On Oskar Lange's Theoretical Positions on Equilibrium and Capital in Some 1930s and 1940s Writings

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Abstract: The paper addresses the hypothesis according to which the traditional equilibrium notion as a centre of gravitation (that is, a position the economy tends to realise over sufficient time under time-consuming adjustments and that forcibly needs a value specification of capital) still prevailed in neoclassical theory even after John Hicks's 1939 temporary equilibria approach became dominant within that theory. This is shown by examining Oskar Lange's work on equilibrium and capital, which has led us to conclude that despite him adopting the Hicksian approach in his 1944 essay, Lange continued to reason in terms of those traditional conceptions of equilibrium and capital.

1   Introduction

Specialised literature on the history of economic analysis agrees that, up to the 1930s, neoclassical authors conceived of equilibrium prices as a centre of gravitation of market values, reached by trial and error and over a sufficient interval of time. Equilibrium thus conceived must be stable and, above all, persistent (Garegnani 2002: 395). The latter condition emerges because the attainment of equilibrium does take time and, therefore, a realistic description of time-consuming adjustments requires that the speed of change of the data is of a lower order of magnitude than that of the change in equilibrium prices. These features of equilibrium have been traditionally justified by having recourse to a value specification of the capital endowment to the economic system among the theory's data that allows the physical composition of the capital stock to be *endogenously* determined. First, the theory could in this way determine 'normal' or cost-of-production relative prices – that is prices yielding a *uniform* return on the supply prices of the capital goods – conceived of as being the result of the persistence of dominant forces such as to consider the inevitable day-by-day changes in market values as accidental deviations from their normal trends. Second, while traditional neoclassical economists certainly admit that the position of rest will eventually be modified due to capital accumulation, this process is considered to be sufficiently slow so as to legitimately neglect it by *assuming* zero net accumulation when studying the main determinants of equilibrium prices and distribution. Finally, by treating the factor capital as a single scalar capable of changing form, the theory derives sufficiently elastic factor demand curves that plausibly justify the tendency towards full employment of resources.

It became apparent with the 1960s capital theory debates that the value specification of the capital endowment to the economic system entails insurmountable logical difficulties. Indeed, recognising this problem earlier, some economists of the Austrian School had in the 1930s began to specify the capital endowment as a collection of heterogeneous capital goods which by the 1950s became the norm in neoclassical general equilibrium analysis. This has led to a fundamental change in neoclassical equilibrium theory (Garegnani 1976): one in which the conception of a neo-Walrasian equilibrium as a position that the

economy tends to realise after sufficient time can no longer be plausibly justified. A central reason for this is that neo-Walrasian equilibria, which include both the 'temporary equilibrium' and the 'intertemporal equilibrium' models, suffer from insufficient persistence because the vectorial endowment of capital goods (and expectation functions in the case of temporary equilibria theory) is (are) bound to change very fast during the process of equilibration. The theory is forced to study the tendency towards equilibrium by introducing the fairytale assumption that the equilibrium set of prices is reached instantaneously through an auctioneer-guided process of equilibration. As real-time adjustments are time-consuming, these stability results, positive or negative, are devoid of implications for the real world.

Moreover, there are other problems which seem to seriously jeopardise the interpretability of neo-Walrasian equilibrium as a centre of gravitation. First, when each capital good is treated as a distinct factor of production, there will be insufficient factor *substitutability* (Garegnani 1990: 56-7): factor demand curves will be highly inelastic, which implies that, if the neoclassical assumption that factor rentals vary in accordance with their excess demands is adopted, some equilibrium rentals will likely be zero and also the wage rate may be zero, something that deprives the theory of plausibility, revealing the implausibility of that assumption itself. Second, without resorting to the notion of capital as a single factor, modern theory has no way of justifying why the interest rate can accommodate investment decisions with full-employment savings; accordingly, there is no reason to expect that the supply-and-demand forces will bring the economy towards a position of full employment. Third and finally, the inclusion of expectations among the data in the temporary general equilibrium (hereafter, TGE) versions of the theory introduces a serious element of *indeterminacy* into the theory.

On the other hand, however, it seems hardly likely that this abstract idea of an equilibrium that is instantaneously reached would have become dominant among neoclassical economists had they been fully aware of the above-mentioned shortcomings of neo-Walrasian equilibria. It seems much more likely, even after the Walrasian specification of capital started to become dominant in the profession, that the traditional conception of equilibrium and the corresponding belief in the long-run tendencies traditionally postulated by neoclassical authors still prevailed among them. As a result, the equilibrium determined on the basis of 'Walras capital' became incompatible with the notion of a centre of gravitation (Gehrke 2003; Petri 2004). In the present paper, we aim at exploring this hypothesis by reviewing and examining some of Oskar Lange's works on equilibrium and capital. This is attempted first because, to the best of our knowledge, there has not yet been a systematic examination of Lange's contributions on these important theoretical topics during this period, when the first serious concerns about how to treat capital in the neoclassical price theory were raised.[1] And second, because, although less well known than Hicks's, Hayek's or Lindahl's contributions, Lange's *Price Flexibility and Employment* (1944; hereafter, PFE) also belongs to the first generation of neo-Walrasian works that had considerable impact on subsequent neoclassical theory. For instance, the analytical framework of PFE is subsequently taken up and developed by Lange's pupil, Don Patinkin (1956), whose contributions were no doubt particularly important in spreading and consolidating the so-called 'neoclassical synthesis' in conventional macroeconomics.

In order to illustrate Lange's theoretical evolution, in section 2 we first review some of his 1930s articles, especially 'The Place of Interest in the Theory of Production' (1936a), but also some passages of 'On the Economic Theory of Socialism' (1936b), to show that Lange's early work falls fully within the realm of what we call traditional neoclassical economics. Accordingly, we attempt to show that Lange's approach demonstrates that a supply-and-demand equilibrium is susceptible to having the role of a centre of gravitation of actual variables *only* if the endowment of capital is specified in value terms. In sections 3 and 4 we examine PFE, where Lange analyses the stability of the neoclassical equilibrium by adopting the TGE framework under the influence of Hick's *Value and Capital* (1946 [1939]). In section 5 we compare Lange's approach in PFE with Keynes's and Patinkin's to further examine and discuss the aims, scope and limitations of Lange's work. In section 6 we draw the main conclusion of our argument, which can be summarised in the following way. Lange moves from a traditional Wicksellian position in his early writings to a neo-Walrasian TGE framework in PFE under the influence of Hicks (1946 [1939]), but without having a clear understanding of the implications of this shift. Despite the change in the datum relative to the capital endowment, Lange still envisages the several capital goods in a very traditional way; that is, as embodiments of a single factor of production that is capable of changing form without changing in quantity. Thus, on the one hand, the survival of this traditional notion of capital explains why Lange still envisages equilibrium as a position that the economy tends to realise only after a sufficiently long period of time. On the other hand, and quite paradoxically indeed, while in PFE Lange endeavours to strengthen Keynes's negative results regarding the stability of the supply-and-demand equilibrium, given that Lange still accepts the notion of capital as a single scalar, he must also accept the traditional neoclassical mechanisms and tendencies that stand or fall with this notion. We show therefore that his objections to the neoclassical claim that full-employment equilibrium is a position that the economy tends to realise are on weak grounds, since Lange's reasoning is essentially based on market failure stemming from purely subjective elements such as uncertainty, expectations and systematic mistakes, and, thereby, his critique loses much of its force.

2   Equilibrium as a Centre of Gravitation in Lange's Early Writings

In his early writings Lange envisages a supply-and-demand equilibrium as a centre of gravitation. Accordingly, the capital endowment is specified in value terms in his general equilibrium equations. The author provides three main reasons for this to be the case. First, the capital endowment must be so measured because the uniform return condition requires an endogenous determination of the composition of the capital stock. Second, because an endogenous determination of the composition of the capital stock also appears necessary to allow for enough substitutability among factors of production. Third, because Lange's model shows that the value measurement of capital is *necessary* to determine a position of rest that is persistent but not strictly stationary (that is, not stationary in a secular sense).

