\bar{\psi}_2\left(\frac{\vec{\tau}}{2}\cdot\vec{\varphi}_2\right)\psi_2. \tag{C16}$$

Now the first scalar is singlet, the second is triplet. Obviously λ_{12} cannot appear now. We will again call the remaining mixed constant $\lambda_{11} = \lambda$ in this section.

The fixed flow RGE are now

$$\tilde{\alpha}_g = 2b_0\tilde{\alpha}_g^2, \tag{C17}$$

$$-\tilde{\alpha}_{y_1} = 14\tilde{\alpha}_{y_1}^2 - 9\tilde{\alpha}_g\tilde{\alpha}_{y_1}, \tag{C18}$$

$$-\tilde{\alpha}_{y_2} = \frac{5}{2}\tilde{\alpha}_{y_2}^2 - 9\tilde{\alpha}_g\tilde{\alpha}_{y_2}, \tag{C19}$$

$$-\tilde{\alpha}_{\lambda_1} = 18\tilde{\alpha}_{\lambda_1}^2 + 6\tilde{\alpha}_{\lambda}^2 - 16\tilde{\alpha}_{y_1}^2 + 16\tilde{\alpha}_{\lambda_1}\tilde{\alpha}_{y_1}, \tag{C20}$$

$$-\tilde{\alpha}_{\lambda_2} = 12\tilde{\alpha}_g^2 - 24\tilde{\alpha}_g\tilde{\alpha}_{\lambda_2} + 2\tilde{\alpha}_{\lambda}^2 + 22\tilde{\alpha}_{\lambda_2}^2 - \tilde{\alpha}_{y_2}^2 + 4\tilde{\alpha}_{\lambda_2}\tilde{\alpha}_{y_2}, \tag{C21}$$

$$\tilde{\alpha}_{\lambda} = 12\tilde{\alpha}_g\tilde{\alpha}_{\lambda} - 6\tilde{\alpha}_{\lambda_1}\tilde{\alpha}_{\lambda} + 8\tilde{\alpha}_{\lambda}^2 - 10\tilde{\alpha}_{\lambda}\tilde{\alpha}_{\lambda_2} - 8\tilde{\alpha}_{\lambda}\left(\tilde{\alpha}_{y_1} + \frac{1}{4}\tilde{\alpha}_{y_2}\right) \tag{C22}$$

while the thermal masses are

$$m_1^2(T) = (4\pi)^2\frac{T^2}{12\log T}(4\tilde{\alpha}_{y_1} + 3\tilde{\alpha}_{\lambda_1} - 3\tilde{\alpha}_{\lambda}), \tag{C23}$$

$$m_2^2(T) = (4\pi)^2\frac{T^2}{12\log T}(\tilde{\alpha}_{y_2} + 5\tilde{\alpha}_{\lambda_2} + 6\tilde{\alpha}_g - \tilde{\alpha}_{\lambda}). \tag{C24}$$

The gauge beta function is

$$b_0 = \frac{22}{3} - \frac{2}{3}(N_{f_0} + 2) - \frac{1}{3} = \frac{17 - 2N_{f_0}}{3} \to \tilde{\alpha}_g = \frac{3}{2(17 - 2N_{f_0})} \tag{C25}$$

from where, to get $\tilde{\alpha}_g > 1/9$, see (C18) or (C19), we need

$$2 \leq N_{f_0} \leq 8. \tag{C26}$$

We find only two solutions:

$$N_{f_0} = 8: (\tilde{\alpha}_g, \tilde{\alpha}_{y_1}, \tilde{\alpha}_{y_2}, \tilde{\alpha}_{\lambda_1}, \tilde{\alpha}_{\lambda_2}, \tilde{\alpha}_{\lambda}) = (1.5, 0.893, 5.0, 0.518, 0.182, 0) \tag{C27}$$

$$N_{f_0} = 8: (\tilde{\alpha}_g, \tilde{\alpha}_{y_1}, \tilde{\alpha}_{y_2}, \tilde{\alpha}_{\lambda_1}, \tilde{\alpha}_{\lambda_2}, \tilde{\alpha}_{\lambda}) = (1.5, 0.893, 5.0, 0.518, 0.5, 0) \tag{C28}$$

Since both have $\tilde{\alpha}_{\lambda} = 0$, symmetry is always restored at high enough T.