'Money-capital' as accumulated savings

It is the aim of Lange's (1936a) paper to shed light on the relation between general price theory and the theory of interest in neoclassical economics. Lange considers a competitive economy in which there are two industries: industry I, which produces the consumption good, 'wood' (x), and industry II, which produces the capital good

or 'real capital', 'axes' ($\bar{m}$). Production is assumed to take place in yearly cycles, and all capital is circulating. Wood is produced by direct labour (l) and axes (m), while axes are produced by axes (m') and what Lange calls 'indirect' labour (l'). Both the consumption and the capital goods are produced under well-behaved, constant-returns-to-scale production functions:[2]

$$x = F(m,l) \tag{1}$$

$$\bar{m} = \phi(m',l') \tag{2}$$

The assumption that the economy is in a self-replacing state implies that

$$\phi(m',l') = m + m' \tag{3}$$

Each entrepreneur maximises her own profits, but, in order to simplify the analysis further, Lange (1936a: 171, n. 1) only considers aggregate profits within each sector. He subsequently observes that the optimal methods of production chosen by firms are constrained by their disposal of 'money capital'. This is the necessary amount of value that will enable producers to purchase the required amounts of physical factors of production – axes and labour, since wages are assumed to be paid in advance -- which are determined by the optimal methods. Then, sector I (respectively, sector II) maximises profits π (respectively, π') subject to its availability of capital disposal κ (respectively, κ'):

$$\pi - \lambda\kappa = xp_x - mp_m - lp_l - \lambda(mp_m + lp_l) \tag{4}$$

$$\pi' - \lambda'\kappa' = (m+m')p_m - m'p_m(1+\lambda') - l'p_l(1+\lambda') \tag{5}$$

Where p_x stands for the price of wood, p_m for the price of axes, p_l for the wage rate and $\kappa = p_m m + p_l l$ (respectively, $\kappa' = p_m m' + p_l l'$) represents the quantity of 'money capital' available for sector I (sector II); λ and λ' are the Lagrange multipliers whose interpretation is discussed below. Under the admitted influence of Schumpeter (1934: 116-17), Lange characterises κ and κ' as 'a general "command over means of production"' (Lange 1936a: 173); but as he clarifies at the outset of the article:

> By 'money capital' in this paper the purchasing power at the disposal of entrepreneurs is meant and the only way to increase it is, according to our assumptions, through saving, i.e. through refraining from spending purchasing power for consumption and through transferring it to the disposal of entrepreneurs. (Lange 1936a: 160)

Like Schumpeter, Lange's conception of capital is a traditional one. Money capital simply represents *past* or *accumulated savings* that are, so to speak, *incorporated* in the 'real capital', that is, axes -- and, in Lange's example, also in advanced wages. Capital being heterogeneous must therefore be measured in the same units as are savings, some composite unit of consumption that, as the passage suggests,

individuals 'refrain' from consuming when they save. In the quotation above we thus find the idea that the services that could be used in the production of consumption goods, and are instead saved and devoted to the production of capital goods, have the same value as those consumption goods that are not produced.

While Lange initially assumes entrepreneurs from all industries have an exogenously given amount of money capital at their disposal, he ultimately suggests that it is actually '*the total amount of money capital in the whole economic system which is fixed and constant*' (Lange 1936a: 176; emphasis added). Thus, Lange's assertion amounts to saying that the capital endowment will be specified as an *aggregate sum of value* for the determination of normal prices. This means that,

$$K = \kappa + \kappa' \tag{6}$$

where K represents the value capital endowment of the economy and is a *datum* of the system. While the endowment of capital could be directly measured in technical units in the case of only one capital good, Lange *explicitly* remarks that this particular case:

> allows a considerable simplification ... while a generalisation of the theory to the case of production of... many real capital goods does not encounter any logical difficulty. (1936a: 159)

Then, in the general case, K must be necessarily measured as an amount of value. The first order conditions (FOC) of the maximisation problem for the firms in industry I are:

$$p_m = \frac{F_m p_x}{1 + \lambda} \tag{7}$$

$$p_l = \frac{F_l p_x}{1 + \lambda} \tag{8}$$

Respectively, for the firms in industry II:

$$1 = \frac{\phi_{m'}}{1 + \lambda'} \tag{9}$$

$$p_l = \frac{\phi_{l'} p_m}{1 + \lambda'} \tag{10}$$

The meaning of the FOC is postponed to the following section, since we must first interpret the multipliers λ and λ'. By simple manipulations we obtain:[3]

$$\lambda = \frac{d\pi}{d\kappa} \tag{11}$$

$$\lambda' = \frac{d\pi'}{d\kappa'} \tag{11'}$$

These multipliers are termed by Lange 'the marginal profitableness of the money capital invested' (Lange 1936a: 176). Indeed, given that an increase in k (κ') by

$\Delta \kappa$ ($\Delta \kappa'$) makes firms belonging to sector I (II) increase their profits by an amount $\frac{d\pi}{d\kappa}\Delta k$ ($\frac{d\pi}{d\kappa'}\Delta k'$), the multipliers measure the rate of increase of profits due to an increase in the amount of capital at the firms' disposal. In this way, the rate of interest that the firm would be willing to pay in order to borrow the extra sum $\Delta \kappa$ is determined. Hence, $\frac{d\pi}{d\kappa}$:

> is then the rate of interest paid for money capital. The Lagrange multiplier [λ] is thus the rate of interest on the money capital employed by firm I. (Lange 1936a: 176)[4]

Lange then argues that, through the force of competition among firms and lenders of money capital:

> the total fund of money capital is distributed among the firms so that each firm pays the same rate of interest, the marginal profitableness of money capital is the same for each firm. (Lange 1936a: 176)

This condition is formalised by

$$\lambda = \lambda' \tag{12}$$

Therefore, at a theoretical level, a first reason emerges as to why the capital endowment must be specified as a given quantity of value: equilibrium prices must yield a uniform return on the total amount of capital invested (K), which in this case equals the cost of axes plus the value of advanced wages. For this condition to be satisfied within the system, the quantity of capital goods – but also their form, when more than one kind of capital good is considered (recall that Lange considers the one-capital-good case as a simplifying assumption only) – and hence the quantity of labour used in the production both of axes (indirect labour) and of the consumption good (direct labour), must be endogenous variables, which are a *result* of the adjustment process. We add the full-employment condition

$$l + l' = L \tag{13}$$

and by setting $p_x = 1$, we have ten independent equations, namely, 1-3, 6-10, 12, 13 to determine the ten unknowns: $x, \overline{m}, m, m', p_l, p_m, l, l', \lambda, \lambda'$.[5]

The role of the direct and indirect labour

Lange then proceeds to investigate the effect of a decrease or 'shortage' in the quantity of capital (K) on the interest rate (λ) and hence on the profit-maximising methods, by examining how a change in λ affects the distribution of the given endowment of labour between its *direct* and *indirect* employment. We must first notice, however, that, according to the conditions (7)-(10) determined above, equilibrium requires that 'the prices of the factors of production are equal to the discounted value of their marginal product' (Lange 1936a: 176). Given that wages are paid in advance, this equilibrium condition implies that the marginal product of direct labour is discounted once while indirect labour is discounted twice at the uniform rate of interest λ in order to be made equal to the wage. This can be readily

seen by inserting (10) into (7) to obtain $p_l = \frac{p_x F_m \phi_{l'}}{(1+\lambda)^2}$, where the right-hand side of the equation measures the value marginal product of indirect labour ($p_x F_m \phi_{l'}$) which is discounted twice at the rate λ. On the other hand, the equality between the wage rate and the value marginal product of direct labour ($p_x F_l$), which is discounted only once, is directly given by equation (7), that is, $p_l = \frac{p_x F_l}{1+\lambda}$. Equilibrium thus requires that:

$$\frac{p_x F_m \phi_{l'}}{(1+\lambda)^2} = \frac{p_x F_l}{1+\lambda} \tag{14}$$

in which the discounted value marginal product of direct labour equals the discounted value marginal product of indirect labour. Lange thus goes on to argue that a decrease in K raises λ and, because of (14), the marginal product of direct labour will rise *relative* to the marginal product of indirect labour (Lange 1936a: 180). To restore equilibrium, his argument goes, it is necessary that 'a shift of the labour resources towards their direct use' (Lange 1936a: 180) takes place. (The contrary is the case when the rate of interest falls after an increase in the quantity of capital).

The general validity of this adjustment process does not concern us here.[6] The point we wish to make is that the description of the working of the substitution mechanisms in terms of direct and indirect labour inputs clearly shows the Wicksellian influence on Lange's work (as Lange himself admits in several parts of his essay; see Lange 1936a: 169, n. 4)[7]. His recurrent reliance on the traditional substitution mechanisms makes it easier to understand why Lange is forced to determine the composition of the capital stock endogenously. On the one hand, given that the (discounted) marginal product of labour must be the same in its direct and indirect employments, the distribution of these labour inputs in the different industries must be endogenously determined. And in the many-capital goods case,[8] this requires that not only the quantities but also the *forms* of the capital goods which are used and reproduced in equilibrium by means of indirect labour, be endogenously determined magnitudes. Moreover, there is a second, independent, reason for this to be the case: enough substitutability among productive factors requires an endogenous determination of the composition of the capital stock because different productive methods will usually call for use of different kinds of capital goods. Although this point is not explicitly discussed in the 1936a article, the issue does emerge in Lange's (1937b) reply to Knight's (1937) review of that contribution. Knight objects that 'the only use of capital admitted in Dr. Lange's set-up is that of increasing the number of capital instruments of a given kind (axes in his illustration)' (Knight 1937: 224). But Knight correctly notices that a more realistic model must consider that:

> the addition of new capital to a system will generally involve both new forms of equipment for making old products and new forms of final product. (Knight 1937: 224)

In his reply, Lange (1937b: 234) admits that 'additional capital may be used, as Professor Knight is quite right in observing ... to make new forms of products', but he goes on to argue that the main insights of his simplified model would not be

substantially modified under this more realistic set-up: given that an increase in capital in any case implies a decrease in the rate of interest, the re-establishment of equilibrium still requires that factors are shifted from their direct to their indirect employments, so that their respective value marginal products can be re-equalised among industries. All this suggests that Lange is aware that an endogenous determination of the capital stock is also necessary to justify, in a plausible way, the working of the factor substitution mechanisms.

A persistent, non-stationary equilibrium

In the last section of his 1936a article, Lange (189-92) addresses a third and last feature of the centre-of-gravitation notion of equilibrium that explains why a value capital endowment must be among the data of the equilibrium rather than being endogenously determined. In long-period equilibrium, he writes:

> the amount of capital is assumed as fixed and interest is deduced from the assumption that this amount is less than the amount required to saturate production with capital. (Lange 1936a: 190)

On the other hand, when the secular conditions of accumulation are examined:

> The amount of capital is itself a variable to be determined by all the equations of economic equilibrium. In long-period equilibrium all adjustments are accomplished and the economic system becomes stationary. (Lange 1936a: 190)

It is important to notice that, as Lange explicitly states (Lange 1936a: 191, n. 1), the expression 'long-period equilibrium' refers to a *secular* equilibrium. What is more, this notion is clearly distinguished from the 'static' long-period equilibrium in the sense used by traditional neoclassical authors as a situation where the quantity of capital is given while its 'form' changes so that a uniform return on the capital goods' supply prices is obtained. As also clarified by Lange (1936a), this notion of 'static' long-period equilibrium is the one he uses in the 1936a article. Therefore, while in long-period equilibrium in this latter sense the 'amount of capital is assumed to be fixed', when the secular conditions of accumulation are examined this variable 'is determined by all the equations of economic equilibrium'; that is, it becomes an endogenous magnitude. Lange subsequently explains that:

> The way towards a long-period [secular] equilibrium with regard to interest is necessarily a slow one. For capital accumulation adds per annum but a small fraction to the existing stock of capital. As the annual accumulation is but a small fraction of the existing stock of capital the movements towards a long-period equilibrium in interest must be of a secular character, too. (1936a: 191)

And he concludes that the study of these secular trends of accumulation 'is a topic for a separate study' (Lange 1936a: 191, n. 2). In line with this approach, in his intervention in the socialist calculation debate with the Austrian School, Lange (1936b: 58) clears up his argument further and, after remarking that the 'ultimate resources' are 'labour, capital and natural resources', notes that:

> As to capital, its amount may be regarded in the short-period as constant, whereas in the long run the rate of interest certainly affects savings. In long-period [secular] equilibrium the amount of capital is determined by the

> condition that the rate of its marginal net productivity (the interest rate) is equal to the time preference of the individuals. (1936b: 58, n. 3)[9]

These pieces of evidence show that Lange's approach is fully traditional; he thus shares the idea held by traditional authors that the process of capital accumulation is 'slow' enough so that, as a first analytical approximation, the theory can neglect this issue – by *assuming* zero net savings, as implied by his writing 'constant' – when explaining normal prices and distribution. The persistent, slowly changing quantity of value capital implies that those secular movements related to significant changes in the capital disposal of the economy could be safely dealt with later on, in a subsequent analytical stage. Additionally, Lange openly acknowledges in the latter citation that the assumed constancy of the factor capital is perfectly compatible with capital accumulation over time, and hence that the equilibrium thus determined should not necessarily be stationary. Indeed, according to Lange, capital accumulation would come to a halt only when the marginal productivity of capital would equal the time preference of the individuals. Yet, in the shorter run, whatever the discrepancy between these magnitudes, capital accumulation 'adds per annum but a small fraction to the existing capital stock', so equilibrium prices determined under the assumption of zero net accumulation (that is, of a *given* quantity of value capital whatever prices and distribution) could be safely assumed to describe the centre of gravitation of market prices over a sufficient interval of time. Moreover, because actual economic conditions are far from being stationary, Lange might have rightly argued that a characterisation of a secular equilibrium would give no indication of the actual trend of real economies. This position of Lange is fully traditional.[10]

In a nutshell, Lange's aim, on the one hand, is to characterise a persistent position of rest by determining relative normal prices and, on the other hand, his awareness that a secular equilibrium would be achieved, if at all, only in the very long run, forcibly led him to place interest rate theory in the realm of the traditional, static or long-period notion of equilibrium. Thus, we submit, although Lange is aware that, when dealing with secular conditions, the 'amount of capital is itself a variable to be determined', still, in his equilibrium system, the author has to resort to a specification of capital as a single, exogenously given magnitude in value for the determination of prices and distributive variables (wages and interest rates).

3   The Shift in Method in Lange's *Price Flexibility and Employment* (1944): Significant Change in Form, Minor Change in Content

We now turn to examine Lange's approach to the problem of equilibrium and capital as dealt with in PFE. The aim of PFE is to assess the neoclassical claim that, if not disturbed, the forces of supply and demand will eventually push the economy towards a position of full employment. Under this view, Lange explains, unemployment can only be the result of the 'rigidity of factor prices' (PFE: 1). However, Lange (PFE: 1) well recognises that this vision has been 'subjected to serious criticism', for instance, by Keynes in the *General Theory* (1936). Indeed, it is well known that in chapter 19 of his masterpiece, Keynes examines the possible effects on unemployment of a decrease of money wages, and he concludes that the effects are likely to be negative. PFE can therefore be read as an attempt at re-examining these results, but resorting to the method of temporary equilibrium developed by Hicks (1946 [1939]).[11]

Lange in fact underlines that Hicks's *Value and Capital* (Hicks 1946 [1939]) is the 'most up-to-date formulation' of the theory of general equilibrium (PFE: Preface). Lange thus abandons the traditional neoclassical framework used in his early writings and adopts, instead, Hicks's TGE method. Accordingly, he specifies the capital endowment in physical terms, along with the expectation functions, among the data. Yet, it seems relevant to recall that, in the first article where Hicks seriously attempts to develop a TGE, 'Wages and Interest: the Dynamic Problem' (1935), he justifies the relative convenience of the TGE approach over the traditional concept of equilibrium on the grounds that the latter approach would have fallen under a 'condemnation', that is to say, it would have been 'quite incompetent to deal properly with capital, or interest' outside secular equilibria (Hicks 1935: 456-7; 1946 [1939]: 116). Hicks does not object to the internal consistency of the theory since he freely admits that the traditional neoclassical theory of capital is a 'plausible theory for the stationary state' (1946 [1939]: 119). Having wrongly restricted traditional neoclassical economics to the realms of secular stationary states, Hicks only disputes the relevance of the traditional concept of equilibrium for understanding the trends of prices and distribution ruling in actual economies. However, our perusal of Lange's 1936 articles reveals that the Polish author was at that time well aware that a long-period position is *not* a secular equilibrium.