3. $SU(N_c)$ with two scalar fundamentals

The potential is

$$V = \frac{\lambda_1}{2}(\vec{\varphi}_1^* \cdot \vec{\varphi}_1)^2 + \frac{\lambda_2}{2}(\vec{\varphi}_2^* \cdot \vec{\varphi}_2)^2 + \lambda_3(\vec{\varphi}_1^* \cdot \vec{\varphi}_1)(\vec{\varphi}_2^* \cdot \vec{\varphi}_2) + \lambda_4(\vec{\varphi}_1^* \cdot \vec{\varphi}_2)(\vec{\varphi}_2^* \cdot \vec{\varphi}_1) + \frac{\lambda_5}{2}(\vec{\varphi}_1^* \cdot \vec{\varphi}_2)^2 + \frac{\lambda_5^*}{2}(\vec{\varphi}_2^* \cdot \vec{\varphi}_1)^2 \tag{C29}$$

with $\lambda_{1,2,3,4}$ real and in general λ_5 complex.

The relation between the complex and the real basis is as usual

$$\varphi_{\alpha k} = \frac{1}{\sqrt{2}}(R_{\alpha k} + iI_{\alpha k}), \qquad \alpha = 1, 2, \quad k = 1, \ldots, N_c \tag{C30}$$

so that

$$\phi^a = (R_1^k, I_1^k, R_2^k, I_2^k)^T. \tag{C31}$$

We get

$$\begin{aligned} V_{\Lambda^2} &= \frac{1}{2}\frac{\partial^2 V}{\partial\phi^a\partial\phi^b}\frac{\partial^2 V}{\partial\phi^b\partial\phi^a} \\ &= \sum_{\alpha,\beta=1}^{2}\sum_{k,l=1}^{N_c}\left(\frac{\partial^2 V}{\partial\varphi_\alpha{}^k\partial\varphi_{\beta l}^*}\frac{\partial^2 V}{\partial\varphi_\beta{}^l\partial\varphi_{\alpha k}^*} + \frac{\partial^2 V}{\partial\varphi_\alpha{}^k\partial\varphi_\beta{}^l}\frac{\partial^2 V}{\partial\varphi_{\beta l}{}^*\partial\varphi_{\alpha k}^*}\right) \\ &= \mathrm{Tr}(M_1 M_1^\dagger + 2M_2 M_2^\dagger + M_3 M_3^\dagger + N_1 N_1^\dagger + 2N_2 N_2^\dagger + N_3 N_3^\dagger) \end{aligned} \tag{C32}$$

with

$$M_1 = (\lambda_1(\vec{\varphi}_1{}^* \cdot \vec{\varphi}_1) + \lambda_3(\vec{\varphi}_2^* \cdot \vec{\varphi}_2))\mathbb{1} + \lambda_1\vec{\varphi}_1^* \otimes \vec{\varphi}_1 + \lambda_4\vec{\varphi}_2^* \otimes \vec{\varphi}_2, \tag{C33}$$

$$M_2 = (\lambda_4(\vec{\varphi}_1^* \cdot \vec{\varphi}_2) + \lambda_5^*(\vec{\varphi}_2^* \cdot \vec{\varphi}_1))\mathbb{1} + \lambda_3\vec{\varphi}_1^* \otimes \vec{\varphi}_2 + \lambda_5^*\vec{\varphi}_2^* \otimes \vec{\varphi}_1, \tag{C34}$$

$$M_3 = (\lambda_2(\vec{\varphi}_2^* \cdot \vec{\varphi}_2) + \lambda_3(\vec{\varphi}_1^* \cdot \vec{\varphi}_1))\mathbb{1} + \lambda_4\vec{\varphi}_1^* \otimes \vec{\varphi}_1 + \lambda_2\vec{\varphi}_2^* \otimes \vec{\varphi}_2, \tag{C35}$$

$$N_1 = \lambda_1\vec{\varphi}_1 \otimes \vec{\varphi}_1 + \lambda_5\vec{\varphi}_2 \otimes \vec{\varphi}_2, \tag{C36}$$

$$N_2 = \lambda_3\vec{\varphi}_1 \otimes \vec{\varphi}_2 + \lambda_4\vec{\varphi}_2 \otimes \vec{\varphi}_1, \tag{C37}$$

$$N_3 = \lambda_2\vec{\varphi}_2 \otimes \vec{\varphi}_2 + \lambda_5^*\vec{\varphi}_1 \otimes \vec{\varphi}_1. \tag{C38}$$