Why, then, in 1944 does Lange finally come to adopt the TGE as *the* central concept of equilibrium? Although Lange does never explicitly justify his advocacy of the TGE method, we presently suggest three main reasons. First, because the TGE approach allows him to explicitly introduce price and interest rate expectations into the analysis which, after Keynes (1936), had become a central topic in economic theory. Second, because by 1944 Lange seems to be more aware of the difficulties behind the specification of the capital endowment as a given amount of value. Finally and we submit, more importantly, because Lange still reasons as if the changes in the data with respect to the factor capital do not cause major modifications of the theoretical structure of the supply-and-demand approach.[12]

The problem relative to measuring the capital endowment in value terms: the investment function

A first indication that Lange is aware of the difficulties of conceiving the capital endowment as a single factor of production can be found in his assessment of the stability of the temporary equilibrium; in particular, in the way he justifies the negative elasticity of investment demand. From the outset of his work, Lange points out that traditional neoclassical theory failed to properly deal with the role of money, and hence that it is the substitution among money, commodities and bonds which 'provides the key for understanding the equilibrating as well as the disequilibrating processes of the economy' (PFE: Preface). According to Lange, if unemployment causes wages and prices to fall, a position of full employment will eventually be re-established if individuals attempt to decrease their money holdings, or cash balances (CB), when prices (P) fall, that is to say, if they buy commodities or bonds. In the first case, aggregate demand for commodities increases and the subsequent attempt of producers to satisfy the demand forthcoming ends up by raising labour demand and wages. The price level eventually returns to its old equilibrium position, compatible with the full employment of resources. If, instead, individuals spend their excess of money holdings on buying bonds, the rate of interest will decrease and investment demand will increase. Therefore, equilibrium

will be re-established if, for a given money stock (M'), the ratio of M' to CB must then rise. Following Lange (PFE: 7), we shall say that when this condition is satisfied, the monetary effect is positive. The case of a positive monetary effect is, according to Lange, the normal, established neoclassical adjustment mechanism. If, on the contrary, this ratio decreases as prices fall, the monetary effect is negative. We thus have:

$$\begin{cases} \textit{Positive monetary effect} : \nabla P \Rightarrow \Delta\left(\dfrac{M}{CB}\right) \\[2ex] \textit{Negative monetary effect} : \nabla P \Rightarrow \nabla\left(\dfrac{M}{CB}\right) \end{cases}$$

As will be shown in section 4 below, Lange argues that the monetary effect may be negative such that equilibrium can be unstable, but only owing to the possible disturbing effects of price and interest rate expectations and uncertainty. Accordingly, Lange does not object to the validity of the neoclassical mechanisms if the monetary effect is positive; in particular, he does not question the traditional role of the rate of interest in bringing into equilibrium investment and saving decisions.

What we want to stress is that Lange does not justify the negative interest-elasticity of investment demand in the traditional way, namely, by arguing that a decrease in the rate of interest induces firms to adopt more capital-intensive methods of production. Instead he assumes that expected prices are independent of the rate of interest. Essentially under the influence of Hicks (1946 [1939]: chapter XVII), Lange argues that as the interest rate decreases, for given undiscounted (expected) capital goods prices, the present value of their expected market price increases relative to their current market price. In this way, profit-maximising firms are induced to 'tilt' their production plan to the right: they substitute inputs *over time* by shifting their purchases of capital goods to the present in order to increase their future production of output and, accordingly, increase current investment demand.

Now, on the one hand, it is certainly true that this justification of the negative interest elasticity of investment is foreign to traditional neoclassical thinking. Neglected by Lange, if intended to represent the action of persistent forces at work, the negative relation between investment and the interest rate must be derived under the assumption of cost-of-production relative prices (Ackley 1978: 623-34).[13] However, on the other hand, we wish to argue that the traditional neoclassical conception of capital as a single value-magnitude, which is embodied in the several physical capital goods and inversely varies with the rate of interest, is *still* present in Lange's analysis of investment. First, although it takes a different form, the traditional Wicksellian claim that it is possible to order the different production methods according to their degree of 'roundaboutness' is still there: when the rate of interest decreases, the 'tilting' (Hicks 1946 [1939]: 216-17) of the production plan to the right causes the *average* time-lag between inputs ('investment goods' in capital), and outputs, to increase.

But in order to confirm that in his analysis of investment Lange still envisages the different capital goods as embodiments of a single factor, we must turn to consider Lange's (PFE, chapter IX) analysis of the potential effects of a decrease (increase) in the aggregate marginal propensity to consume (or save). Not only does Lange argue that the *level* of current consumption is reduced as a result of the shift in individuals' preferences, but he also notices that the *composition* of consumption will generally change.[14] In addition, since consumption goods are produced by

capital goods, changes in consumption composition will eventually be reflected in the dynamics of the savings-investment market. First, the aggregate level of savings becomes higher than aggregate investment; but second, the *composition* itself of investment will sooner or later start changing as entrepreneurs attempt, in due course, to satisfy the new kinds of consumption goods desired by the community. Lange (PFE: 53, n. 11) in fact admitted earlier that it may well happen that decreases in the aggregate marginal propensity to consume 'may be associated' with *increases* in the demand for some consumption goods. And, therefore, owing to what he calls 'the principle of derived demand' (PFE: 53, n. 13), the demand for the investment goods used in the production of those consumption goods whose demand has increased, will increase as well.[15] Lange (PFE: 55, n. 19) goes on to argue that, as long as the money rate of interest remains constant, there will be a 'downward Wicksellian process' because the excess of aggregate savings over investment will cause the 'natural rate' of interest to decrease. He writes:

> the excess supply of investment goods, resulting from a propensity to consume [that is below the] equilibrium requirement, lowers the 'natural rate'. (PFE: 55, n. 19)

Note first that, while Lange does not explicitly refer to the determinants of the natural rate, the reference to Wicksell makes it clear that the natural rate must be determined by the marginal product of capital, the single homogeneous magnitude, which in Wicksell (1934 [1901]) is unambiguously measured as an amount of value. In any case, although Lange speaks of an 'excess supply of investment goods', it seems to be sufficiently clear that these 'investment goods' are envisaged as embodiments of a single factor 'capital'. It is the additional savings, namely, the supply of capital as a flow, and not the relative abundance of the single 'investment goods', which pushes the 'natural' rate downwards. In fact, given that Lange openly admits that changes in the aggregate marginal propensity to consume will change the composition of investment unpredictably, unless he treats each single capital good as an embodiment of a single factor, he can reach no general conclusion regarding the effects of changes in the forms and quantities of the investment goods on the rate of interest.[16]

The analysis of accumulation

It is to Lange's analysis of accumulation that we must now turn to find the author of PFE *explicitly* admitting the problem behind the measurement of the factor capital as a single magnitude, albeit the issue is confined to a footnote. Consider what Lange writes when he deals with the effects of capital accumulation on employment:

> It is thus assumed in the text that none of the investment goods decreases in stock. Actually, capital accumulation need not imply this restrictive assumption. All that is necessary is an increase of the aggregate real value of the stock of investment goods in the community. The latter, however, leads straight into the tricky subject of real aggregates, namely the problem: what is to be meant by the aggregate quantity of capital in the community? The whole difficulty is avoided by the assumption made in the text, without loss of any significant aspect of the problem under discussion. (PFE: 67, n. 4)

On the one hand, the passage openly shows that, by 1944, Lange is aware that a value specification of the capital endowment is problematic for the theory. On the other hand, however, closer inspection of his analysis of accumulation clearly suggests that Lange believes that the results can easily be proven on the basis of the assumption that 'none of the investment goods decreases in quantity' is sufficiently general; that is, 'without loss of any significant aspect of the problem under discussion'. In other terms, the author envisages the process of accumulation *as if* things would continue working in the way justified by traditional theory, that considers the different capital goods as embodiments of a single factor, *value* capital.

Lange claims that decreasing returns must cause the marginal value productivity of the whole capital stock to decrease. Since Lange argues that 'the demand for an investment good, like that for any other factor of production, is determined by the equalization of the marginal value productivity with the price of the good', then, due to the law of diminishing returns, 'an increase in the stock of (some or all) investment goods that is not accompanied by a proportional increase in the supply of primary factors [labour or land] leads to a decline in the marginal physical productivity of the former' (PFE: 67-8). For given selling prices of the capital goods, 'this implies a decrease in their marginal value productivity and also ... in the demand for them' (PFE: 68). But then, Lange proceeds, spare capacity means that there is also labour unemployment, money wages thus fall and this 'is bound to reduce the cost of production, and thus also the prices of investment goods to an extent that compensates for the decline in their marginal productivity' (PFE: 68). Therefore, in the new equilibrium, the new capital stock and the labour force can both be fully employed.

In PFE then, Lange essentially accepts the traditional neoclassical idea of a decreasing marginal productivity of capital, which is here derived by assuming that 'some or all' of the capital goods increase in quantity with capital accumulation. This implicitly assumes that the changes in the kinds of capital goods in existence, generally brought about by changes in income distribution, can be neglected because they entail no difference relative to a situation in which there is no change in the physical composition of capital, which amounts to treating the different capital goods as portions of a single factor. But Lange must proceed in this way because, as capital accumulation takes place, there is no *a priori* reason to presume that none of the quantities of the single capital goods will fall. It will generally happen that some capital goods will increase in quantity, some others will fall, some will appear that were previously not utilised at all and others will disappear completely. In fact, we have seen that in Lange (1936a) a plausible foundation for the factor demand curves requires that the substitution mechanisms should encompass an endogenous determination of the composition of the capital stock when the total amount of capital in the economy changes, as is the case with capital accumulation. Then, while it is clear that an explicit specification of the capital endowment in value terms would have directly made Lange face 'the tricky subject of real aggregates' (an issue which, as we have seen, Lange tries by all means to bypass), he continues to think of the different capital goods as representing a homogenous factor of production.