This gives

$$\begin{aligned} V_{\Lambda^2} = &((2N_c + 8)\lambda_1^2 + 2N_c\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2|\lambda_5|^2)\frac{1}{2}(\vec{\varphi}_1^* \cdot \vec{\varphi}_1)^2 \\ &+ ((2N_c + 8)\lambda_2^2 + 2N_c\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2|\lambda_5|^2)\frac{1}{2}(\vec{\varphi}_2^* \cdot \vec{\varphi}_2)^2 \\ &+ (2(N_c + 1)(\lambda_1 + \lambda_2)\lambda_3 + 4\lambda_3^2 + 2(\lambda_1 + \lambda_2)\lambda_4 + 2\lambda_4^2 + 2|\lambda_5|^2)(\vec{\varphi}_1^* \cdot \vec{\varphi}_1)(\vec{\varphi}_2^* \cdot \vec{\varphi}_2) \\ &+ (2(\lambda_1 + \lambda_2)\lambda_4 + 8\lambda_3\lambda_4 + 2N_c\lambda_4^2 + (4 + 2N_c)|\lambda_5|^2)(\vec{\varphi}_1^* \cdot \vec{\varphi}_2)(\vec{\varphi}_2^* \cdot \vec{\varphi}_1) \\ &+ 2(\lambda_1 + \lambda_2 + 4\lambda_3 + 2(N_c + 1)\lambda_4)\lambda_5\frac{1}{2}(\vec{\varphi}_1^* \cdot \vec{\varphi}_2)^2 + 2(\lambda_1 + \lambda_2 + 4\lambda_3 + 2(N_c + 1)\lambda_4)\lambda_5^*\frac{1}{2}(\vec{\varphi}_2^* \cdot \vec{\varphi}_1)^2. \end{aligned} \tag{C39}$$

We easily find

$$V_{\Lambda^S} = -3g^2\phi^a(T^A(S)T^A(S))_{ab}\frac{\partial V}{\partial\phi^a} = -3\frac{N_c^2 - 1}{2N_c}g^2\left(\varphi_\alpha^k\frac{\partial V}{\partial\varphi_\alpha^k} + \varphi_{\alpha k}^*\frac{\partial V}{\partial\varphi_{\alpha k}^*}\right) = -6\frac{N_c^2 - 1}{N_c}g^2 V. \tag{C40}$$

Using

$$\phi^a\{T^A(S),T^B(S)\}_{ab}\phi^b = 2g^2\varphi^*_{\alpha a}\{T^A,T^B\}^a{}_b\varphi^b_\alpha \tag{C41}$$

and the usual

$$(T^A)^a{}_b(T^A)^c{}_d = \frac{1}{2}\left(\delta^a{}_d\delta^c{}_b - \frac{1}{N_c}\delta^a{}_b\delta^c{}_d\right) \tag{C42}$$

we get

$$\begin{aligned}V_A =& \frac{3}{4}g^4\frac{N_c^2+2}{N_c^2}(\vec\varphi_1^*\cdot\vec\varphi_1+\vec\varphi_2^*\cdot\vec\varphi_2)^2\\ &+\frac{3}{4}g^4\frac{N_c^2-4}{N_c}((\vec\varphi_1^*\cdot\vec\varphi_1)^2+2(\vec\varphi_1^*\vec\varphi_2)(\vec\varphi_2^*\vec\varphi_1)+(\vec\varphi_2^*\cdot\vec\varphi_2)^2)\\ =& \frac{3}{4}g^4\frac{N_c^3+N_c^2-4N_c+2}{N_c^2}((\vec\varphi_1^*\cdot\vec\varphi_1)^2+(\vec\varphi_2^*\cdot\vec\varphi_2)^2)\\ &+\frac{3}{2}g^4\frac{N_c^2+2}{N_c^2}(\vec\varphi_1^*\cdot\vec\varphi_1)(\vec\varphi_2^*\cdot\vec\varphi_2)\\ &+\frac{3}{2}g^4\frac{N_c^2-4}{N_c}(\vec\varphi_1^*\vec\varphi_2)(\vec\varphi_2^*\vec\varphi_1).\end{aligned} \tag{C43}$$

By taking

$$g^2 = \frac{16\pi^2\tilde\alpha}{N_c t}, \qquad \lambda_i = \frac{16\pi^2\tilde\lambda_i}{N_c t} \tag{C44}$$

we get for constant α,λ_i in the large N_c limit the following fixed flow RGE:

$$-\tilde\lambda_1 = 2\tilde\lambda_1^2+2\tilde\lambda_3^2-6\tilde\alpha\tilde\lambda_1+\frac{3}{2}\tilde\alpha^2, \tag{C45}$$

$$-\tilde\lambda_2 = 2\tilde\lambda_2^2+2\tilde\lambda_3^2-6\tilde\alpha\tilde\lambda_2+\frac{3}{2}\tilde\alpha^2, \tag{C46}$$

$$-\tilde\lambda_3 = 2(\tilde\lambda_1+\tilde\lambda_2)\tilde\lambda_3-6\tilde\alpha\tilde\lambda_3, \tag{C47}$$

$$-\tilde\lambda_4 = 2\tilde\lambda_4^2+2|\tilde\lambda_5|^2-6\tilde\alpha\tilde\lambda_4+\frac{3}{2}\tilde\alpha^2, \tag{C48}$$

$$-\tilde\lambda_5 = 4\tilde\lambda_4\tilde\lambda_5-6\tilde\alpha\tilde\lambda_5. \tag{C49}$$