The main outcome of the assumption that 'some or all investment goods increase in quantity as accumulation proceeds' is that the traditional conclusion that accumulation must decrease the marginal product of capital is preserved. That is to say, Lange reasons as if the more relevant outcomes of traditional theory – the negative relationship between the demand for productive factors and their rental prices – are preserved, and therefore also preserved is the notion that the economy

gravitates around a full-employment-growth path, despite his attempt to abandon the notion of capital as a single factor of production. Otherwise he could not conclude that the assumption regarding the evolution of the form of the existing capital stock entails no 'loss of any significant aspect of the problem under discussion'. In other words, he denies or simply forgets that no general conclusion can be reached regarding the effects on marginal products and on prices of the evolution of the existing forms of the capital goods as accumulation takes place. To arrive at this result, Lange must assume that these effects are the same as if the several capital goods could be treated as elements of a single factor 'capital', like he had assumed in his earlier writings where this factor 'capital' was a quantity of value. So, Lange is conscious that, unless he makes some restrictive assumption ensuring that things work out as if capital were a single factor, he cannot go on reasoning in the old, Wicksellian, way. This implies that he is aware that there is no guarantee that things work out as he wishes in the absence of the restrictive assumption; but then he is implicitly admitting that he has no right to make the restrictive assumption! And yet he makes it, evidently because of a faith that it cannot be that things work differently.

Some 'methodological' difficulties ignored by Lange in PFE

In the previous sub-section we showed that despite the formal change in the data relative to the factor capital, in PFE the physical capital goods are still seen as a single composite quantity of capital. This may help us explain why Lange does not show any concern with respect to the impermanence problem, nor with respect to the substitutability problem.[17] In fact, Lange's aim to represent plausible, time-consuming adjustments leads him to consider a balancing process where actual transactions and productions may generally take place at out-of-equilibrium prices. Consider, for instance, his description of the adjustment process to the new conditions after the aggregate marginal propensity to consume decreases. He says that:

> at the old output, the demand price for (at least) some final products is reduced, while the demand schedules for all other final products (and direct services) remains unchanged. As long as the prices of the factors of production and of all other goods (including securities) are unchanged, this results, under conditions of perfect competition … in a contraction of output of the products for which there has been a decrease in demand. In consequence, the demand for the factors of production used in making these products also decreases. This causes excess supply of … these factors and a fall of their prices. Substitution of these factors for other factors and expansion of output of commodities produced with them is attempted and causes a decline of the prices of the other factors. At the same time, marginal costs are reduced on account of lower prices. This causes an attempt to expand the output of (at least some) products. (PFE: 53)

The passage suggests that Lange is thinking of mechanisms which only make sense in a long-period framework in which it takes a sufficient interval of time for firms to distinguish between transitory and persistent shifts in tastes, and also to realise which specific consumption goods have permanently decreased in demand. It may also take considerable time for rentals of unemployed factors to decrease. In turn, only after sufficient time will the firms' initial 'attempt' to utilise more intensively those factors whose prices have decreased effectively materialise, and also the 'attempt' to expand the outputs of those consumption goods whose demand

increases on account of their lower prices. During the adjustment process to the new conditions, mistakes on the part of firms usually occur, and transactions and production decisions generally take place at 'false prices'.

That Lange is thinking of mechanisms that may take considerable time to assert their effects fully is openly admitted in the last chapter of PFE, when the author relies on the TGE framework developed in the previous chapters to explain the behaviour of actual economies over the last two centuries. He thus argues that during the period from the 1840s until 1914 'there are good reasons to believe that [the conditions that ensure the tendency towards the full employment of resources according neoclassical theory] were approximately realized in the long-run' (PFE: 83). Accordingly, while during the 'short-run' the economy was subjected 'to strong fluctuations of employment, output, and prices' (PFE: 84), still 'price flexibility operated as a long-run stabilizing force in the economy' (PFE: 84). Lange thus openly accepts that, granting stability, 'price flexibility' allows equilibrium to assert itself only after sufficient time. Then, this mistaken idea that, if stability can be assumed, the TGE framework can easily accommodate a realistic, time-consuming, balancing process as the one that may take place in real economies, helps explain why in PFE 'false' production and trading are not explicitly forbidden. This contrasts with Hicks's approach in *Value and Capital*, which *de facto* considers that equilibrium is instantaneously reached by assuming an economy that is 'always in equilibrium' (Hicks 1946 [1939]: 131).[18]

Therefore, there is some ground for believing that Lange does not notice that his assumption of time-consuming adjustments and, accordingly, of 'false-price' trading, is in sharp contradiction with the TGE approach of a given vectorial endowment of capital goods and given expectation functions, due to the lack of persistence of these latter variables. In other words, Lange's lack of awareness of the potential problems of path dependency, and hence of the indeterminacy of the equilibrium, must certainly be an important reason why he still assumes time-consuming adjustments, despite that the change in the data relative to the factor capital no longer allows him to proceed that way. Moreover, it is also worth pointing out that, despite his careful analysis of the substitution mechanisms within the neoclassical approach in his early articles (see section 2 above), and which generally entails a change in the composition of the existing capital stock, Lange does not seem to notice that the possibilities of substitution between capital inputs and labour will actually be very limited, due to the specific character of the capital goods that are arbitrarily given for the first periods of the TGE. This, in turn, would mean implausibly low-value equilibrium factor rentals, possibly zero.

4   Lange's Objections to the Self-adjusting Nature of the Market

As we have seen in section 2 above, Lange argues that it is only under a 'positive monetary effect' that equilibrium is stable. In Lange's view, neoclassical authors concluded that the monetary effect is positive because they implicitly relied on special assumptions with respect to: i) the role of the monetary authorities in controlling the stock of money; ii) the price and interest rate expectations functions of the individuals; iii) risk and uncertainty. Taking these three features into account, Lange's objections can be summarised in four main arguments.

First, Lange argues that, even in an economy without bonds, when P falls, the monetary effect may be negative: although the desire for CB decreases, the monetary authorities may well decide to decrease the existing M^s by a greater

proportion. However, Lange immediately recognises that, while this possibility cannot be excluded *a priori*, neoclassical authors assumed that M^s remained constant and, therefore, the conclusion that equilibrium is stable 'appears to be fully justified' (PFE: 14). It is thus to the remaining objections that we must now turn to understand the crucial reasons why Lange questions the general validity of the above-discussed mechanisms.

In his second and third strands of criticism, Lange points out that neoclassical theory overlooked the crucial role played by price and interest rate expectations. When this role is duly considered, Lange believes the monetary effect may well be negative. The author thus asserts that a positive monetary effect relies on the implicit assumption of *static* price expectations (PFE: 20) in the sense that current prices are assumed to hold also in the further future.[19] Then, the second destabilising cause may arise due to elastic commodity-price expectations. If the value taken by these expectations is higher than one, when the average price level falls, agents expect the future average-price level to fall even further, and hence, by shifting their purchasing decisions, both on consumption and on investment goods, from the present to the future, they find it optimal to *increase*, rather than to decrease, their current money holdings as stability of equilibrium required.[20] Planned sales, on the other hand, are shifted from the future to the present (PFE: 22). Assuming the stock of money to be constant, shifts in both purchases and sales therefore imply a negative monetary effect. Only when the elasticity of price expectations is equal to or smaller than one does a decrease in the average price level raise the ratio of the monetary stock to the desired cash balances.[21]

The third possible destabilising force may come from the action of expectations over the interest rate. If, when money prices fall, individuals attempt to decrease their cash balances by buying bonds, the current interest rate will fall and, for *given* (expected) consumption good and input (that is, capital goods) prices, this will increase the price of inputs used in the immediate future relative to the price of current inputs, and also raise future output prices relative to their present prices. Therefore, firms will attempt to tilt their production plan to the right: on the one hand, aggregate planned purchases of capital goods will be shifted to the present whilst, on the other hand, planned sales will be shifted to the future.[22] The overall effect will thus be a decrease in current money holdings as the tendency towards equilibrium requires (PFE: 28). However, Lange warns us that, if interest rate expectations are sufficiently inelastic (zero in the limit), the expected interest rates in the more distant future and, therefore, the discounted prices ruling in those more distant periods, will hardly vary relative to present prices; the tilting of the production plan to the right due to the process of intertemporal substitution will be negligible in practice, and investment demand will not sufficiently increase so as to re-establish full employment.