The thermal potential is

$$\begin{aligned}\Delta V_T &= \frac{T^2}{48}\left(2\frac{\partial^2 V}{\partial\phi^a\partial\phi^a}+6\phi^a(T^A(S)T^A(S))_{ab}\phi^b\right)\\ &= \frac{T^2}{48}\sum_{i=1}^{2}\sum_{a=1}^{N_c}\left(4\frac{\partial^2 V}{\partial\varphi_i^a\partial\varphi^*_{ia}}+6g^2\frac{N_c^2-1}{N_c}\varphi_i^a\varphi^*_{ia}\right).\end{aligned} \tag{C50}$$

Using (C33) and (C35)

$$\begin{aligned}\sum_{i=1}^{2}\sum_{a=1}^{N_c}\frac{\partial^2 V}{\partial\varphi_i^a\partial\varphi^*_{ia}} &= \mathrm{Tr}(M_1+M_3)\\ &= ((N_c+1)\lambda_1+N_c\lambda_3+\lambda_4)(\vec\varphi_1^*\cdot\vec\varphi_1)\\ &\quad+((N_c+1)\lambda_2+N_c\lambda_3+\lambda_4)(\vec\varphi_2^*\cdot\vec\varphi_2).\end{aligned} \tag{C51}$$

At large N_c

$$\begin{aligned}\Delta V_T =& (4\pi)^2\frac{T^2}{24\log T}((2(\tilde\lambda_1+\tilde\lambda_3)+3\tilde\alpha)(\vec\varphi_1^*\cdot\vec\varphi_1)\\ &+(2(\tilde\lambda_2+\tilde\lambda_3)+3\tilde\alpha)(\vec\varphi_2^*\cdot\vec\varphi_2)).\end{aligned} \tag{C52}$$

4. $SU(N_{c_1})\times SU(N_{c_2})$ with two scalar fundamentals

The model we will study now is similar to the previous one, but now we have two simple groups, $SU(N_{c_1})\times SU(N_{c_2})$, so that each φ_i is in a fundamental representation of its $SU(N_{ci})$ and a singlet under the other one. The most general potential is

$$V = \frac{\lambda_1}{2}(\vec\varphi_1^*\cdot\vec\varphi_1)^2+\frac{\lambda_2}{2}(\vec\varphi_2^*\cdot\vec\varphi_2)^2-\lambda(\vec\varphi_1^*\cdot\vec\varphi_1)(\vec\varphi_2^*\cdot\vec\varphi_2). \tag{C53}$$

As before we derive the various pieces of the RGE using (A20), (A23), (A24):

$$\begin{aligned}V_{\Lambda^2} =& ((2N_{c1}+8)\lambda_1^2+2N_{c2}\lambda^2)\frac{1}{2}(\vec\varphi_1^*\cdot\vec\varphi_1)^2\\ &+((2N_{c2}+8)\lambda_2^2+2N_{c1}\lambda^2)\frac{1}{2}(\vec\varphi_2^*\cdot\vec\varphi_2)^2\\ &-(2(N_{c1}\lambda_1+N_{c2}\lambda_2)\lambda\\ &+2(\lambda_1+\lambda_2)\lambda-4\lambda^2)(\vec\varphi_1^*\cdot\vec\varphi_1)(\vec\varphi_2^*\cdot\vec\varphi_2),\end{aligned} \tag{C54}$$

$$\begin{aligned}V_{\Lambda^S} =& -6\frac{N_{c1}^2-1}{N_{c1}}g_1^2\left(\frac{\lambda_1}{2}(\vec\varphi_1^*\cdot\vec\varphi_1)^2-\frac{\lambda}{2}(\vec\varphi_1^*\cdot\vec\varphi_1)(\vec\varphi_2^*\cdot\vec\varphi_2)\right)\\ &-6\frac{N_{c2}^2-1}{N_{c2}}g_2^2\left(\frac{\lambda_2}{2}(\vec\varphi_2^*\cdot\vec\varphi_2)^2-\frac{\lambda}{2}(\vec\varphi_1^*\cdot\vec\varphi_1)(\vec\varphi_2^*\cdot\vec\varphi_2)\right),\end{aligned} \tag{C55}$$

$$\begin{aligned}V_A =& \frac{3}{4}g_1^4\frac{N_{c1}^3+N_{c1}^2-4N_{c1}+2}{N_{c1}^2}(\vec\varphi_1^*\cdot\vec\varphi_1)^2\\ &+\frac{3}{4}g_2^4\frac{N_{c2}^3+N_{c2}^2-4N_{c2}+2}{N_{c2}^2}(\vec\varphi_2^*\cdot\vec\varphi_2)^2.\end{aligned} \tag{C56}$$

Defining

$$i = 1,2:\ g_i^2 = \frac{16\pi^2\tilde{\alpha}_i}{N_{ci}t}, \qquad \lambda_i = \frac{16\pi^2\tilde{\lambda}_i}{N_{ci}t}, \tag{C57}$$

$$\lambda = \frac{16\pi^2\tilde{\lambda}}{\sqrt{N_{c1}N_{c2}}t} \tag{C58}$$

with constant we get for the RG equations at large N_{ci}

$$-\tilde{\lambda}_1 = 2\tilde{\lambda}_1^2 + 2\tilde{\lambda}^2 - 6\tilde{\alpha}_1\tilde{\lambda}_1 + \frac{3}{2}\tilde{\alpha}_1^2, \tag{C59}$$

$$-\tilde{\lambda}_2 = 2\tilde{\lambda}_2^2 + 2\tilde{\lambda}^2 - 6\tilde{\alpha}_2\tilde{\lambda}_2 + \frac{3}{2}\tilde{\alpha}_2^2, \tag{C60}$$

$$-\tilde{\lambda} = 2(\tilde{\lambda}_1 + \tilde{\lambda}_2)\tilde{\lambda} - 3(\tilde{\alpha}_1 + \tilde{\alpha}_2)\tilde{\lambda}. \tag{C61}$$

The thermal effective potential

$$\Delta V_T = \frac{T^2}{48}\sum_{i=1}^{2}\sum_{a=1}^{N_{ci}}\left(4\frac{\partial^2 V}{\partial\varphi_i^a\partial\varphi_{ia}^*} + 6g_i^2\frac{N_{ci}^2-1}{N_{ci}}\varphi_i^a\varphi_{ia}^*\right) \tag{C62}$$

becomes at large N_{ci}

$$\Delta V_T = (4\pi)^2\frac{T^2}{24\log T}\left(\left(2\left(\tilde{\lambda}_1 - \sqrt{\frac{N_{c2}}{N_{c1}}}\tilde{\lambda}\right) + 3\tilde{\alpha}_1\right)(\vec{\varphi}_1^*\cdot\vec{\varphi}_1) + \left(2\left(\tilde{\lambda}_2 - \sqrt{\frac{N_{c1}}{N_{c2}}}\tilde{\lambda}\right) + 3\tilde{\alpha}_2\right)(\vec{\varphi}_2^*\cdot\vec{\varphi}_2)\right). \tag{C63}$$

5. $SU(N_{c_1})\times SU(N_{c_2})$ with two scalar adjoints

We now present a model again with two simple gauge groups, $SU(N_{c_1})\times SU(N_{c_2})$, and one adjoint for each gauge group. The potential is parametrized by

$$V = \frac{\lambda_1'}{4}Tr\Sigma_1^4 + \frac{\lambda_2'}{4}Tr\Sigma_2^4 + \frac{\lambda_1}{4}(Tr\Sigma_1^2)^2 + \frac{\lambda_2}{4}(Tr\Sigma_2^2)^2 - \frac{\lambda}{2}Tr\Sigma_1^2 Tr\Sigma_2^2. \tag{C64}$$

The one-loop corrections are (A20), (A23), (A24):

$$\begin{aligned} V_{\Lambda^2} = \frac{1}{8}\Bigg(&Tr\Sigma_1^4\left(12\lambda_1\lambda_1' + \lambda_1'^2\frac{2N_{c_1}^2-18}{N_{c_1}}\right) - Tr\Sigma_1^2Tr\Sigma_2^2\left(\lambda_1\lambda(2N_{c_1}^2+2) + \lambda\lambda_1'\frac{4N_{c_1}^2-6}{N_{c_1}}\right)\\ &+ (Tr\Sigma_1^2)^2\left(\lambda_1^2(N_{c_1}^2+7) + \lambda_1\lambda_1'\frac{4N_{c_1}^2-6}{N_{c_1}} + \lambda_1'^2\frac{3N_{c_1}^2+9}{N_{c_1}^2}\right) + (Tr\Sigma_2^2)^2\lambda^2(N_{c_1}^2-1)\Bigg) + Tr\Sigma_1^2Tr\Sigma_2^2\lambda^2\\ &+\frac{1}{8}\Bigg(Tr\Sigma_2^4\left(12\lambda_2\lambda_2' + \lambda_2'^2\frac{2N_{c_2}^2-18}{N_{c_2}}\right) - Tr\Sigma_1^2Tr\Sigma_2^2\left(\lambda_2\lambda(2N_{c_2}^2+2) + \lambda\lambda_2'\frac{4N_{c_2}^2-6}{N_{c_2}}\right)\\ &+ (Tr\Sigma_2^2)^2\left(\lambda_2^2(N_{c_2}^2+7) + \lambda_2\lambda_2'\frac{4N_{c_2}^2-6}{N_{c_2}} + \lambda_2'^2\frac{3N_{c_2}^2+9}{N_{c_2}^2}\right) + (Tr\Sigma_1^2)^2\lambda^2(N_{c_2}^2-1)\Bigg), \end{aligned} \tag{C65}$$