In his fourth and last objection, Lange (PFE: 55-9) argues that even under the assumption of a positive monetary effect, full employment may not be spontaneously reached by market forces due to the destabilising roles of risk and uncertainty, since 'the intertemporal substitution resulting from the fall in interest rates may be very small and practically even negligible' (PFE: 60). Firstly, uncertainty shortens the 'economic horizon of the firm' (PFE: 60) and, since in Lange's view (PFE: 61) the discounted values of the expected prices in the near future are hardly affected by changes in expected interest rates, this implies that 'the intertemporal substitutions that fall out are the most important ones' (PFE: 61). Secondly, due to uncertainty, the excess of money holdings might be well directed

towards short-term rather than long-term bonds, and, again, given that investment decisions are spread over long periods, Lange stresses that in order for intertemporal substitution to work properly 'it is necessary that the corresponding long-term rates of interest ... decline' (PFE: 62). Finally, if the increase in the demand for bonds has a short-term bias and, accordingly, fails to induce long-term interest rates to fall, the effect on real investment would be very limited.

5   Some Remarks on Lange's Equilibrium and Stability Analysis: Lange, Keynes and Patinkin

The aim of this section is twofold. First, given the similarity between Lange's and Keynes's objections to neoclassical theory, a brief comparison between these two authors is useful to further understand Lange's position on equilibrium theory. Second, given that Lange (1944) reasons as if the notion of capital as a single quantity holds, and hence he accepts the pillars of neoclassical theory, we argue that Lange's critiques of the neoclassical approach are weak. To sustain our claim, we show how these critiques were subsequently overcome by the so-called neoclassical synthesis by reference to Patinkin's classic *Money, Interest and Prices* (1956).

Lange and Keynes

As hinted in section 3 above, most of Lange's negative results regarding the tendency towards a supply-and-demand equilibrium are anticipated in chapter 19 of the *General Theory*.[23] The main difference between Keynes and Lange lies in the way these scholars interpret these outcomes. As noted by Garegnani (1988: 220; also Aspromourgos 1997: 123), from the instability results with flexible wages and prices, Keynes draws the *conclusion* that wages must be rigid downwards in the sense that the forces of supply and demand are unable to account for the trends in income distribution in market economies and, hence, there must be other factors that persistently explain these trends. Otherwise one would reach the absurd result that wages and prices would indefinitely fall in the presence of unemployment, something that has no correspondence with observed facts. This point is summarised by Klein (1947):

> Within the framework of Keynesian economics wage flexibility does not correct unemployment and leads merely to hyper-deflation if carried to its logical conclusion. But in the real world one observes neither hyper-deflation nor full employment. The explanation is that wages *are* sticky; they are not flexible... Because workers do not bid against one another, we do not experience the hopeless downward spiral. (Klein 1947: 90)

This Klein passage traces the line that divides Keynes from Lange: unlike Keynes, in PFE, Lange does not carry his results to their 'logical conclusion', he only limits himself to adopt the *normative* position that wages should be sticky as a matter of policy (PFE: 87-8). It never occurs to him that the instability results he reaches in his work are in fact an *expression* of a more fundamental issue, namely, that the forces that determine income distribution *are not* supply-and-demand forces. By not reaching this conclusion, Lange implicitly reveals his faith in the neoclassical approach. And in fact, Lange expresses this faith when he argues that, under its own premises regarding expectations and uncertainty, neoclassical theory 'is perfectly valid' (PFE: 65).[24]

Lange, Patinkin and the neoclassical synthesis

We now proceed to assess the strength and limitations of Lange's critiques of neoclassical theory owing to the possible destabilising effects of expectations. It may be interesting to bring Patinkin's *Money, Interest and Prices* (1956) into the analysis. As argued in our introduction, Patinkin's considerations are particularly relevant for our discussion because, as a student of Lange, he takes up and develops the analytical framework of PFE. In his influential 1956 contribution, Patinkin argues that 'expectations are not pulled out of the air, but are related to past price experience' (1956: 311); he therefore denies that interest rate expectations may exert an autonomous and long-lasting disturbing influence on the economy because these subjective elements eventually accommodate to variations in the objective factors of the theory such as the marginal productivity of capital. This is how Patinkin interprets the 'liquidity trap': as soon as the current interest rate falls, individuals expect it to return to its old level in the future (whereas in Lange's PFE, the 'trap' corresponds to the case of zero-interest-rate expectations). But Patinkin (1956: 354) goes on to argue that there is an objective reason for this to be the case: the minimum rate of interest below which individuals are unwilling to hold their wealth in the form of bonds 'is not an absolute constant but reflects the public's state of expectations at the point of time in question'. And:

> Since expectations, in turn, are based on historical experience, it follows ...
> that the rate of interest ... can be more appropriately interpreted as reflecting
> a real phenomenon: namely, the fact that the productivity of capital has
> historically been higher than (say) 2 per cent, so that when the interest falls to
> the neighbourhood of this level the public anticipates a subsequent rise and
> acts accordingly. (Patinkin 1956: 354)

Given that over sufficient time interest-rate expectations will vary in accordance with the marginal productivity of capital, inelastic interest rate expectations are viewed as an exceptional case (Patinkin 1956: 349) since, following Lange, Patinkin argues that the marginal productivity schedule is, at least over long periods, sufficiently elastic with respect to the rate of interest.[25]

We now turn to the possible instabilities caused by elastic price expectations. Here we find Patinkin arguing that, even if elastic price expectations initially trigger an increase in current money prices, the negative real-balance effect caused by the continuous rise in these prices will sooner or later be sufficiently important to the extent that individuals:

> just do not have the means by which they can indefinitely increase their
> demands in accordance with their expectations. Hence, after a certain point,
> these expectations will cease to be self-justifying; and accordingly, after a still
> further point, they will be replaced by more stable ones which reflect the
> levelling-off of prices. In brief, the presence of inflationary expectations may
> well make the price level rise above its ... equilibrium level at some stage of
> the dynamic process; but the real-balance effect ... will ultimately push it
> downwards again. (Patinkin 1956: 311)[26]

But interestingly enough, even Keynes admits that his negative conclusions regarding the alleged self-adjusting nature of market economies would be seriously

impaired if they had to depend on the possible destabilising role of expectations. He writes to Kalecki:

> I hope you are not right in thinking that my *General Theory* depends on an
> assumption that the immediate reaction of a capitalist is of a particular kind
> ... I regard behaviour as arrived at by trial and error, and no theory can be
> regarded as sound which depends on the *initial* reaction being of a particular
> kind. (Keynes 1973-79, vol. XII, p. 797, also noticed by Petri 1997: 28, n. 32)

Keynes thus acknowledges that individuals' initial expectations are essentially irrelevant to understanding the behaviour of the system over sufficient time, because these expectations are modified later on in the light of experience, and hence they must be considered as endogenous magnitudes if the theory is to arrive at definite conclusions.

These considerations by Patinkin and Keynes thus help to clarify why subsequent neoclassical literature managed to incorporate Lange's kind of concerns into orthodoxy. Given that the analysis of PFE does not question the pillars of neoclassical theory, in particular, the notion of capital as a single factor, and on which the tendency towards the full employment of resources is ultimately assumed to rest, the possible destabilising role of those subjective factors referred to by Lange were easily dismissed by subsequent literature as being of secondary importance. In this sense, as stressed by Patinkin and Keynes, it is experience over the underlying, persistent conditions of the market that eventually makes individuals endogenously adapt their expectations. So, when these conditions change, when, for example, the 'natural' rate of interest decreases because the aggregate propensity to consume decreases (something that, as seen in section 2, Lange accepts), it usually takes considerable time for individuals to adapt to the new ruling conditions. Accordingly, Patinkin and the so-called 'neoclassical synthesis' in general, do not deny that over a shorter interval of time, this lack of experience may cause other more transitory elements such as political news, the media, and possible future conditions, to influence the current state of expectations, and destabilise the economy in the way suggested by Lange. However, those scholars also point out that, if those other more persistent causes that according to neoclassical theory ensure the tendency towards the full employment of resources (and whose action, we may insist, is not denied by Lange) are given sufficient time to assert themselves, then those more passing disturbing factors like elastic price expectations, or inelastic interest-rate expectations, will eventually accommodate to the new conditions. They will not play a truly *autonomous* destabilising role in the economy.

Lange acknowledges that the destabilising force of expectations is actually an *endogenous* response to situations that can make individuals lose their notion of what is 'normal' (PFE: 85; the two World Wars and the Great Depression are given as possible examples). But then, one would expect that once these events cease to exert a disturbing influence on the economy, also expectations will be subsequently corrected, and accommodate to the new 'normal' conditions. Lange indeed does not deny that this will be the case. He simply points out that this adjustment process may take 'a long time' (PFE: 85) and, hence, exhorts government to accelerate it by direct intervention. In view of the foregoing considerations, it seems likely to conjecture that a traditional neoclassical economist, or, indeed, an economist belonging to the neoclassical synthesis, would have essentially agreed with Lange's

policy prescriptions. At a theoretical level however, the possible disturbing effect of expectations would be confined to the short period when, under more particular assumptions, the possible causes of the trade cycle around a full-employment position are dealt with. To this, we may add that this is essentially the method Lange himself pursued in his early writings.