$$\begin{aligned} V_{\Lambda^S} = &-3g_1^2N_{c_1}(\lambda_1' Tr\Sigma_1^4 + \lambda_1(Tr\Sigma_1^2)^2 - \lambda Tr\Sigma_1^2Tr\Sigma_2^2)\\ &- 3g_2^2N_{c_2}(\lambda_2' Tr\Sigma_2^4 + \lambda_2(Tr\Sigma_2^2)^2 - \lambda Tr\Sigma_1^2Tr\Sigma_2^2), \end{aligned} \tag{C66}$$

$$\begin{aligned} V_{\Lambda_A} = &3g_1^4(N_{c_1}Tr\Sigma_1^4 + 3(Tr\Sigma_1^2)^2)\\ &+ 3g_2^4(N_{c_2}Tr\Sigma_2^4 + 3(Tr\Sigma_2^2)^2). \end{aligned} \tag{C67}$$

We redefine the constants as

$$\lambda_{1,2}' = (4\pi)^2\frac{\tilde{\lambda}_{1,2}'}{N_{c_{1,2}}}\times\frac{1}{t}, \quad \lambda_{1,2} = (4\pi)^2\frac{\tilde{\lambda}_{1,2}}{N_{c_{1,2}}^2}\times\frac{1}{t}, \tag{C68}$$

$$\lambda = (4\pi)^2\frac{\tilde{\lambda}}{N_{c_1}N_{c_2}}\times\frac{1}{t}, \quad g_{1,2}^2 = (4\pi)^2\frac{\tilde{\alpha}_{1,2}}{N_{c_{1,2}}}\times\frac{1}{t}, \tag{C69}$$

with all tilded quantities constants, and eventually took the large $N_{c_{1,2}}$ limit.

In the Veneziano limit the RGE are

$$-\tilde{\lambda}_1 = \frac{1}{2}\tilde{\lambda}_1^2 + 2\tilde{\lambda}_1\tilde{\lambda}_1' + \frac{3}{2}\tilde{\lambda}_1'^2 + \frac{1}{2}\tilde{\lambda}^2 - 12\tilde{\alpha}_1\tilde{\lambda}_1 + 36\tilde{\alpha}_1^2, \tag{C70}$$

$$-\tilde{\lambda}_2 = \frac{1}{2}\tilde{\lambda}_2^2 + 2\tilde{\lambda}_2\tilde{\lambda}_2' + \frac{3}{2}\tilde{\lambda}_2'^2 + \frac{1}{2}\tilde{\lambda}^2 - 12\tilde{\alpha}_2\tilde{\lambda}_2 + 36\tilde{\alpha}_2^2, \tag{C71}$$

$$-\tilde{\lambda} = \tilde{\lambda}\left(\frac{1}{2}(\tilde{\lambda}_1 + \tilde{\lambda}_2) + \tilde{\lambda}_1' + \tilde{\lambda}_2' - 6(\tilde{\alpha}_1 + \tilde{\alpha}_2)\right), \tag{C72}$$

$$-\tilde{\lambda}_1' = \tilde{\lambda}_1'^2 - 12\tilde{\alpha}_1\tilde{\lambda}_1' + 12\tilde{\alpha}_1^2, \tag{C73}$$

$$-\tilde{\lambda}_2' = \tilde{\lambda}_2'^2 - 12\tilde{\alpha}_2\tilde{\lambda}_2' + 12\tilde{\alpha}_2^2. \tag{C74}$$

The thermal mass is

$$V_T = \frac{T^2}{48}\left(\left(\lambda_1(N_{c_1}^2+1)+\lambda_1'\frac{2N_{c_1}^2-3}{N_{c_1}}-\lambda(N_{c_2}^2-1)+12N_{c_1}g_1^2\right)Tr\Sigma_1^2 \times\left(\lambda_2(N_{c_2}^2+1)+\lambda_2'\frac{2N_{c_2}^2-3}{N_{c_2}}-\lambda(N_{c_1}^2-1)+12N_{c_2}g_2^2\right)Tr\Sigma_2^2\right) \tag{C75}$$

and becomes in the Veneziano limit

$$V_T = (4\pi)^2\frac{T^2}{48\log T}\left(\left(\tilde\lambda_1+2\tilde\lambda_1'-\frac{N_{c_2}}{N_{c_1}}\tilde\lambda+12\tilde\alpha_1\right)Tr\Sigma_1^2+\left(\tilde\lambda_2+2\tilde\lambda_2'-\frac{N_{c_1}}{N_{c_2}}\tilde\lambda+12\tilde\alpha_2\right)Tr\Sigma_2^2\right). \tag{C76}$$