The role of expectation functions in PFE allows us to trace two other drawbacks in Lange's analysis. First, the author does not notice that the TGE assumption of exogenously given expectation functions introduces a serious element of indeterminacy into the theory. For instance, Friedman (1946) pointed out in his assessment of PFE that:

> [Lange] seeks to enumerate all possible economic systems to which these functions [excess demand functions which depend, in turn, on price expectation functions] could give rise. The kind of economic system and the results in that system will depend on the specific character of the functions and their interrelations, and there *clearly are a very large number of permutations and combinations* (1946: 618; emphasis added).

Secondly, given that the introduction of expectation functions allowed Lange to provide an *ad hoc* justification for both stable and unstable equilibria, in his framework it is always possible to reconcile economic phenomena with the theory; the latter thus becomes tautological. This lack of definite conclusions only allows Lange to argue *ex post* that, given an *actual* situation of economic stability (instability), the parameters of the fundamentals must have been such so as to generate a stable (unstable) equilibrium (for example, Lange's documentation of the trends in the capitalist system since the 1870s in PFE: 83-5.) As Patinkin (1956: 257) summarises it, 'expectations ... introduce many additional "degrees of freedom" [and hence] we can obtain any conclusion we might desire'. But also Friedman noticed this second shortcoming, and claimed: 'A theory that has no implications that facts, potentially capable of being observed, can contradict is useless for prediction: *if all possible occurrences are consistent with it, it cannot furnish a basis for selecting those that are likely*' (1946: 618; emphasis added). He further remarked of Lange's 1944 work:

> For the most part, the crucial question, 'What observed facts would contradict the generalization suggested, and what operations could be followed to observe such critical facts?' is never asked; and the theory is so set up that it could seldom be answered if it were asked. The theory provides formal models of imaginary worlds, not generalizations about the real world. (Friedman 1946: 618)[27]

6   Concluding Remarks

Under the influence of Hicks (1946 [1939]), Lange moves from an essentially Wicksellian position in his 1936 writings to a neo-Walrasian temporary equilibrium framework in PFE, without showing a clear understanding of the implications, both theoretical and methodological. At a methodological level, Lange seems to be less conscious than the other forerunners of the neo-Walrasian approach, especially Hicks, of the new problems that appear when the set of capital goods is specified among the data of the equilibrium. Confirmation of this is that, unlike Hicks, and

despite Lange's considerations regarding the need to specify the capital endowment in value terms examined in his 1936 writings, in PFE he does not show any concern with respect to the 'impermanence' and the 'substitutability' problems, nor with the indeterminacy of results due to expectations. The importance of these issues should not be underestimated: as argued, Lange's analytical framework has been subsequently adopted and developed by his pupil Patinkin (1956), whose contributions have been no doubt highly influential in the subsequent development of neoclassical theory. In view of this it can be plausibly conjectured that Lange's vivid discussion of the equilibration process towards a temporary equilibrium, as if this framework could easily accommodate time-consuming, realistic adjustments, may have had a considerable influence on the subsequent development of the theory, by easing the spread of the mistaken idea that, if stability could be assumed, the TGE (or, rather, the sequence of TGEs) could be susceptible of having the role of a centre of gravitation of actually observed prices and quantities.

At a more theoretical level, an examination of PFE gives a clear indication that the change in the specification of the factor capital does not prevent Lange, nor subsequent neoclassical economists, from continuing to believe in the traditional notion of capital as a single factor and, hence, in the basic traditional mechanisms that stand or fall with that notion. Paradoxically, it is the faith in these traditional tendencies which seems to have prevented Lange from developing fully convincing reasons to question the tendency towards the full employment of resources. In other words, given that in PFE the pillars of neoclassical theory are not questioned, Lange is forced to confine the possible destabilising causes to *purely subjective* elements such as uncertainty, expectations and systematic mistakes, which lose importance over longer periods relative to those more objective factors that are supposed to regulate the profitability of investment in neoclassical theory (for example, the marginal product of capital), and that ultimately justify the tendency towards the full employment of resources. In a nutshell, Lange's inability to abandon traditional neoclassical ways of thinking, even in a framework like the TGE that is completely inhospitable to them, helps in understanding why subsequent contributors to the 'neoclassical synthesis' such as Patinkin (1956) easily managed to dismiss the relevance of Lange's sort of critiques by interpreting them as *frictions* or *imperfections* that, although capable of explaining the short-run fluctuations of the economy, would vanish over longer periods of time.

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Notes

1   Although there have recently been some contributions on Lange's work during the 1930s and 1940s (Rubin 2011; Lampa 2011), they do not focus their attention on the relation between Lange's conception of equilibrium and capital theory.

2   It seems useful to distinguish between the role of axes as outputs ($\overline{m}$) and their role as inputs (m and m'). This distinction however does not appear in Lange's original work; that is, the variable $\overline{m}$ is not explicitly specified in Lange (1936a). This however will not alter the results.

3   From (4) we have $d\pi = p_x dx - p_m d_m - p_l d_l$. Since from (1) we have that $dx = F_m dm + F_l dl$, if we substitute this last condition for the expression dx in the right-hand side of the previous equation, and by employing FOC (8) and (9), we obtain: $d\pi = \lambda(P_m dm + P_l dl)$. But from the condition $\kappa = p_m m + p_l l$ we know that $d\kappa = p_m dm + p_l dl$. We thus obtain (11). A similar procedure allows us to determine (11$'$).

4   The same condition holds for firms of sector II in the case of λ'.

5   In Lange's original model $\overline{m}$ is not an explicit unknown, so he has one equation less (Lange does not include equation 2). In his original model these equations are: (1)-(3), (15)-(18), (23)-(24).

6   The reader is referred to Hatta (1976) for a detailed critique of these kinds of adjustment processes. Notice however that Lange (1936a: 180) considers the possibility that, due to complementarity among inputs, an increase in the rate of interest *increases* rather than decreases the quantity of indirect labour employed, as the tendency towards equilibrium requires. Without justification he dismisses this case, probably because this would imply an unstable equilibrium.

7   As Lange himself notes in the referred footnote, the main difference between his and Wicksell's approach is that the former's analysis is a generalisation of the latter's, since Lange considers the possibility of basic goods (axes), which are excluded by Wicksell.

8   We remind the reader that Lange considers his example as a valid simplification of the more general situation in which many capital goods and many consumption goods are produced.

9   Here short period must be interpreted as 'static' long-period since it is contrasted with a 'long-period equilibrium' which is clearly a secular one.

10   In this connection, it is worth adding that the slow change – slow relative to the process of equilibration – that the data determining prices and distribution may be undergoing over time is no doubt the reason why Lange, in interventions in the socialist calculation debate (Lange 1936b; 1937a), does not feel the need to have recourse to an auctioneer-guided process of equilibration. Although he admits that, when conditions change, 'a change in the quantity supplied [of commodities] requires a period of time' (Lange 1936b: 60, n. 1), he argues anyway that the equilibrium outcome would be slowly reached by trial and error, through a 'Marshallian process of adaptation' (*ibid.*) such that the proportions among factors employed in each industry are gradually adjusted to the conditions of static long-period equilibrium. Moreover, in his description of the working of the price mechanism under a socialist organisation of production, Lange argues that 'the process of price determination is quite analogous to that in a competitive economy' (Lange 1936b: 64) and he thus assumes time-consuming adjustments. Accordingly, the 'Central Planning Board' would correct the originally announced prices only after a physical shortage or surplus of the several commodities reveal themselves at the end of the relevant period *after* transactions and production at disequilibrium prices have actually taken place. Lange (1936b: 64) then proceeds to

argue that, after successive corrections, the equilibrium position assumed by him to be unique is eventually reached.

11   As we shall see in section 4 below, the conclusions that Lange reaches on the tendency towards a full employment of resources are very similar to those reached by Keynes in chapter 19 of the *General Theory*. The main difference between Keynes and Lange is addressed in section 5.

12   It is as if, despite the change in the specification of the capital endowment as a set of physically heterogeneous capital goods, it were still possible to characterise the neoclassical equilibrium as a centre of gravitation of actually observed variables.

13   It could perhaps be objected to the above claims that this investment function aims at determining investment decisions at a particular point of time; hence the validity of the assumption of given expected prices. However, at every instant of time, expectations are conditioned by a myriad of accidental events, which can prevent the theory from arriving at any definite result regarding the relationship between investment and the rate of interest (Petri 2004).

14   Lange writes: 'Not only a change in the level (i.e. the total amount of expenditure) of the propensity to consume, but also a change in its composition (i.e. the direction of expenditure) causes disequilibrium. The equilibrium propensity to consume therefore implies not only a definite level but also a definite direction of expenditure' (PFE: 55, n. 18).

15   This is important for what follows, because here Lange is in fact admitting that no general conclusion can be reached on the evolution of the single capital goods as the marginal propensity to save changes.

16   It should come as no surprise to us that Lange argues in these terms because in an earlier article (Lange 1938), he also assumes a short-period framework and accordingly specifies the capital endowment in physical terms. However, Lange (1938: 13, n. 2), explicitly follows Lerner's approach to investment, arguing that 'the investment function is based on the theorem that the amount of investment per unit of time is such as to equalise the rate of net return on that investment … to the rate of interest' and he thus asserts that the rate of return on investment is derived from the rate of net return (marginal efficiency) on capital. Lerner (1944) does speak of capital as a single factor. (For a detailed analysis of this approach, see Petri 2004: 273-6, and Ackley 1978: 629). So, despite the assumption of a given endowment of physical capital goods, in Lange's (1938) analysis the negative relationship between investment – the demand for capital as a *flow* – and the rate of interest is ultimately derived from the negative relationship between the rate of interest and the demand for the factor capital as a *stock*. The physical capital goods are accordingly seen as crystallisations of this single magnitude.

17   The indefiniteness problem due to expectations is discussed in section 4 below.

18   Lange only limits himself to warning the reader of the possibility that 'when considerable friction is present in the economy' (PFE: 19) the working of the adjustment processes, 'although to be expected according to pure theory, may be hampered to such a degree by friction as to be negligible in practice' (PFE: 19). So, it seems evident that, having considered the possibility that the economy presents 'considerable friction', Lange cannot have been assuming that equilibrium is instantaneously reached. In this connection, we can notice that in a footnote appended to the Introduction of PFE, Lange (PFE; 1, n. 1) accepts that the 'degree of realism' of the analysis could 'be increased' by considering the 'possible lags in reaction'. So he comes close to admitting that the time-span considered within a single TGE is not sufficiently long for equilibrium to emerge as the result of repeated interactions in the market. However, the issue is simply dismissed by Lange as being of secondary importance. The very different attitude taken by Hicks in *Value and Capital* (1946 [1939]: 212), and which indicates Hicks is much more aware than Lange of the

difficulties entailed by the notion of TGE, is expressed in his remark that 'a theory which leaves out the probability of input lags is likely to be *gravely* misleading'. While Lange simply points out that the degree of realism 'could be increased' by incorporating lags in reactions, Hicks argues that ignoring this issue is 'gravely misleading'.

19   Lange (1944: 22) states that static expectations are a particular case of elasticity of expectations being equal to zero.

20   As noticed above, Lange here echoes Hicks (1946 [1939]) and labels the substitution among commodity bundles of different periods as *intertemporal* substitution or substitution *over time*, to be distinguished from the *intratemporal* substitution that takes place among bundles of the same period.

21   If price expectations are equal to one, there is no intertemporal substitution, and only intratemporal substitution takes place. As prices fall, individuals decrease their money holdings and, assuming a constant stock of money, the monetary effect is positive.

22   While Lange considers the possibility that a decrease in interest rates may also induce households to increase their current purchases of consumption goods, he argues in several instances (PFE: 17, 27-8, 57) that the most important effect of a fall in the interest rate is on aggregate investment.

23   When examining whether or not a decrease in the demand for money due to a fall in prices could affect the interest rate and therefore stimulate investment, Keynes concludes that if the quantity of money shrinks along with money income, 'there is ... nothing to hope in this direction' (1936: 266). Keynes however denies that, when the quantity of money increases in wage units, full employment could be achieved, because an increase in the quantity of money could have a 'disturbing effect on confidence' (1936: 266-7). The case of elastic price expectations is addressed by Keynes (1936: 263, 265) and the so-called 'liquidity trap' can be interpreted as the case of completely inelastic interest rate expectations. An important qualification must be made, however: Keynes studies the possible disturbing effects caused only by long-run expectations. In his framework, short-run expectations cannot exert a disturbing influence in the economy because they are endogenously determined and hence are *correct* in equilibrium. Lange, on the other hand, by adopting the TGE method, must extend his analysis to include short-run expectations as exogenous variables.

24   In this connection, we cannot omit mentioning Modigliani's (1944) contribution on this issue and the similarities and differences between the approach of this author and Lange's. Although neither Lange nor Modigliani use Keynes's negative results regarding the tendency towards the full employment of resources to question the explanation of distribution in terms of supply-and-demand forces, for the reasons discussed in the main text (see sections 3 and 4; see also Rubin 2011) it is clear that Lange is aware that the negative results regarding the tendency towards full employment reached by Keynes in the *General Theory* are *not* due to the assumption of rigid wages, which is instead the well-known interpretation of Keynes (1936) put forward by Modigliani.

25   Patinkin (1956: 380) for instance writes: 'Variations in the average long-term rate of interest ... have originated primarily in technological changes which have affected the marginal productivity of capital'. The reader must notice that also Patinkin, although formally adopting the TGE framework, relies on neoclassical gravitational ways of reasoning that presuppose the traditional notion of capital as single factor of production.

26   Patinkin (1956: 311) also notes that in order for price expectations to exert a destabilising influence, the elasticity of expectations should be greater than one. If they are equal to one, then 'there will be no further increase in current demand as a result of

inter-temporal substitution'. And at the same time, there is the 'dampening pressure' of a negative real-balance that implies that 'the stability of the system would be assured'.

27   Friedman was not alone in criticising the methodology of PFE. It must be noticed in this connection that PFE was also criticised by Harrod (1946) (see also Timlin 1946). At any rate, as argued in the main text, PFE would gain recognition when taken up by Patinkin (1956).

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Conflicting Views of the Entrepreneur in Turn-of-the-Century Vienna

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Abstract: Joseph Schumpeter's theory of the entrepreneur is the most well-known approach to be developed in continental Europe at the turn of the century, and has exercised considerable influence on the literature. However, this paper shows that alternative theories of the entrepreneur were developed by scholars close in time and space to Schumpeter's *The Theory of Economic Development*. First, the contribution of Rudolf Hilferding is unknown in the literature on entrepreneurship and is important due to the absence of entrepreneurial theory in the Marxist literature upon which Hilferding built. Second, while the later contributions of Ludwig von Mises to entrepreneurial theory have been discussed, the early comments of his 1912 treatise are often passed over, and are a significant foil to Schumpeter's views. Discussing the early writing of Mises on entrepreneurship also helps show the ambitious scope of Mises's first book, and furthermore, allows us to properly position his writings in the history of economic thought.

1   Introduction

At the turn of the twentieth century Vienna experienced a renaissance in the arts and sciences in which economics also experienced a revolutionary change (Janik and Toulmin 1973; Schulak and Unterköfler 2011). In economics one field in which there was significant progress was entrepreneurial theory. While Joseph Schumpeter's theory of the entrepreneur is perhaps the most important and certainly the best known developed in this period, this paper will show that other scholars in Vienna were simultaneously producing different theories of entrepreneurship which merit attention. In particular, our paper will be concerned with examining major contributions to entrepreneurship theory by Rudolf Hilferding, on the one hand, and Ludwig von Mises, on the other hand, and compare them to that of Schumpeter's theory.

The distinct aspects of the entrepreneurial function have sometimes become lost or blurred in the history of economic thought. Especially in the British classical tradition, entrepreneurial theory was largely neglected in favour of a narrower view of economic development which conflated any entrepreneurial function with that of the capitalist, to the neglect of the former (Hébert and Link 1988; Kirzner 1979). In the early twentieth century, however, Viennese economists, working in several different traditions in economic thought, considered entrepreneurship an important area of study. The founder of the Austrian School, Carl Menger, wrote only briefly on the subject (Gunning 1997; Martin 1979; Kirzner 1979: 53-75). Nevertheless, Menger's teaching inspired his students, particularly Victor Mataja, to continue work on entrepreneurship within the Austrian tradition (Schulak and Unterköfler 2011: 56-7). Eugen von Böhm-Bawerk also touched on the subject of entrepreneurship, although his theory remained largely nascent (McCaffrey and Salerno 2013). Thus, young scholars in Vienna intent on developing the theory of the entrepreneur had significant recent contributions upon which to draw, though to be drawn principally from a critical perspective. This intellectual setting in Vienna