[1] G. Senjanović, arXiv:hep-ph/9805361.
[2] B. Bajc, arXiv:hep-ph/0002187.
[3] S. Weinberg, Phys. Rev. D **9**, 3357 (1974).
[4] R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **42**, 1651 (1979).
[5] R. N. Mohapatra and G. Senjanović, Phys. Rev. D **20**, 3390 (1979).
[6] P. Langacker and S. Y. Pi, Phys. Rev. Lett. **45**, 1 (1980).
[7] P. Salomonson, B. S. Skagerstam, and A. Stern, Phys. Lett. **151B**, 243 (1985).
[8] G. R. Dvali, A. Melfo, and G. Senjanović, Phys. Rev. Lett. **75**, 4559 (1995).
[9] G. R. Dvali and G. Senjanović, Phys. Rev. Lett. **74**, 5178 (1995).
[10] R. N. Mohapatra and G. Senjanović, Phys. Rev. D **21**, 3470 (1980).
[11] V. A. Kuzmin, M. E. Shaposhnikov, and I. I. Tkachev, Phys. Lett. **105B**, 159 (1981).
[12] V. A. Kuzmin, M. E. Shaposhnikov, and I. I. Tkachev, Phys. Lett. **105B**, 167 (1981).
[13] V. A. Kuzmin, M. E. Shaposhnikov, and I. I. Tkachev, Nucl. Phys. **B196**, 29 (1982); **B202**, 543 (1982).
[14] S. Dodelson and L. M. Widrow, Phys. Rev. Lett. **64**, 340 (1990).
[15] S. Dodelson, B. R. Greene, and L. M. Widrow, Nucl. Phys. **B372**, 467 (1992).
[16] J. w. Lee and I. g. Koh, Phys. Rev. D **54**, 7153 (1996).
[17] A. D. Linde, Phys. Rev. D **14**, 3345 (1976).
[18] H. E. Haber and H. A. Weldon, Phys. Rev. D **25**, 502 (1982).
[19] K. M. Benson, J. Bernstein, and S. Dodelson, Phys. Rev. D **44**, 2480 (1991).
[20] A. Riotto and G. Senjanović, Phys. Rev. Lett. **79**, 349 (1997).
[21] J. Liu and G. Segre, Phys. Lett. B **338**, 259 (1994).
[22] B. Bajc, A. Riotto, and G. Senjanović, Phys. Rev. Lett. **81**, 1355 (1998).
[23] B. Bajc, A. Riotto, and G. Senjanović, Mod. Phys. Lett. A **13**, 2955 (1998).
[24] B. Bajc and G. Senjanović, Phys. Lett. B **472**, 373 (2000).
[25] H. S. Kang and G. Steigman, Nucl. Phys. **B372**, 494 (1992).
[26] W. H. Kinney and A. Riotto, Phys. Rev. Lett. **83**, 3366 (1999).
[27] J. Lesgourgues and S. Pastor, Phys. Rev. D **60**, 103521 (1999).
[28] G. Barenboim, W. H. Kinney, and W. I. Park, Phys. Rev. D **95**, 043506 (2017).
[29] G. Barenboim, W. H. Kinney, and W. I. Park, Eur. Phys. J. C **77**, 590 (2017).
[30] G. Barenboim and W. I. Park, J. Cosmol. Astropart. Phys. 04 (2017) 048.
[31] H. E. Haber, Phys. Rev. D **26**, 1317 (1982).
[32] M. L. Mangano, Phys. Lett. **147B**, 307 (1984).
[33] B. Bajc, A. Melfo, and G. Senjanović, Phys. Lett. B **387**, 796 (1996).
[34] G. R. Dvali and L. M. Krauss, arXiv:hep-ph/9811298.
[35] B. Bajc and G. Senjanović, Phys. Rev. D **61**, 103506 (2000).
[36] G. Bimonte and G. Lozano, Phys. Lett. B **366**, 248 (1996).
[37] J. Orloff, Phys. Lett. B **403**, 309 (1997).
[38] T. G. Roos, Phys. Rev. D **54**, 2944 (1996).
[39] M. Pietroni, N. Rius, and N. Tetradis, Phys. Lett. B **397**, 119 (1997).
[40] G. Amelino-Camelia, Phys. Lett. B **388**, 776 (1996).
[41] M. B. Pinto and R. O. Ramos, Phys. Rev. D **61**, 125016 (2000).
[42] G. Bimonte and G. Lozano, Nucl. Phys. **B460**, 155 (1996).
[43] Y. Fujimoto and S. Sakakibara, Phys. Lett. **151B**, 260 (1985).
[44] K. G. Klimenko, Theor. Math. Phys. **80**, 929 (1989).
[45] M. P. Grabowski, Z. Phys. C **48**, 505 (1990).
[46] M. B. Gavela, O. Pene, N. Rius, and S. Vargas-Castrillon, Phys. Rev. D **59**, 025008 (1998).
[47] G. Bimonte, D. Iniguez, A. Tarancon, and C. L. Ullod, Phys. Rev. Lett. **81**, 750 (1998).
[48] S. I. Hong and J. B. Kogut, Phys. Rev. D **63**, 085014 (2001).
[49] N. Chai, S. Chaudhuri, C. Choi, Z. Komargodski, E. Rabinovici, and M. Smolkin, Phys. Rev. D **102**, 065014 (2020).
[50] A. Buchel, arXiv:2005.07833.
[51] A. Buchel, Phys. Rev. D **103**, 026008 (2021).
[52] K. G. Wilson, Phys. Rev. B **4**, 3174 (1971).

[53] K. G. Wilson, Phys. Rev. B **4**, 3184 (1971).
[54] S. Weinberg, in *General Relativity: An Einstein Centenary Survey*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, 1979), pp. 790–831.
[55] D. F. Litim and F. Sannino, J. High Energy Phys. 12 (2014) 178.
[56] D. F. Litim, M. Mojaza, and F. Sannino, J. High Energy Phys. 01 (2016) 081.
[57] K. Intriligator and F. Sannino, J. High Energy Phys. 11 (2015) 023.
[58] B. Bajc and F. Sannino, J. High Energy Phys. 12 (2016) 141.
[59] B. Bajc, N. A. Dondi, and F. Sannino, J. High Energy Phys. 03 (2018) 005.
[60] D. Orlando, S. Reffert, and F. Sannino, J. High Energy Phys. 08 (2019) 164.
[61] S. Abel and F. Sannino, Phys. Rev. D **96**, 056028 (2017).
[62] R. B. Mann, J. R. Meffe, F. Sannino, T. G. Steele, Z. W. Wang, and C. Zhang, Phys. Rev. Lett. **119**, 261802 (2017).
[63] G. M. Pelaggi, A. D. Plascencia, A. Salvio, F. Sannino, J. Smirnov, and A. Strumia, Phys. Rev. D **97**, 095013 (2018).
[64] F. Sannino, J. Smirnov, and Z. W. Wang, Phys. Rev. D **100**, 075009 (2019).
[65] S. Abel and F. Sannino, Phys. Rev. D **96**, 055021 (2017).
[66] A. D. Bond, G. Hiller, K. Kowalska, and D. F. Litim, J. High Energy Phys. 08 (2017) 004.
[67] G. Cacciapaglia, T. Ma, S. Vatani, and Y. Wu, Eur. Phys. J. C **80**, 1088 (2020).
[68] G. Cacciapaglia, T. Ma, and Y. Wu, J. Phys. Conf. Ser. **1271**, 012014 (2019).
[69] E. Molinaro, F. Sannino, and Z. W. Wang, Phys. Rev. D **98**, 115007 (2018).
[70] M. Fabbrichesi, C. M. Nieto, A. Tonero, and A. Ugolotti, arXiv:2012.03987.
[71] S. Abel, E. Mølgaard, and F. Sannino, Phys. Rev. D **99**, 035030 (2019).
[72] G. F. Giudice, G. Isidori, A. Salvio, and A. Strumia, J. High Energy Phys. 02 (2015) 137.
[73] D. D. Dietrich and F. Sannino, Phys. Rev. D **75**, 085018 (2007).
[74] S. Chaudhuri, C. Choi, and E. Rabinovici, arXiv:2011.13981.
[75] F. F. Hansen, T. Janowski, K. Langæble, R. B. Mann, F. Sannino, T. G. Steele, and Z. W. Wang, Phys. Rev. D **97**, 065014 (2018).
[76] E. Kiritsis and V. Niarchos, Nucl. Phys. **B812**, 488 (2009).
[77] A. Salvio and A. Strumia, J. High Energy Phys. 06 (2014) 080.
[78] M. E. Machacek and M. T. Vaughn, Nucl. Phys. **B236**, 221 (1984).
[79] M. E. Machacek and M. T. Vaughn, Nucl. Phys. **B249**, 70 (1985).
[80] L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974).